

Title: Introduction to Machine Learning

Speakers: Lauren Hayward

Date: October 29, 2019 - 1:00 PM

URL: <http://pirsa.org/19100089>

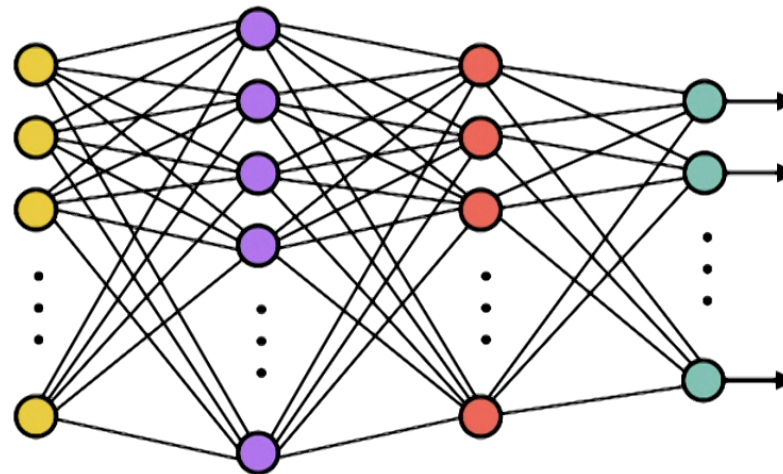
Abstract: Machine learning has led to recent advancements in image processing, language translation, finance, robotics, musical and visual arts, and medical diagnosis. In this session, we will explore how machine learning can be applied within fields of physics. We will introduce fundamental concepts in machine learning such as neural networks and supervised vs. unsupervised learning, and then proceed to learn to use tools from Python's TensorFlow library.

Bring your laptop. You can attend remotely via Zoom <<https://zoom.us/j/154009181>>.

Introduction to Machine Learning

Lauren Hayward

October 29, 2019



Computational Tutorial Series



github.com / eschneitt / ML

Machine learning popularity

Interest over time

Google Trends



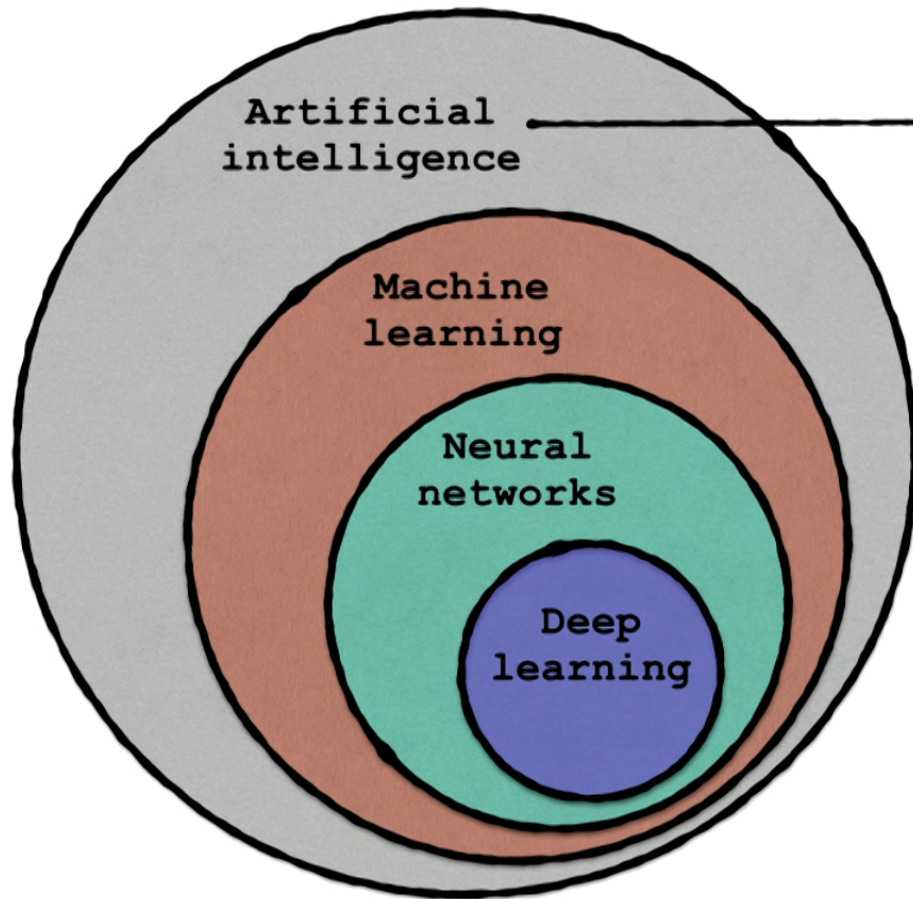
What is machine learning?

“Machine learning is a field of computer science that uses statistical techniques to give computer systems the ability to “learn” (i.e., progressively improve performance on a specific task) with data, without being explicitly programmed.”

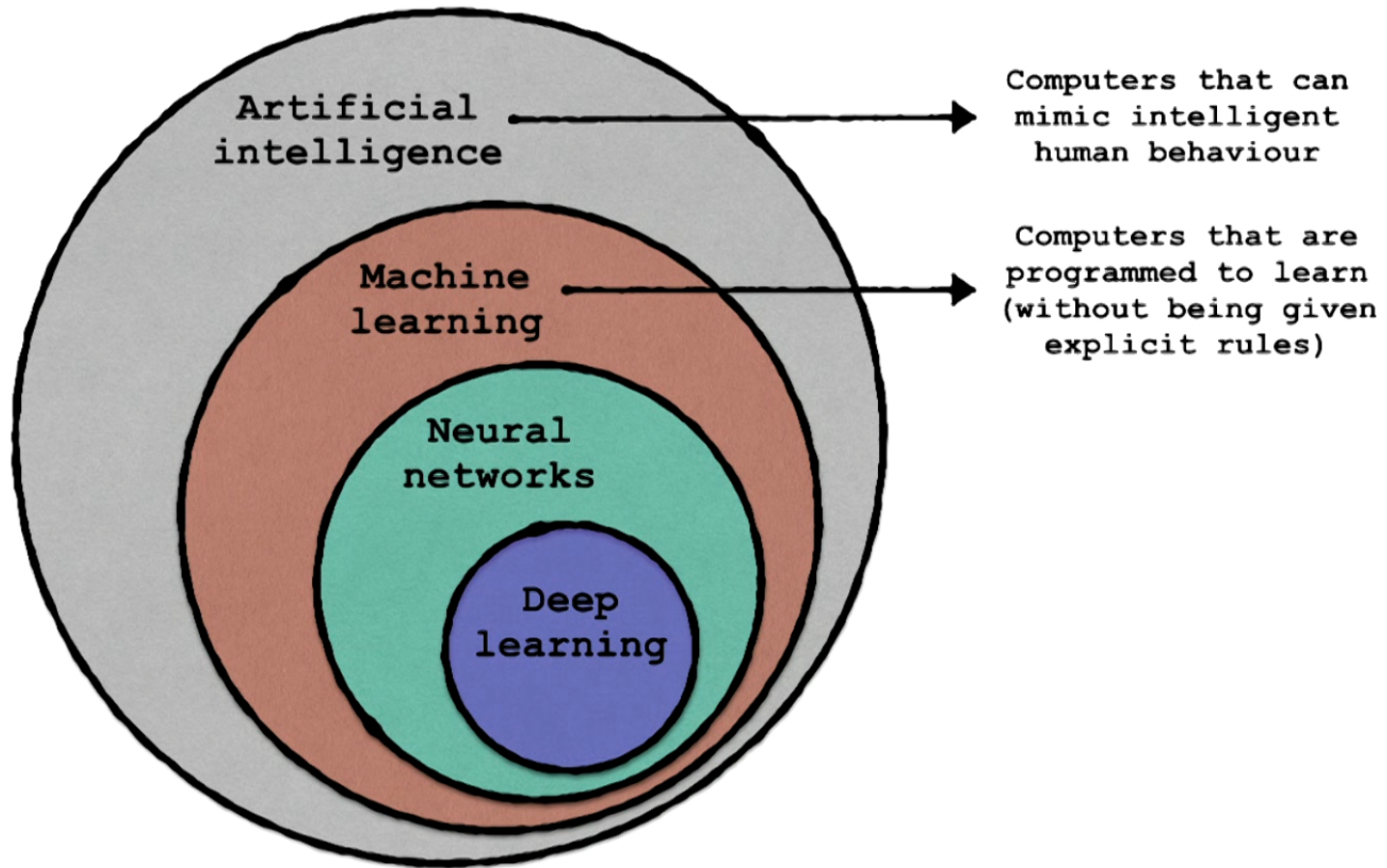
<https://en.wikipedia.org>

“[Machine learning] is about finding out regularities in data and making use of them for fun and profit.”

L.-G. Liu, S.-H. Li and L. Wang, <http://wangleiphy.github.io>



Computers that can
mimic intelligent
human behaviour



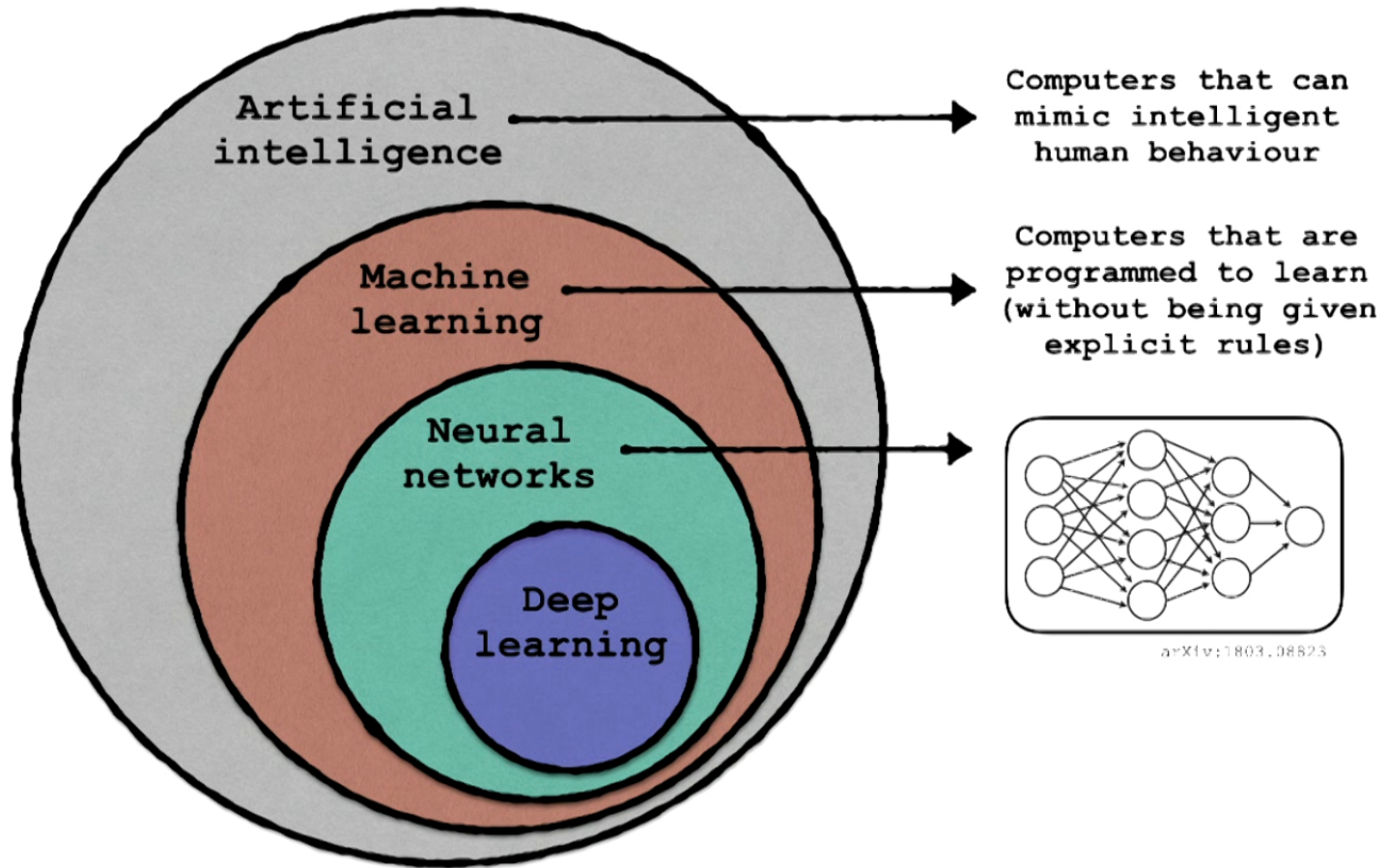


Image classification

ImageNet Classification with Deep Convolutional Neural Networks

Alex Krizhevsky
 University of Toronto
 kriz@cs.utoronto.ca

Ilya Sutskever
 University of Toronto
 ilya@cs.utoronto.ca

Geoffrey E. Hinton
 University of Toronto
 hinton@cs.utoronto.ca

Abstract

We trained a large, deep convolutional neural network to classify the 1.2 million high-resolution images in the ImageNet ILSVRC-2010 contest into the 1000 different classes. On the test data, we achieved top-1 and top-5 error rates of 37.5% and 17.0% which is considerably better than the previous state-of-the-art. The neural network, which has 60 million parameters and 650,000 neurons, consists of five convolutional layers, some of which are followed by max-pooling layers, and three fully-connected layers with a final 1000-way softmax. To make training faster, we used non-saturating neurons and a very efficient GPU implementation of the convolution operation. To reduce overfitting in the fully-connected layers we employed a recently-developed regularization method called "dropout" that proved to be very effective. We also entered a variant of this model in the ILSVRC-2012 competition and achieved a winning top-5 test error rate of 15.3%, compared to 26.2% achieved by the second-best entry.



