Title: PSI 2019/2020 - Quantum Field Theory II (David) - Lecture 1

Speakers: Francois David

Collection: PSI 2019/2020 - Quantum Field Theory II (David)

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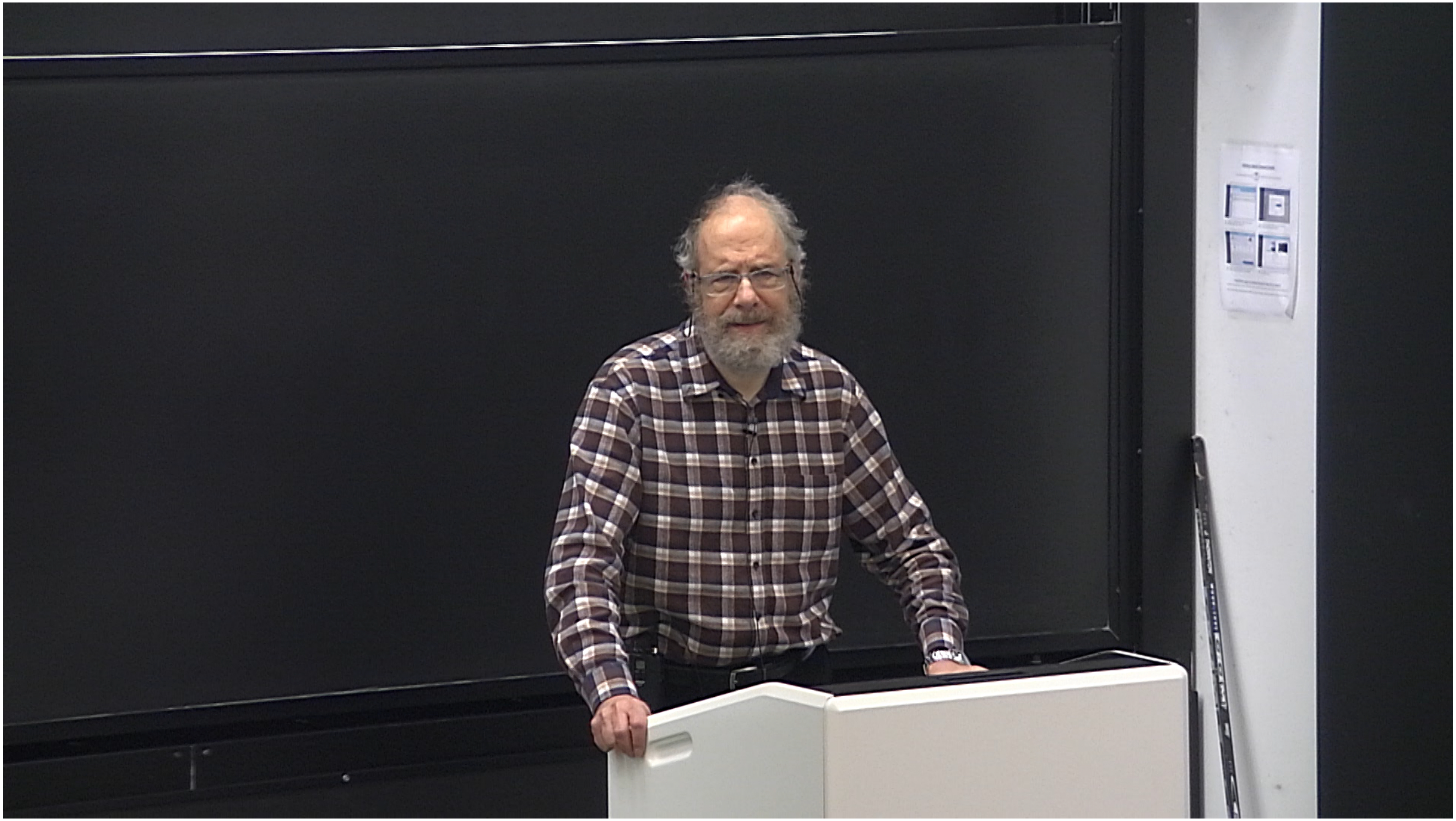
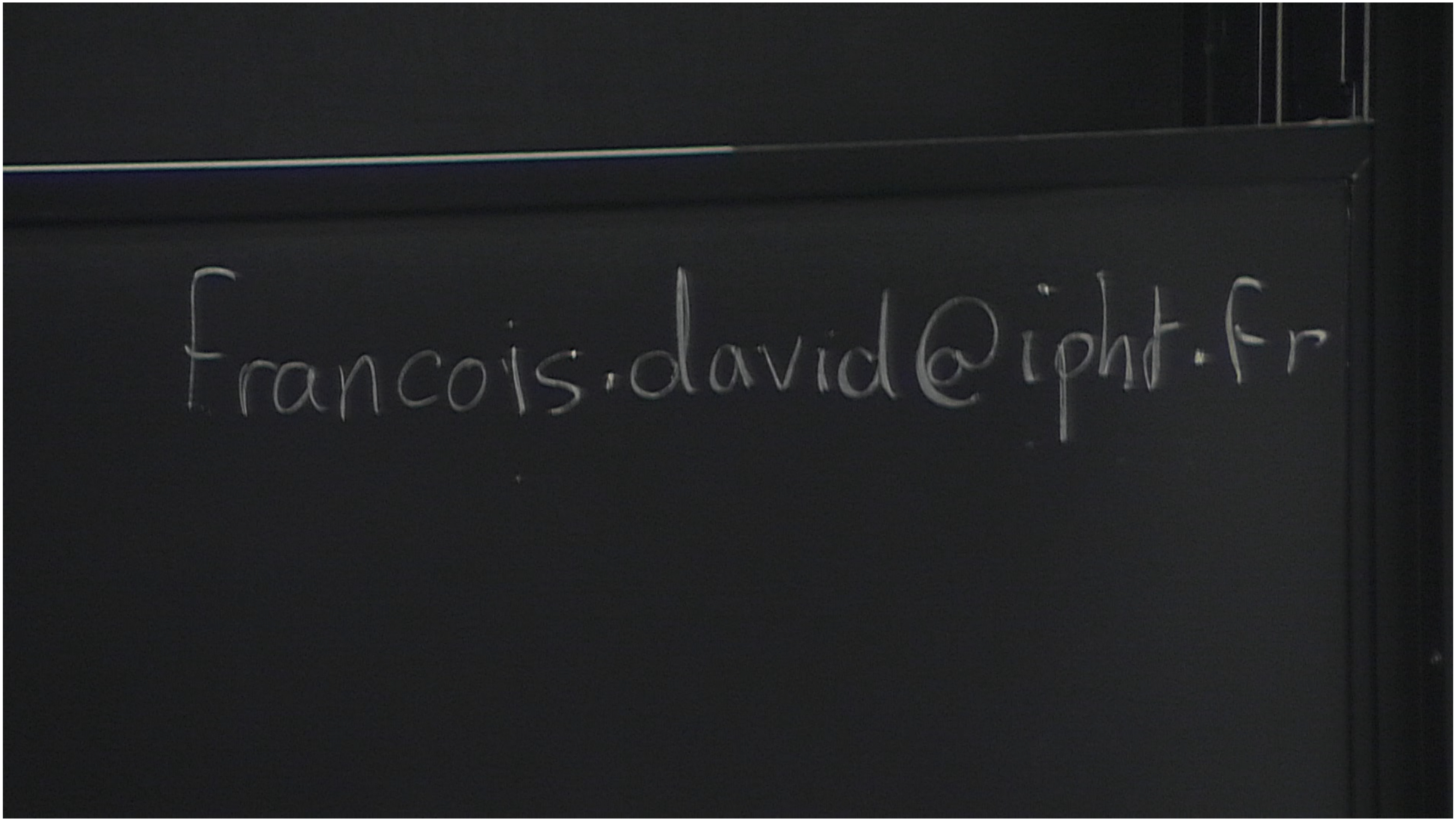
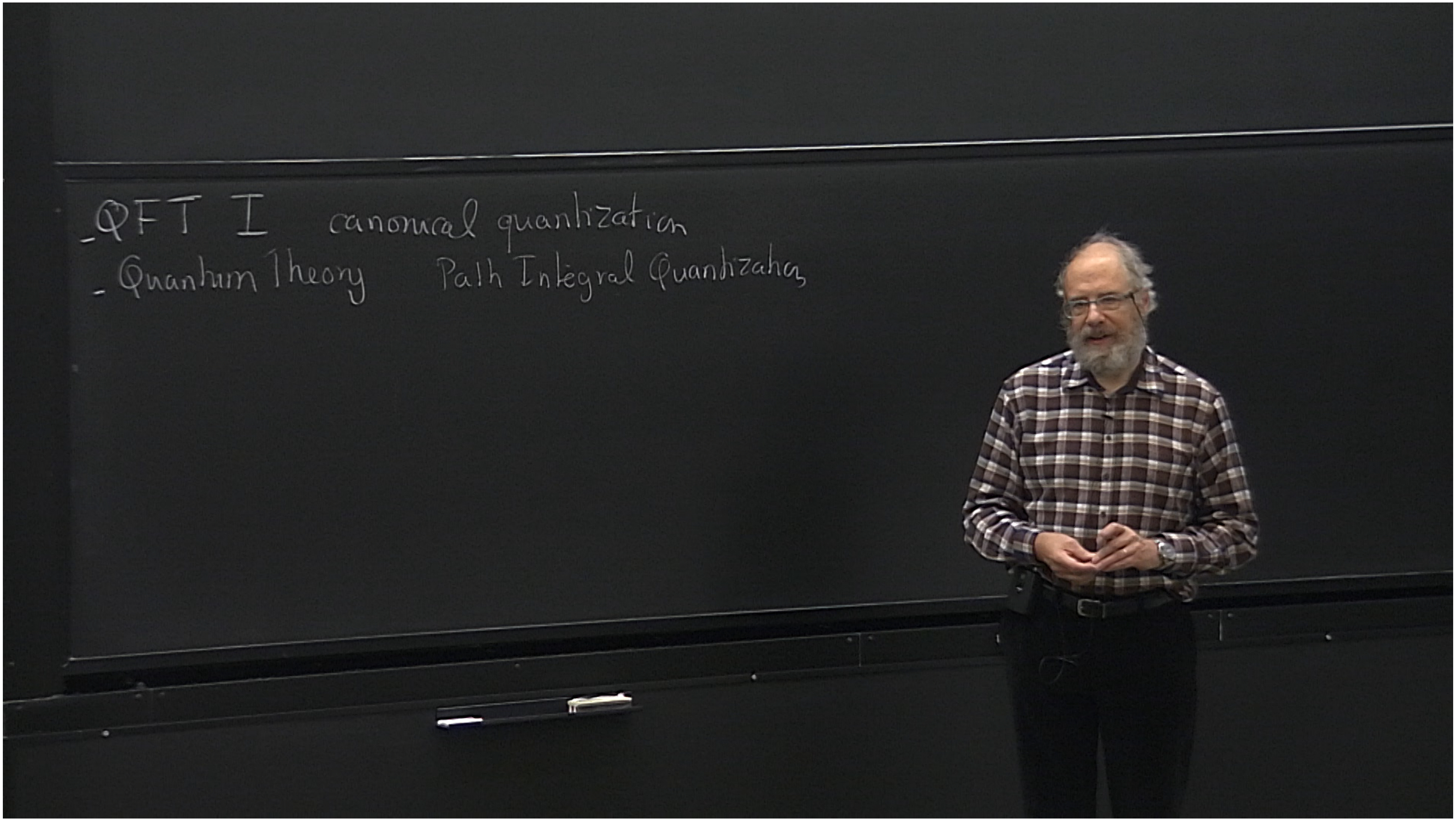



