Title: Hydrodynamics and (Pre)Thermalization in Floquet Systems

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Series: Condensed Matter

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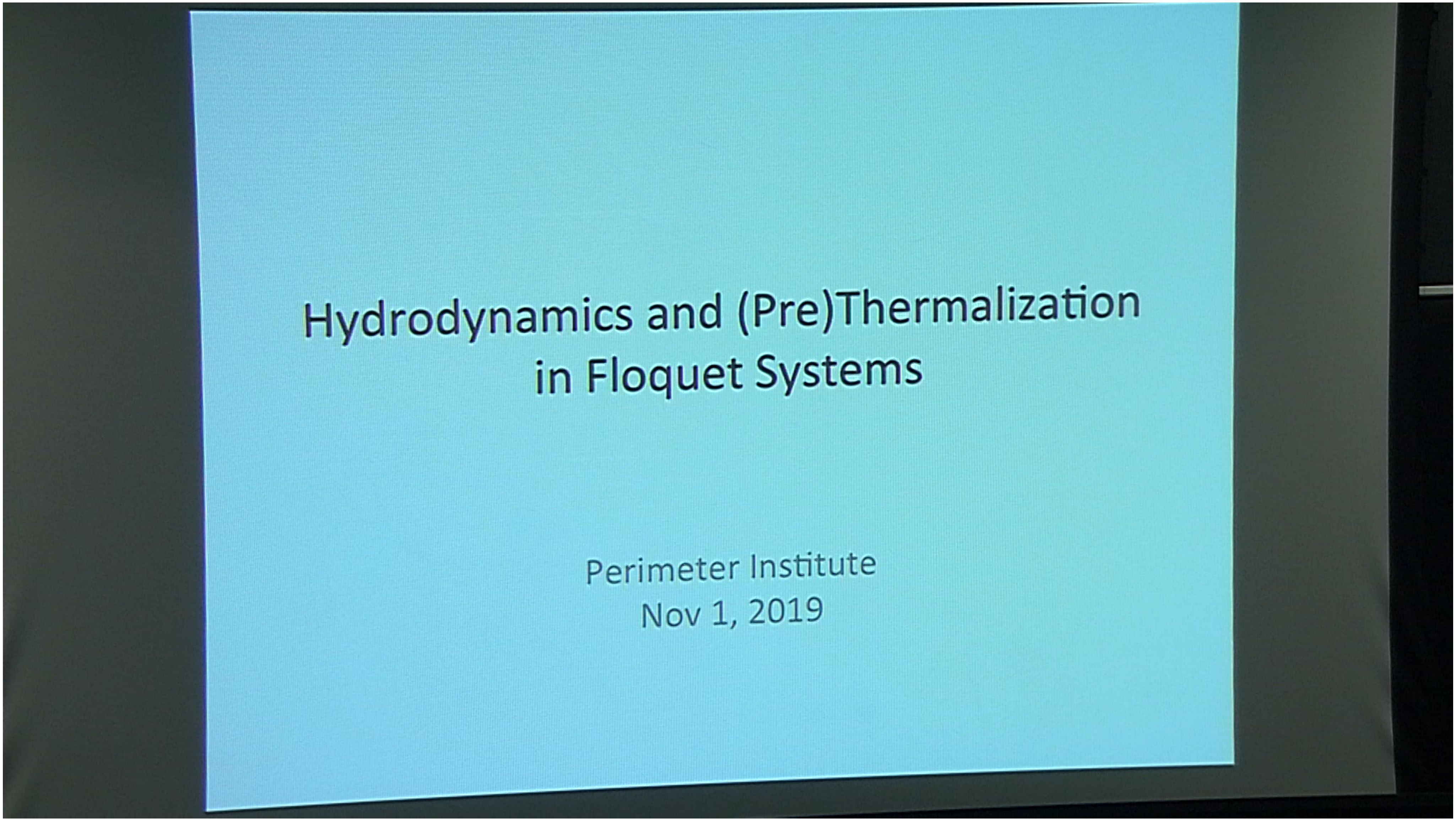
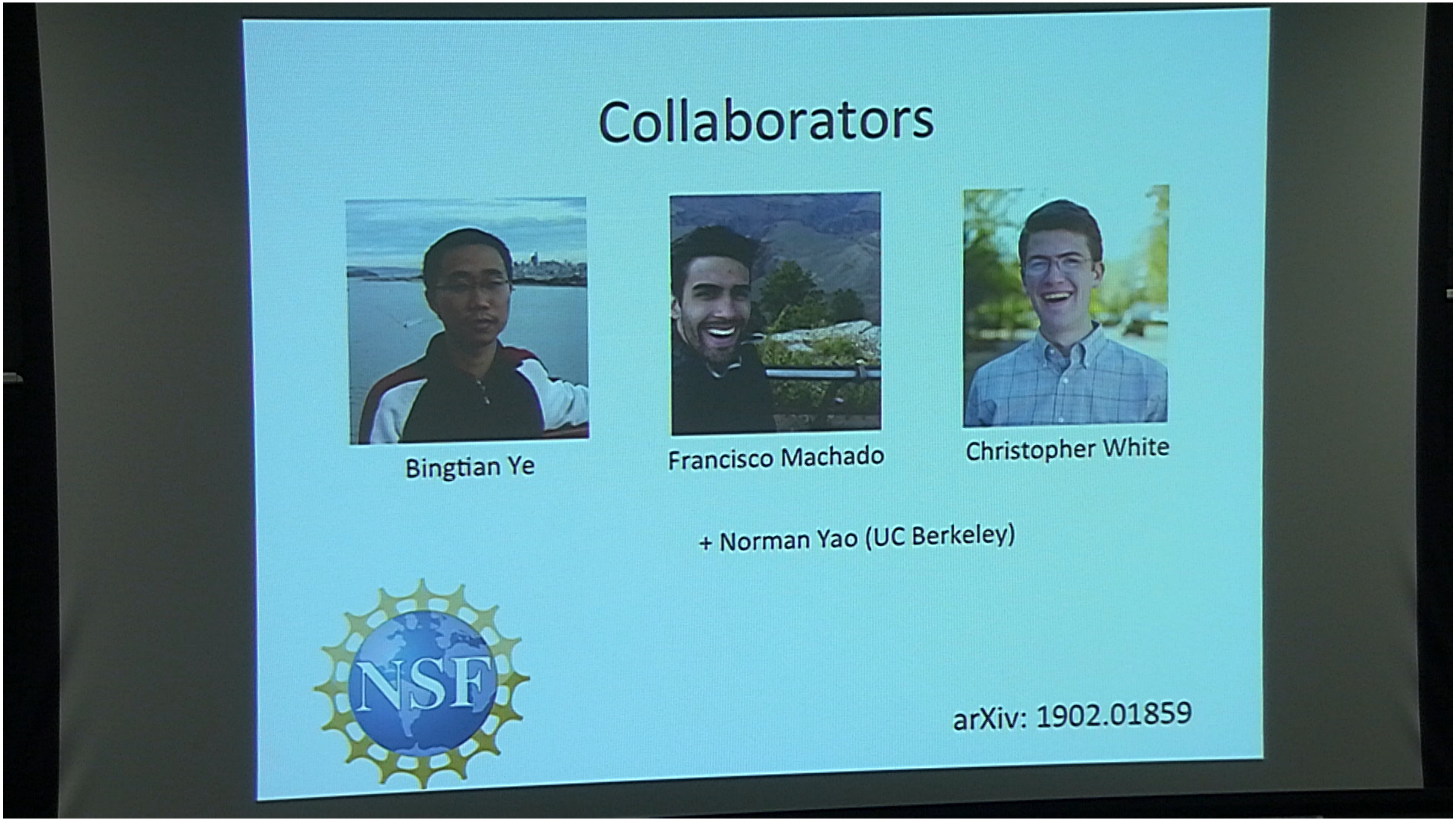
Abstract: A tremendous amount of recent attention has focused on characterizing the dynamical properties of periodically driven many-body systems. Here, we use a novel numerical tool termed â€˜density matrix truncationâ€™ (DMT) to investigate the long-time dynamics of large-scale Floquet systems. By implementing a spatially inhomogeneous drive to a 1D quantum chain, we demonstrate that an interplay between Floquet heating and diffusive transport is crucial to understanding the systemâ€™s dynamics. We find that DMT accurately captures two essential pieces of Floquet physics, namely prethermalization and late-time heating to infinite temperature. Moreover, we show that these two aspects are driven by different microscopic mechanisms.