2012



AlphaGo



Altmetric: 3193 Citations: 569 More detail >>

Article

Mastering the game of Go with deep neural networks and tree search

David Silver¹, Aja Huang, Chris J. Maddison, Arthur Guez, Laurent Sifre, George van den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, Sander Dieleman, Dominik Grewe, John Nham, Nal Kalchbrenner, Ilya Sutskever, Timothy Lillicrap, Madeleine Leach, Koray Kavukcuoglu, Thore Graepel & Demis Hassabis¹

Nature **529**, 484–489 (28 January 2016)
doi:10.1038/nature16961
Download Citation

Received: 11 November 2015
Accepted: 05 January 2016
Published: 27 January 2016

Abstract

The game of Go has long been viewed as the most challenging of classic games for artificial intelligence owing to its enormous search space and the difficulty of evaluating board positions and moves. Here we introduce a new approach to computer Go that uses 'value networks' to evaluate board positions and 'policy networks' to select moves. These deep neural networks are trained by a novel combination of supervised learning from human expert games, and reinforcement learning from games of self-play. Without any lookahead search, the neural networks play Go at the level of state-of-the-art Monte Carlo tree search programs that simulate thousands of random games of self-play. We also introduce a new search algorithm that combines Monte Carlo simulation with value and policy networks. Using this search algorithm, our program AlphaGo achieved a 99.8% winning rate against other Go programs, and defeated the human European Go champion by 5 games to 0. This is the first time that a computer program has defeated a human professional player in the full-sized game of Go, a feat previously thought to be at least a decade away.

AlphaGo seals 4-1 victory over Go grandmaster Lee Sedol

DeepMind's artificial intelligence astonishes fans to defeat human opponent and offers evidence computer software has mastered a major challenge

Steven Borowiec

Tue 15 Mar 2016 10:12 GMT



<https://www.theguardian.com>

2016

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AlphaGo Zero

▼ nature

Article · Published: 18 October 2017

Mastering the game of Go without human knowledge

David Silver[✉], Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, Yutian Chen, Timothy Lillicrap, Fan Hui, Laurent Sifre, George van den Driessche, Thore Graepel & Demis Hassabis

Nature **550**, 354–359 (19 October 2017)

Abstract

A long-standing goal of artificial intelligence is an algorithm that learns, *tabula rasa*, superhuman proficiency in challenging domains. Recently, AlphaGo became the first program to defeat a world champion in the game of Go. The tree search in AlphaGo evaluated positions and selected moves using deep neural networks. These neural networks were trained by supervised learning from human expert moves, and by reinforcement learning from self-play. Here we introduce an algorithm based solely on reinforcement learning, without human data, guidance or domain knowledge beyond game rules. AlphaGo becomes its own teacher: a neural network is trained to predict AlphaGo's own move selections and also the winner of AlphaGo's games. This neural network improves the strength of the tree search, resulting in higher quality move selection and stronger self-play in the next iteration. **Starting *tabula rasa*, our new program AlphaGo Zero achieved superhuman performance, winning 100–0 against the previously published, champion-defeating AlphaGo.**

2017

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Self-driving cars

End to End Learning for Self-Driving Cars

Mariusz Bojarski NVIDIA Corporation Holmdel, NJ 07735	Davide Del Testa NVIDIA Corporation Holmdel, NJ 07735	Daniel Dworakowski NVIDIA Corporation Holmdel, NJ 07735	Bernhard Ermer NVIDIA Corporation Holmdel, NJ 07735
Beat Flepp NVIDIA Corporation Holmdel, NJ 07735	Prasoon Goyal NVIDIA Corporation Holmdel, NJ 07735	Lawrence D. Jackel NVIDIA Corporation Holmdel, NJ 07735	Mathew Monfort NVIDIA Corporation Holmdel, NJ 07735
Urs Muller NVIDIA Corporation Holmdel, NJ 07735	Jiakui Zhang NVIDIA Corporation Holmdel, NJ 07735	Xin Zhang NVIDIA Corporation Holmdel, NJ 07735	Jake Zhao NVIDIA Corporation Holmdel, NJ 07735

Karol Zieba
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Abstract

We trained a convolutional neural network (CNN) to map raw pixels from a single front-facing camera directly to steering commands. This end-to-end approach proved surprisingly powerful. With minimum training data from humans the system learns to drive in traffic on local roads with or without lane markings and on highways. It also operates in areas with unclear visual guidance such as in parking lots and on unpaved roads.

The system automatically learns internal representations of the necessary processing steps such as detecting useful road features with only the human steering angle as the training signal. We never explicitly trained it to detect, for example, the outline of roads.

Compared to explicit decomposition of the problem, such as lane marking detection, path planning, and control, our end-to-end system optimizes all processing steps simultaneously. We argue that this will eventually lead to better performance and smaller systems. Better performance will result because the internal components self-optimize to maximize overall system performance, instead of optimizing human-selected intermediate criteria, e.g., lane detection. Such criteria understandably are selected for ease of human interpretation which doesn't automatically guarantee maximum system performance. Smaller networks are possible because the system learns to solve the problem with the minimal number of processing steps.

We used an NVIDIA DevBox and Torch 7 for training and an NVIDIA DRIVE™ PX self-driving car computer also running Torch 7 for determining where to drive. The system operates at 30 frames per second (FPS).

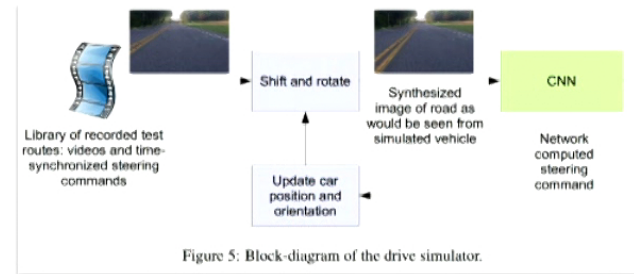


Figure 5: Block-diagram of the drive simulator.

arXiv:1604.07316v1 [cs.CV] 25 Apr 2016

2016



Language translation

Google's Neural Machine Translation System: Bridging the Gap between Human and Machine Translation

Yonghui Wu, Mike Schuster, Zhifeng Chen, Quoc V. Le, Mohammad Norouzi
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Wolfgang Macherey, Maxim Krikun, Yuan Cao, Qin Gao, Klaus Macherey,
Jeff Klingner, Apurva Shah, Melyin Johnson, Xiaobing Liu, Lukasz Kaiser,
Stephan Gouws, Yoshikiyo Kato, Taku Kudo, Hideto Kazawa, Keith Stevens,
George Kurian, Nishant Patil, Wei Wang, Cliff Young, Jason Smith, Jason Riesa,
Alex Rudnick, Oriol Vinyals, Greg Corrado, Macduff Hughes, Jeffrey Dean

Abstract

Neural Machine Translation (NMT) is an end-to-end learning approach for automated translation, with the potential to overcome many of the weaknesses of conventional phrase-based translation systems. Unfortunately, NMT systems are known to be computationally expensive both in training and in translation inference – sometimes prohibitively so in the case of very large data sets and large models. Several authors have also charged that NMT systems lack robustness, particularly when input sentences contain rare words. These issues have hindered NMT's use in practical deployments and services, where both accuracy and speed are essential. In this work, we present GNMT, Google's Neural Machine Translation system, which attempts to address many of these issues. Our model consists of a deep LSTM network with 8 encoder and 8 decoder layers using residual connections as well as attention connections from the decoder network to the encoder. To improve parallelism and therefore decrease training time, our attention mechanism connects the bottom layer of the decoder to the top layer of the encoder. To accelerate the final translation speed, we employ low-precision arithmetic during inference computations. To improve handling of rare words, we divide words into a limited set of common sub-word units ("wordpieces") for both input and output. This method provides a good balance between the flexibility of "character"-delimited models and the efficiency of "word"-delimited models, naturally handles translation of rare words, and ultimately improves the overall accuracy of the system. Our beam search technique employs a length-normalization procedure and uses a coverage penalty, which encourages generation of an output sentence that is most likely to cover all the words in the source sentence. To directly optimize the translation BLEU scores, we consider refining the models by using reinforcement learning, but we found that the improvement in the BLEU scores did not reflect in the human evaluation. On the WMT'14 English-to-French and English-to-German benchmarks, GNMT achieves competitive results to state-of-the-art. Using a human side-by-side evaluation on a set of isolated simple sentences, it reduces translation errors by an average of 60% compared to Google's phrase-based production system.

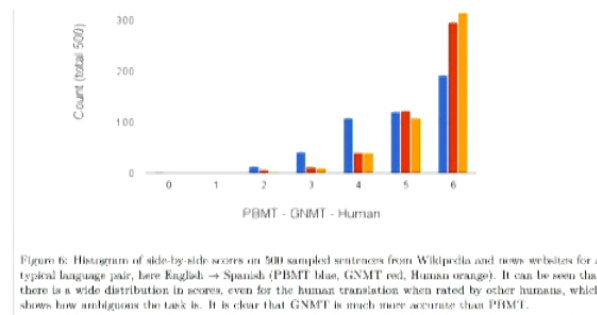


Figure 6: Histogram of side-by-side scores on 500 sampled sentences from Wikipedia and news websites for a typical language pair, here English -> Spanish (PBMT blue, GNMT red, Human orange). It can be seen that there is a wide distribution in scores, even for the human translation when rated by other humans, which shows how ambiguous the task is. It is clear that GNMT is much more accurate than PBMT.

arXiv:1609.08144v2 [cs.CL] 8 Oct 2016

2016

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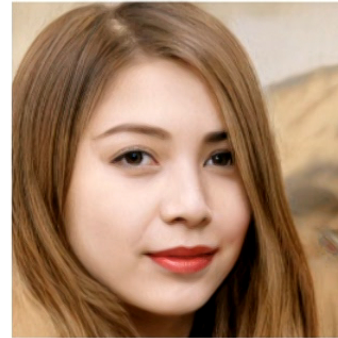
Generating art

Robbie Barrat
<https://robbiebarrat.github.io>



Computer-generated people

<https://thispersondoesnotexist.com>



2018



Medical diagnosis

CheXNet: Radiologist-Level Pneumonia Detection on Chest X-Rays with Deep Learning

Pranav Rajpurkar¹ Jeremy Irvin² Kaylie Zhu¹ Brandon Yang¹ Hershel Mehta¹
Tony Duan¹ Daisy Ding¹ Aarti Bagul¹ Robyn L. Ball² Curtis Langlotz³ Katie Shpankaya³
Matthew P. Lungren³ Andrew Y. Ng¹

Abstract

We develop an algorithm that can detect pneumonia from chest X-rays at a level exceeding practicing radiologists. Our algorithm, CheXNet, is a 121-layer convolutional neural network trained on ChestX-ray14, currently the largest publicly available chest X-ray dataset, containing over 100,000 frontal-view X-ray images with 14 diseases. Four practicing academic radiologists annotate a test set, on which we compare the performance of CheXNet to that of radiologists. We find that CheXNet exceeds average radiologist performance on the F1 metric. We extend CheXNet to detect all 14 diseases in ChestX-ray14 and achieve state-of-the-art results on all 14 diseases.