francois.david@ipht.fr


- QFT I   canonical quantization
- Quantum Theory   Path Integral Quantization

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- Quantum Theory   Path Integral Quantization

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* QFT   Functional Integral Quantization
* Renormalization Theory ← specific to QFT   K. Wilson

  Stat. Mech & Cond. Matter

- QFT I canonical quantization
- Quantum Theory Path Integral Quantization

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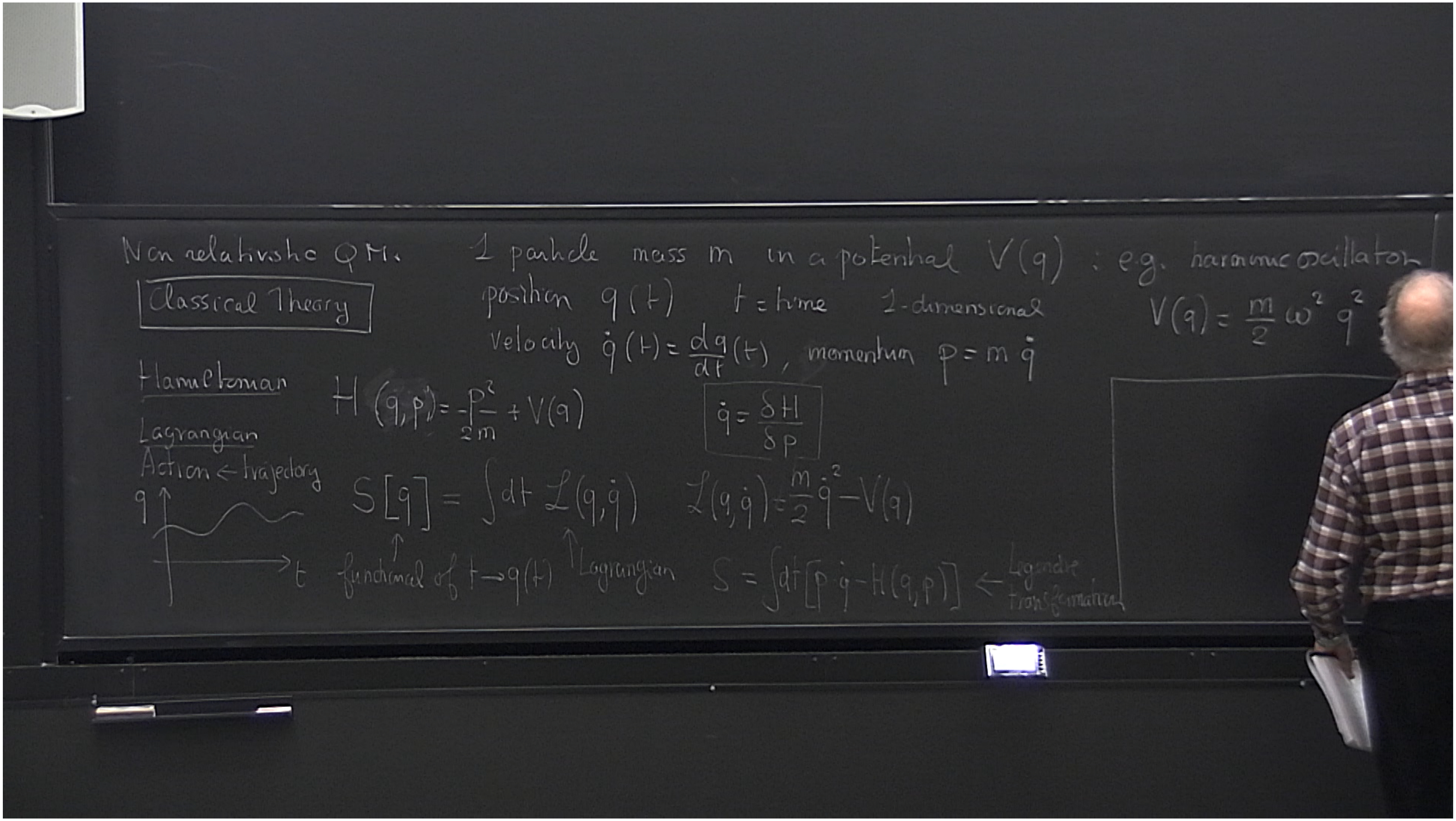
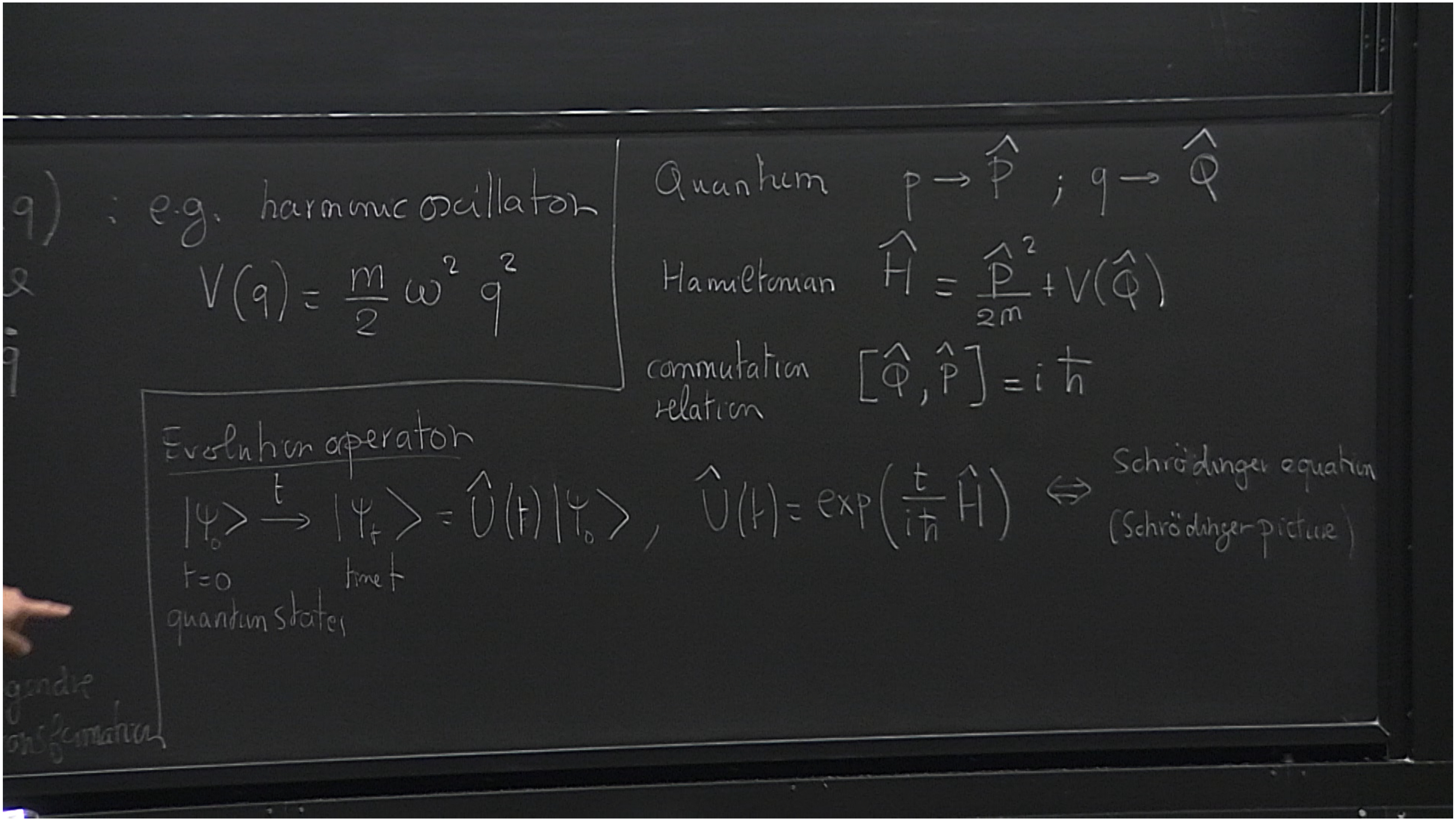
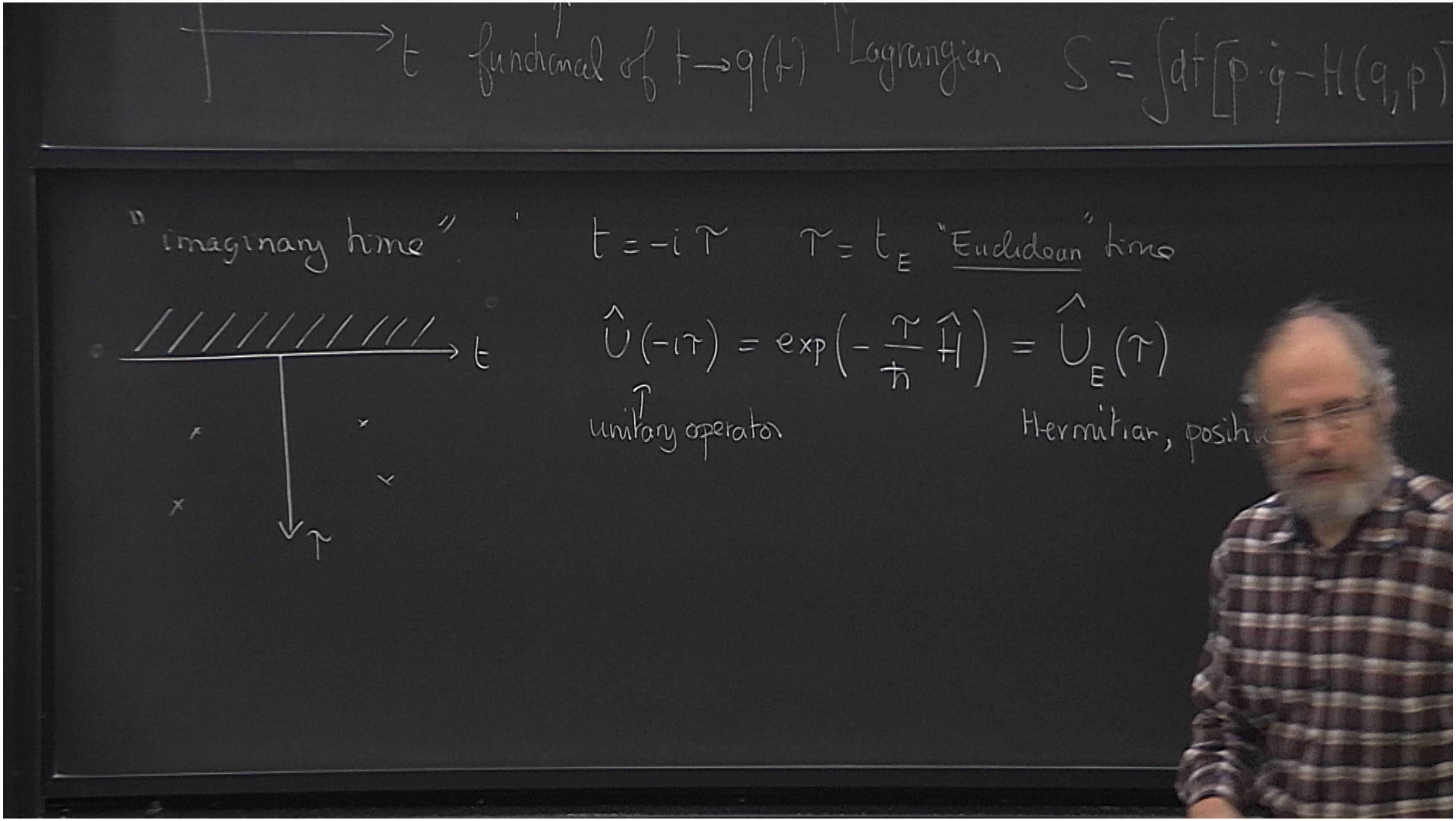
* QFT Functional Integral Quantization
* Renormalization Theory ← specific to QFT K. Wilson
Wilsonian Ren. Group
Stat. Mech & Cond. Matter
* Gauge Theories Non-abelian gauge theories → Higgs Mechanism

