Title: Toy Models of Holographic Duality between local Hamiltonians

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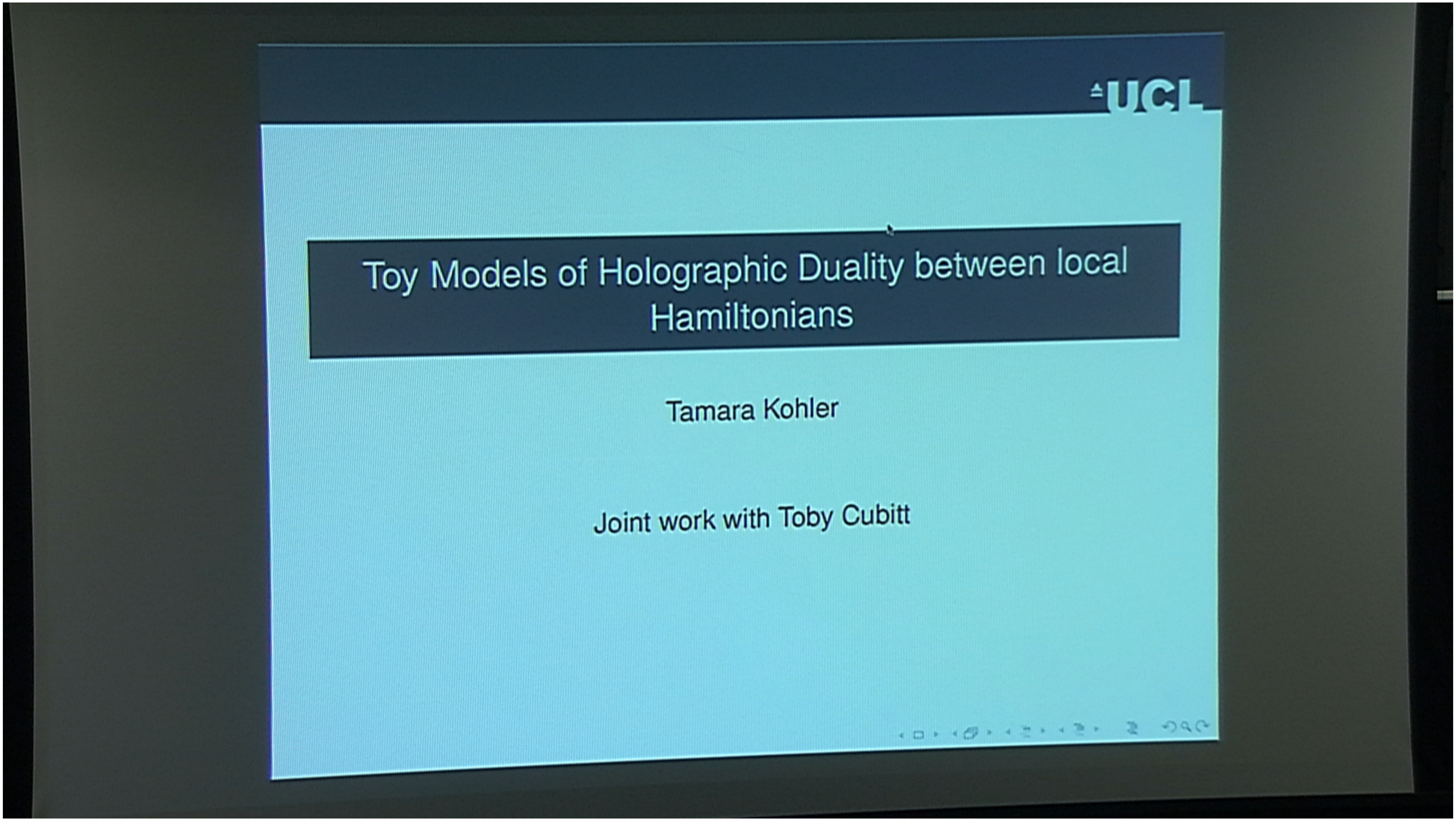
Series: Perimeter Institute Quantum Discussions

Date: October 30, 2019 - 4:00 PM

URL: http://pirsa.org/19100087

Abstract: Holographic quantum error correcting codes (HQECC) have been proposed as toy models for the AdS/CFT correspondence, and exhibit many of the features of the duality. HQECC give a mapping of states and observables. However, they do not map local bulk Hamiltonians to local Hamiltonians on the boundary. In this work, we combine HQECC with Hamiltonian simulation theory to construct a bulk-boundary mapping between local Hamiltonians, whilst retaining all the features of the HQECC duality. This allows us to construct a duality between models, encompassing the relationship between bulk and boundary energy scales and time dynamics.

It also allows us to construct a map in the reverse direction: from local boundary Hamiltonians to the corresponding local Hamiltonian in the bulk. Under this boundary-to- bulk mapping, the bulk geometry emerges as an approximate, low-energy, effective theory living in the code-space of an (approximate) HQECC on the boundary. At higher energy scales, this emergent bulk geometry is modified in a way that matches the toy models of black holes proposed previously for HQECC. Moreover, the duality on the level of dynamics shows how these toy-model black holes can form dynamically.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Toy Models of Holographic Duality between local Hamiltonians
Tamara Kohler
Joint work with Toby Cubitt

Why are we interested in holographic dualities?

Holographic principle: a quantum gravity theory in $(d+1)$-dimensional spacetime is equivalent to a many body system defined on its boundary

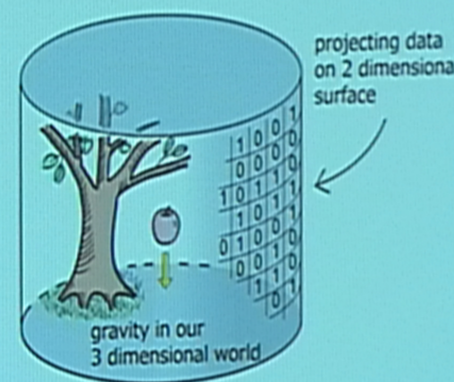

Figure credit: Hirosi Ooguri

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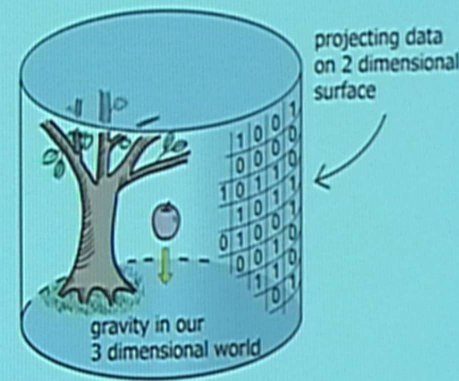
- AdS/CFT is the key example of the holographic principle



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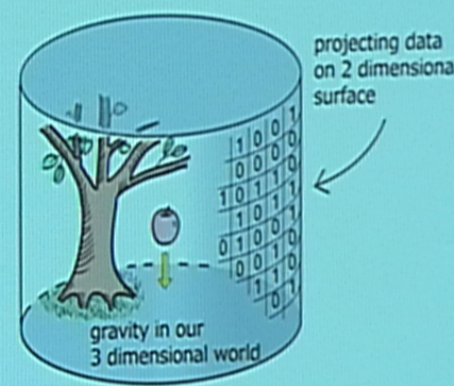
- A number of toy models of holographic duality which exhibit features of AdS/CFT have been proposed



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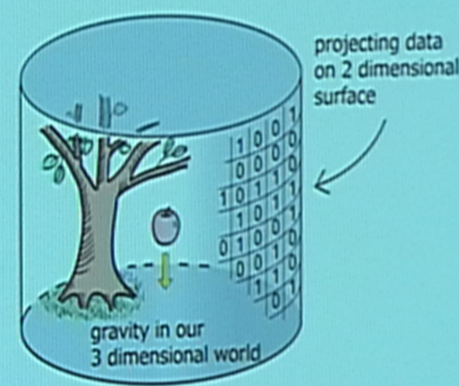
- AdS/CFT is the key example of the holographic principle
- A number of toy models of holographic duality which exhibit features of AdS/CFT have been proposed
- This is the first toy model which is a duality on the level of *local Hamiltonians*



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Why are we interested in holographic dualities?