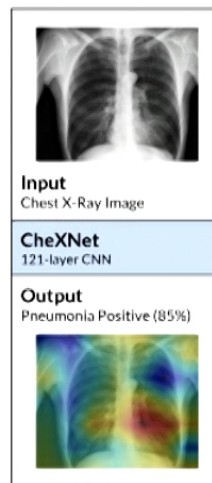


Figure 1. CheXNet is a 121-layer convolutional neural network that takes a chest X-ray image as input, and outputs the probability of a pathology. On this example, CheXNet correctly detects pneumonia and also localizes areas in the image most indicative of the pathology.

arXiv:1711.05225v3 [cs.CV] 25 Dec 2017

2017



Machine Learning for Physics

nature
communications

Article | Published: 02 July 2014

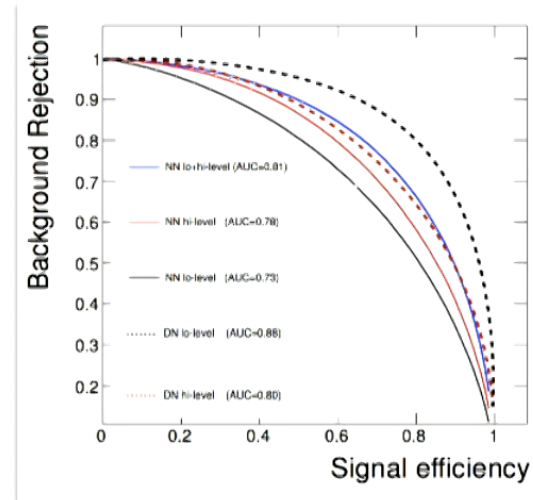
Searching for exotic particles in high-energy physics with deep learning

P. Baldi , P. Sadowski & D. Whiteson 

Nature Communications **5**, Article number: 4308 (2014)

Abstract

Collisions at high-energy particle colliders are a traditionally fruitful source of exotic particle discoveries. Finding these rare particles requires solving difficult signal-versus-background classification problems, hence machine-learning approaches are often used. Standard approaches have relied on 'shallow' machine-learning models that have a limited capacity to learn complex nonlinear functions of the inputs, and rely on a painstaking search through manually constructed nonlinear features. Progress on this problem has slowed, as a variety of techniques have shown equivalent performance. Recent advances in the field of deep learning make it possible to learn more complex functions and better discriminate between signal and background classes. Here, using benchmark data sets, we show that deep-learning methods need no manually constructed inputs and yet improve the classification metric by as much as 8% over the best current approaches. This demonstrates that deep-learning approaches can improve the power of collider searches for exotic particles.

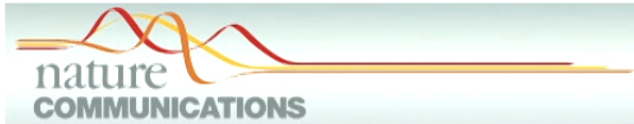


arXiv:1806.11484

2014

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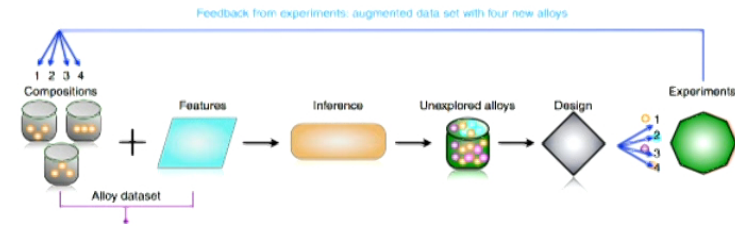
Machine Learning for Physics



Accelerated search for materials with targeted properties by adaptive design

Dezhen Xue^{1,2}, Prasanna V. Balachandran¹, John Hogden³, James Theiler⁴, Deqing Xue² & Turab Lookman¹

Finding new materials with targeted properties has traditionally been guided by intuition, and trial and error. With increasing chemical complexity, the combinatorial possibilities are too large for an Edisonian approach to be practical. Here we show how an adaptive design strategy, tightly coupled with experiments, can accelerate the discovery process by sequentially identifying the next experiments or calculations, to effectively navigate the complex search space. Our strategy uses inference and global optimization to balance the trade-off between exploitation and exploration of the search space. We demonstrate this by finding very low thermal hysteresis (ΔT) NiTi-based shape memory alloys, with $\text{Ti}_{50.0}\text{Ni}_{46.7}\text{Cu}_{0.8}\text{Fe}_{2.3}\text{Pd}_{0.2}$ possessing the smallest ΔT (1.84 K). We synthesize and characterize 36 predicted compositions (9 feedback loops) from a potential space of $\sim 800,000$ compositions. Of these, 14 had smaller ΔT than any of the 22 in the original data set.



2016



Machine Learning for Physics

nature physics

Letter Published: 13 February 2017

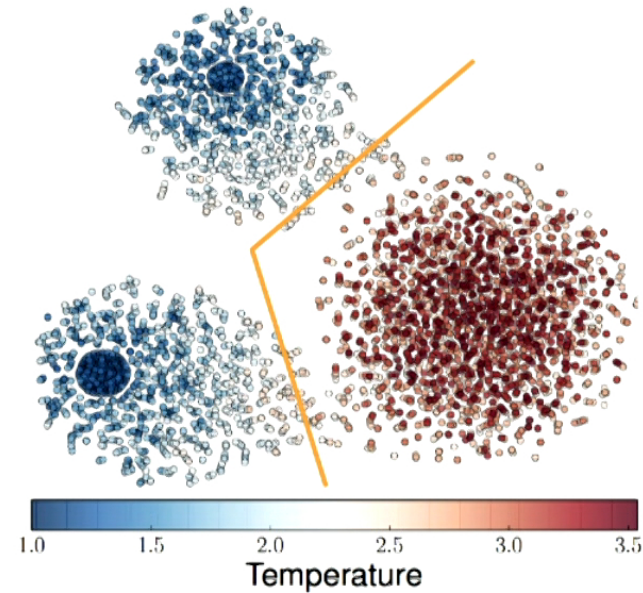
Machine learning phases of matter

Juan Carrasquilla & Roger G. Melko

Nature Physics **13**, 431–434 (2017)

Abstract

Condensed-matter physics is the study of the collective behaviour of infinitely complex assemblies of electrons, nuclei, magnetic moments, atoms or qubits¹. This complexity is reflected in the size of the state space, which grows exponentially with the number of particles, reminiscent of the 'curse of dimensionality' commonly encountered in machine learning². Despite this curse, the machine learning community has developed techniques with remarkable abilities to recognize, classify, and characterize complex sets of data. Here, we show that modern machine learning architectures, such as fully connected and convolutional neural networks³, can identify phases and phase transitions in a variety of condensed-matter Hamiltonians. Readily programmable through modern software libraries^{4,5}, neural networks can be trained to detect multiple types of order parameter, as well as highly non-trivial states with no conventional order, directly from raw state configurations sampled with Monte Carlo^{6,7}.



2016

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Machine Learning for Physics



QuCumber: wavefunction reconstruction with neural networks

Matthew J. S. Beach^{1,2}, Isaac De Vlucht², Anna Golubeva^{1,2}, Patrick Huembel^{1,3},
Bohdan Kulchytskyi^{1,2}, Xiuzhe Luo², Roger G. Melko^{1,2*}, Ejaaz Merali²,
Giacomo Torlai^{1,2,4}

¹ Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

² Department of Physics and Astronomy, University of Waterloo, Ontario N2L 3G1, Canada

³ ICFO-Institut de Ciències Fotoniques, Barcelona Institute of Science and Technology,
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York, NY 10010, USA

* rgmelko@uwaterloo.ca

December 27, 2018

Abstract

As we enter a new era of quantum technology, it is increasingly important to develop methods to aid in the accurate preparation of quantum states for a variety of materials, matter, and devices. Computational techniques can be used to reconstruct a state from data, however the growing number of qubits demands ongoing algorithmic advances in order to keep pace with experiments. In this paper, we present an open-source software package called QuCumber that uses machine learning to reconstruct a quantum state consistent with a set of projective measurements. QuCumber uses a restricted Boltzmann machine to efficiently represent the quantum wavefunction for a large number of qubits. New measurements can be generated from the machine to obtain physical observables not easily accessible from the original data.

arXiv:1812.09329v1 [quant-ph] 21 Dec 2018

2018

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Machine Learning for Physics: Resources

- ◆ Michael Nielsen, “Neural networks and deep learning”, neuralnetworksanddeeplearning.com
- ◆ Goodfellow, Bengio and Courville, “Deep learning”, MIT Press (2016), deeplearningbook.org
- ◆ Liu, Li and Wang, “Lecture note on deep learning and quantum many-body computation”, <http://wangleiphy.github.io/lectures/DL.pdf>
- ◆ Mehta, Bukov, Wang, Day, Richardson, Fisher, and Schwab, “A high-bias, low-variance introduction to machine learning for physicists”, [arXiv:1803.08823](https://arxiv.org/abs/1803.08823)
- ◆ Carleo, Cirac, Cranmer, Daudet, Schuld, Tishby, Vogt-Maranto, and Zdeborová, “Machine learning and the physical sciences”, [arXiv:1903.10563](https://arxiv.org/abs/1903.10563)
- ◆ Guest, Cranmer, and Whiteson, “Deep learning and its application to LHC physics”, [arXiv:1806.11484](https://arxiv.org/abs/1806.11484)
- ◆ Torlai and Melko, “Machine learning quantum states in the NISQ era”, [arXiv:1905.04312](https://arxiv.org/abs/1905.04312)
- ◆ physicsml.github.io

Machine Learning (ML)

ML: Training computers to detect and characterize features from data

Categories of algorithms:

1. Supervised learning (SL)

Given a dataset $\mathcal{D} = \{ \vec{x}, \vec{y} \}$ of data points \vec{x} and labels \vec{y} ,
fit a function $\vec{f}(\vec{x})$ to \vec{y} .

2. Unsupervised learning (UL)

3. Reinforcement learning (RL)

Machine Learning (ML)

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2. Unsupervised learning (UL)

Given an unlabelled dataset $\mathcal{D} = \{ \vec{x} \}$, efficiently represent the data's underlying probability distribution $p(\vec{x})$.

3. Reinforcement learning (RL)

Machine Learning (ML)

ML: Training computers to detect and characterize features from data

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Given an unlabelled dataset $\mathcal{D} = \{ \vec{x} \}$, efficiently represent the data's underlying probability distribution $p(\vec{x})$.

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Given an environment, take an action such that the resulting reward will be maximized.

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TODAY: SL using neural networks

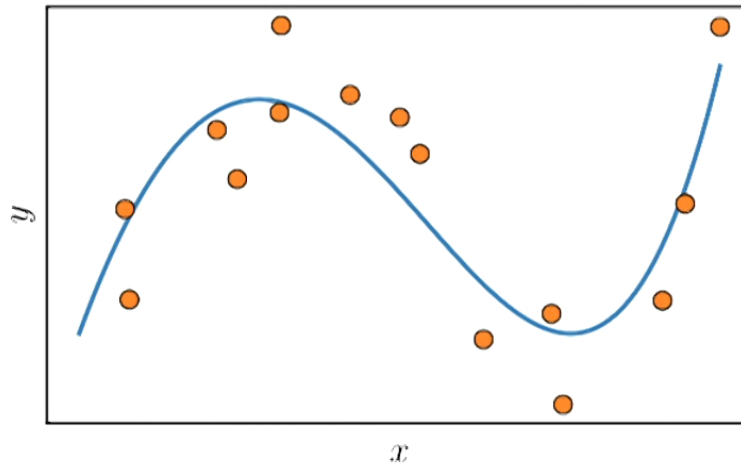
2. Unsupervised learning (UL)

Given an unlabelled dataset $\mathcal{D} = \{\vec{x}\}$, efficiently represent the data's underlying probability distribution $p(\vec{x})$.