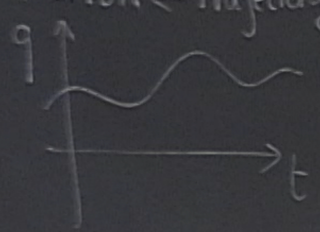

Non relativistic QM.
1 particle mass m in a potential $V(q)$: e.g. harmonic oscillator
Classical Theory
position $q(t)$ t = time   1-dimensional
$V(q) = \frac{m}{2}\omega^2 q^2$
velocity $\dot{q}(t) = \frac{dq}{dt}(t)$, momentum $p = m\dot{q}$
Hamiltonian
$H(q,p) = \frac{p^2}{2m} + V(q)$
$\boxed{\dot{q} = \frac{\delta H}{\delta p}}$
Lagrangian
Action ← trajectory
$S[q] = \int dt\, \mathcal{L}(q,\dot{q})$
$\mathcal{L}(q,\dot{q}) = \frac{m}{2}\dot{q}^2 - V(q)$
q
t
functional of $t \to q(t)$
Lagrangian
$S = \int dt\,[p\dot{q} - H(q,p)]$ ← Legendre transformation



q) : e.g. harmonic oscillator
V(q) = m/2 ω² q²
Quantum p → P̂ ; q → Q̂
Hamiltonian Ĥ = P̂²/2m + V(Q̂)
commutation relation [Q̂, P̂] = iħ
Evolution operator
|ψ₀> →t |ψ_t> = Û(t)|ψ₀>, Û(t) = exp(t/iħ Ĥ) ⇔ Schrödinger equation
t=0 time t
quantum states
(Schrödinger picture)
gendre
ansformation


t
functional of $t \to q(t)$
Lagrangian
$S = \int dt\,[p\cdot\dot{q} - H(q,p)]$
"imaginary time"
$t = -i\tau$
$\tau = t_E$ "Euclidean" time
$\hat{U}(-i\tau) = \exp\left(-\frac{\tau}{\hbar}\hat{H}\right) = \hat{U}_E(\tau)$
unitary operator
Hermitian, positi
t
τ

...tion ← trajectory



$S[q] = \int dt\, \mathcal{L}(q,\dot q)$ $\qquad \mathcal{L}(q,\dot q) = \frac{m}{2}\dot q^2 - V(q)$

functional of $t \to q(t)$ $\qquad$ Lagrangian

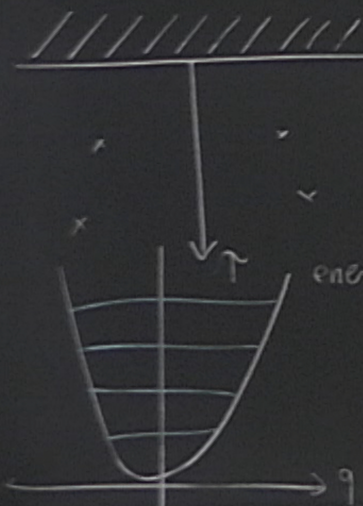
$S = \int dt\,[p\cdot\dot q - H(q,p)]$ ← Legendre transformation

$t=0$

quantum state

Physic...