Hydrodynamics and (Pre)Thermalization
in Floquet Systems
Perimeter Institute
Nov 1, 2019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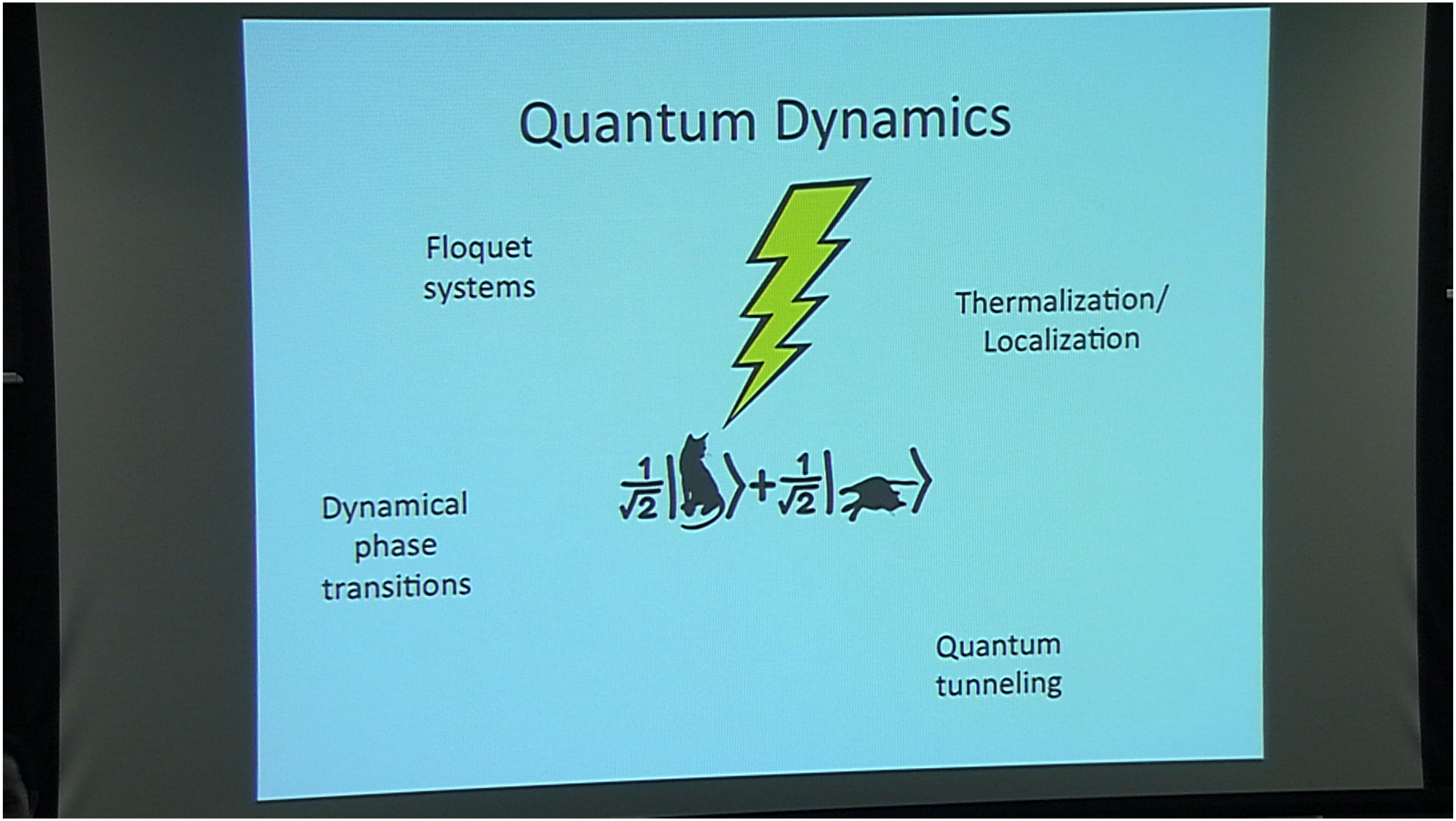
arXiv: 1902.01859


Quantum Dynamics
Floquet systems
Thermalization/ Localization
Dynamical phase transitions
Quantum tunneling

Thermalization

- Does the system reach thermal equilibrium (with itself)?
  - Does it become a Gibbs state? $\frac{1}{Z}e^{-\beta H}$
- Quantum quench
  - Suddenly takes system out of equilibrium
- Transport measurements
  - Do quasiparticles thermalize with their environment?

Thermalization

- Examples with thermalization
  - Interacting fermion chain (no disorder)
  - Generic strongly-interacting, low disorder systems

- Examples that do not thermalize
  - Integrable systems (e.g. harmonic oscillator)
  - Free (non-interacting) fermions
  - Many body localization

Hydrodynamics

- Conserved quantities governed by classical equations of motion.
  - Energy, charge, etc. behaves like a fluid
- All observables are determined by classical variables
  - Local thermalization

$$\rho = \frac{1}{Z} e^{\beta(x) H}$$

Floquet Physics

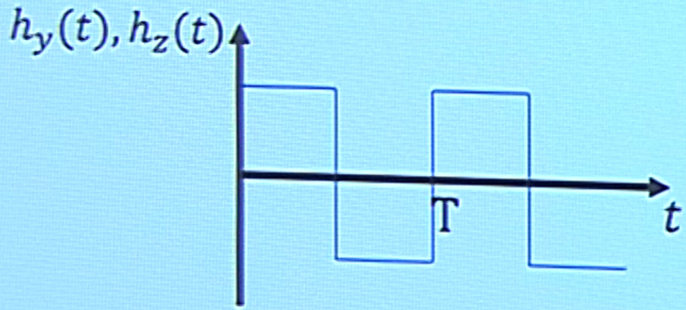
- Periodically driven from external source
  - Unitary evolution (does not couple to bath)
- Examples
  - Classical light (pump drive)
  - Periodic trap modulation

Floquet Hamiltonian

$$H(t) = H_{\text{static}} + H_{\text{drive}}(t)$$

$$H_{\text{static}} = \sum_x \left[J Z_x Z_{x+1} + J_x X_x X_{x+1} + h_x X_x \right]$$

$$H_{\text{drive}}(t) = \sum_x \left[h_y(t) Y_x + h_z(t) Z_x \right]$$



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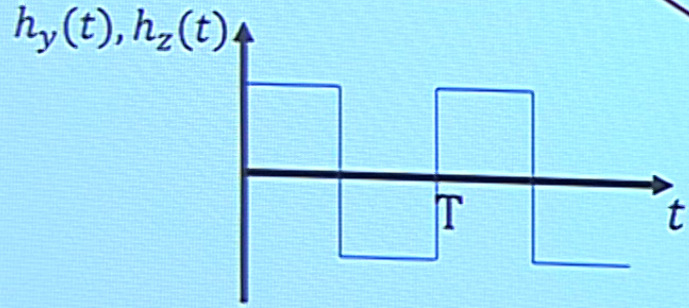
Local Interactions



Drive induces transitions between states with energy ω apart.

$$\omega = \frac{2\pi}{T}$$

Floquet Physics

- Periodically driven from external source
  - Unitary evolution (does not couple to bath)
- Examples
  - Classical light (pump drive)
  - Periodic trap modulation
- Energy is not conserved
  - External drive creates excitations
  - Typically drive system to infinite temperature

Floquet (Pre)thermalization

- Rate of heating << Rate of thermalization
- System appears to be a Gibbs state at some finite temperature

Floquet (Pre)thermalization

- Rate of heating << Rate of thermalization
- System appears to be a Gibbs state at some finite temperature
- Exponentially slow heating
  - Driving frequency >> single-particle excitation energies
  - Need to create multiple particles at the same time
  - Heating rate $\propto e^{-\omega/J}$

[Abanin et al. PRL 2015; Mori et al. PRL 2016; Kuwahara et al. Ann. Phys. 2016; Abanin et al. PRB 2017]

Floquet Hamiltonian

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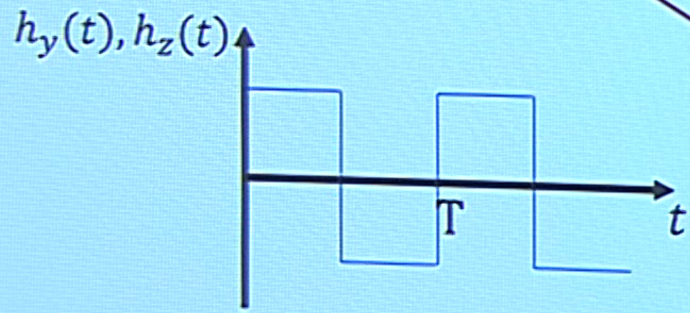
Local Interactions



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Floquet Prethermalization

- Heating rate $\propto e^{-\omega/J}$
- Drive frequency >> single-body energy

$$\omega \gg J$$

Floquet Prethermalization

- Heating rate $\propto e^{-\omega/J}$
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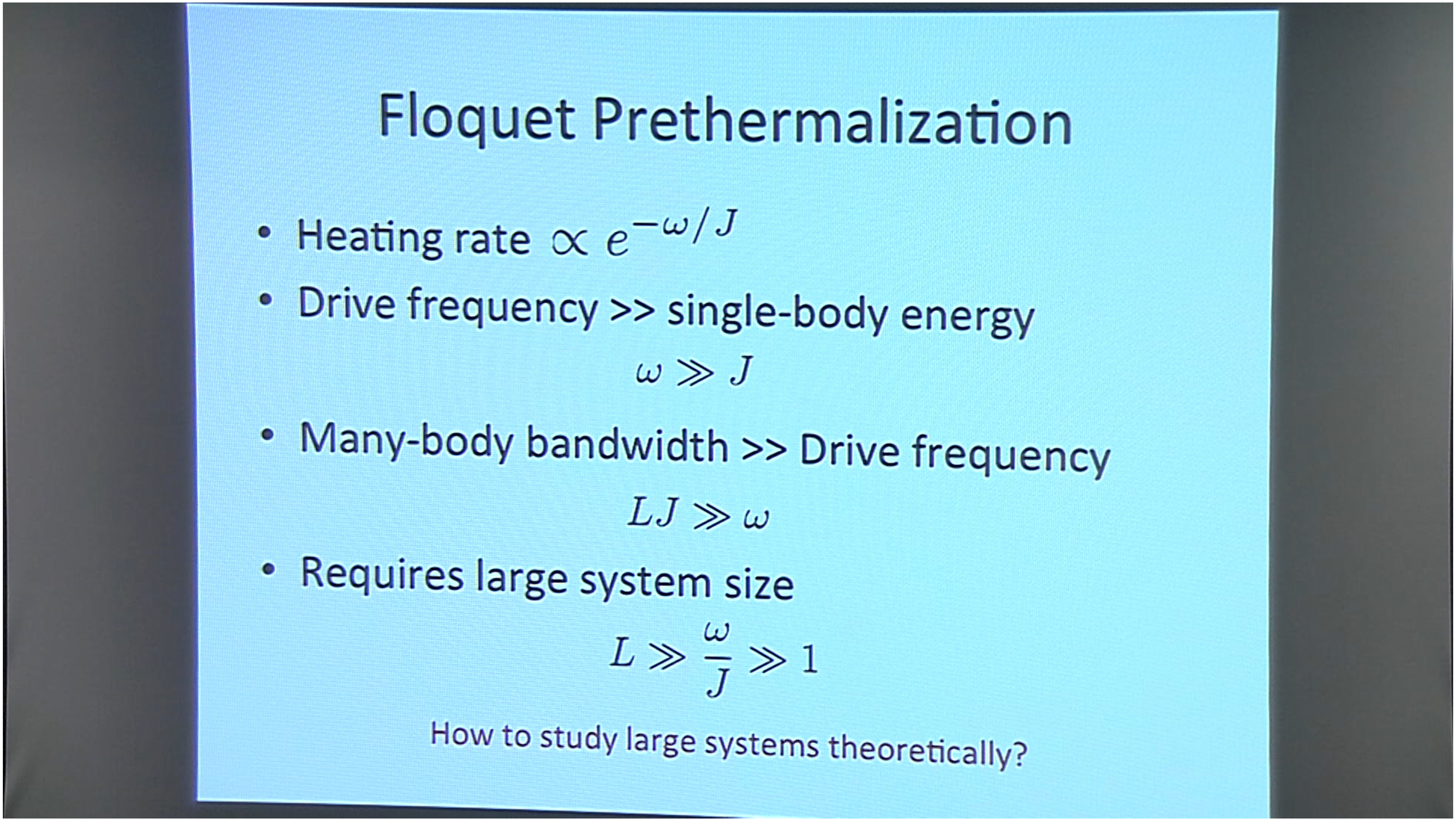
$$\omega \gg J$$