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Given an environment, take an action such that the resulting reward will be maximized.

Supervised Learning Example: 1D Regression



The data points x and labels y are both 1D coordinates. The goal is to find a function $f(x)$ (such as the blue curve) that describes the data.

Supervised Learning Example: Classifying handwritten digits

$$\vec{x}_1 = \boxed{5} \quad y_1 = 5$$

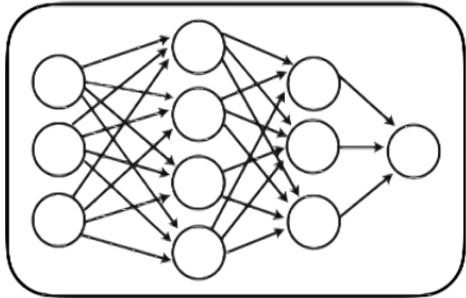
$$\vec{x}_2 = \boxed{0} \quad y_2 = 0$$

$$\vec{x}_3 = \boxed{4} \quad y_3 = 4$$

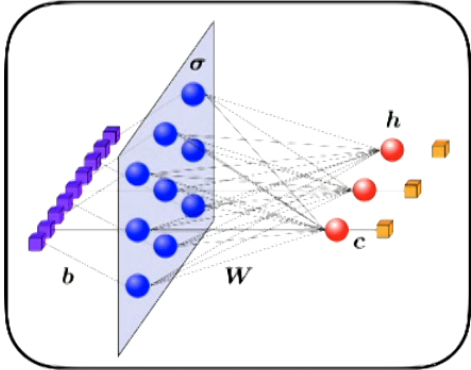
$$\vec{x}_4 = \boxed{9} \quad y_4 = 9$$

Data points taken from the MNIST database

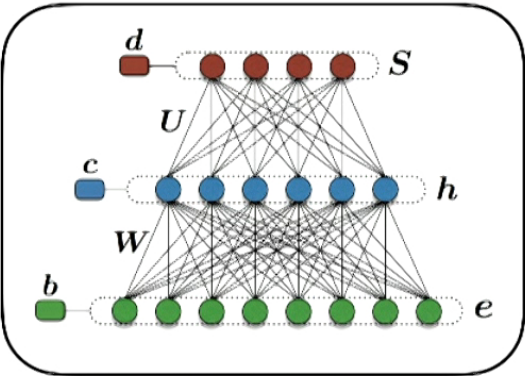
Artificial neural networks



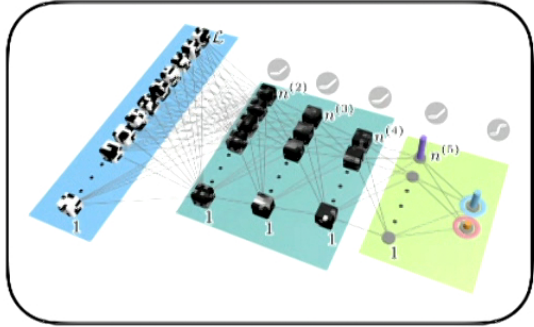
arXiv:1803.08823



arXiv:1606.02718

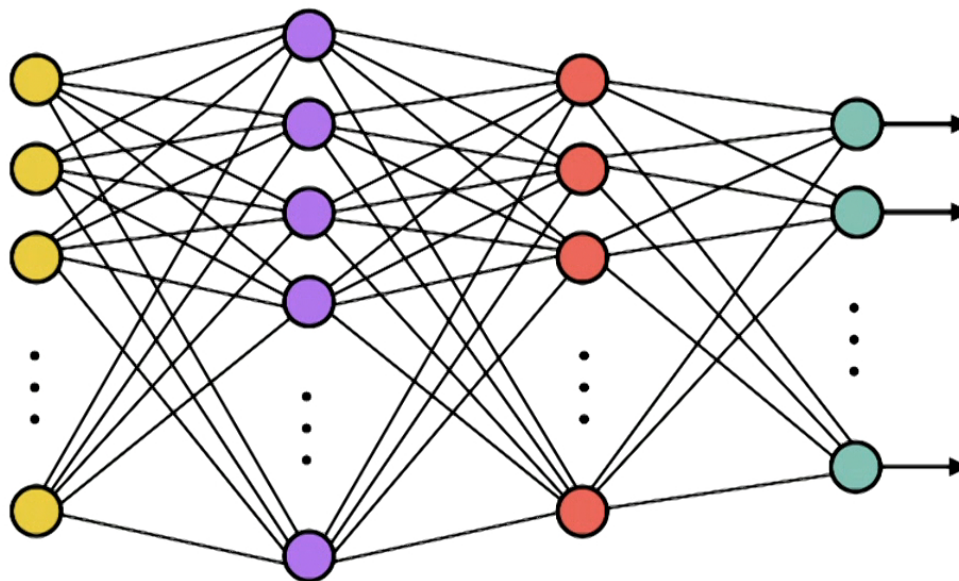


arXiv:1610.04238

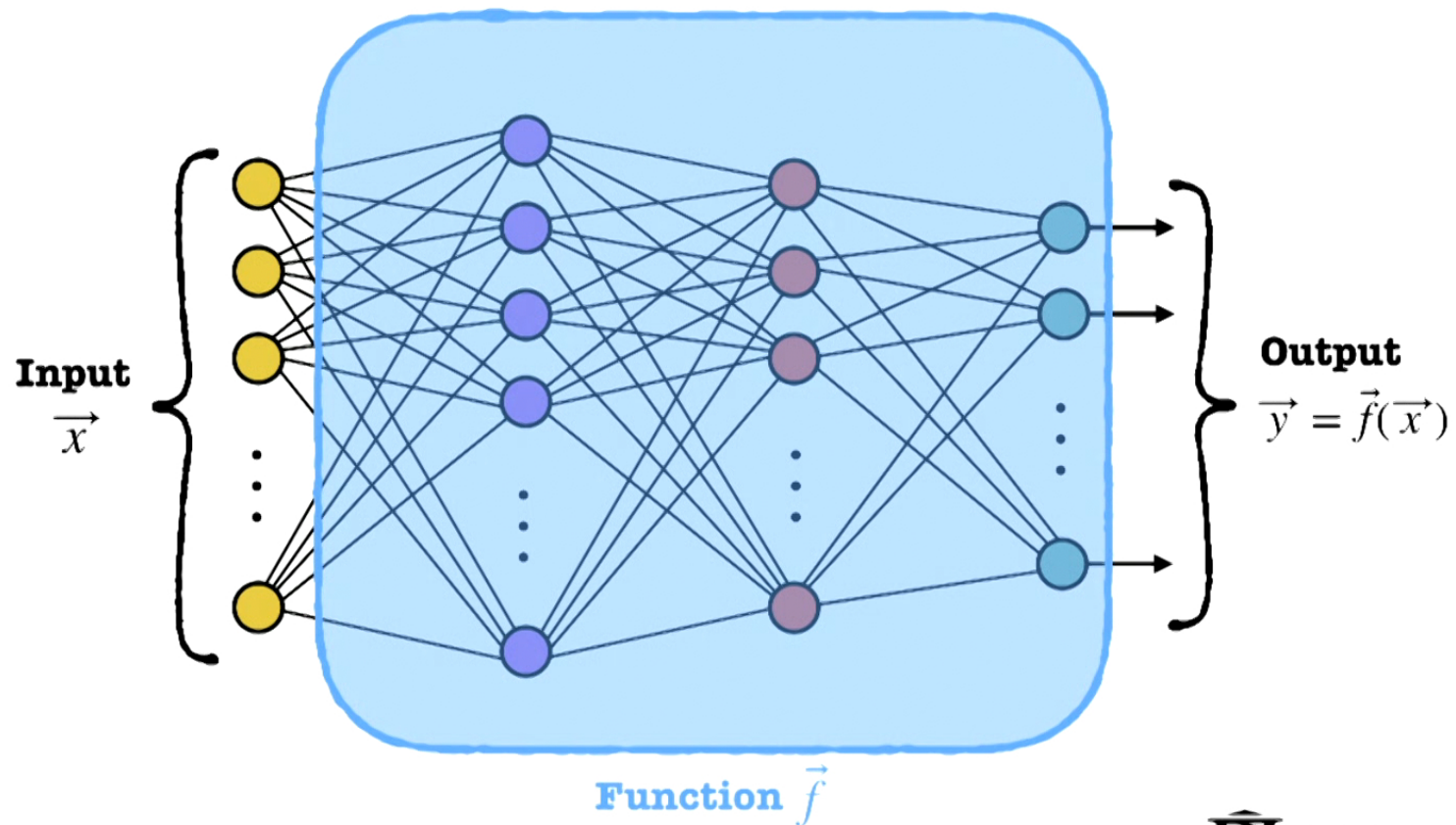


arXiv:1609.02552

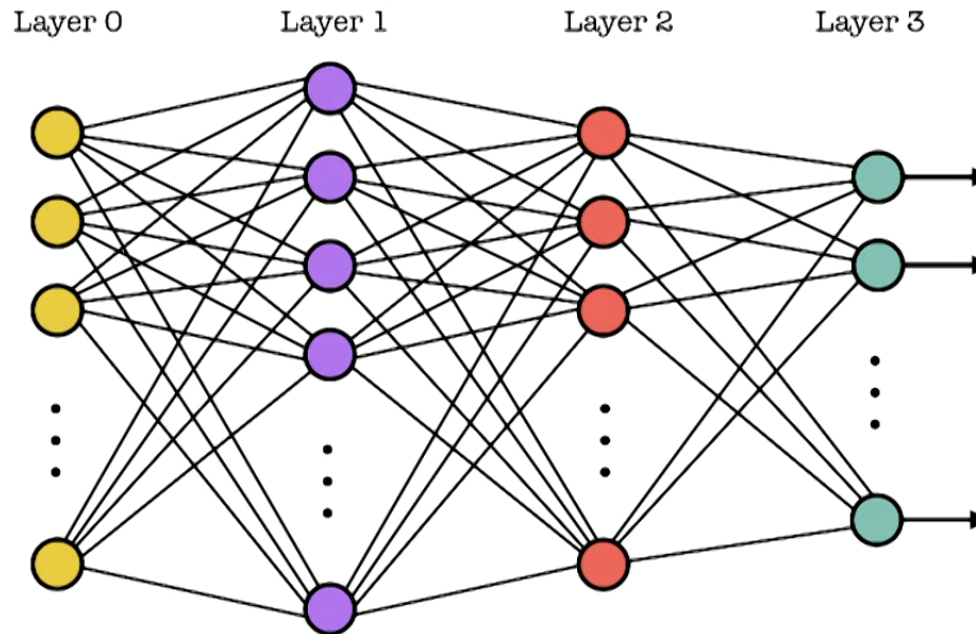
Feedforward neural networks



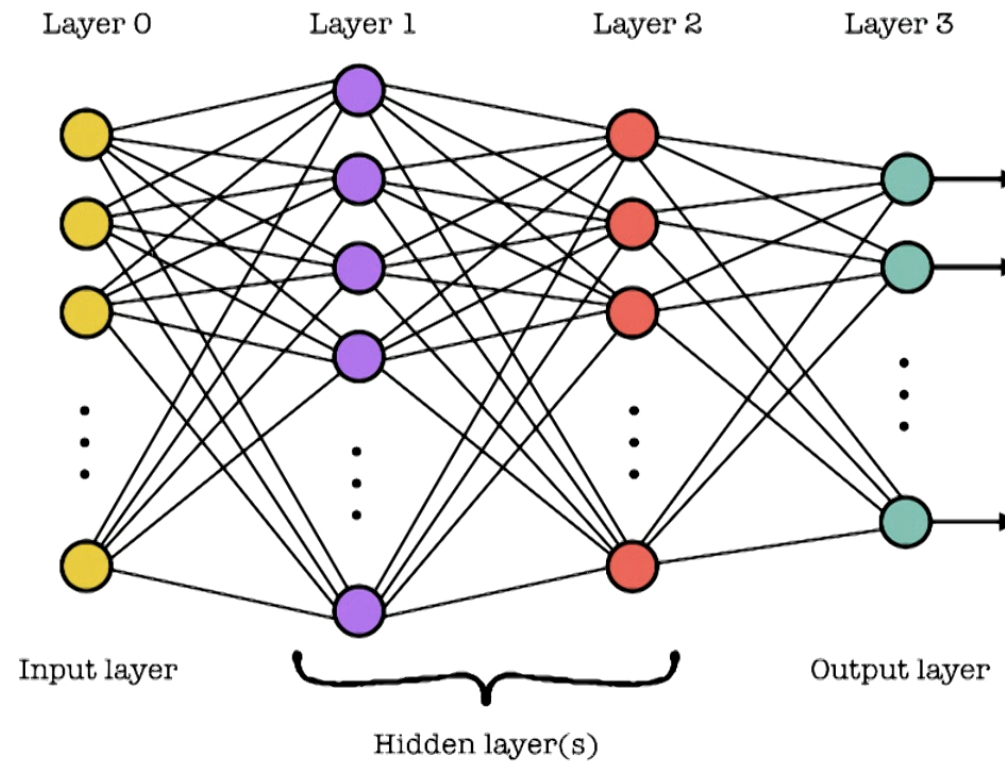
Feedforward neural networks



Feedforward neural networks

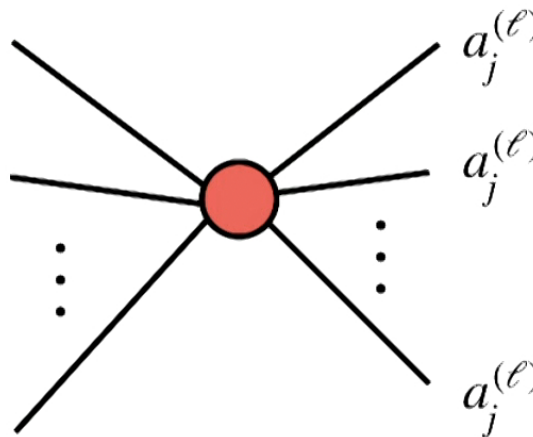


Feedforward neural networks



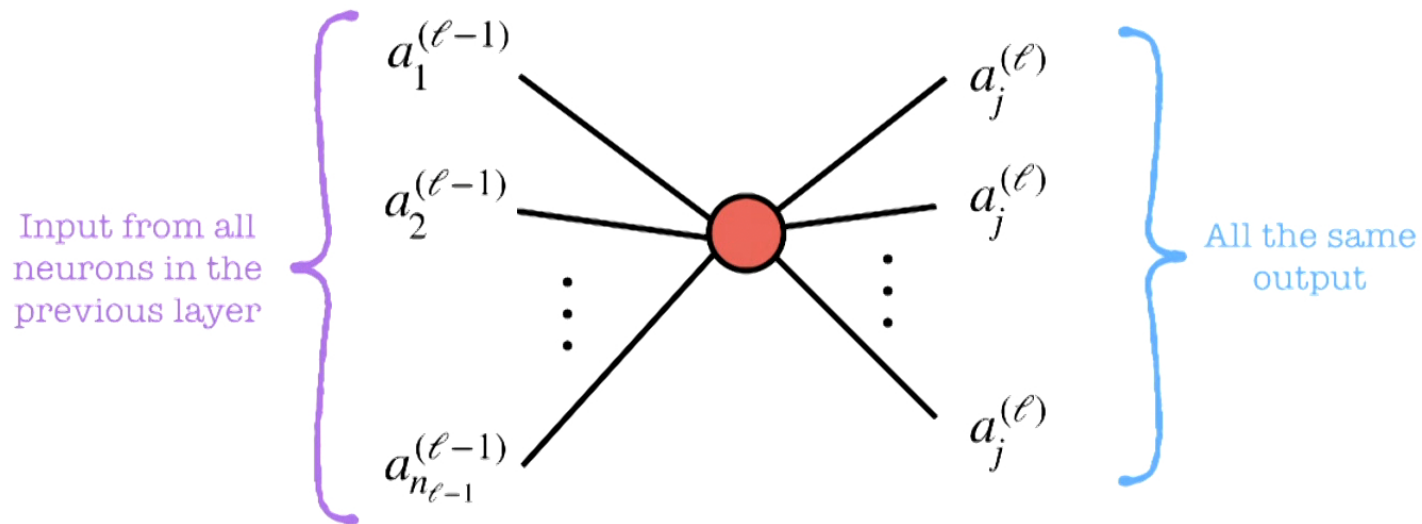
Neuron output

Let's zoom in on the j^{th} neuron in layer $\ell > 0$



Neuron output

Let's zoom in on the j^{th} neuron in layer $\ell > 0$

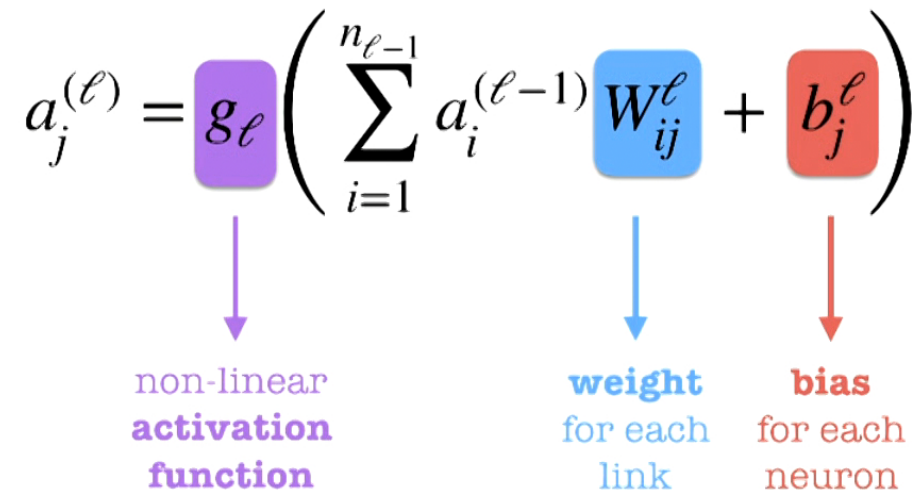


Neuron output

$$a_j^{(\ell)} = g_\ell \left(\sum_{i=1}^{n_{\ell-1}} a_i^{(\ell-1)} W_{ij}^\ell + b_j^\ell \right)$$

Neuron output

$$a_j^{(\ell)} = g_\ell \left(\sum_{i=1}^{n_{\ell-1}} a_i^{(\ell-1)} W_{ij}^\ell + b_j^\ell \right)$$