"imaginary time" $\qquad t = -i\tau \qquad \tau = t_E$ "Euclidean" time $\qquad \tau \geq 0$



$$\hat U(-i\tau) = \exp\left(-\frac{\tau}{\hbar}\hat H\right) = \hat U_E(\tau) = \sum_{i=0}^{\infty} \exp\left(-\frac{\tau}{\hbar}E_i\right)|i\rangle\langle i|$$

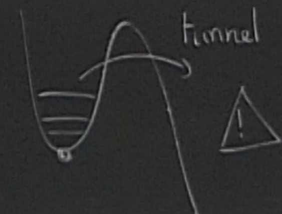
unitary operator $\qquad$ Hermitian, positive definite $\qquad$ smaller and smaller as long as $\tau \geq 0$

$\hat H$ spectrum bounded from below, and infinite

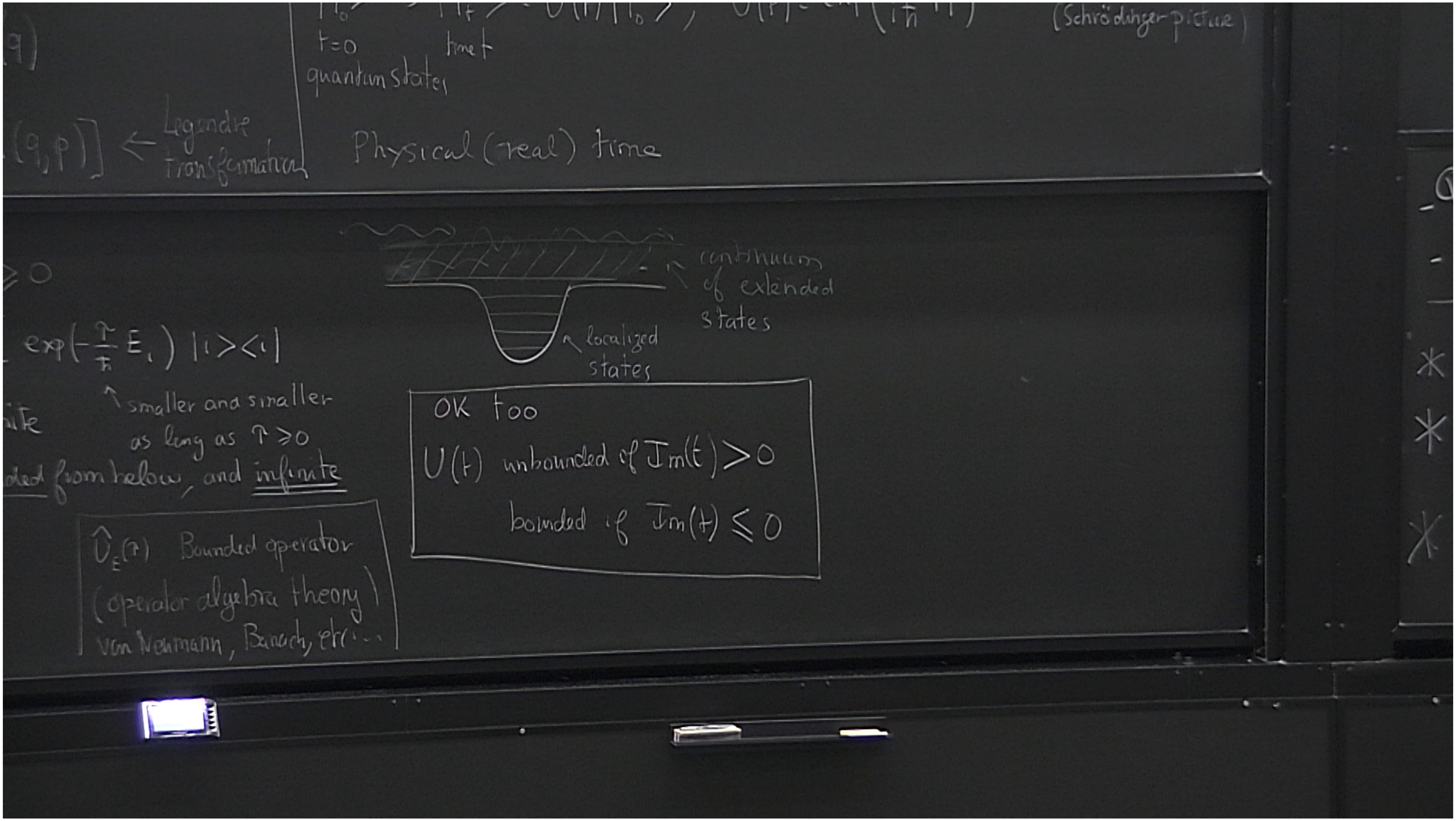
energy e. states $|n\rangle$ $\quad n=0,1,\dots$

$E_n \qquad E_0<E_1<E_2<\dots$

$\hat H = \sum_i E_i |i\rangle\langle i|$



$\hat U_E(\tau)$ Bounded operator
(operator algebra theory)
von Neumann, Banach, etc...


(Schrödinger picture)
t=0
time t
quantum states
← Legendre transformation
Physical (real) time
continuum of extended states
localized states
exp(−τ/ħ E_i) |i><i|
smaller and smaller as long as τ ≥ 0
from below, and infinite
Û_E(τ) Bounded operator
(operator algebra theory)
von Neumann, Banach, etc...
OK too
U(t) unbounded if Im(t) > 0
bounded if Im(t) ≤ 0

$E_n \qquad E_0 < E_1 < E_2 < \dots$

$$\hat{H} = \sum_i E_i |i\rangle\langle i|$$

(operator

Von Neuma

Path integrals (Maïté) Feynman

probability amplitude

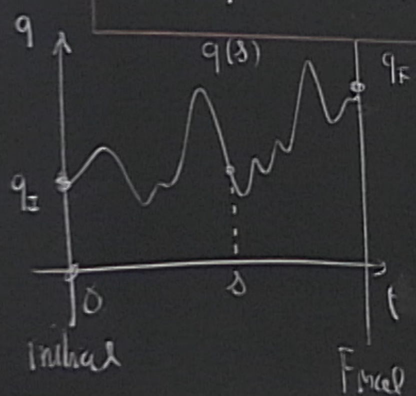
$$\boxed{\langle q_F | U(t) | q_i \rangle = \int D[q(s)] \exp\left(\frac{i}{\hbar} S[q]\right)}$$

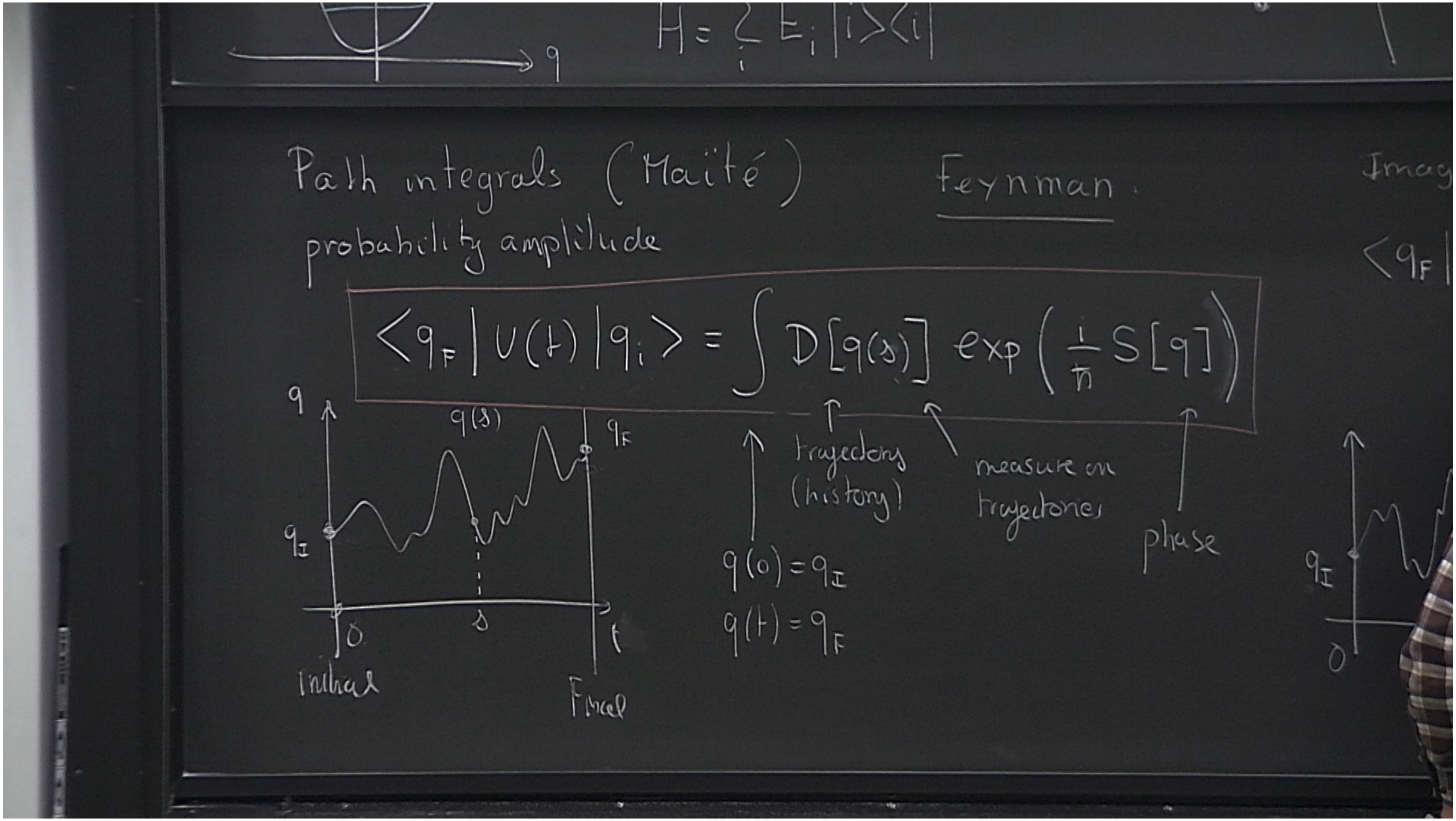
trajectory (history)

measure on trajectories

$q(0) = q_I$

$q(t) = q_F$




H = Σ_i E_i |i><i|
Path integrals (Maïté)
Feynman
probability amplitude
<q_F | U(t) | q_i> = ∫ D[q(s)] exp((i/ħ) S[q])
q(0) = q_I
q(t) = q_F
trajectory
(history)
measure on
trajectories
phase
q
q(s)
q_F
q_I
0
s
t
Initial
Final

(operator algebra theory)
Von Neumann, Banach, etc...