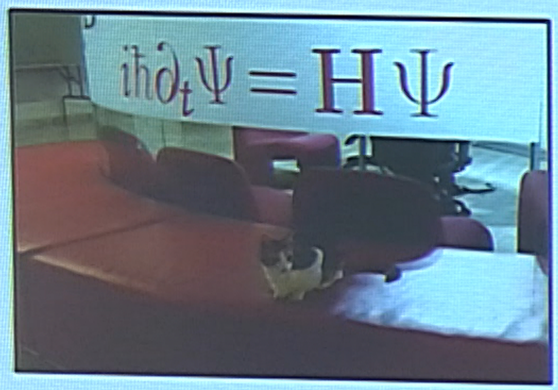
- Many-body bandwidth >> Drive frequency

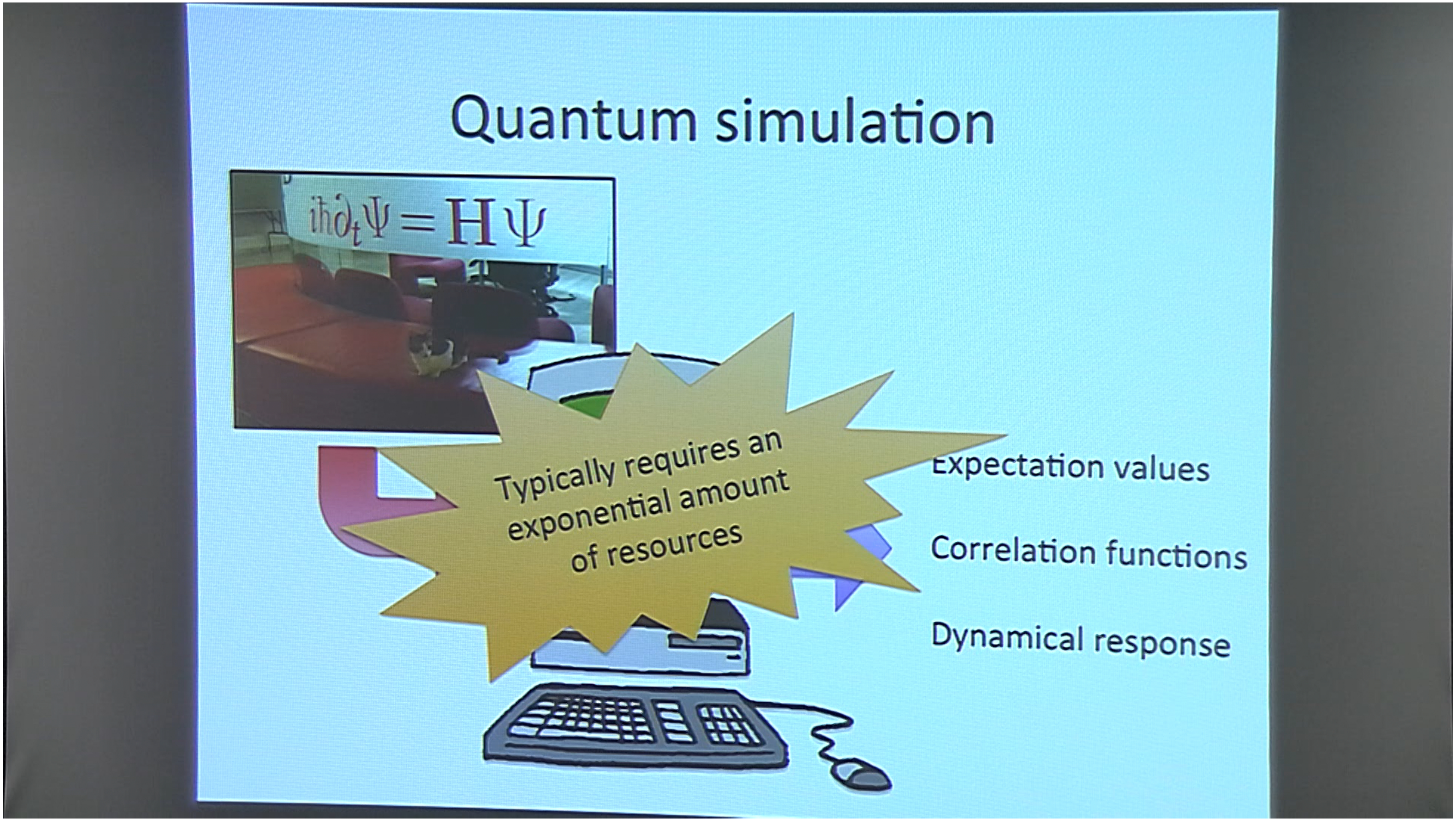
$$LJ \gg \omega$$

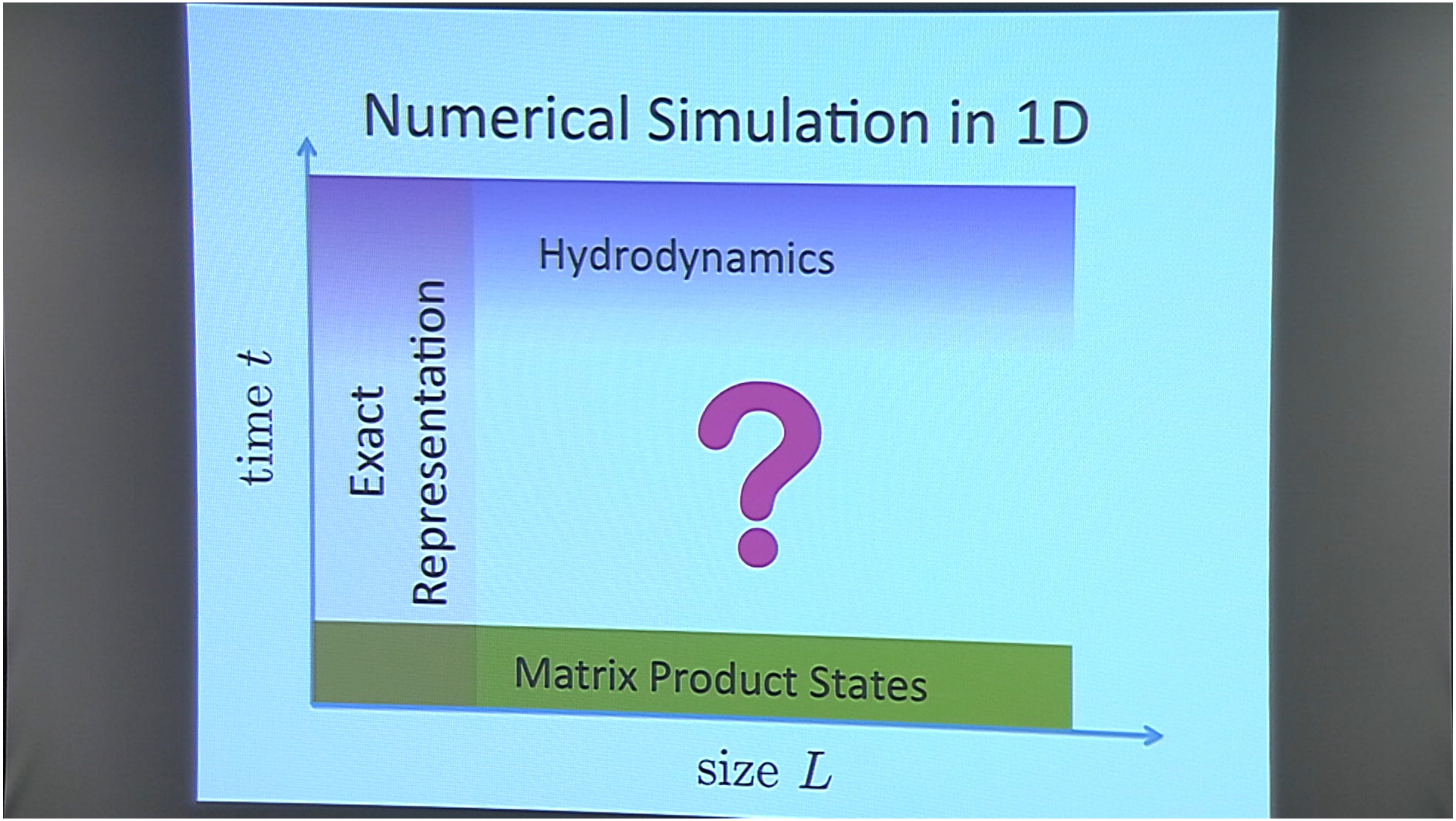


Floquet Prethermalization
• Heating rate $\propto e^{-\omega/J}$
• Drive frequency >> single-body energy
$\omega \gg J$
• Many-body bandwidth >> Drive frequency
$LJ \gg \omega$
• Requires large system size
$L \gg \frac{\omega}{J} \gg 1$
How to study large systems theoretically?