The diagram illustrates the neuron output equation. The equation is $a_j^{(\ell)} = g_\ell \left(\sum_{i=1}^{n_{\ell-1}} a_i^{(\ell-1)} W_{ij}^\ell + b_j^\ell \right)$. The components are labeled as follows: g_ℓ is labeled as "non-linear activation function" (purple text); W_{ij}^ℓ is labeled as "weight for each link" (blue text); and b_j^ℓ is labeled as "bias for each neuron" (red text). Arrows point from each component to its respective label.

Neuron output

$$a_j^{(\ell)} = g_\ell \left(\sum_{i=1}^{n_{\ell-1}} a_i^{(\ell-1)} W_{ij}^\ell + b_j^\ell \right) \equiv z_j^{(\ell)}$$

non-linear
activation
function

weight
for each
link

bias
for each
neuron

Neuron output

$$a_j^{(\ell)} = g_\ell \left(\sum_{i=1}^{n_{\ell-1}} a_i^{(\ell-1)} W_{ij}^\ell + b_j^\ell \right) \equiv z_j^{(\ell)}$$

non-linear
activation
function

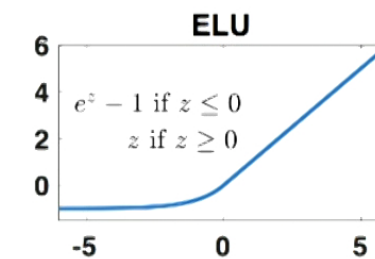
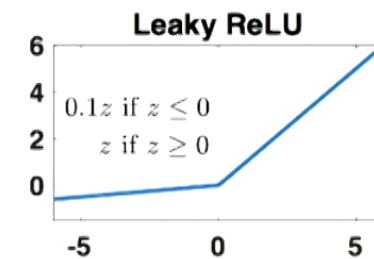
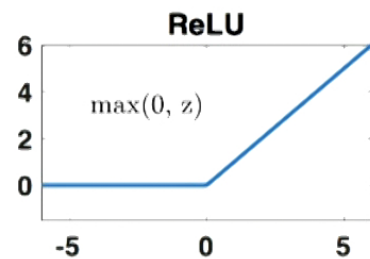
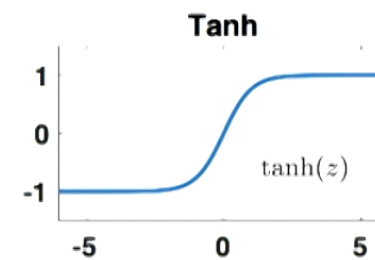
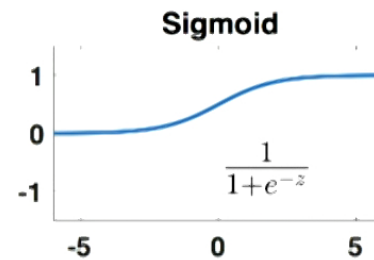
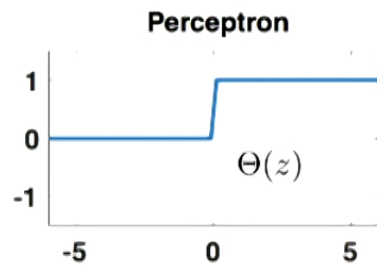
weight
for each
link

bias
for each
neuron

The weights and biases are adjusted as the network **learns**.

Activation functions

We can choose various non-linear activation functions g_ℓ , such as:



arXiv:1803.08823

Cost functions

Our goal is to find weights and biases such that when \vec{x} is the input, the network's output $\vec{a}^{(L)}(\vec{x}) = \vec{f}(\vec{x})$ is close to the label \vec{y} .

We use a **cost function** to measure how well the neural network is approximating the labels.

Cost functions

Our goal is to find weights and biases such that when \vec{x} is the input, the network's output $\vec{a}^{(L)}(\vec{x}) = \vec{f}(\vec{x})$ is close to the label \vec{y} .

We use a **cost function** to measure how well the neural network is approximating the labels.