Imaginary time

$$\langle q_F | U_E(\tau) | q_I \rangle = \int D[q(\sigma)] \; \exp\left(-\frac{1}{\hbar} S_E[q]\right),$$

positive weight

Euclidean time

Euclidean Action

$q(0) = q_I$

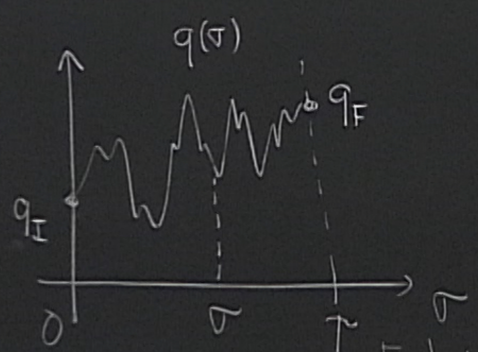
$q(\tau) = q_F$

Euclidean action

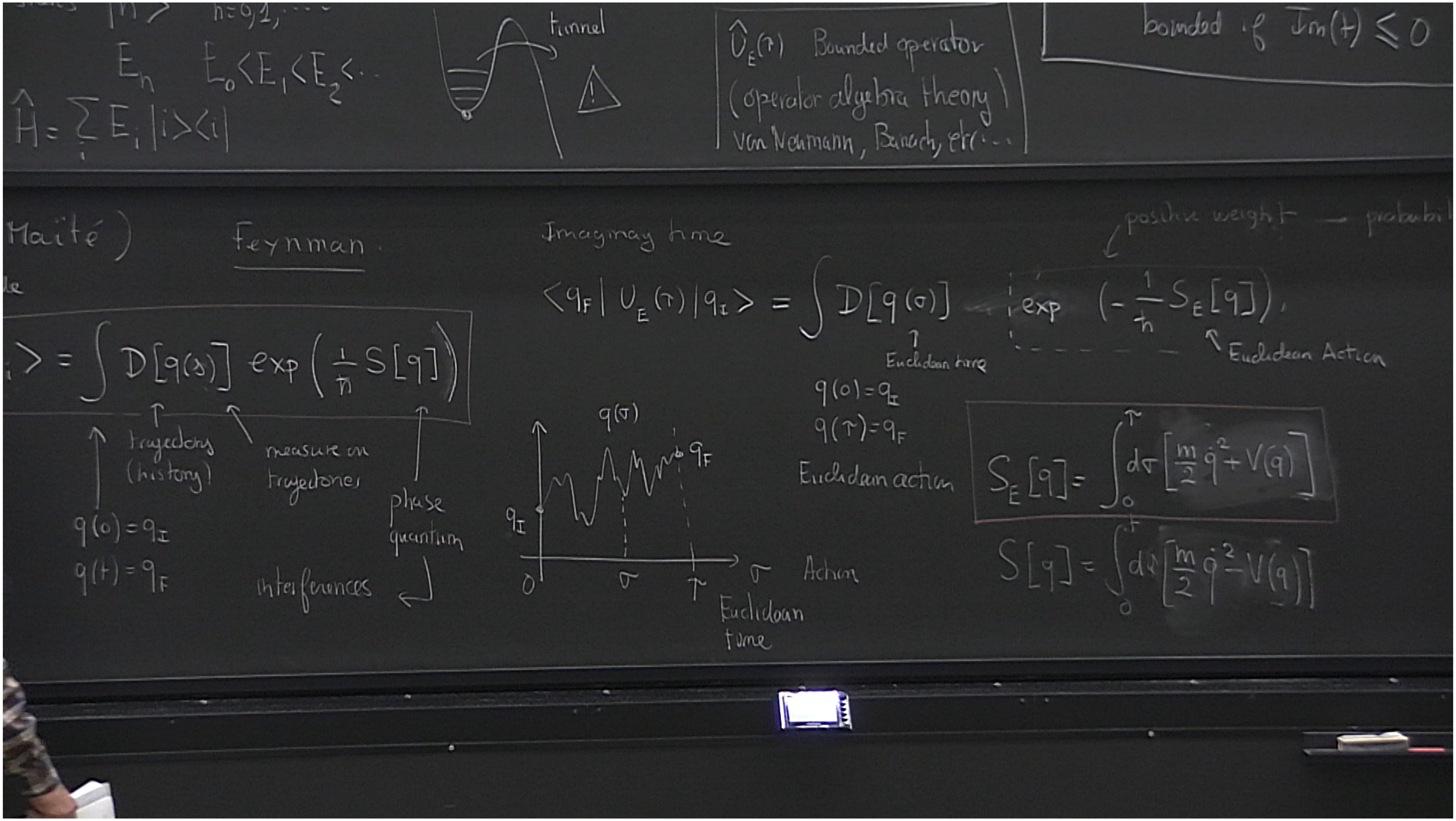
$$\boxed{S_E[q] = \int_0^\tau d\sigma \left[\frac{m}{2}\dot{q}^2 + V(q)\right]}$$

$$S[q] = \int_0^t dt \left[\frac{m}{2}\dot{q}^2 - V(q)\right]$$

Action



phase
quantum


$|n\rangle$ $n=0,1,\dots$
E_n $E_0<E_1<E_2<\dots$
$\hat{H}=\sum_i E_i |i\rangle\langle i|$
tunnel
$\hat{U}_E(\tau)$ Bounded operator
(operator algebra theory)
Von Neumann, Banach, etc...
bounded if $\mathrm{Im}(t)\leq 0$
Maïté)
Feynman.
Imaginary time
positive weight → probabil
$\langle q_F | U_E(\tau) | q_I\rangle = \int D[q(\sigma)]\ \exp\left(-\frac{1}{\hbar}S_E[q]\right)$,
Euclidean time
Euclidean Action
$\rangle = \int D[q(s)]\ \exp\left(\frac{i}{\hbar}S[q]\right)$
trajectory (history)
measure on trajectories
phase quantum
interferences
$q(0)=q_I$
$q(t)=q_F$
$q(\sigma)$
q_I
q_F
0 σ τ σ
Euclidean time
$q(0)=q_I$
$q(\tau)=q_F$
Euclidean action
$S_E[q]=\int_0^\tau d\sigma\left[\frac{m}{2}\dot{q}^2+V(q)\right]$
Action
$S[q]=\int_0^t ds\left[\frac{m}{2}\dot{q}^2-V(q)\right]$

$\ldots_E(\tau)$ Bounded operator

(operator algebra theory)

von Neumann, Banach, etc...