Why is this important?

- The duality on the level of local Hamiltonians allows us to extend the toy models of holography to encompass energy scales and time dynamics.

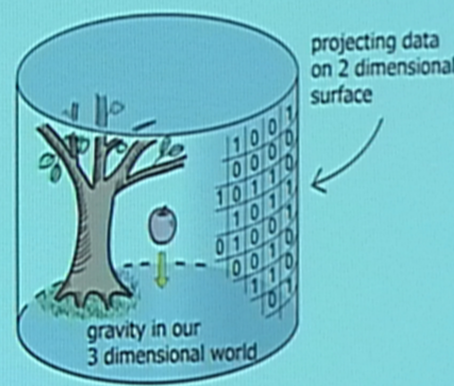

Figure credit: Hirosi Ooguri

Outline

- Background
  - The AdS/CFT correspondence
  - The HaPPY code
  - Hamiltonian simulation
- Overview of technical details
- Applications of the construction

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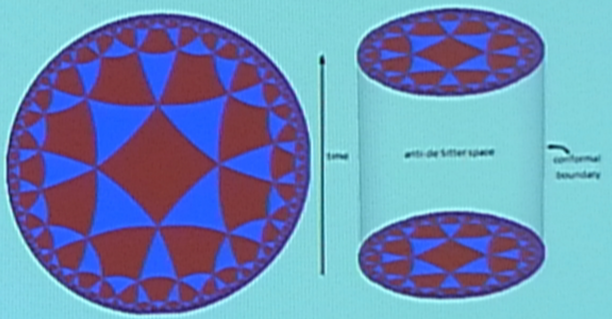
The AdS / CFT correspondence

Duality between a gravity theory in AdS_{d+1} and a conformal field theory on the boundary:

- $|\psi\rangle_{\text{bulk}} \leftrightarrow |\psi\rangle_{\text{boundary}}$
- $\lim_{r\to\infty} r^{\Delta}\phi(r,x) = \mathcal{O}(x)$
- Quantum error correcting code



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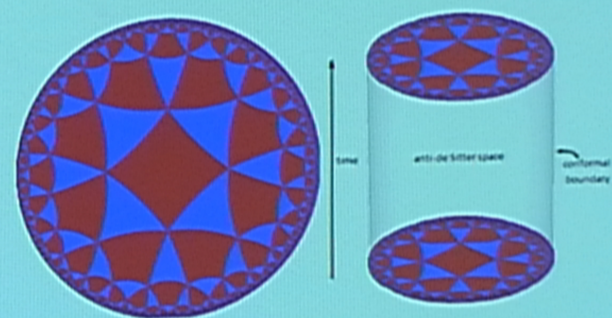

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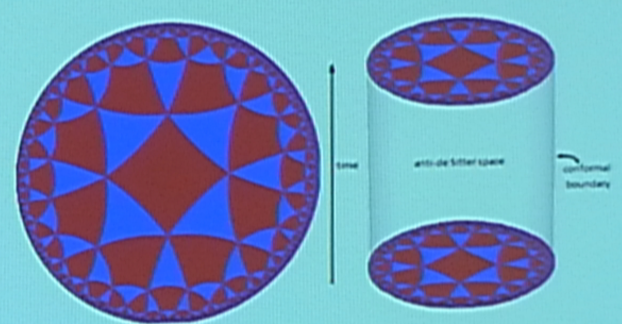
The AdS / CFT correspondence

AdS/CFT as a quantum error correcting code:

- $\mathscr{H}_{\text{logical}} = \mathscr{H}_{\text{bulk}}$
- $\mathscr{H}_{\text{physical}} = \mathscr{H}_{\text{boundary}}$
- Operators near the centre of the bulk are better protected against erasure than operators living near the boundary



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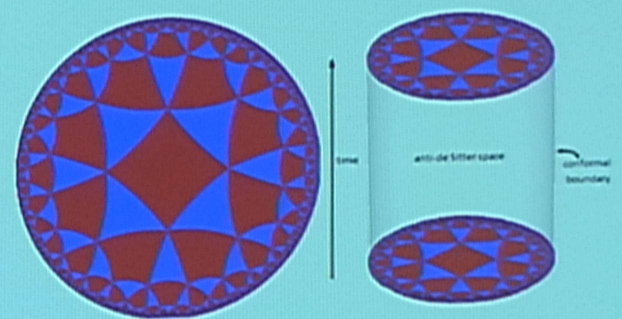

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The HaPPY code

Perfect tensors

Definition (Perfect tensors, definition 2 from (Pastawski et al. 2015))

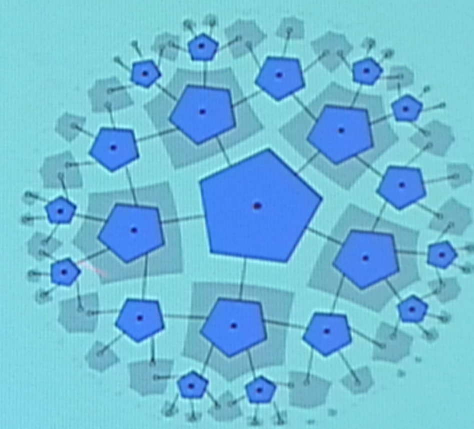
A $2m$-index tensor $T_{a_1 a_2 \ldots a_{2m}}$ is a perfect tensor if, for any bipartition of its indices into a set A and a complementary set A^c with $|A| \leq |A^c|$, T is proportional to an isometric tensor from A to A^c.

- Perfect tensors are the encoding isometries for optimal quantum error correcting codes.

HaPPY code

Tessellate the hyperbolic plane, and place a 6-leg perfect tensor in each pentagon (Pastawski et al. 2015):

- Five legs are contracted
- The sixth leg is the bulk index
- The tensor network is an isometry from bulk to boundary



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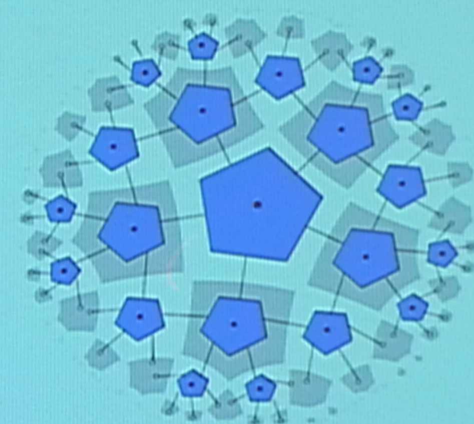
HaPPY code

Properties

The resulting tensor network has many of the structural features of AdS/CFT (Pastawski et al. 2015):

- It is an error-correcting code
- Operators near the centre better protected than operators near the boundary
- But a local bulk Hamiltonian maps to a global boundary Hamiltonian



What is Hamiltonian simulation?