Possible cost functions include:

♦ **Mean-squared error:**
$$C_{\text{MSE}} = \frac{1}{2|\mathcal{D}|} \sum_{\vec{x} \in \mathcal{D}} \sum_{i=1}^{n_L} \left[a_i^{(L)}(\vec{x}) - y_i(\vec{x}) \right]^2$$

♦ **Cross entropy:**
$$C_{\text{CE}} = -\frac{1}{|\mathcal{D}|} \sum_{\vec{x} \in \mathcal{D}} \sum_{i=1}^{n_L} \left[y_i(\vec{x}) \log a_i^{(L)}(\vec{x}) + (1 - y_i(\vec{x})) \log(1 - a_i^{(L)}(\vec{x})) \right]$$

Cost functions

We would like to minimize the cost function over all possible weights and biases in each layer such that

$$\left. \begin{aligned} \frac{\partial C}{\partial W_{ij}^{(\ell)}} &= 0 \\ \frac{\partial C}{\partial b_j^{(\ell)}} &= 0 \end{aligned} \right\} \text{For all } i, j, \ell$$

Learning algorithms

◆ Gradient descent:

$$W_{ij}^{(\ell)} \rightarrow W_{ij}^{(\ell)} - \eta \frac{\partial C}{\partial W_{ij}^{(\ell)}}$$

$$b_j^{(\ell)} \rightarrow b_j^{(\ell)} - \eta \frac{\partial C}{\partial b_j^{(\ell)}}$$

Learning algorithms

◆ Gradient descent:

$$W_{ij}^{(\ell)} \rightarrow W_{ij}^{(\ell)} - \eta \frac{\partial C}{\partial W_{ij}^{(\ell)}}$$

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η : Learning rate

Learning algorithms

- ◆ **Gradient descent:**

$$W_{ij}^{(\ell)} \rightarrow W_{ij}^{(\ell)} - \eta \frac{\partial C}{\partial W_{ij}^{(\ell)}}$$

$$b_j^{(\ell)} \rightarrow b_j^{(\ell)} - \eta \frac{\partial C}{\partial b_j^{(\ell)}}$$

η : Learning rate

- ◆ **Stochastic gradient descent**

- ◆ **RMSProp**

- ◆ **Adam optimizer**

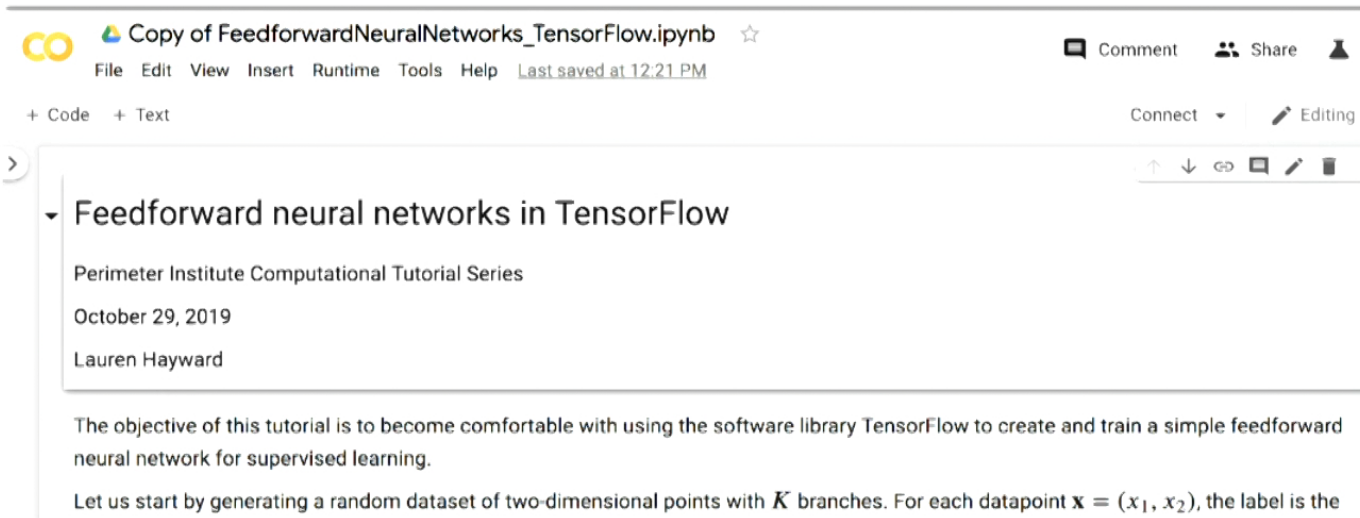
⋮

Feedforward Neural Networks in TensorFlow

Go to:

<https://drive.google.com/file/d/17CihZKb04U2TRwDXQwwwMDYODFXMfbCf/view?usp=sharing>

- ◆ Open with Google Colaboratory
- ◆ Choose 'Open in Playground'
- ◆ Choose the option 'Copy to Drive'



The screenshot shows the Google Colaboratory interface for a notebook titled "Copy of FeedforwardNeuralNetworks_TensorFlow.ipynb". The notebook is open in "Editing" mode. The main content area displays the title "Feedforward neural networks in TensorFlow" and the following text:

Perimeter Institute Computational Tutorial Series
October 29, 2019
Lauren Hayward

The objective of this tutorial is to become comfortable with using the software library TensorFlow to create and train a simple feedforward neural network for supervised learning.

Let us start by generating a random dataset of two-dimensional points with K branches. For each datapoint $\mathbf{x} = (x_1, x_2)$, the label is the

FeedforwardNeuralNetworks_T x

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FeedforwardNeuralNetworks_TensorFlow.ipynb

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- Suggested third-party apps
- Mindmap
- Connect more apps

```

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Computational Tutorial Series\n","\n","October 29, 2019\n
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np\n","\n","##### CREATE
AND PLOT THE DATA SET
#####\n","#####\n","\n","N = 50 #
number of points per branch\n","K = 3 # number of branches\n","\n","N_train = N*K # total number of points in the training
set\n","x_train = np.zeros((N_train,2)) # matrix containing the 2-dimensional datapoints\n","y_train = np.zeros(N_train,
dtype='uint8') # labels (not in one-hot representation)\n","\n","mag_noise = 0.3 # controls how much noise gets added to the
data\n","dTheta = 4 # difference in theta in each branch\n","\n","### Data generation: ###\n","for j in range(K):\n","
ix = range(N*j,N*(j+1))\n"," r = np.linspace(0.01,1,N) # radius\n"," t = np.linspace(j*(2*np.pi)/K,j*(2*np.pi)/K + dTheta,N)
+ np.random.randn(N)*mag_noise # theta\n"," x_train[ix] = np.c_[r*np.cos(t), r*np.sin(t)]\n"," y_train[ix] = j\n","\n","###
Plot the data set: ###\n","fig = plt.figure(1, figsize=(5,5))\n","plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, s=40)#,
cmap=plt.cm.Spectral)\n","plt.xlim(-1,1)\n","plt.ylim(-1,1)\n","plt.xlabel(r'$x_1$')\n","plt.ylabel(r'$x_2$')\n","plt.show
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corresponding one-hot encoding is a $K$-dimensional vector with all entries zero \n","except for the $k^{th}$ entry
(which has value 1).\n","So, for example, when $K=3$ the one-hot encodings for the labels are\n","\begin{equation*}\n","0
\rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad\n","1 \rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad\n","2 \rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.\n","\end{equation*}"],
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which first defines the structure of the neural network \n","and then uses the dataset to train this network. \n","Look at how
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time\n","\n","##### DEFINE
THE NETWORK ARCHITECTURE
#####\n","\n","### Create
placeholders for the input data and labels ###\n","### Create input actual values when we ask TensorFlow to run an actual
training step ###\n","x = tf.placeholder(tf.float32, [None, 2], name='x')\n","y = tf.placeholder(tf.int32, [None, 1], name='y')

```

FeedforwardNeuralNetworks_T x FeedforwardNeuralNetworks_T x

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Feedforward neural networks in TensorFlow

Perimeter Institute Computational Tutorial Series
October 29, 2019
Lauren Hayward

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import numpy as np

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##### CREATE AND PLOT THE DATA SET #####
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N = 50 # number of points per branch
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N_train = N*K # total number of points in the training set
x_train = np.zeros((N_train,2)) # matrix containing the 2-dimensional datapoints
y_train = np.zeros(N_train, dtype='uint8') # labels (not in one-hot representation)

mag_noise = 0.1 # controls how much noise gets added to the data
```

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Feedforward neural networks in TensorFlow

Perimeter Institute Computational Tutorial Series

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The screenshot shows a Google Colab notebook interface. The browser tabs at the top include 'FeedforwardNeuralNetworks_T...', 'FeedforwardNeuralNetworks_T...', and 'Copy of FeedforwardNeuralNet...'. The address bar shows the URL 'https://colab.research.google.com/drive/1CQVrD1_3wRS_48-By2lQ3vGqx7LCp9bN'. The notebook title is 'Copy of FeedforwardNeuralNetworks_TensorFlow.ipynb', last saved at 1:39 PM. The notebook content includes a title 'Feedforward neural networks in TensorFlow', a subtitle 'Perimeter Institute Computational Tutorial Series', the date 'October 29, 2019', and the author 'Lauren Hayward'. The main text states: 'The objective of this tutorial is to become comfortable with using the software library TensorFlow to create and train a simple feedforward neural network for supervised learning. Let us start by generating a random dataset of two-dimensional points with K branches. For each datapoint $\mathbf{x} = (x_1, x_2)$, the label is the branch index such that $y = 0, 1, \dots, K - 2$ or $K - 1$. Our goal is to implement a neural network capable of classifying the branches.'

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```

At the bottom of the screen, a taskbar shows the URL 'http://10.30.14.191', the port '1199', a user icon, the time '1:40 PM', and a green circular icon.

FeedforwardNeuralNetworks_T x FeedforwardNeuralNetworks_T x Copy of FeedforwardNeuralNet x +

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neural network for supervised learning.

Let us start by generating a random dataset of two-dimensional points with K branches. For each datapoint $\mathbf{x} = (x_1, x_2)$, the label is the branch index such that $y = 0, 1, \dots, K - 2$ or $K - 1$. Our goal is to implement a neural network capable of classifying the branches.

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dTheta = 4 # difference in theta in each branch

### Data generation: ###
for j in range(K):
    ix = range(N*j,N*(j+1))
    r = np.linspace(0.01,1,N) # radius
    t = np.linspace(j*(2*np.pi)/K,j*(2*np.pi)/K + dTheta,N) + np.random.randn(N)*mag_noise # theta
    x_train[ix] = np.c_[r*np.cos(t), r*np.sin(t)]
    y_train[ix] = j

### Plot the data set: ###
fig = plt.figure(1, figsize=(5,5))
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, s=40)#, cmap=plt.cm.Spectral)
```



FeedforwardNeuralNetworks_T x FeedforwardNeuralNetworks_T x Copy of FeedforwardNeuralNet x

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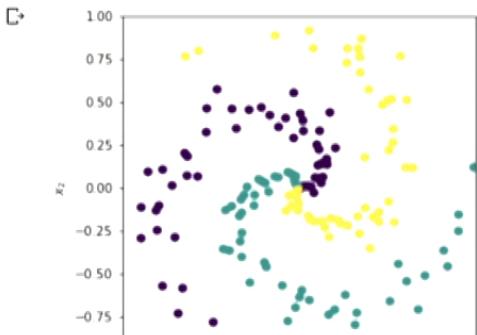
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plt.xlim([-1,1])
plt.ylim([-1,1])
plt.xlabel(r'$x_1$')
plt.ylabel(r'$x_2$')
plt.show()
```



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FeedforwardNeuralNetworks_T x | FeedforwardNeuralNetworks_T x | Copy of FeedforwardNeuralNet x

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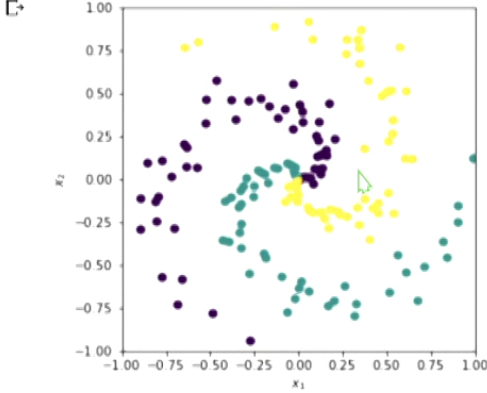
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```

t = np.linspace(j*(2*np.pi)/K, j*(2*np.pi)/K + 2*np.pi, N) + np.random.randn(N)*mag_noise # theta
x_train[ix] = np.c_[r*np.cos(t), r*np.sin(t)]
y_train[ix] = j

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plt.xlim([-1,1])
plt.ylim([-1,1])
plt.xlabel(r'$x_1$')
plt.ylabel(r'$x_2$')
plt.show()

```



This network will compare its output with labels in the so-called *one-hot encoding*. For a given label $y = k$, the corresponding one-hot encoding is a K -dimensional vector with all entries zero except for the k^{th} entry (which has value 1). So for example when $K = 3$ the one-hot

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Exercise #1: Run the code below, which first defines the structure of the neural network and then uses the dataset to train this network. Look at how this code attempts to classify the two-dimensional space. You should find that the resulting classifier separates the two-dimensional space using lines, and thus does a poor job of representing the data.

```

%matplotlib inline
from IPython import display
import tensorflow as tf
import time

#####
##### DEFINE THE NETWORK ARCHITECTURE #####
#####

### Create placeholders for the input data and labels ###
### (we'll input actual values when we ask TensorFlow to run an actual computation later) ###
x = tf.placeholder(tf.float32, [None, 2]) # input data
y = tf.placeholder(tf.int32,[None])      # labels

### Layer 1: ###
W1 = tf.Variable( tf.random_normal([2, K], mean=0.0, stddev=0.01, dtype=tf.float32) )
b1 = tf.Variable( tf.zeros([K]) )
z1 = tf.matmul(x, W1) + b1
a1 = tf.nn.sigmoid( z1 )

### Network output: ###
aL = a1

### Cost function: ###
### (measures how far off our model is from the labels) ###
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot )*tf.log(1.0-aL+eps) , reduction_indices=[1]))