positive weight → probability theory on trajectories

$$\rangle = \int D[q(\sigma)] \; \exp\left(-\frac{1}{\hbar} S_E[q]\right),$$

Euclidean time

Euclidean Action

$q(0) = q_I$

$q(\tau) = q_F$

Euclidean action

$$\boxed{S_E[q] = \int_0^\tau d\tau \left[\frac{m}{2}\dot{q}^2 + V(q)\right]}$$

$$S[q] = \int_0^t dt \left[\frac{m}{2}\dot{q}^2 - V(q)\right]$$

$\to \tau$ Action

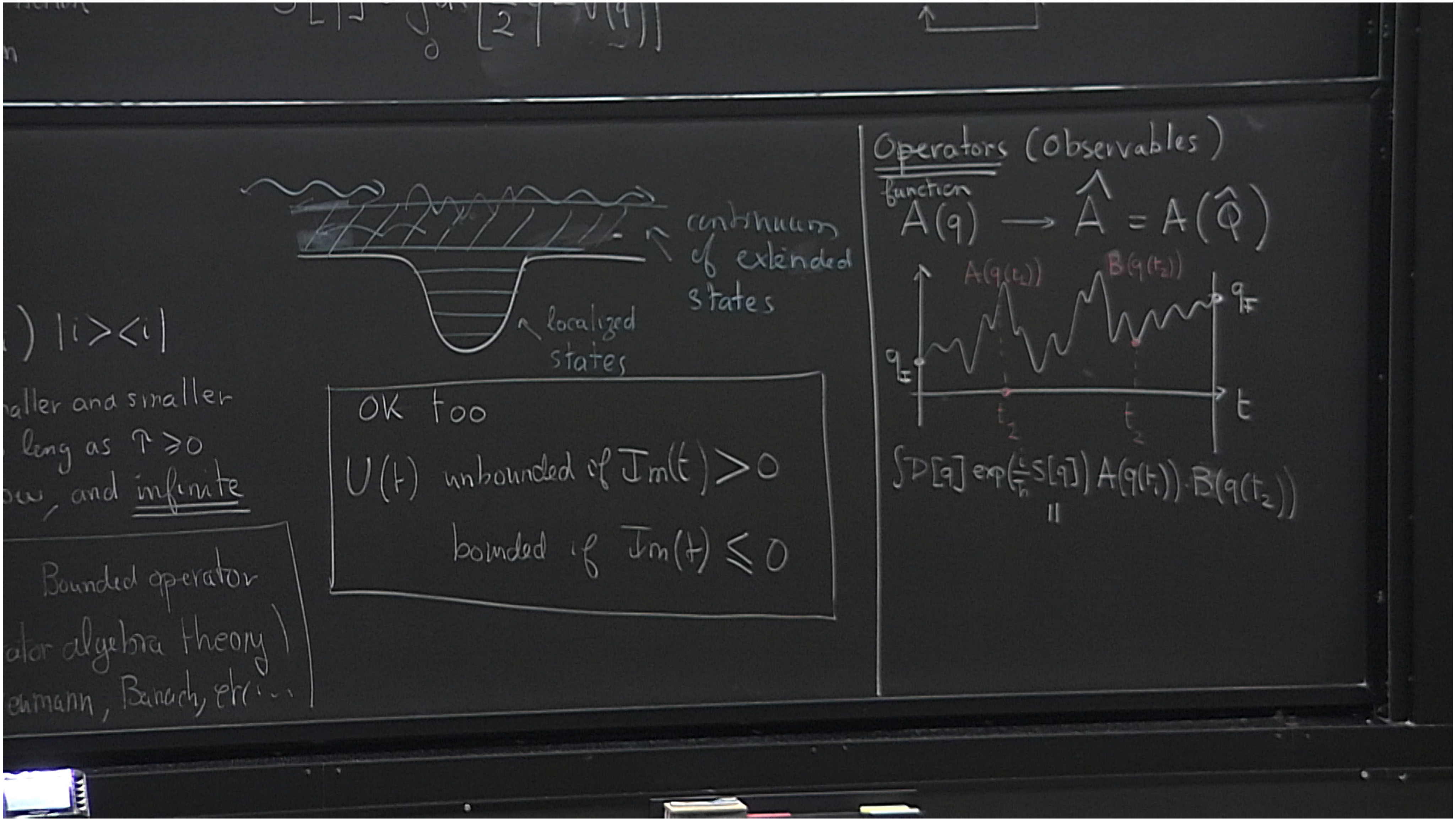
Euclidean time

$\Downarrow$

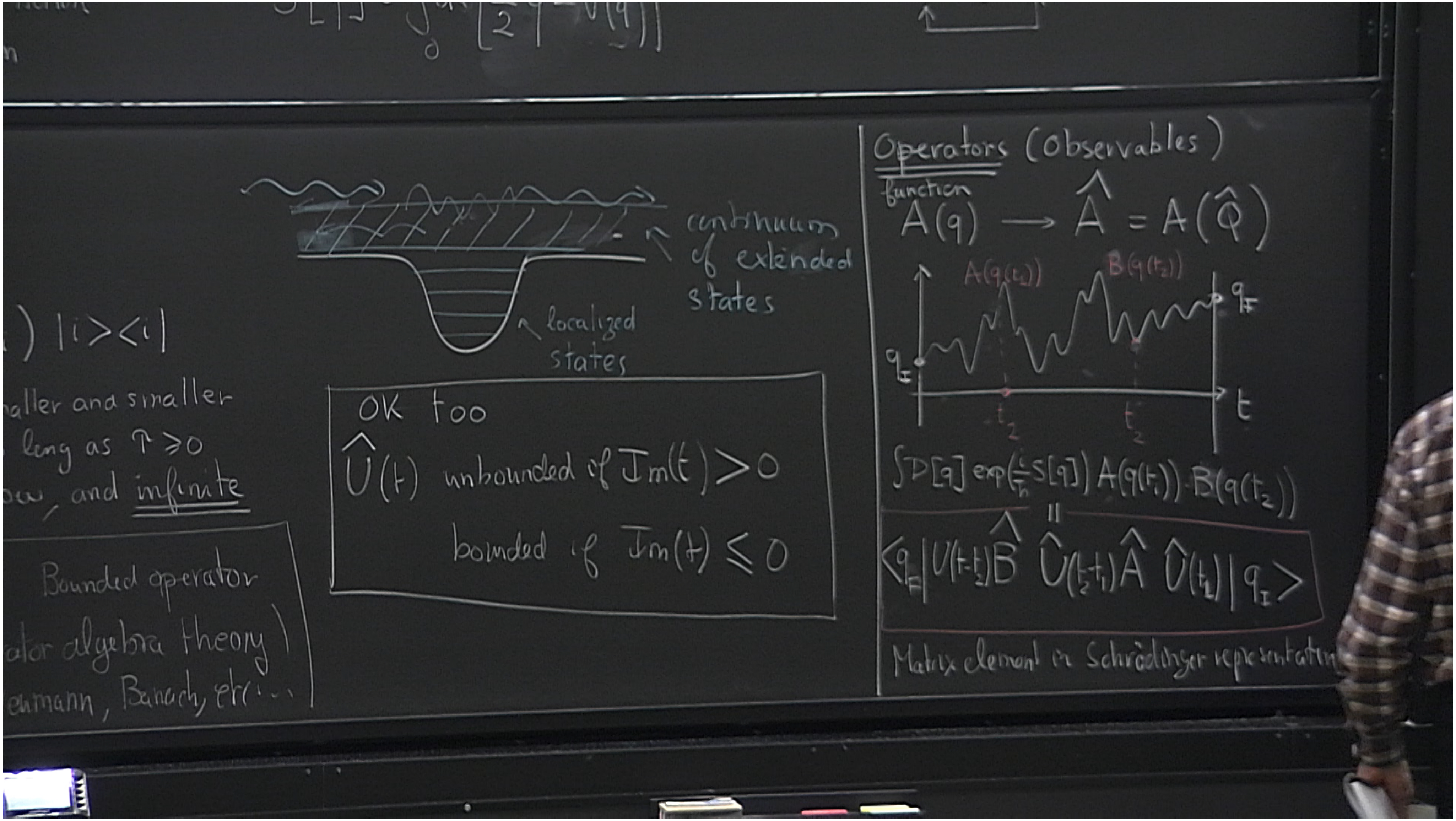
Random Walks & Brownian motions

$\downarrow$ QFT

Gaussian Free Field



Operators (Observables)
function
A(q) → Â = A(Q̂)
A(q(t1))
B(q(t2))
q_F
q_I
t
t1
t2
∫D[q] exp(i/ħ S[q]) A(q(t1)) · B(q(t2))
continuum of extended states
localized states
OK too
U(t) unbounded if Im(t) > 0
bounded if Im(t) ≤ 0
) |i><i|
aller and smaller
long as τ ≥ 0
and infinite
Bounded operator
ator algebra theory)
eumann, Banach, etc...