In order for H' to simulate H we need to 'encode' the physics of H in this new system:

$$H' = \mathcal{E}(H) \tag{1}$$

What properties do we need $\mathcal{E}(H)$ to have so that H' replicates the physics of H?

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What is Hamiltonian simulation?

What properties should $\mathcal{E}(H)$ have?

1. Must be a Hamiltonian! $\mathcal{E}(H)^\dagger = \mathcal{E}(H)$

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4. Time evolution: $\mathrm{tr}(\mathcal{E}(M)e^{-i\mathcal{E}(H)t}\mathcal{E}(\rho)e^{i\mathcal{E}(H)t}) = \mathrm{tr}(Me^{-iHt}\rho e^{iHt})$

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5. Preserve the partition function: $Z_{\mathcal{E}(H)}(\beta) = cZ_H(\beta)$
6. Real linear: $\mathcal{E}(pA + (1-p)B) = \mathcal{E}(pA) + \mathcal{E}((1-p)B)$, $p \in [0,1]$

What is Hamiltonian simulation?

In Cubitt, Montanaro, and Piddock 2018 it's shown that any map $\mathcal{E}(H)$ satisfying:

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Must be of the form:

$$\mathcal{E}(H) = V\left(H^{\otimes p} \oplus (H^\dagger)^{\otimes q}\right)V^\dagger \quad (2)$$

And any map of this form preserves measurement statistics, time dynamics and partition functions.

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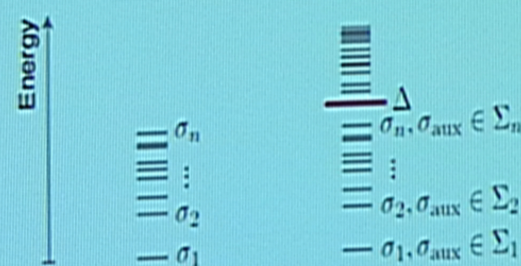
What is Hamiltonian simulation?

Definition (Perfect simulation Cubitt, Montanaro, and Piddock 2018)

We say that H' perfectly simulates H below energy Δ if there is an encoding $\mathcal{E}$ into the subspace $S_{\mathcal{E}}$ such that

1. $S_{\mathcal{E}} = S_{\leq \Delta(H')}$
2. $H'|_{\leq \Delta} = \mathcal{E}(H)|_{S_{\mathcal{E}}}$

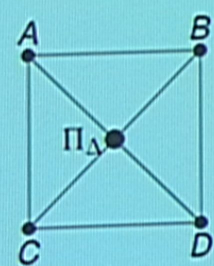


What about approximate simulations?

In Cubitt, Montanaro, and Piddock 2018 it's shown that *approximate* simulation *approximately* preserves all physical quantities.
Can use *perturbation gadgets* to construct approximate simulations:

$$H' = \Delta H_0 + H_1 \tag{3}$$

Generate effective interaction: $||H'|_{<\Delta} - H_{\text{eff}}|| \leq \epsilon$



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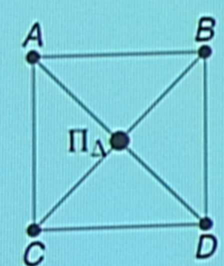

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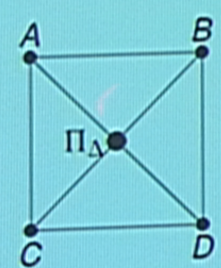

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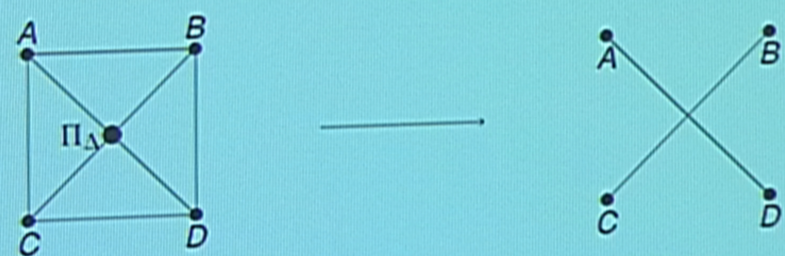


Hamiltonian simulation

Results from (Cubitt, Montanaro, and Piddock 2018) show that *all* Hamiltonians can be (approximately) simulated by certain simple 2d spin lattice models using *perturbation gadgets*.

Can we use these simulation techniques, along with a holographic quantum error correcting code from a 3D bulk to a 2D boundary, to construct a toy model of holographic duality between local Hamiltonians?

Outline

- Background
- Overview of technical details
  - Tessellations of $\mathbb{H}^3$
  - Tensor network construction
  - Boundary Hamiltonian
- Applications of the construction

How do we generalise the HaPPY code to higher dimensions?

Embed a perfect tensor network in a tessellation of $\mathbb{H}^3$ - but how do we analyse the properties of the tensor network?

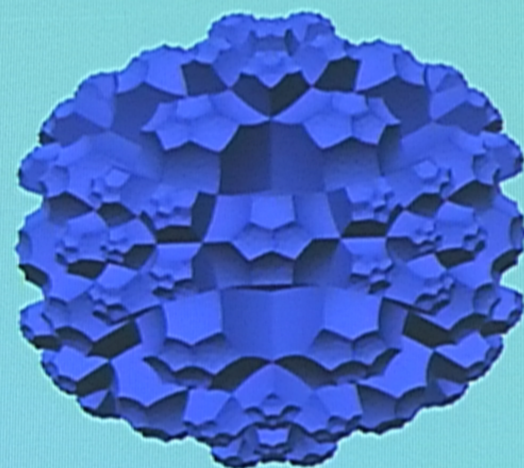


Figure: Tessellation of $\mathbb{H}^3$ by right angled dodecahedron

Hyperbolic Coxeter groups

Definition (Coxeter groups)

Let $S = \{s_i\}_{i \in I}$ be a finite set. Let $M = (m_{ij})_{i,j \in I}$ be a symmetric $n \times n$ matrix with entries from $\mathbb{N} \cup \{\infty\}$ such that $m_{ii} = 1$ for all $i \in [n]$ and $m_{ij} > 1$ whenever $i \neq j$. The associated *Coxeter group* $W = W(M)$ is defined by the presentation:

$$W = \langle S \mid (s_i s_j)^{m_{ij}} \rangle \qquad (5)$$

The pair (W, S) is called a *Coxeter system.*

Hyperbolic Coxeter groups

Coxeter polytopes

Definition (Coxeter polytope)

A polytope $P \in \mathbb{S}^n$, $\mathbb{E}^n$ or $\mathbb{H}^n$ is a Coxeter polytope iff all its dihedral angles are submultiples of π.

A Coxeter polytope $P \in \mathbb{X}^n$ tiles $\mathbb{X}^n$.