```



FeedforwardNeuralNetworks_T x FeedforwardNeuralNetworks_T x Copy of FeedforwardNeuralNet x

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```

... update the plot of the resulting classification ...
fig = plt.figure(2,figsize=(10,5))
fig.subplots_adjust(hspace=.3,wspace=.3)
plt.clf()
updatePlot()
display.display(plt.gcf())
display.clear_output(wait=True)
#time.sleep(0.1) #Uncomment this line if you want to slow down the rate of plot updates

```

Exercise #2: Look through the section of code marked DEFINE THE NETWORK ARCHITECTURE . On paper, draw the neural network corresponding to the one in the code for the case of K branches. Pay particular attention to the number of neurons in each layer.

Exercise #3: Add in a hidden layer with 4 neurons and study how this hidden layer changes the output. On paper, draw the neural network in this case.



FeedforwardNeuralNetworks_T x | FeedforwardNeuralNetworks_T x | Copy of FeedforwardNeuralNet x +

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Exercise #3: Add in a hidden layer with 4 neurons and study how this hidden layer changes the output. On paper, draw the neural network in this case.

Exercise #4: Replace the sigmoid activation function on the first layer with a rectified linear unit (ReLU), and study how the choice of activation function changes the output.

Exercise #5: Change the cost function so that it is computed using the mean-squared error (MSE) instead of the cross-entropy, and study how the choice of cost function changes the output.

Exercise #6: Study the effects of increasing and decreasing the `learning_rate` hyperparameter. Examine these effects using both the cross-entropy and mean-squared error cost functions.

Exercise #7: Explain why the K -dimensional one-hot encoding is useful. What do you think would happen if you used a one-dimensional label (such that $y = 0, 1, \dots, K - 1$ or K) instead?

Exercise #8: Study how the neural network's accuracy changes as a function of:

- the number of neurons in the hidden layer
- `mag_noise` (the magnitude of noise in the data)
- the number of different labels x

[]




```
FeedforwardNeuralNetworks_T x | FeedforwardNeuralNetworks_T x | Copy of FeedforwardNeuralNet x +
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### Generate coordinates covering the whole plane: ###
padding = 0.1
spacing = 0.02
x1_min, x1_max = x_train[:, 0].min() - padding, x_train[:, 0].max() + padding
x2_min, x2_max = x_train[:, 1].min() - padding, x_train[:, 1].max() + padding
x1_grid, x2_grid = np.meshgrid(np.arange(x1_min, x1_max, spacing),
                               np.arange(x2_min, x2_max, spacing))

NN_output      = sess.run(aL, feed_dict={x:np.c_[x1_grid.ravel(), x2_grid.ravel()]})
predicted_class = np.argmax(NN_output, axis=1)

### Plot the classifier: ###
plt.subplot(121)
plt.contourf(x1_grid, x2_grid, predicted_class.reshape(x1_grid.shape), K, alpha=0.8)
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, s=40)
plt.xlim(x1_grid.min(), x1_grid.max())
plt.ylim(x2_grid.min(), x2_grid.max())
plt.xlabel(r'$x_1$')
plt.ylabel(r'$x_2$')

### Plot the cost function during training: ###
plt.subplot(222)
plt.plot(epoch_list, cost_training, 'o-')
plt.xlabel('Epoch')
plt.ylabel('Training cost')

### Plot the training accuracy: ###
plt.subplot(224)
plt.plot(epoch_list, acc_training, 'o-')
plt.xlabel('Epoch')
plt.ylabel('Training accuracy')
##### End of plotting function #####
```

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```

epoch_list.append(epoch)
cost_training.append(cost)
acc_training.append(accuracy)

### Update the plot of the resulting classifier: ###
fig = plt.figure(2,figsize=(10,5))
fig.subplots_adjust(hspace=.3,wspace=.3)
plt.clf()
updatePlot()
display.display(plt.gcf())
display.clear_output(wait=True)
#time.sleep(0.1) #Uncomment this line if you want to slow down the rate of plot updates

```

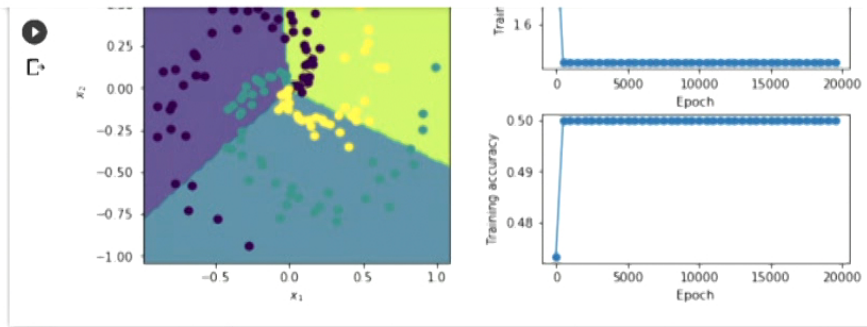
Exercise #2: Look through the section of code marked DEFINE THE NETWORK ARCHITECTURE . On paper, draw the neural network corresponding to the one in the code for the case of K branches. Pay particular attention to the number of neurons in each layer.

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- Exercise #2:** Look through the section of code marked `DEFINE THE NETWORK ARCHITECTURE`. On paper, draw the neural network corresponding to the one in the code for the case of K branches. Pay particular attention to the number of neurons in each layer.
- Exercise #3:** Add in a hidden layer with 4 neurons and study how this hidden layer changes the output. On paper, draw the neural network in this case.
- Exercise #4:** Replace the sigmoid activation function on the first layer with a rectified linear unit (ReLU), and study how the choice of activation function changes the output.
- Exercise #5:** Change the cost function so that it is computed using the mean-squared error (MSE) instead of the cross-entropy, and study how the choice of cost function changes the output.
- Exercise #6:** Study the effects of increasing and decreasing the `learning_rate` hyperparameter. Examine these effects using both the cross-entropy and mean-squared error cost functions.
- Exercise #7:** Explain why the K -dimensional one-hot encoding is useful. What do you think would happen if you used a one-dimensional label (such that $y = 0, 1, \dots, K - 1$ or K) instead?

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## Create placeholders for the input data and labels ##
## (we'll input actual values when we ask TensorFlow to run an actual computation later) ##
x = tf.placeholder(tf.float32, [None, 2]) # input data
y = tf.placeholder(tf.int32,[None]) # labels

## Layer 1: ##
W1 = tf.Variable( tf.random_normal([2, K], mean=0.0, stddev=0.01, dtype=tf.float32) )
b1 = tf.Variable( tf.zeros([K]) )
z1 = tf.matmul(x, W1) + b1
a1 = tf.nn.sigmoid( z1 )

## Network output: ##
aL = a1

## Cost function: ##
## (measures how far off our model is from the labels) ##
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot) * tf.log(1.0-aL+eps) , reduction_indices=[1]))
cost_func = cross_entropy

## Use backpropagation to minimize the cost function using the gradient descent algorithm: ##
learning_rate = 1.0 # hyperparameter
train_step = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost_func)

N_epochs = 20000 # number of times to run gradient descent

#####
##### TRAINING #####
#####

sess = tf.Session()
sess.run(tf.global_variables_initializer())

epoch list = []
```



```
FeedforwardNeuralNetworks_T x | FeedforwardNeuralNetworks_T x | Copy of FeedforwardNeuralNet x +
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RAM 100% Disk 100% Editing ^
from IPython import display
import tensorflow as tf
import time

#####
##### DEFINE THE NETWORK ARCHITECTURE #####
#####

### Create placeholders for the input data and labels ###
### (we'll input actual values when we ask TensorFlow to run an actual computation later) ###
x = tf.placeholder(tf.float32, [None, 2]) # input data
y = tf.placeholder(tf.int32,[None])     # labels

### Layer 1: ###
W1 = tf.Variable( tf.random_normal([2, K], mean=0.0, stddev=0.01, dtype=tf.float32) )
b1 = tf.Variable( tf.zeros([K]) )
z1 = tf.matmul(x, W1) + b1
a1 = tf.nn.sigmoid( z1 )

### Network output: ###
aL = a1

### Cost function: ###
### (measures how far off our model is from the labels) ###
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.0000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot )*tf.log(1.0-aL+eps) , reduction_indices=[1]))
cost_func = cross_entropy

### Use backpropagation to minimize the cost function using the gradient descent algorithm: ###
learning_rate = 1.0 # hyperparameter
train_step = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost_func)

N_epochs = 20000 # number of times to run gradient descent
```

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#####
### Create placeholders for the input data and labels ###
### (we'll input actual values when we ask TensorFlow to run an actual computation later) ###
x = tf.placeholder(tf.float32, [None, 2]) # input data
y = tf.placeholder(tf.int32, [None, 1]) # labels
TFModuleWrapper: tf.nn(wrapped, module_name, public_api=None,
deprecation=True, has_lite=False)
W1 = tensorflow.python.util.module_wrapper.TFModuleWrapper instance (float32)
b1 =
z1 = #click text to go to definition.
a1 = tf.nn.sigmoid( z1 )

### Network output: ###
aL = a1

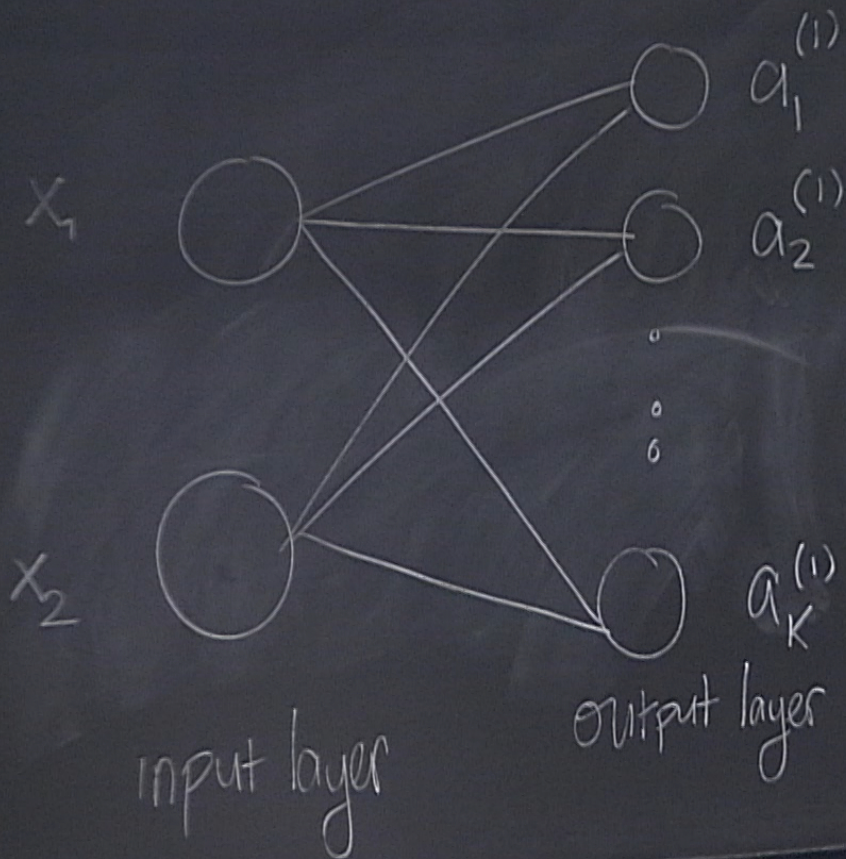
### Cost function: ###
### (measures how far off our model is from the labels) ###
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot )*tf.log(1.0-aL +eps) , reduction_indices=[1]))
cost_func = cross_entropy

### Use backpropagation to minimize the cost function using the gradient descent algorithm: ###
learning_rate = 1.0 # hyperparameter
train_step = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost_func)

N_epochs = 20000 # number of times to run gradient descent

##### TRAINING #####
sess = tf.Session()
sess.run(tf.global_variables_initializer())
```