continuum
of extended
states
localized
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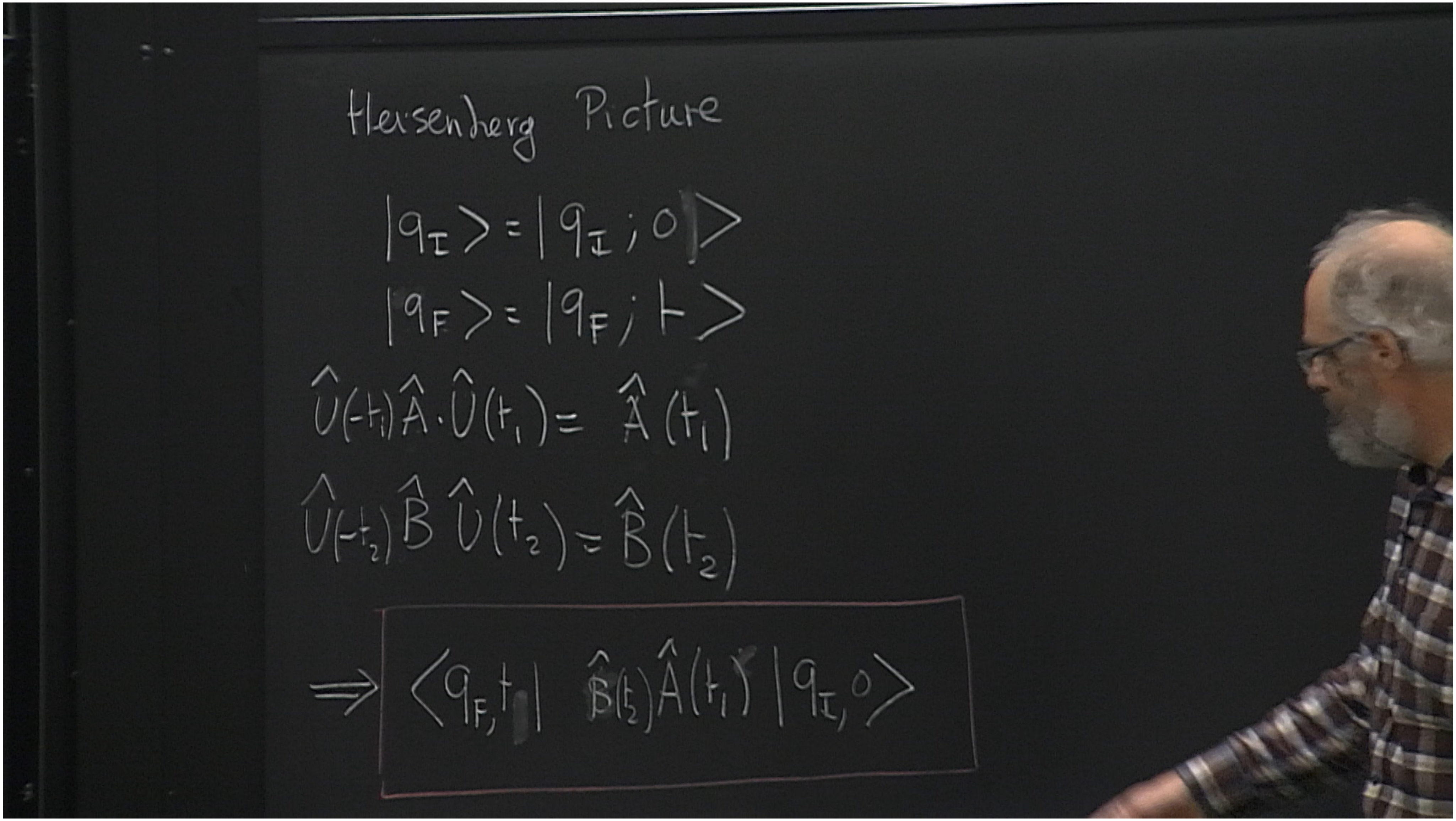
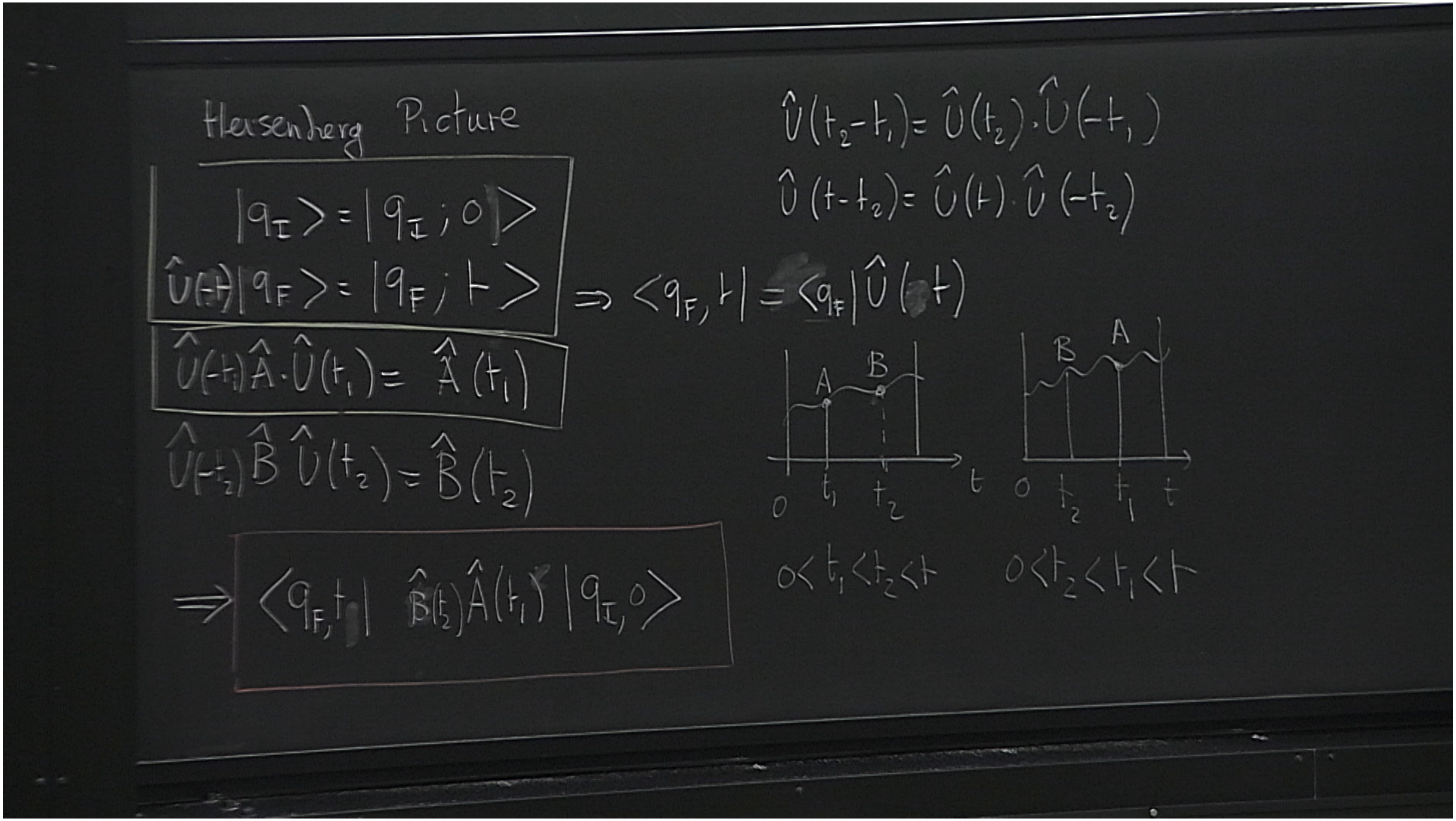
ehmann, Banach, etc...
Operators (Observables)
function
A(q) → Â = A(Q̂)
A(q(t₁))
B(q(t₂))
q_F
q_I
t
t₁
t₂
∫D[q] exp(i/ħ S[q]) A(q(t₁)) · B(q(t₂))
=
<q_F| U(t_F−t₂) B̂ Û(t₂−t₁) Â Û(t₁) |q_I>
Matrix element in Schrödinger representation


Heisenberg Picture
$|q_I\rangle = |q_I ; 0\rangle$
$|q_F\rangle = |q_F ; t\rangle$
$\hat{U}(-t_1)\hat{A}\cdot\hat{U}(t_1) = \hat{A}(t_1)$
$\hat{U}(-t_2)\hat{B}\,\hat{U}(t_2) = \hat{B}(t_2)$
$\Rightarrow \boxed{\langle q_F, t|\ \hat{B}(t_2)\hat{A}(t_1)\,|q_I, 0\rangle}$


Heisenberg Picture
$|q_I\rangle = |q_I; 0\rangle$
$\hat{U}(-t)|q_F\rangle = |q_F; t\rangle \Rightarrow \langle q_F, t| = \langle q_F|\hat{U}(t)$
$\hat{U}(-t_1)\hat{A}\cdot\hat{U}(t_1) = \hat{A}(t_1)$
$\hat{U}(-t_2)\hat{B}\,\hat{U}(t_2) = \hat{B}(t_2)$
$\Rightarrow \boxed{\langle q_F, t|\,\hat{B}(t_2)\hat{A}(t_1)\,|q_I, 0\rangle}$
$\hat{U}(t_2-t_1) = \hat{U}(t_2)\cdot\hat{U}(-t_1)$
$\hat{U}(t-t_2) = \hat{U}(t)\cdot\hat{U}(-t_2)$
A
B
0 t_1 t_2 t
$0<t_1<t_2<t$
B
A
0 t_2 t_1 t
$0<t_2<t_1<t$

Heisenberg Picture

$$\boxed{\begin{aligned} |q_I\rangle &= |q_I;0\rangle \\ \hat U(-t)|q_F\rangle &= |q_F;t\rangle \end{aligned}} \Rightarrow \langle q_F,t| = \langle q_F|\hat U(t)$$

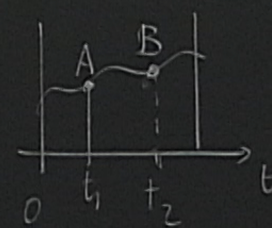
$$\boxed{\hat U(-t_1)\hat A\cdot\hat U(t_1) = \hat A(t_1)}$$