Quantum simulation




Quantum simulation
$i\hbar\partial_t\Psi = \mathrm{H}\,\Psi$
Typically requires an exponential amount of resources
Expectation values
Correlation functions
Dynamical response



Numerical Simulation in 1D
time t
Exact Representation
Hydrodynamics
?
Matrix Product States
size L

Quantum Dynamics

- What happens when we combine
  - Floquet systems
  - Prethermalization
  - Hydrodynamics

- How do we study such system?
  - New numerical technique

"Brute Force" Algorithm

Initial state

$$|\psi(0)\rangle = \sum \psi_{s_1, s_2, \ldots} |s_1, s_2, \ldots\rangle$$

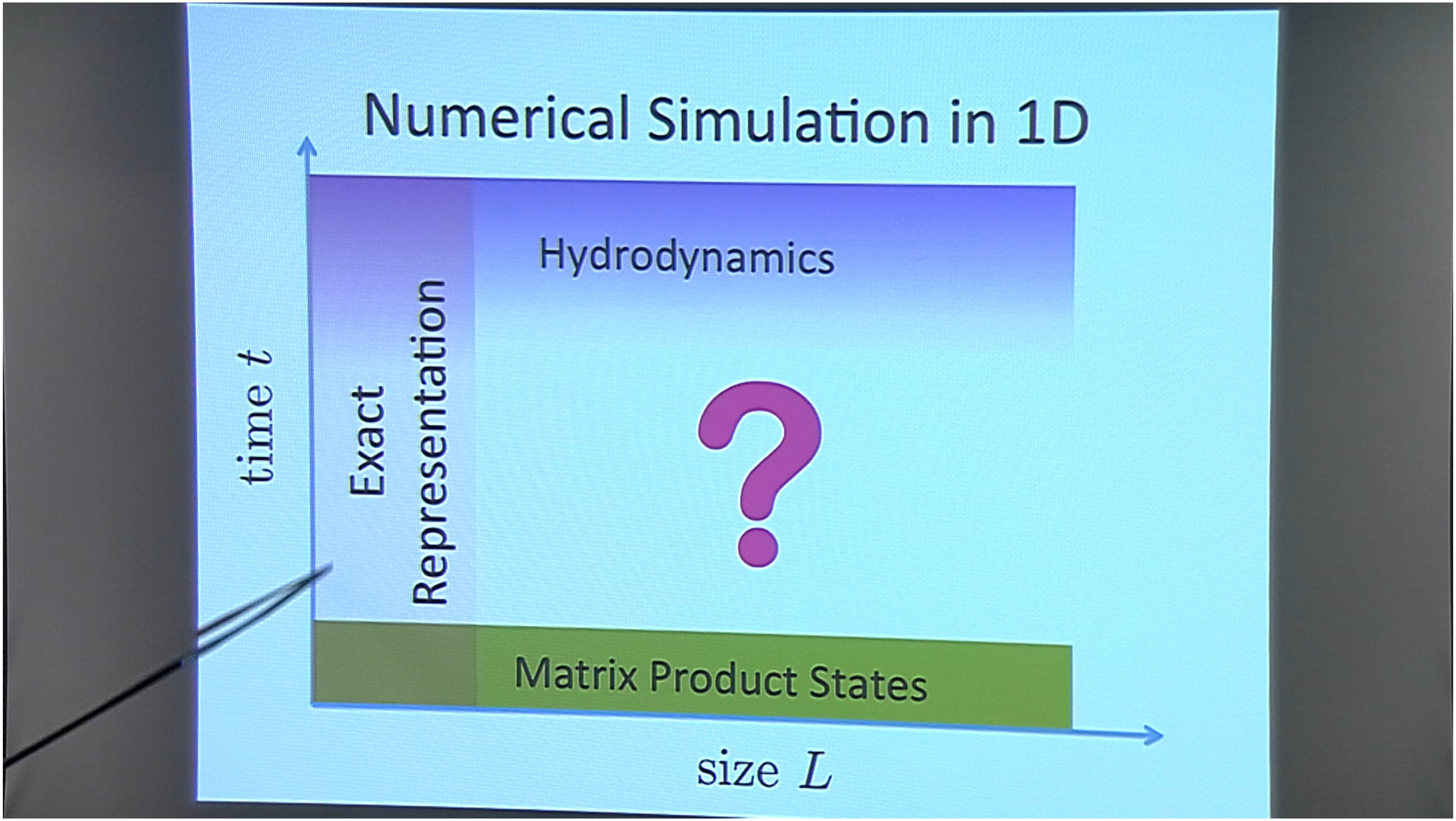
Time-evolution

$$U(\delta t) = \exp\left[-\frac{i}{\hbar} H \delta t\right]$$

$$|\psi(t + \delta t)\rangle = U(\delta t)\, |\psi(t)\rangle$$

Final state

$$|\psi(T)\rangle$$


Numerical Simulation in 1D
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Matrix Product States (MPS)

Product state: $|\psi\rangle = |\varphi^{(1)}\rangle\,|\varphi^{(2)}\rangle \cdots |\varphi^{(L)}\rangle$

MPS: $|\psi\rangle = A^{(1)} A^{(2)} \ldots A^{(L)}$

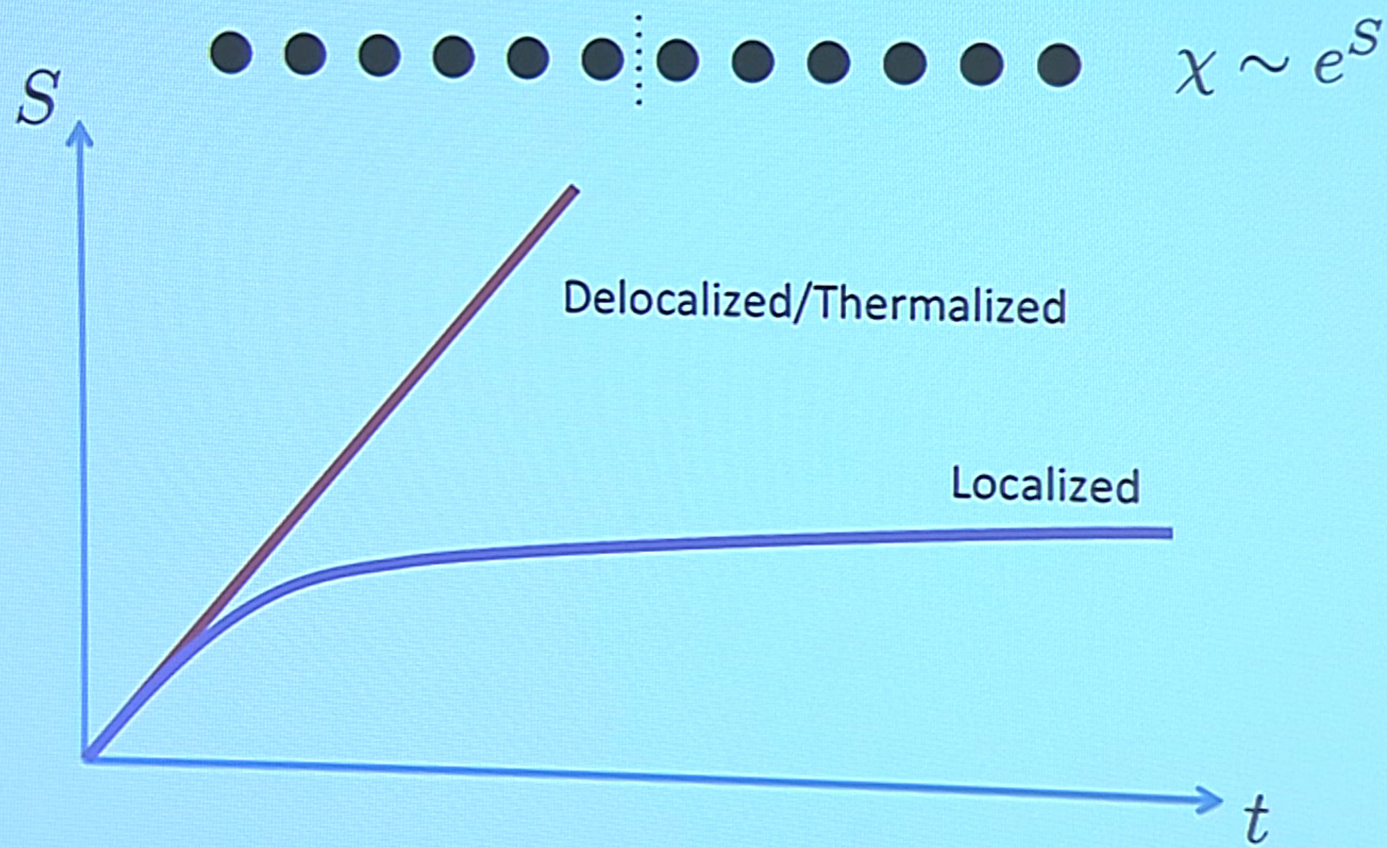
$$A = \begin{bmatrix} |?\rangle & |?\rangle & |?\rangle \\ |?\rangle & |?\rangle & |?\rangle \\ |?\rangle & |?\rangle & |?\rangle \end{bmatrix}$$