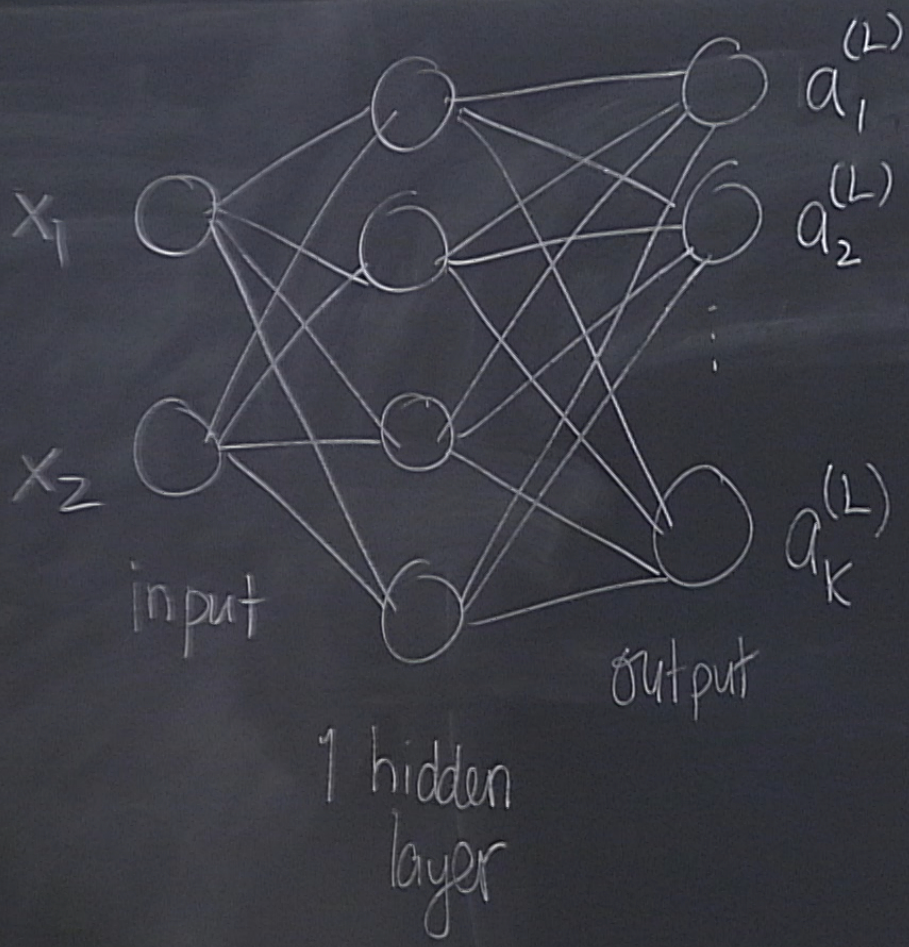
The NN is performing well
when $a_i^{(L)}$ is "close" to y_i

all operators have
mass dimension > 4
are irrelevant/non-renormalizable
(less important in IR)

Ψ interaction
theory

$$\langle \Omega | T \Psi_4 \Psi_3 \Psi_1 \Psi_2 | \Omega \rangle$$

perturbation theory order 2
order 1



$$L=2$$



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##### DEFINE THE NETWORK ARCHITECTURE #####
#####

### Create placeholders for the input data and labels ###
### (we'll input actual values when we ask TensorFlow to run an actual computation later) ###
x = tf.placeholder(tf.float32, [None, 2]) # input data
y = tf.placeholder(tf.int32,[None])      # labels

### Layer 1: ###
W1 = tf.Variable( tf.random_normal([2, K], mean=0.0, stddev=0.01, dtype=tf.float32) )
b1 = tf.Variable( tf.zeros([K]) )
z1 = tf.matmul(x, W1) + b1
a1 = tf.nn.sigmoid( z1 )

### Network output: ###
aL = a1

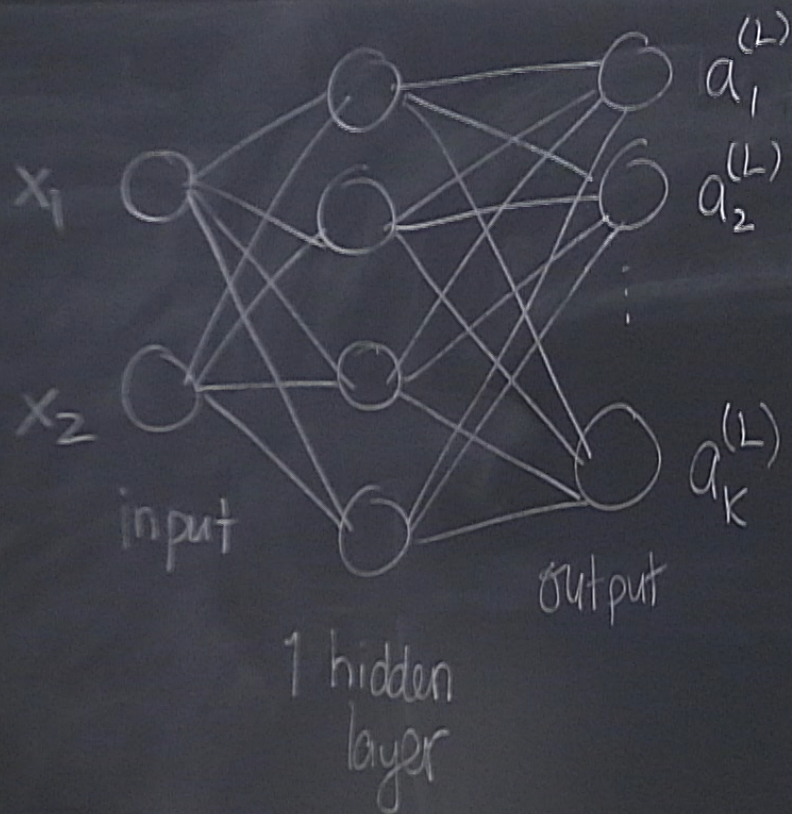
### Cost function: ###
### (measures how far off our model is from the labels) ###
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.0000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot )*tf.log(1.0-aL +eps) , reduction_indices=[1]))
cost_func = cross_entropy

### Use backpropagation to minimize the cost function using the gradient descent algorithm: ###
learning_rate = 1.0 # hyperparameter
train_step = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost_func)

N_epochs = 20000 # number of times to run gradient descent

#####
##### TRAINING #####
#####
sess = tf.Session()
sess.run(tf.global_variables_initializer())
```





$$L=2$$

$$a_j^{(L)} = g_L \left(\sum_{i=1}^{n_{L-1}} a_i^{(L-1)} W_{ij} + b_j^{(L)} \right)$$

b) \vec{x} : 2-dim. vector

FeedforwardNeuralNetworks_T x FeedforwardNeuralNetworks_T x Copy of FeedforwardNeuralNet x

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Exercise #2: Look through the section of code marked `DEFINE THE NETWORK ARCHITECTURE`. On paper, draw the neural network corresponding to the one in the code for the case of K branches. Pay particular attention to the number of neurons in each layer.

Exercise #3: Add in a hidden layer with 4 neurons and study how this hidden layer changes the output. On paper, draw the neural network in this case.

Exercise #4: Replace the sigmoid activation function on the first layer with a rectified linear unit (ReLU), and study how the choice of activation function changes the output.

Exercise #5: Change the cost function so that it is computed using the mean-squared error (MSE) instead of the cross-entropy, and study how the choice of cost function changes the output.

Exercise #6: Study the effects of increasing and decreasing the `learning_rate` hyperparameter. Examine these effects using both the cross-




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https://colab.research.google.com/drive/1CQVrD1_3wRS_48-By2lQ3vGqx7LCp9bN#scrollTo=EG5DlIjISEvB
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CO Copy of FeedforwardNeuralNetworks_TensorFlow.ipynb ☆
File Edit View Insert Runtime Tools Help
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RAM 100% Disk 100% Editing ^
from IPython import display
import tensorflow as tf
import time

#####
##### DEFINE THE NETWORK ARCHITECTURE #####
#####

### Create placeholders for the input data and labels ###
### (we'll input actual values when we ask TensorFlow to run an actual computation later) ###
x = tf.placeholder(tf.float32, [None, 2]) # input data
y = tf.placeholder(tf.int32,[None]) # labels

|
### Layer 1: ###
W1 = tf.Variable( tf.random_normal([2, K], mean=0.0, stddev=0.01, dtype=tf.float32) )
b1 = tf.Variable( tf.zeros([K]) )
z1 = tf.matmul(x, W1) + b1
a1 = tf.nn.sigmoid( z1 )

### Network output: ###
aL = a1

### Cost function: ###
### (measures how far off our model is from the labels) ###
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.0000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot )*tf.log(1.0-aL +eps) , reduction_indices=[1]))
cost_func = cross_entropy

### Use backpropagation to minimize the cost function using the gradient descent algorithm: ###
learning_rate = 1.0 # hyperparameter
train_step = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost_func)

K_epochs = 20000 # number of times to run gradient descent
```

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FeedforwardNeuralNetworks_T x FeedforwardNeuralNetworks_T x Copy of FeedforwardNeuralNet x

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```

epoch_list.append(epoch)
cost_training.append(cost)
acc_training.append(accuracy)

### Update the plot of the resulting classifier: ###
fig = plt.figure(2,figsize=(10,5))
fig.subplots_adjust(hspace=.3,wspace=.3)
plt.clf()
updatePlot()
display.display(plt.gcf())
display.clear_output(wait=True)
#time.sleep(0.1) #Uncomment this line if you want to slow down the rate of plot updates

```

The figure displays three plots related to the training process:

- Scatter Plot:** Shows data points in a 2D space with axes x_1 and x_2 . The plot is divided into three regions (purple, yellow, and blue) representing different classifier outputs.
- Training Cost Plot:** Shows Training cost on the y-axis (ranging from 0.5 to 2.0) versus Epoch on the x-axis (ranging from 0 to 12000). The cost starts at approximately 1.9 and decreases rapidly, stabilizing around 0.7 after about 4000 epochs.
- Training Accuracy Plot:** Shows Training accuracy on the y-axis (ranging from 0.4 to 0.8) versus Epoch on the x-axis (ranging from 0 to 12000). The accuracy starts at approximately 0.3 and increases rapidly, stabilizing around 0.8 after about 4000 epochs.

Exercise #2: Look through the section of code marked DEFINE THE NETWORK ARCHITECTURE . On paper, draw the neural network



```
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RAM 100% Disk 100% Editing ^
nH = 4
### Layer 1: ###
W1 = tf.Variable( tf.random_normal([2, nH], mean=0.0, stddev=0.01, dtype=tf.float32) )
b1 = tf.Variable( tf.zeros([nH]) )
z1 = tf.matmul(x, W1) + b1
a1 = tf.nn.sigmoid( z1 )

### Layer 2: ###
W2 = tf.Variable( tf.random_normal([nH, K], mean=0.0, stddev=0.01, dtype=tf.float32) )
b2 = tf.Variable( tf.zeros([K]) )
z2 = tf.matmul(a1, W2) + b2
a2 = tf.nn.sigmoid( z2 )

### Network output: ###
aL = a2

### Cost function: ###
### (measures how far off our model is from the labels) ###
y_onehot = tf.one_hot(y,depth=K) # labels are converted to one-hot representation
eps=0.000000001 # to prevent the logs from diverging
cross_entropy = tf.reduce_mean(-tf.reduce_sum( y_onehot * tf.log(aL+eps) + (1.0-y_onehot )*tf.log(1.0-aL+eps) , reduction_indices=[1]))
cost_func = cross_entropy

### Use backpropagation to minimize the cost function using the gradient descent algorithm: ###
learning_rate = 1.0 # hyperparameter
train_step = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost_func)

N_epochs = 20000 # number of times to run gradient descent

#####
##### TRAINING #####
#####
sess = tf.Session()
sess.run(tf.global_variables_initializer())
```

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