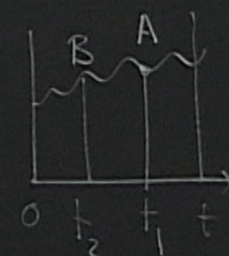
$$\hat U(-t_2)\hat B\,\hat U(t_2) = \hat B(t_2)$$

$$\Rightarrow \boxed{\langle q_F,t|\,T[\hat B(t_2)\hat A(t_1)]\,|q_I,0\rangle}$$

$$\hat U(t_2-t_1) = \hat U(t_2)\cdot\hat U(-t_1)$$

$$\hat U(t-t_2) = \hat U(t)\cdot\hat U(-t_2)$$





$0<t_1<t_2<t$

$\hat B(t_2)\hat A(t_1)$

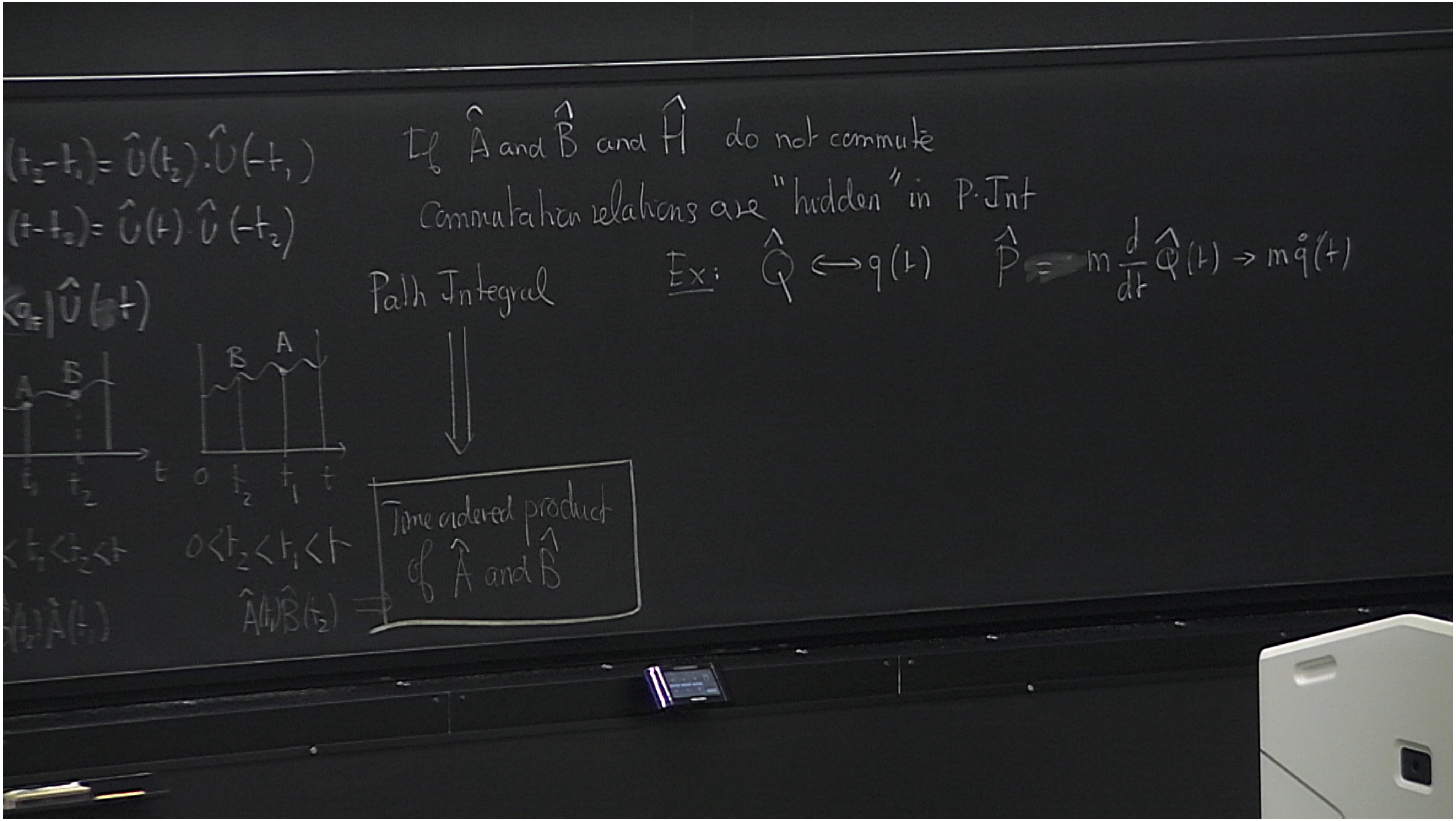
$0<t_2<t_1<t$

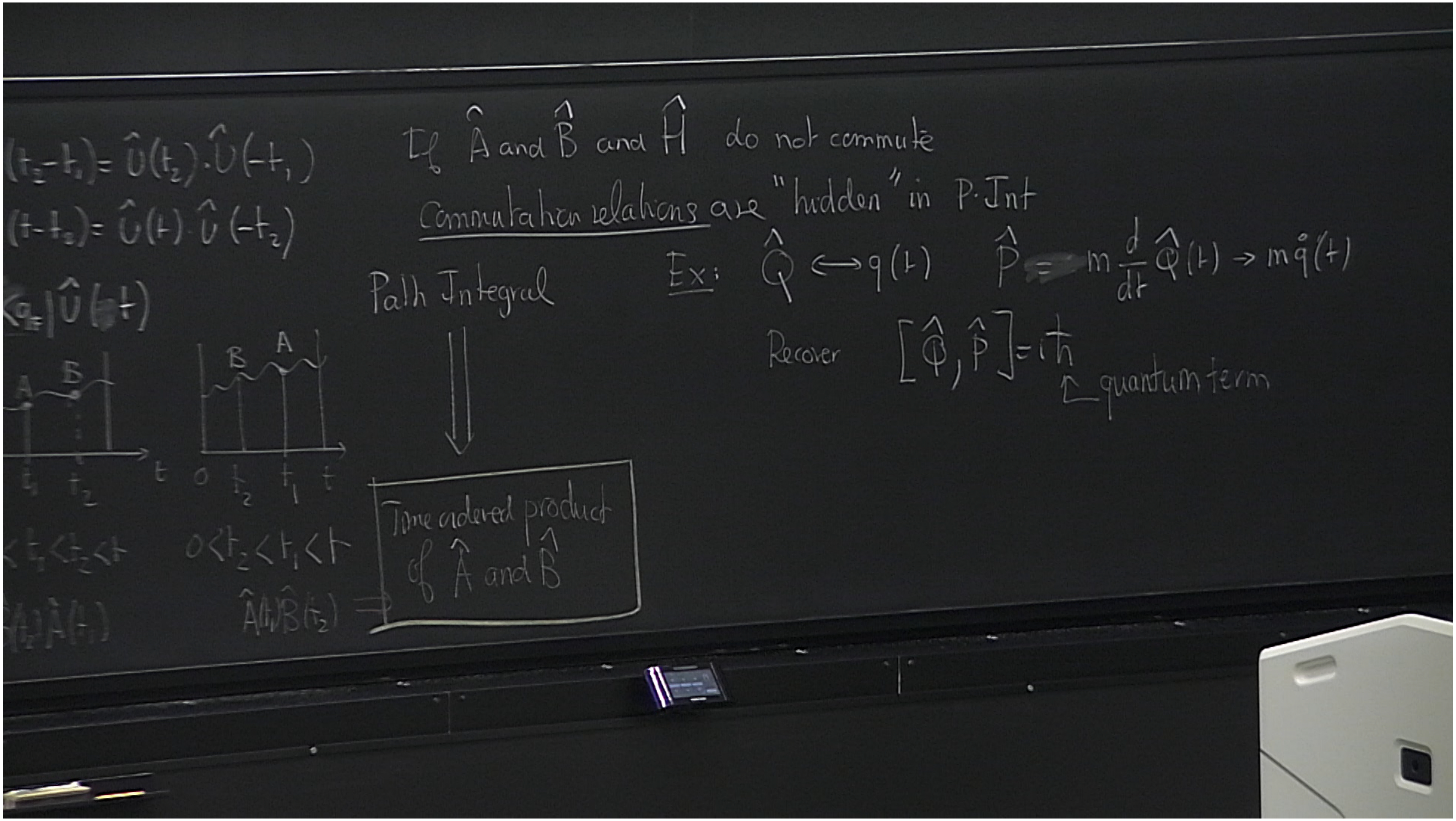
$\hat A(t_1)\hat B(t_2)$ $\Rightarrow$

Path Integral

$\Downarrow$

Time ordered product of $\hat A$ and $\hat B$


If Â and B̂ and Ĥ do not commute
Commutation relations are "hidden" in P. Int
Ex: Q̂ ⟷ q(t)   P̂ = m d/dt Q̂(t) → m q̇(t)
Path Integral
Time ordered product of Â and B̂
0 < t₂ < t₁ < t
Â(t₁)B̂(t₂)


If $\hat{A}$ and $\hat{B}$ and $\hat{H}$ do not commute
Commutation relations are "hidden" in P. Int
Ex: $\hat{Q} \longleftrightarrow q(t)$ $\hat{P} = m \frac{d}{dt}\hat{Q}(t) \to m\dot{q}(t)$
Recover $[\hat{Q}, \hat{P}] = i\hbar$
quantum term
Path Integral
Time ordered product of $\hat{A}$ and $\hat{B}$
$\hat{A}(t_1)\hat{B}(t_2)$
$0 < t_2 < t_1 < t$