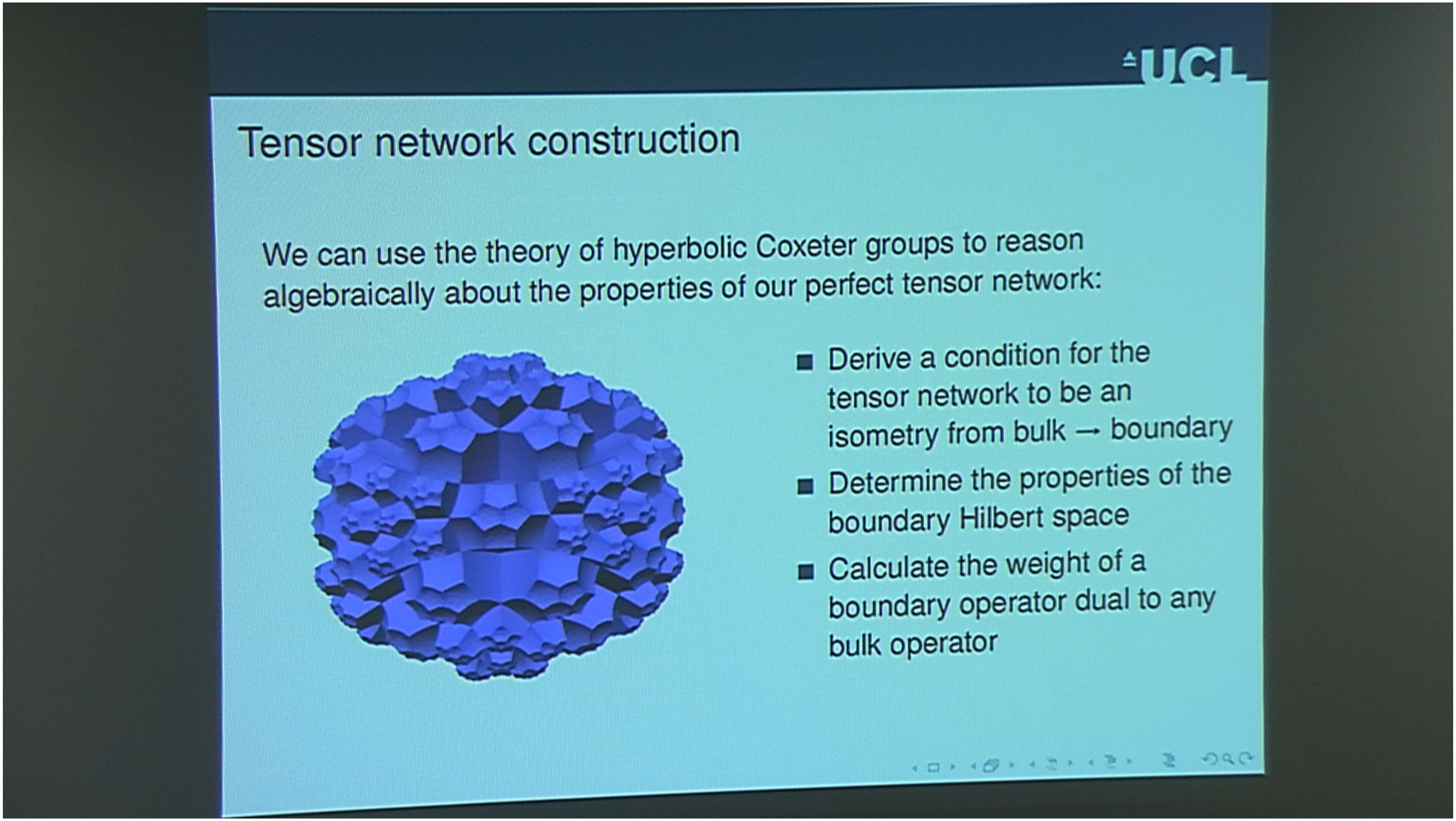
Hyperbolic Coxeter groups

Coxeter systems and tessellations

A Coxeter system (W, S) can be associated to every Coxeter polytope, $P \in \mathbb{X}^n$:

- The generators S are reflections in the facets of P
- $m_{ij} = \frac{\pi}{\alpha_{ij}}$ where α_{ij} is the dihedral angle between the i^{th} and j^{th} facet of P

The properties of the tessellation of $\mathbb{X}^n$ by P can be inferred from the Coxeter system (W, S).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Tensor network construction
We can use the theory of hyperbolic Coxeter groups to reason algebraically about the properties of our perfect tensor network:
Derive a condition for the tensor network to be an isometry from bulk → boundary
Determine the properties of the boundary Hilbert space
Calculate the weight of a boundary operator dual to any bulk operator

Holographic QECC in $\mathbb{H}^3$

We can construct a non-local boundary Hamiltonian which is a perfect simulation of H_{Bulk}.
For each tensor in the network, construct the projector:

$$\Pi_{\mathscr{C}^{(w)}} = \frac{1}{|S^{(w)}|} \sum_{M \in S^{(w)}} M \tag{6}$$

where $S^{(w)}$ is the stabilizer associated with that tensor.
Let

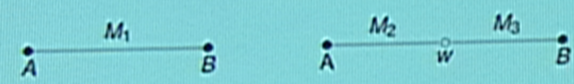
$$H_{\text{non-local}} = V H_{\text{Bulk}} V^{\dagger} + \Delta_S H_S \tag{7}$$

where $H_S = \sum_w (\mathbb{1} - \Pi_{\mathscr{C}^{(w)}})$.
$\Rightarrow H_{\text{non-local}}$ is a perfect simulation of H_{Bulk}

Boundary Hamiltonian

Perturbation gadgets

Subdivision gadget:



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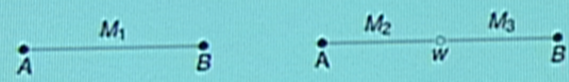
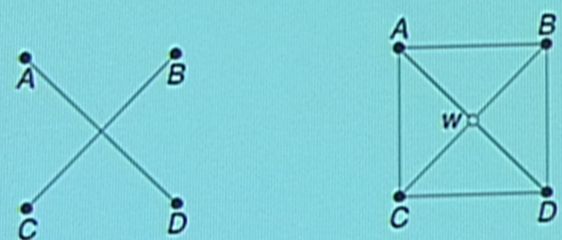


Crossing gadget:



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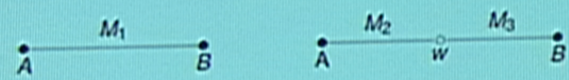

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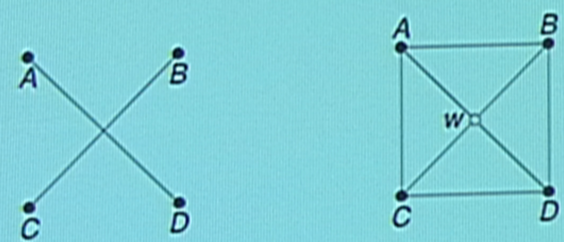
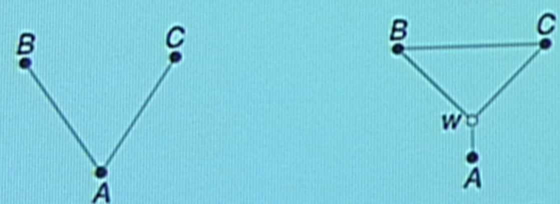
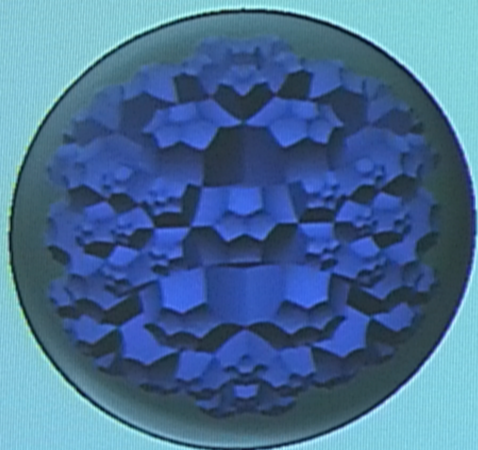