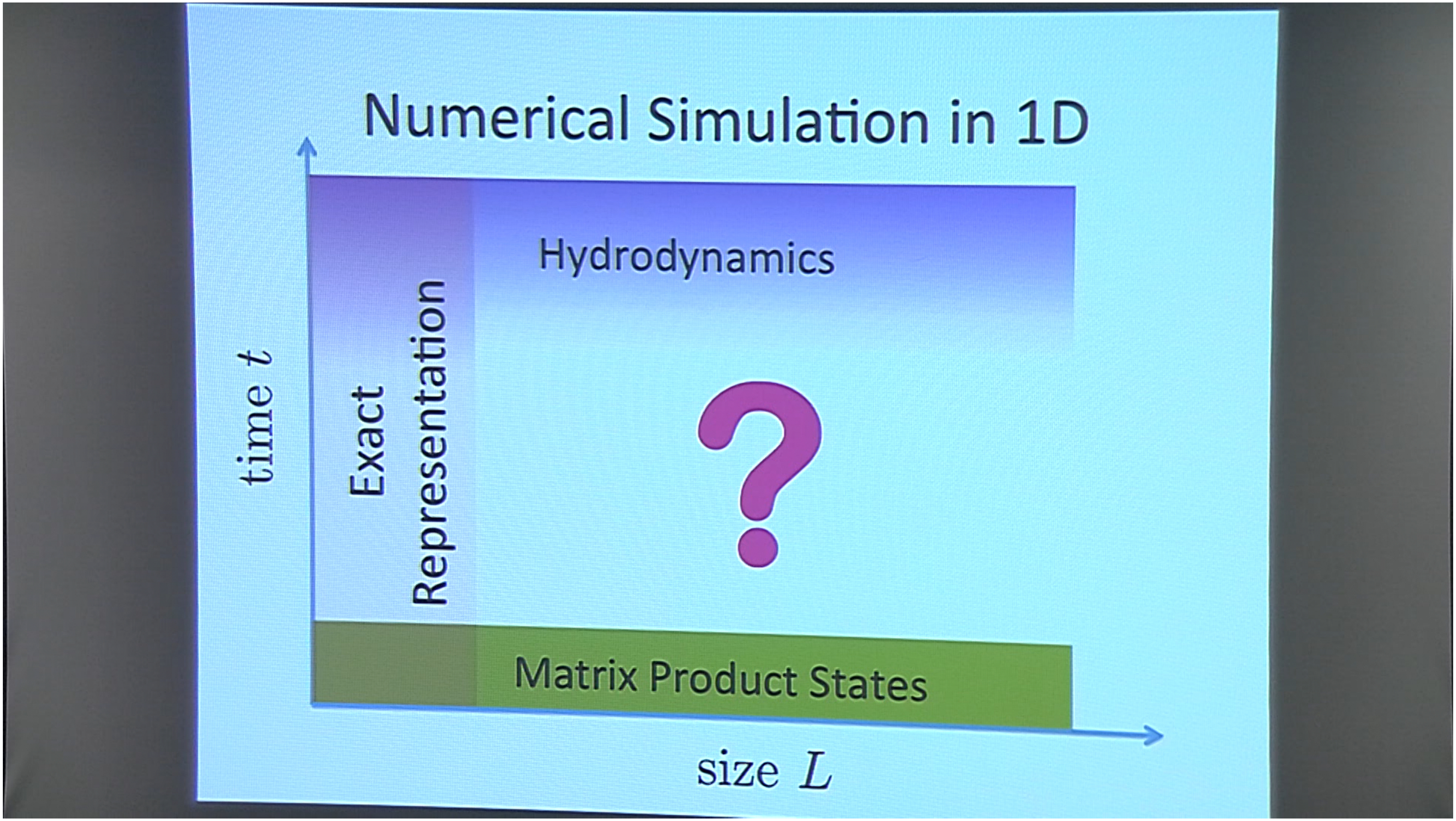
Each matrix has dimensions $\chi \times \chi$

χ : bond dimension

Memory resource $\sim L\chi^2$

Entanglement Growth





Numerical Simulation in 1D
Hydrodynamics
Exact
Representation
?
Matrix Product States
time t
size L

Eigenstate Thermalization Hypothesis

- For a closed system, eigenstates at the same energy density are locally indistinguishable.

- Thermalization criteria (Does not apply to localized systems)

- Local observable: $\langle\varphi|\hat{O}|\varphi\rangle \approx \mathrm{Tr}\left[e^{-\beta H}\hat{O}\right]$

Eigenstate Thermalization Hypothesis

- For a closed system, eigenstates at the same energy density are locally indistinguishable.
- Thermalization criteria (Does not apply to localized systems)
- Local observable: $\langle\varphi|\hat{O}|\varphi\rangle \approx \mathrm{Tr}\left[e^{-\beta H}\hat{O}\right]$
- For a thermalized system,

  long ranged entanglement entropy ≈ thermal entropy

Density Matrix Truncation (DMT)

Idea: Long ranged entanglement → thermal fluctuations

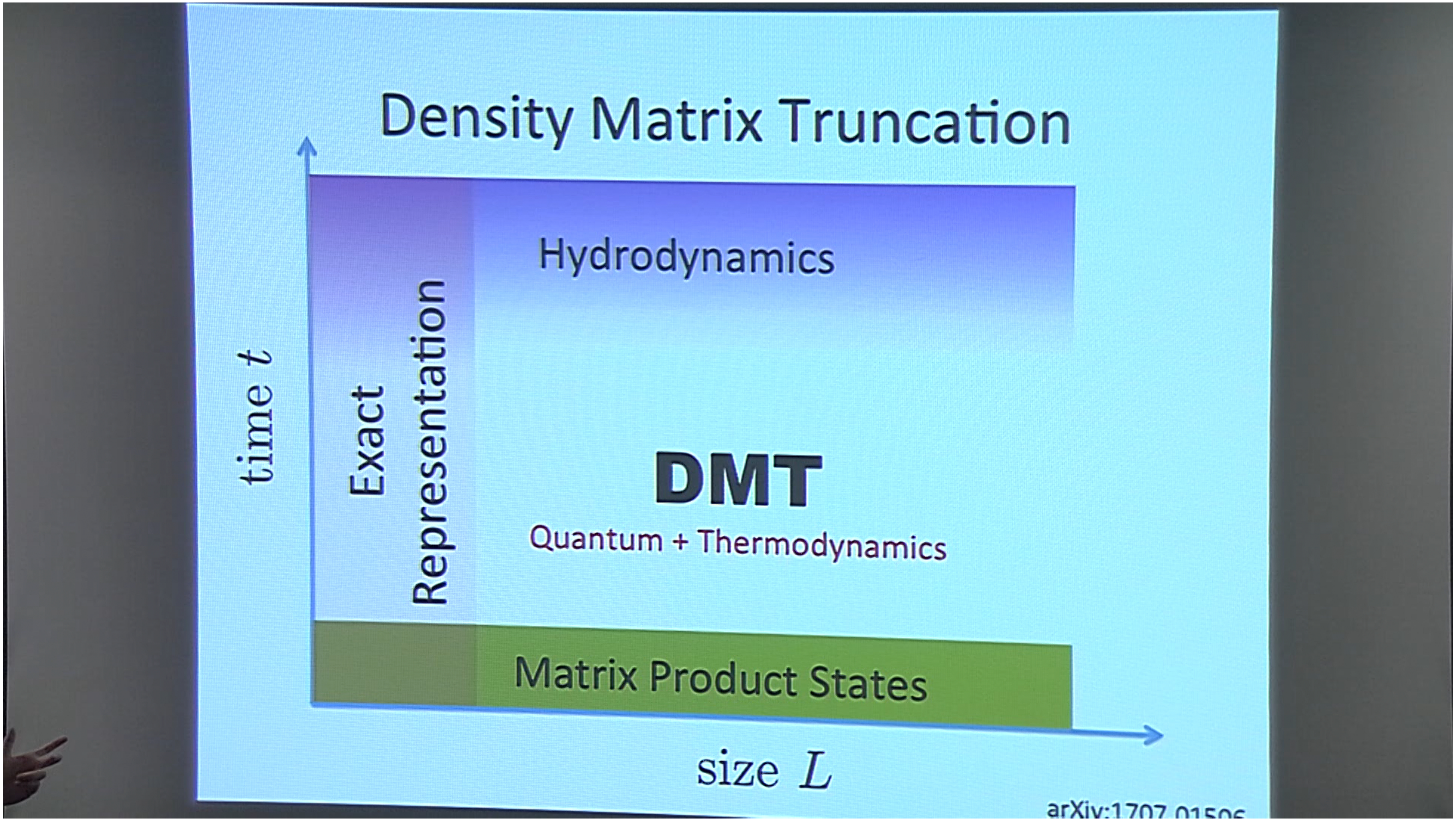
Density matrix representation: $\rho(t)$

Match local observables: $\mathrm{Tr}[\hat{A}\rho] \approx \langle\psi|\hat{A}|\psi\rangle$

Differing global states: $\rho \neq |\psi\rangle\langle\psi|$

With the Matrix-Product-Operator (MPO) Representation:

- Thermalized (Gibbs) states do admit an easy MPO description.
- (Excited eigenstates do not admit an efficient MPS description.)

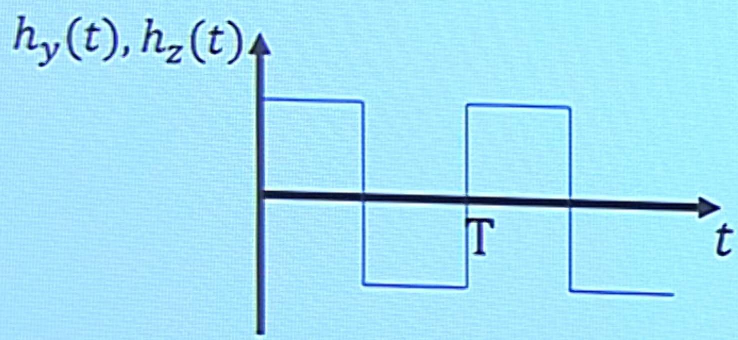

Density Matrix Truncation
Hydrodynamics
time t
Exact
Representation
DMT
Quantum + Thermodynamics
Matrix Product States
size L

Floquet Hamiltonian

$$H = J\sum_{i}^{L}\sigma_i^z\sigma_{i+1}^z + J_x\sum_{i}^{L}\sigma_i^x\sigma_{i+1}^x + \sum_{i}^{L} h_x\sigma_i^x + f(t)(h_y\sigma_i^y + h_z\sigma_i^z)$$

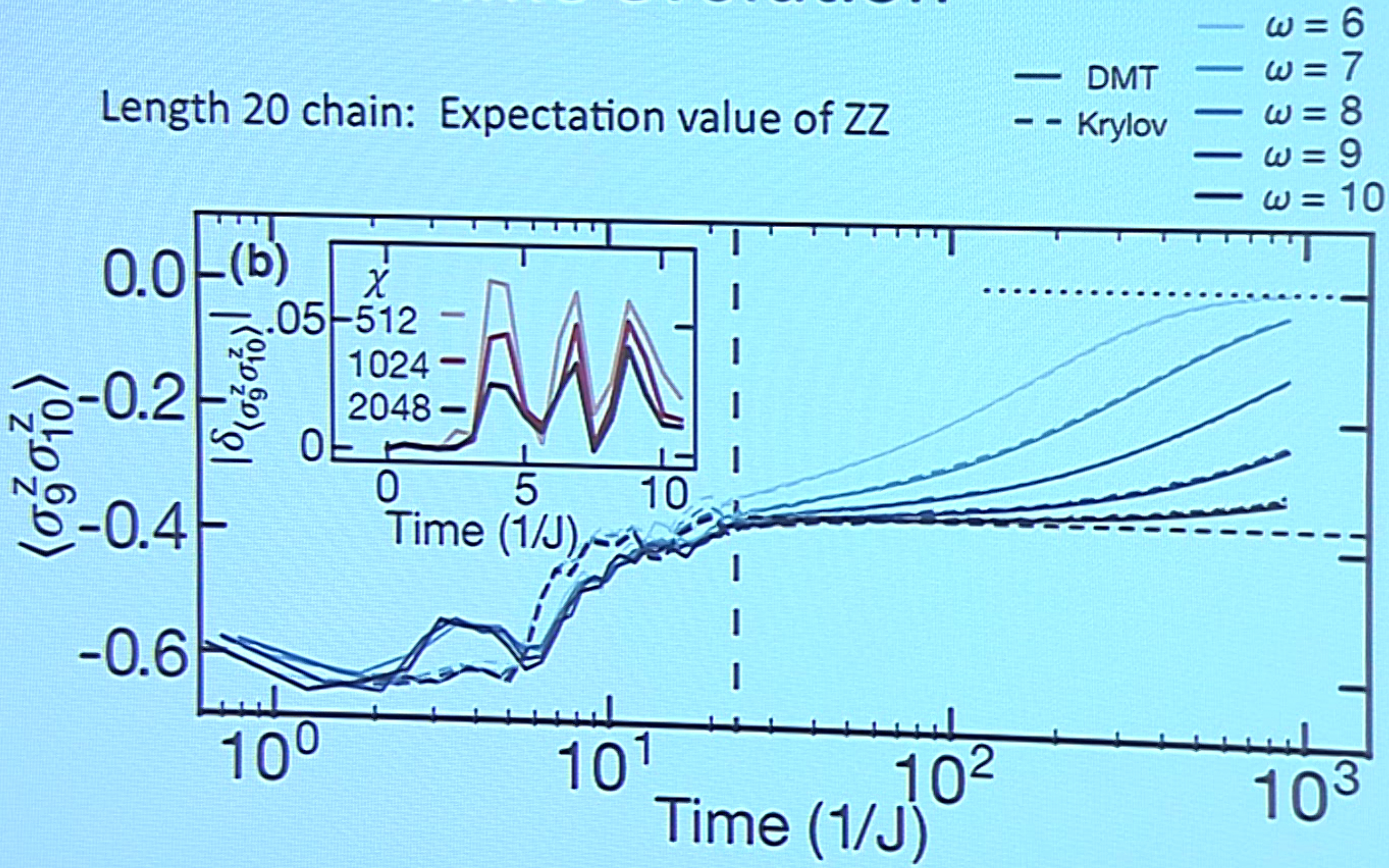
Parameters: $J = 1,\ J_x = 0.75,\ h_x = 0.21,\ h_y = 0.17,\ h_z = 0.13$

Initial state: $|\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\cdots\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\rangle$



Time evolution

Length 20 chain: Expectation value of ZZ



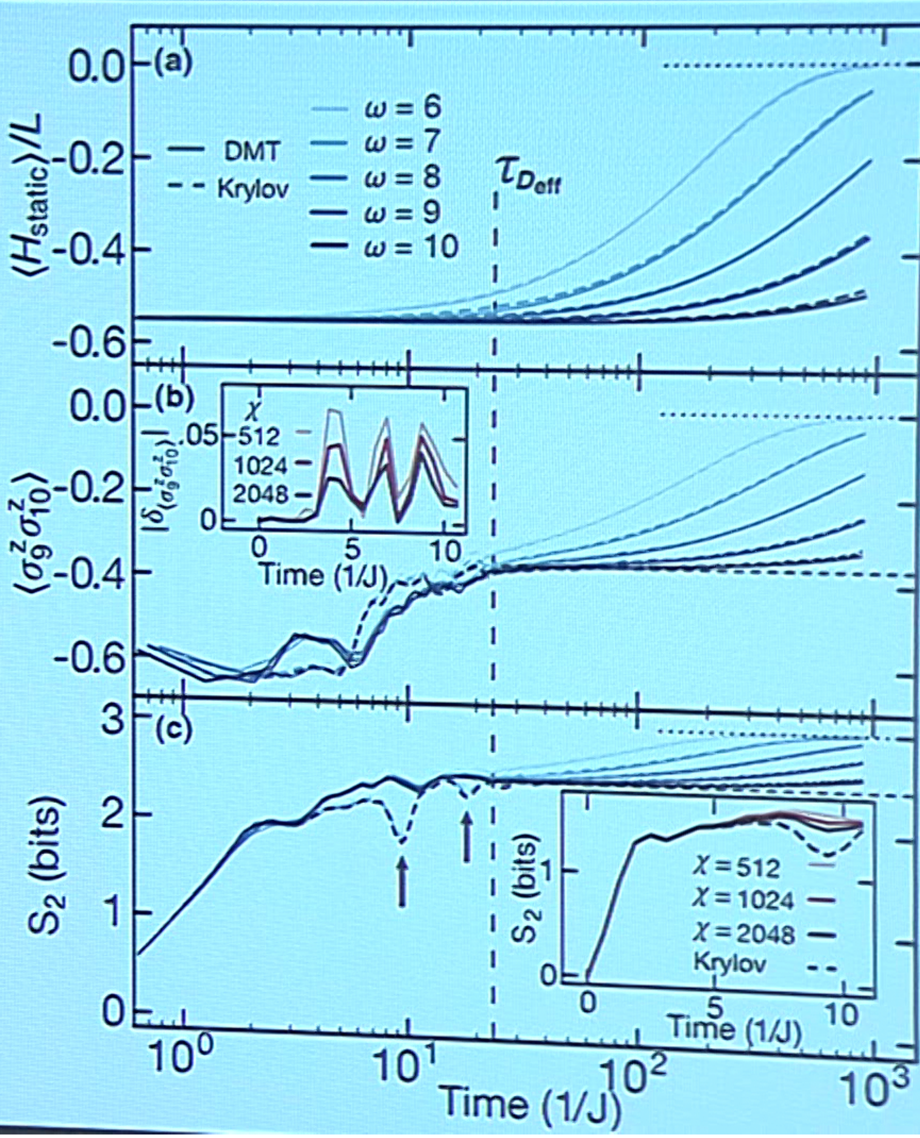
Time scales

$\tau_{D_{\text{eff}}}$: Time scale to reach prethermalization $\propto J^{-1}$

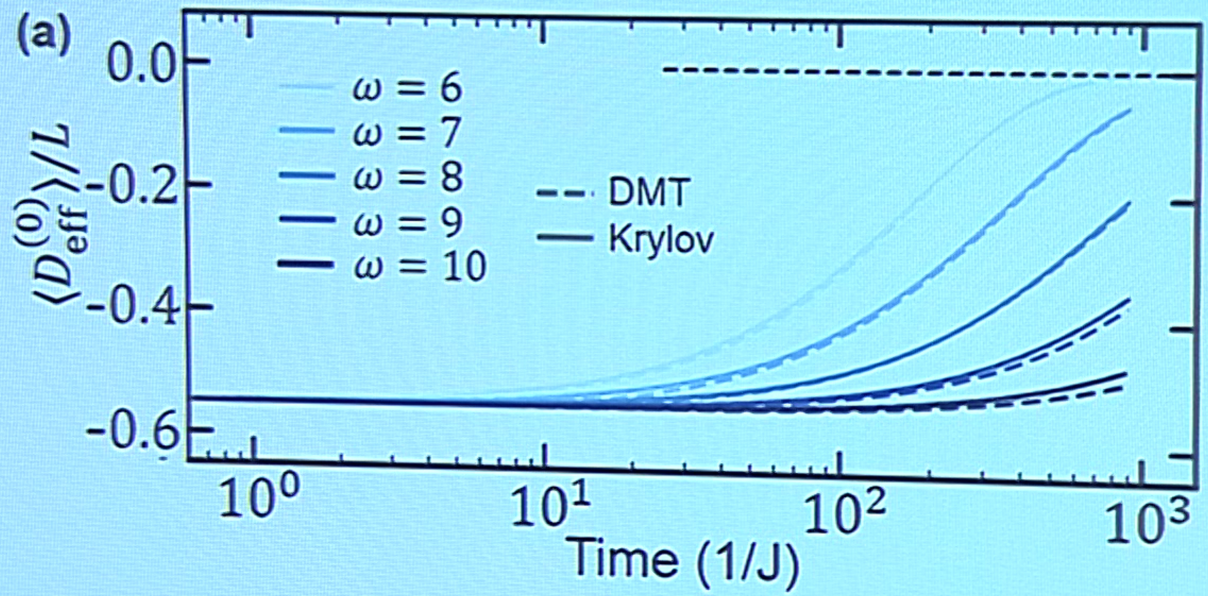
τ^* : Time scale reach infinite temperature $\sim \exp(\omega/J)$

Time evolution

- Exponentially slow heating at late times.
- Near conservation of H_{static}.
- Deviations at early time, but almost identical results at late times.