Fork gadget:



Holographic duality between local Hamiltonians

Use the perturbative simulation techniques to simulate $H_{\text{non-local}}$ by a local boundary model:



- Local boundary Hamiltonian: $H_{\text{bound}} = \sum_{\langle i,j \rangle} \alpha_{ij} (X_i X_j + Y_i Y_j + Z_i Z_j)$
- $\mathcal{H}_{\text{bound}}$ consists of $O(n(\log(n)^4))$ qubits, embedded in an $O(1)$ triangulation of a 2-sphere
- The new 'boundary' is at a distance $O(\log\log(n))$ from the original boundary
- The structural features of AdS/CFT captured by the HaPPY code are still present

Outline

- Background
- Overview of technical details
- Applications of the construction
  - Map from boundary → bulk
  - Energy scales in the duality
  - Black hole formation

Boundary → bulk mapping

The boundary Hamiltonian can be written as:

$$H_{\text{bound}} = \Delta_L H_L + \Delta_S \overline{H}_S + \overline{H}_{\text{Bulk}} \quad (8)$$

$\nearrow$ Locality $\uparrow$ Stabilizers $\nwarrow$ H_{Bulk}

Consider the boundary Hamiltonian which is dual to the zero-Hamiltonian in the bulk:

$$H_{\text{generic}} = \Delta_L H_L + \Delta_S \overline{H}_S \quad (9)$$

We can recover a geometric interpretation of the bulk from Eq. (9).

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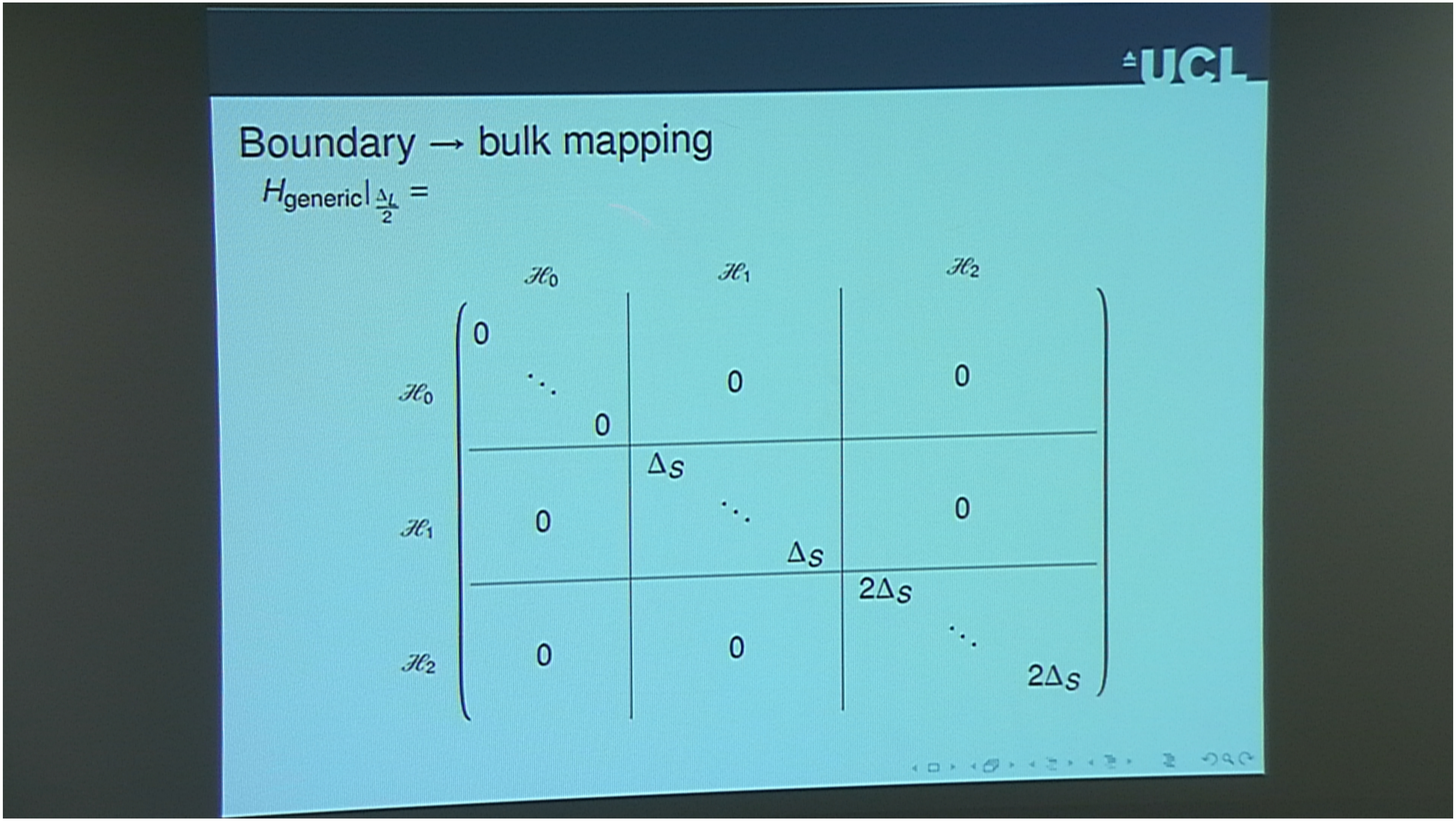
Boundary → bulk mapping

Decompose $\mathscr{H}_{\text{boundary}}|_{\frac{\Delta_L}{2}}$ into subspaces $\mathscr{H}_n$ of energy E where:

$$(n-\frac{1}{2})\Delta_S \leq E \leq (n+\frac{1}{2})\Delta_S \tag{10}$$

$\Rightarrow H_{\text{generic}}|_{\frac{\Delta_L}{2}}$ is block diagonal with respect to the decomposition $\mathscr{H}_{\text{boundary}}|_{\frac{\Delta_L}{2}} = \oplus_n \mathscr{H}_n$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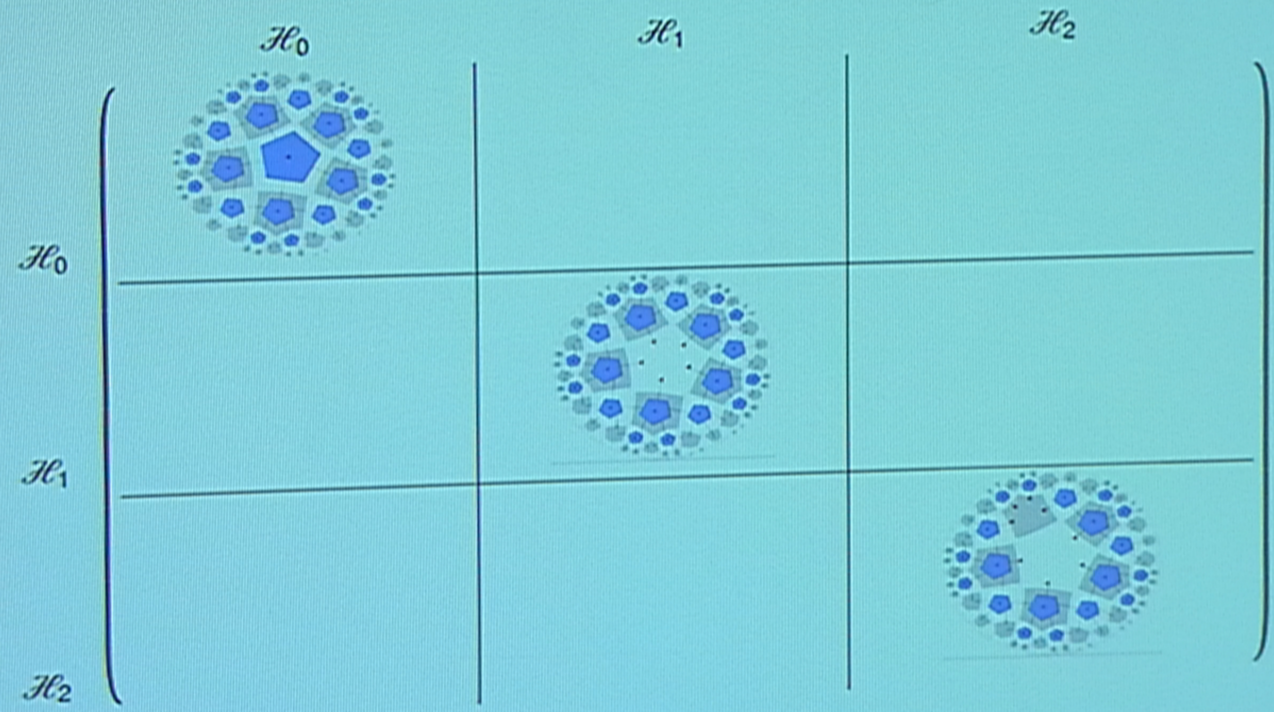
Boundary → bulk mapping
$H_{\text{generic}}|_{\frac{\Delta_L}{2}} =$
$$\begin{array}{c} \\ \mathscr{H}_0 \\ \mathscr{H}_1 \\ \mathscr{H}_2 \end{array} \begin{array}{c} \begin{array}{ccc} \quad\mathscr{H}_0 \quad & \quad \mathscr{H}_1 \quad & \quad \mathscr{H}_2 \quad \end{array} \\ \left(\begin{array}{c|c|c} \begin{matrix} 0 & & \\ & \ddots & \\ & & 0 \end{matrix} & 0 & 0 \\ \hline 0 & \begin{matrix} \Delta_S & & \\ & \ddots & \\ & & \Delta_S \end{matrix} & 0 \\ \hline 0 & 0 & \begin{matrix} 2\Delta_S & & \\ & \ddots & \\ & & 2\Delta_S \end{matrix} \end{array}\right) \end{array}$$

Boundary → bulk mapping

What's the geometric interpretation?



Boundary → bulk mapping

$H_{\text{generic}}|_{\frac{\Delta_L}{2}} =$

$$
\begin{array}{c}
 \\ \mathscr{H}_0 \\ \\ \mathscr{H}_1 \\ \\ \mathscr{H}_2
\end{array}
\overset{\quad\quad\mathscr{H}_0 \quad\quad\quad\quad \mathscr{H}_1 \quad\quad\quad\quad \mathscr{H}_2}{
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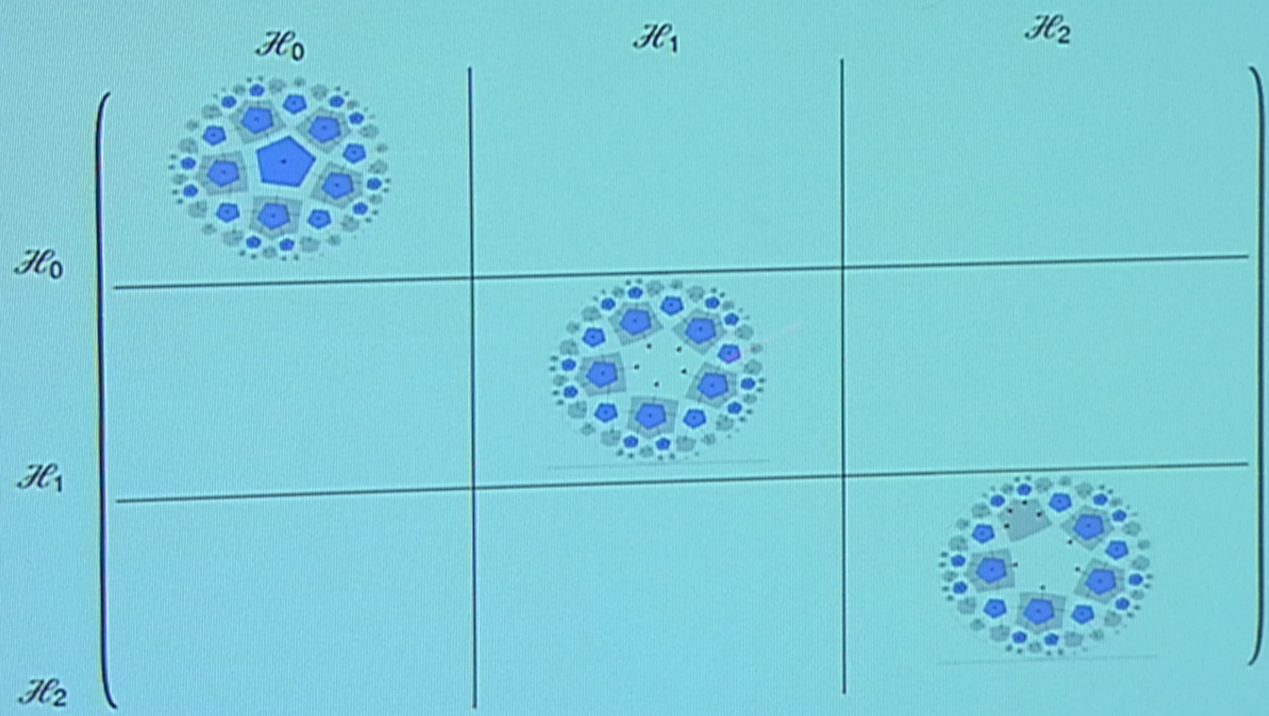
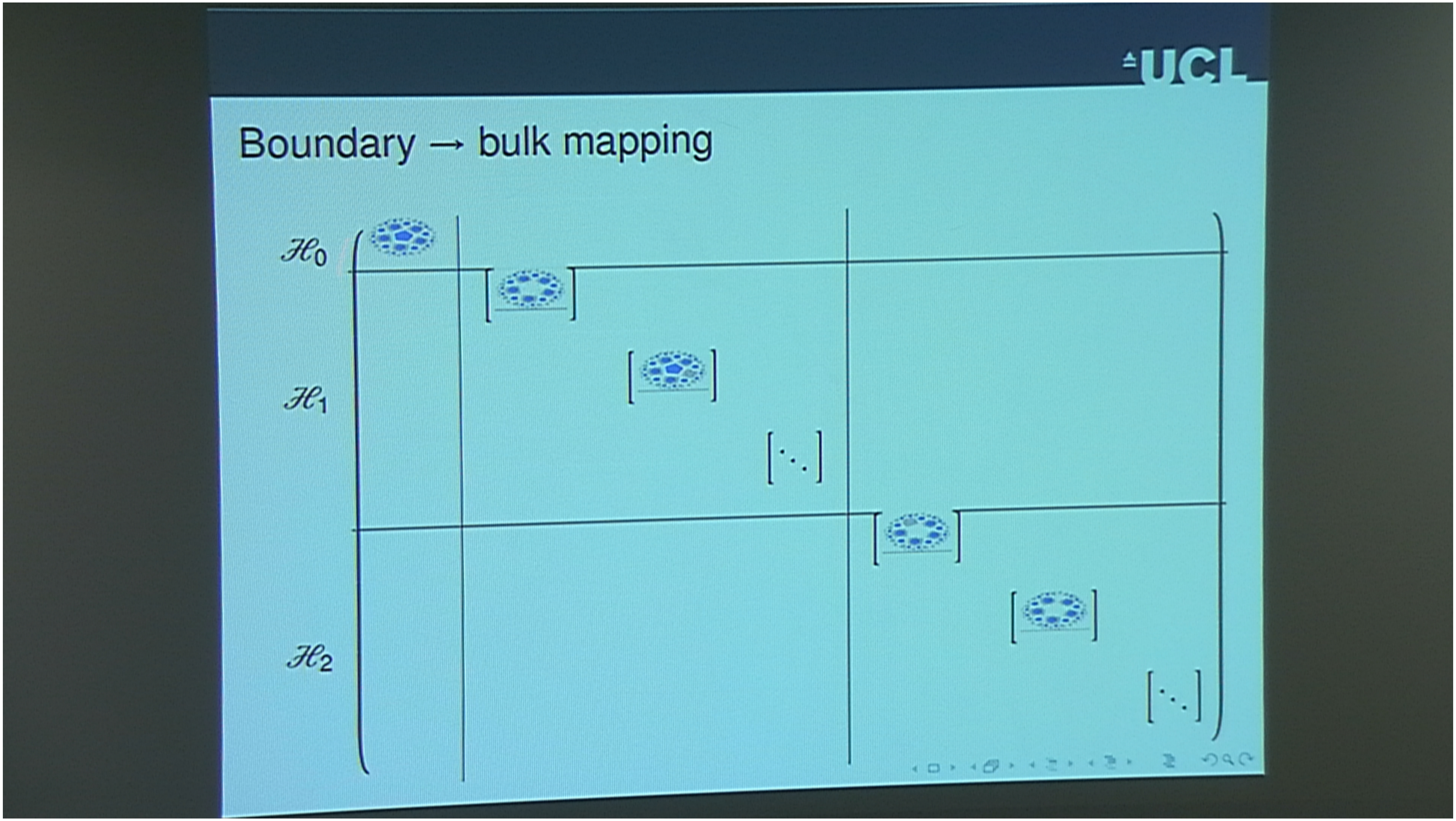
0 & & & & & & & & \\
 & \ddots & & & 0 & & & 0 & \\
 & & 0 & & & & & & \\
\hline
 & & & \Delta_S & & & & & \\
 & 0 & & & \ddots & & & 0 & \\
 & & & & & \Delta_S & & & \\
\hline
 & & & & & & 2\Delta_S & & \\
 & 0 & & & 0 & & & \ddots & \\
 & & & & & & & & 2\Delta_S
\end{array}\right)}
$$

Boundary → bulk mapping

What's the geometric interpretation?




UCL
Boundary → bulk mapping
$\mathcal{H}_0$
$\mathcal{H}_1$
$\mathcal{H}_2$

Boundary → bulk mapping

Boundary Hilbert space:

$$\mathcal{H}_{\text{boundary}}|_{\frac{\Delta_L}{2}} = \oplus_n \oplus_c \mathcal{H}_{n,c} \tag{11}$$

where $\mathcal{H}_{n,c}$= tensor network with n ‘holes’ in configuration c.
Let $V_{n,c}$ be the encoding isometry for the tensor network corresponding to $\mathcal{H}_{n,c}$. Define:

$$U = \oplus_n \oplus_c V_{n,c} \tag{12}$$

Consider a boundary state: $|\psi\rangle_{\text{boundary}} \in \mathcal{H}_{\text{boundary}}|_{\frac{\Delta_L}{2}} \oplus_n \oplus_c \mathcal{H}_{n,c}$.
The corresponding bulk state is given by:

$$|\psi\rangle_{\text{Bulk}} = U^{\dagger} |\psi\rangle_{\text{boundary}} \tag{13}$$

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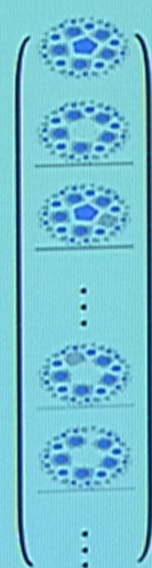
$|\psi\rangle_{\text{Bulk}}$ has a geometric interpretation:

$\Rightarrow |\psi\rangle_{\text{Bulk}} =$

$\mathcal{H}_0$, $\mathcal{H}_{1,c_1}$, $\mathcal{H}_{1,c_2}$, $\vdots$, $\mathcal{H}_{2,c_1}$, $\mathcal{H}_{2,c_2}$, $\vdots$



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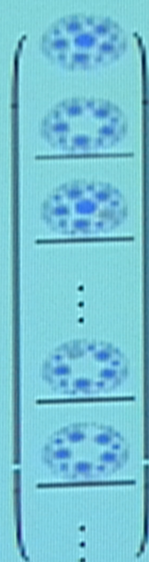

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Boundary → bulk mapping

What about mapping more interesting boundary Hamiltonians to the bulk?

$H_{\text{bound}}|_{\frac{\Delta_L}{2}} =$

$$
\left(\begin{array}{c|cccc|c}
\overline{H}_{\text{Bulk}}|_{\mathscr{H}_0} & 0 & 0 & \cdots & & 0 \\
\hline
0 & \left[\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{1,c_1}}\right] & 0 & \cdots & & 0 \\
0 & 0 & \left[\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{1,c_2}}\right] & \cdots & & 0 \\
\vdots & \vdots & \vdots & \left[\ddots\right] & & \vdots \\
\hline
0 & 0 & 0 & \cdots & & \left[\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{n,c}}\right]
\end{array}\right)
$$

Boundary → bulk mapping

What about mapping more interesting boundary Hamiltonians to the bulk?
⇒ We can calculate the bulk Hamiltonian which acts on the tensor network with n 'holes' in configuration c as:

$$H_{\text{Bulk}}^{(n,c)} = V_{n,c}\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{n,c}}V_{n,c}^{\dagger} \quad (15)$$

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Boundary → bulk mapping

What about mapping non-block-diagonal boundary Hamiltonians to the bulk?

$H_{\text{bound}}|_{\frac{\Delta_L}{2}} =$

$$
\left(\begin{array}{c|cccc|c}
\overline{H}_{\text{Bulk}}|_{\mathscr{H}_0} & \delta_{1,2}^{(c_1)} & \delta_{1,2}^{(c_2)} & \cdots & & \delta_{1,n} \\
\hline
\delta_{2,1}^{(c_1)} & \left[\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{1,c_1}}\right] & \delta_{2,2}^{(c_1,c_2)} & \cdots & & \delta_{2,n}^{(c_1,c)} \\
\delta_{2,1}^{(c_2)} & \delta_{2,2}^{(c_2,c_1)} & \left[\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{1,c_2}}\right] & \cdots & & \delta_{2,n}^{(c_2,c)} \\
\vdots & \vdots & \vdots & \left[\ddots\right] & & \vdots \\
\hline
\delta_{n,1}^{(c)} & \delta_{n,2}^{(c,c_1)} & \delta_{n,2}^{(c,c_2)} & \cdots & & \left[\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{n,c}}\right]
\end{array}\right)
$$

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Boundary → bulk mapping

What about mapping non-block diagonal boundary Hamiltonians to the bulk?
As before:

$$H^{(n,c)}_{\text{Bulk}} = V_{n,c}\overline{H}_{\text{Bulk}}|_{\mathscr{H}_{n,c}} V^{\dagger}_{n,c} \qquad (16)$$

The non-diagonal terms in $H_{\text{bound}}|_{\frac{\Delta_L}{2}}$ correspond to coupling between the different bulk geometries.
⇒ The tensor network description is only an approximation to the underlying bulk physics.