Prethermal "Energy"



$$H = J \sum_{i}^{L} \sigma_i^z \sigma_{i+1}^z + J_x \sum_{i}^{L} \sigma_i^x \sigma_{i+1}^x + \sum_{i}^{L} h_x \sigma_i^x + f(t)(h_y \sigma_i^y + h_z \sigma_i^z)$$

Prethermal Hamiltonian (0th order)

Inhomogeneous drive

$$H(t) = H_{\text{static}} + H_{\text{drive}}(t)$$

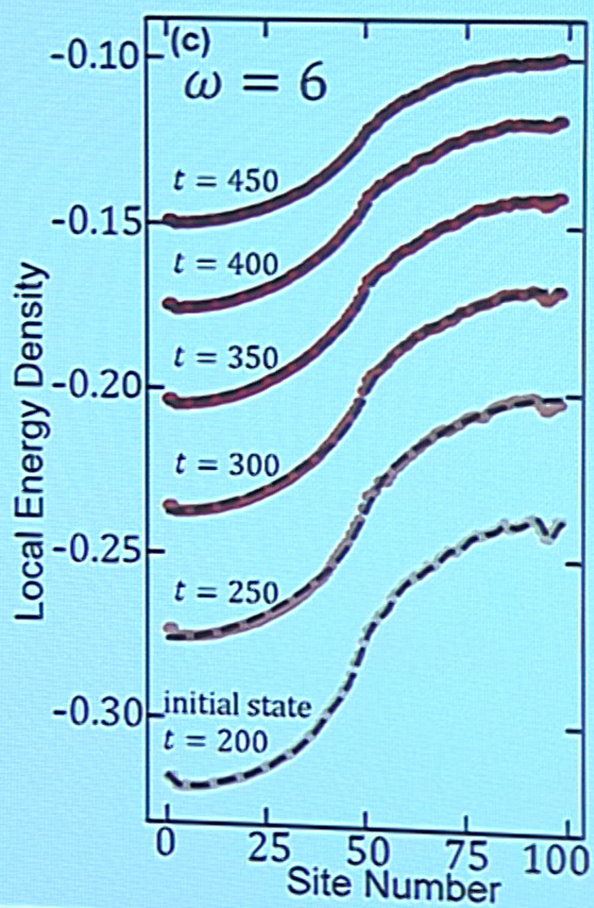
$$H_{\text{static}} = \sum_{x=1}^{L} \left[J Z_x Z_{x+1} + J_x X_x X_{x+1} + h_x X_x \right]$$

$$H_{\text{drive}}(t) = \sum_{x>L/2} \left[h_y(t) Y_x + h_z(t) Z_x \right]$$

Spatial profile: step function

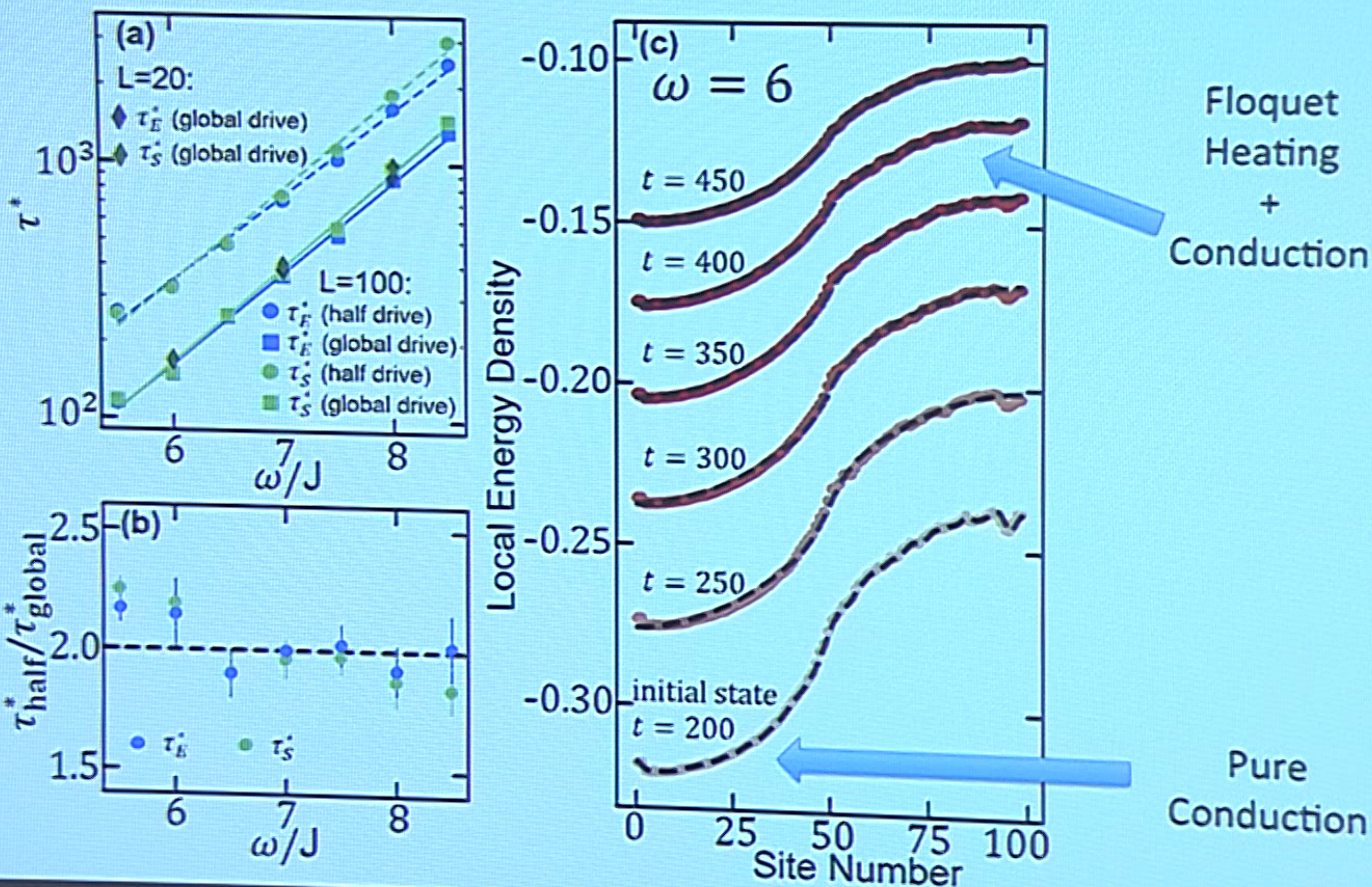
Hydrodynamics + Floquet Heating

Left half: Static Hamiltonian



Right half: Floquet Hamiltonian

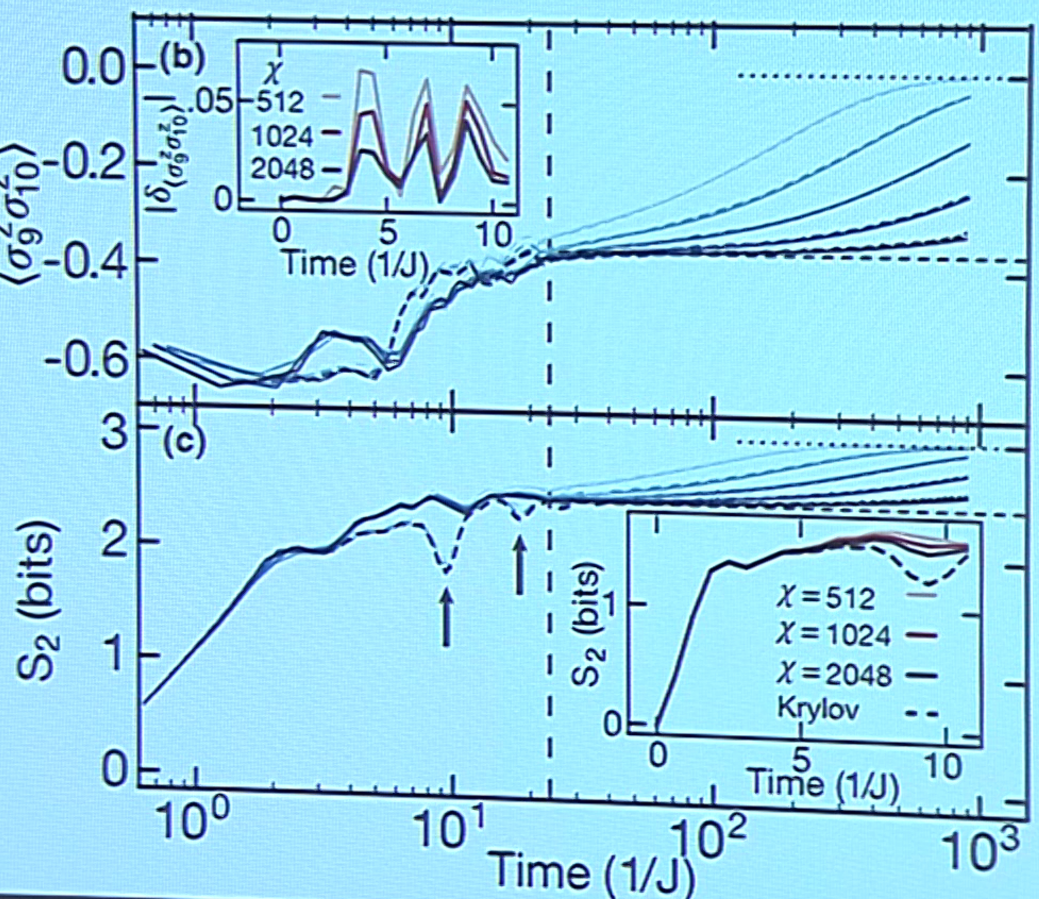
Hydrodynamics + Floquet Heating



Why does DMT work so well?