Boundary → bulk mapping

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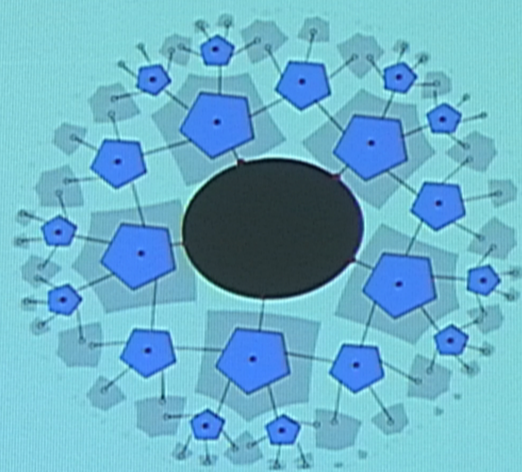
⇒ The tensor network description is only an approximation to the underlying bulk physics.

Energy scales in the duality

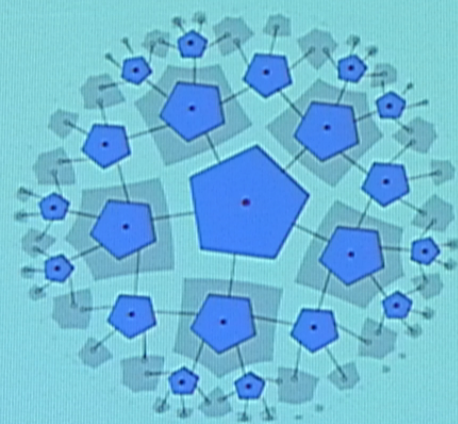
Black holes in the HaPPY code are modelled by removing a tensor from the tensor network:



In our model these are dual to high energy boundary states - as expected in AdS/CFT.

Black hole formation

Bulk Hamiltonian: $H_{\text{bulk}} = \sum_Z h_Z$
Choose Δ_S such that $\Delta_S \gg ||h_Z||$, but $\Delta_S < \sum_Z ||h_Z||$.



Start in the ground state: $|\psi_{\text{vac}}\rangle$

Black hole formation

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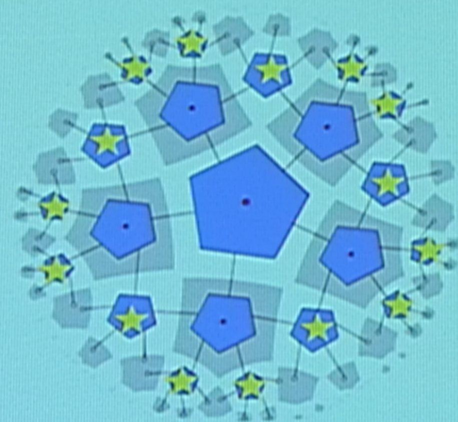
Choose Δ_S such that $\Delta_S \gg ||h_Z||$, but $\Delta_S < \sum_Z ||h_Z||$.



Apply some local bulk excitations: $|\psi_1\rangle = \otimes_x A_x |\psi_{\text{vac}}\rangle$

$$\langle \psi_1 | H_{\text{bulk}} | \psi_1 \rangle > \Delta_S$$

Black hole formation

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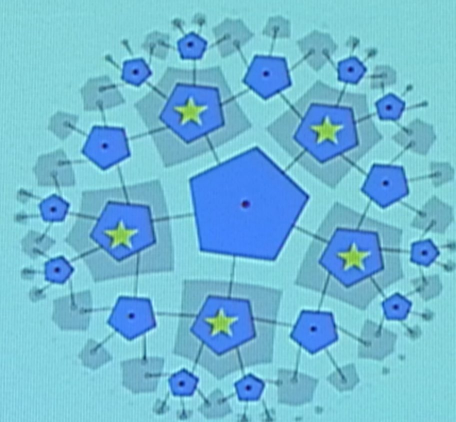
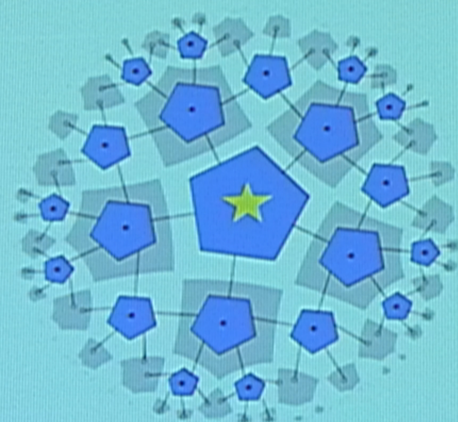


Allow the system to evolve under H_{bulk}, the shell of matter will fall inwards

Black hole formation

Bulk Hamiltonian: $H_{\text{bulk}} = \sum_Z h_Z$

Choose Δ_S such that $\Delta_S \gg \|h_Z\|$, but $\Delta_S < \sum_Z \|h_Z\|$.



Evolution has been unitary, so must still have energy $> \Delta_S$, but can't pick this up from $O(1)$ bulk terms

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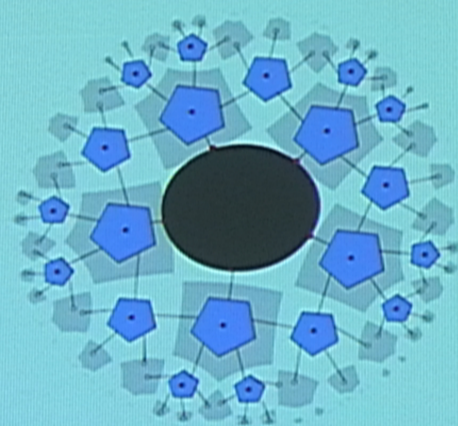

The only way to conserve energy under unitary dynamics is to violate a stabilizer term, i.e. remove a tensor from the network

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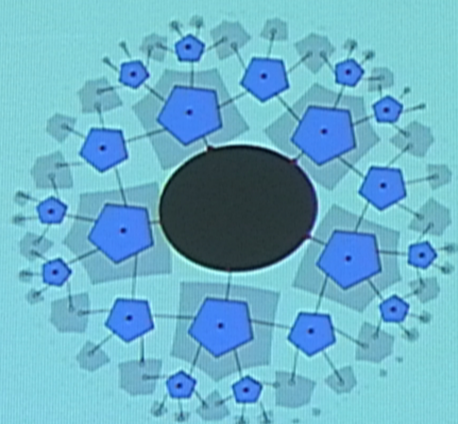

The only way to conserve energy under unitary dynamics is to violate a stabilizer term, i.e. remove a tensor from the network

Conclusions

- We have constructed a toy model of holographic duality where the bulk-boundary mapping is between local Hamiltonians
- This allows us to incorporate energy scales and dynamics in the toy models, and to construct a boundary → bulk mapping
- At what point do these toy models break down?