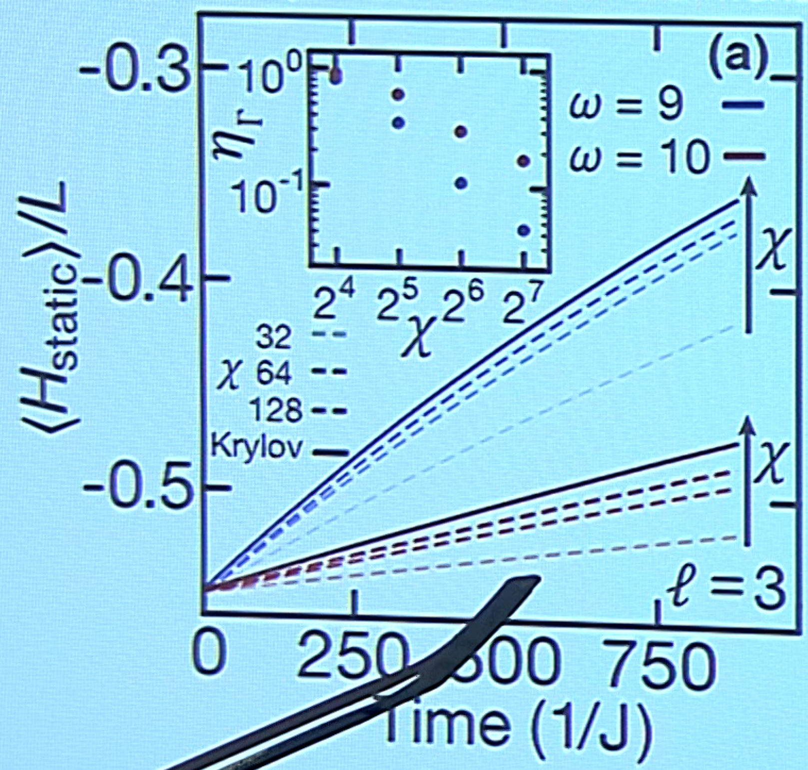
- Algorithm:
  - time-evolution, truncation, time-evolution, truncation, etc.
- DMT is designed to conserve all local observables during truncation step.
  - Guarantees local energy conservation.
- Captures short-distance quantum correlations, but throws away long-distance correlations.

Errors in DMT



Errors in DMT

Energy density for various bond-dimensions



- Larger bond-dimension gives more accurate heating rate
- Larger drive frequency → slower heating rate!

Errors in DMT

- At small/intermediate times, DMT over-thermalizes.
  - Reaches local equilibrium (prethermal plateau).
  - Misses dynamical fluctuations.

- At late times, DMT under-thermalizes.
  - Reaches infinite temperature.
  - Slows Floquet heating.

Errors in DMT

- At small/intermediate times
  - Thermalization proceeds from small distances to long distances.
  - DMT accelerate this process.

- At late times

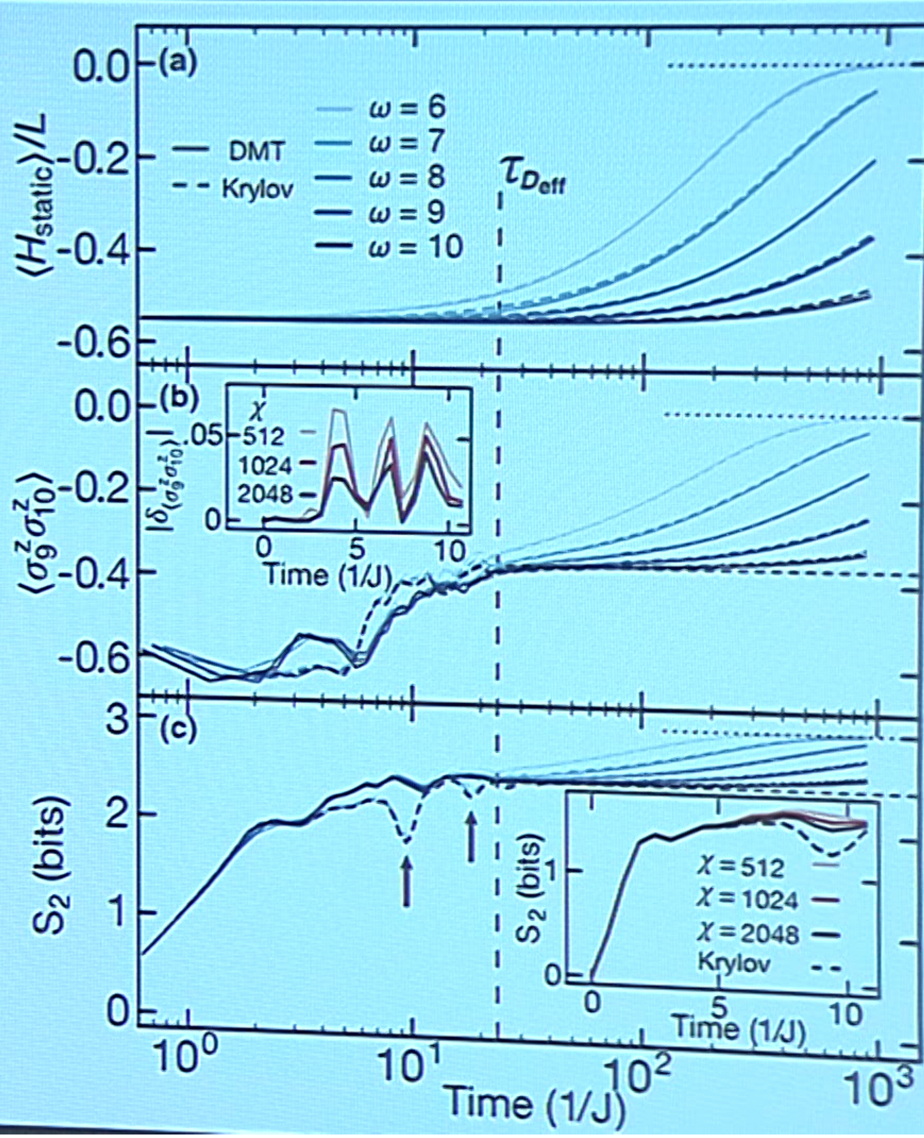
Errors in DMT

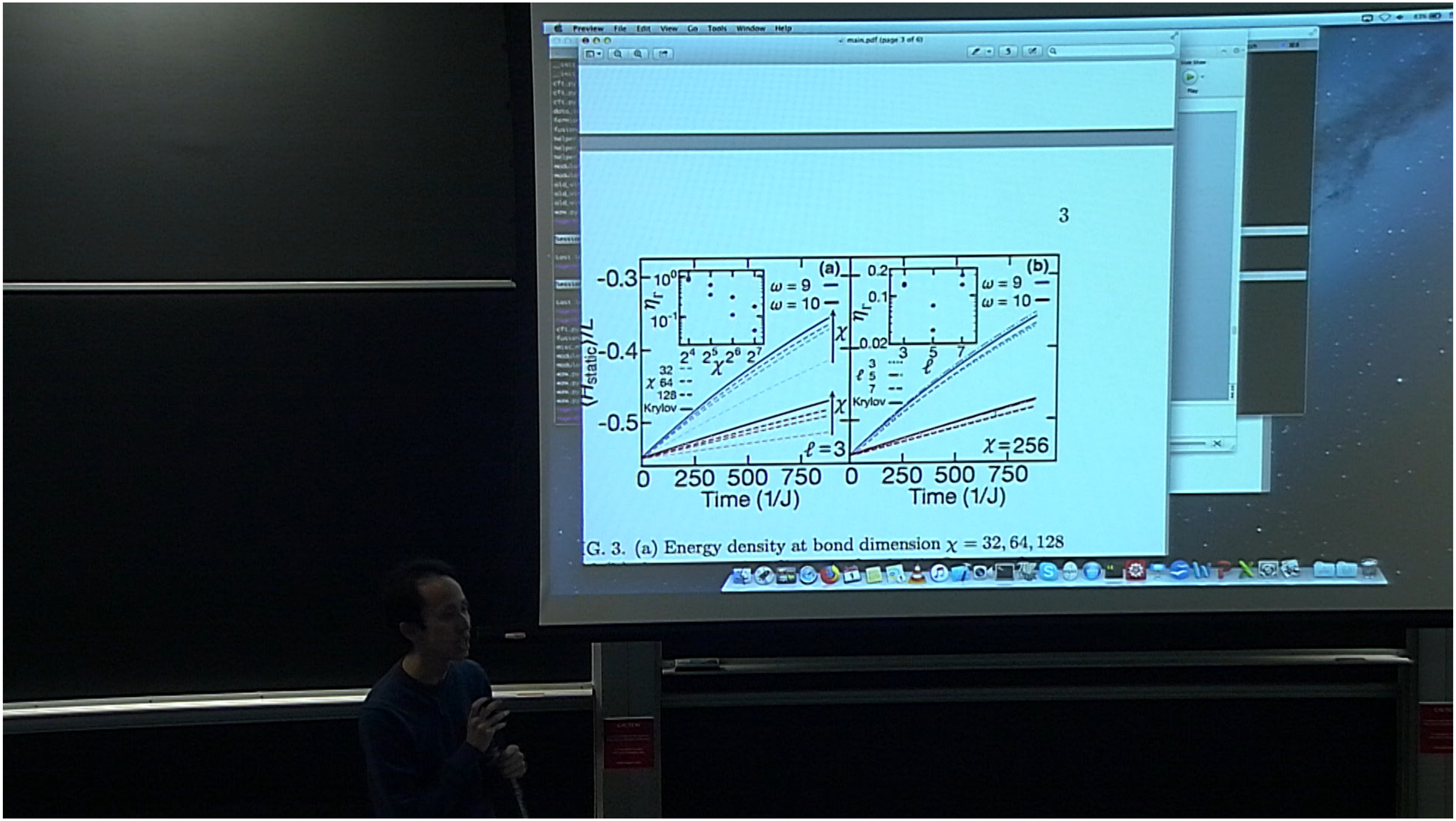
- At small/intermediate times
  - Thermalization proceeds from small distances to long distances.
  - DMT accelerate this process.

- At late times
  - System is already locally-thermal
  - Heating is induced by a many-body process (J/ω)
  - Thermalization proceeds from long distances down to small distances.
  - DMT retards down this process.

Summary

- Combination of Floquet physics and thermalization
- New numerical method (DMT) to study such systems
- The heating mechanism differs between early time and late times.




3
(a)
(b)
ω = 9
ω = 10
χ = 256
ℓ = 3
Krylov
Time (1/J)
0 250 500 750
-0.3
-0.4
-0.5
G. 3. (a) Energy density at bond dimension χ = 32, 64, 128