Title: NMR simulation of topological phases

Speakers: Ling-Yan Hung

Series: Colloquium

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Abstract: We will talk about recent progress in NMR technologies simulating topological phases. We will describe how states are prepared, how they are evolved in time and various tricks that we can play with it, including measurements of topological properties such as modular matrices, and thus potentially applied for identifying phases of matter in future simulations.

NMR simulations of topological phases

Nature Physics
volume 14, 160–165 (2018)

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30th Nov, 2019 Perimeter Institute

Collaborators

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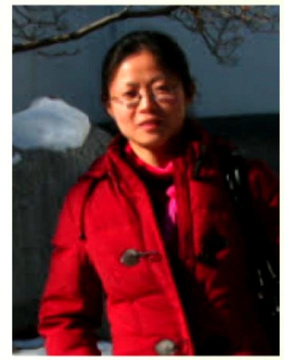
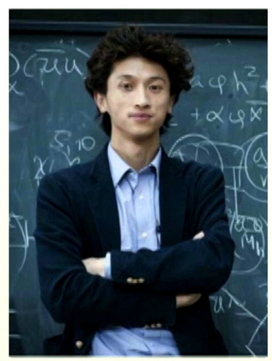


- Fudan Theory
- Yidun Wan



Overview

- 1) Brief overview of quantum simulator – NMR simulator
- 2) What do we want to simulate? Topological order

-- some introduction of topological phases of matter and the toric code model

- 3) How do we do that?
- 4) What lies ahead

Quantum Simulations

Simulating Physics with Computers

Richard P. Feynman

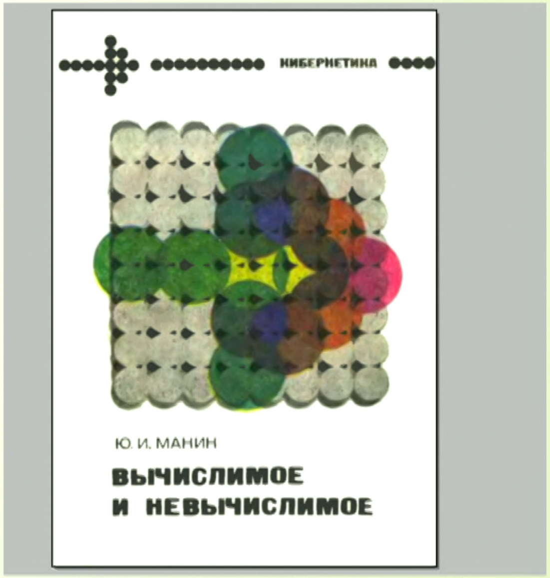
Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I

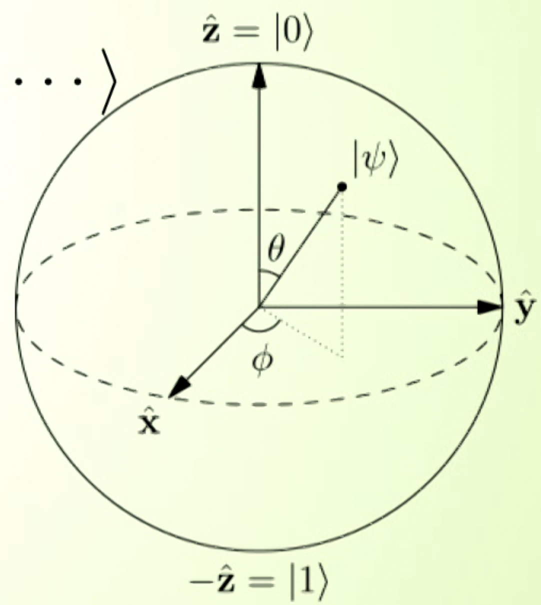
Yuri Ivanovitch Manin



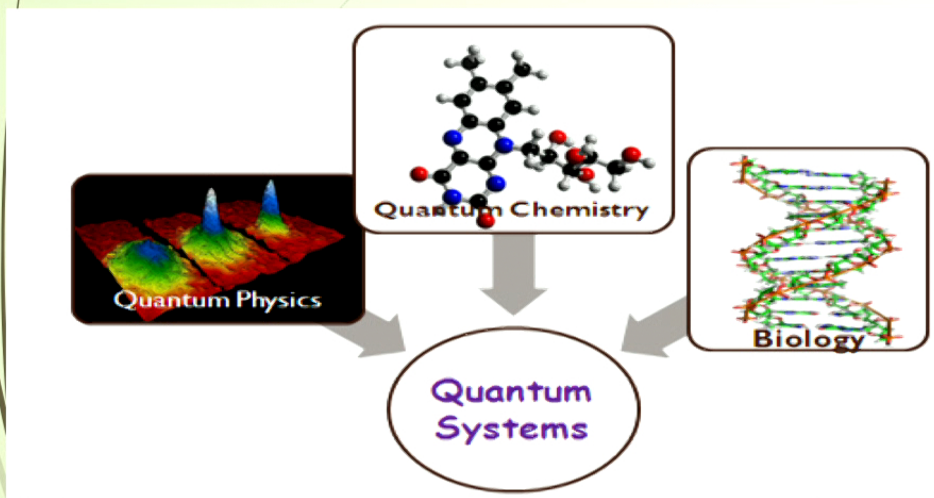
Computable and Noncomputable

Bits vs Qubits

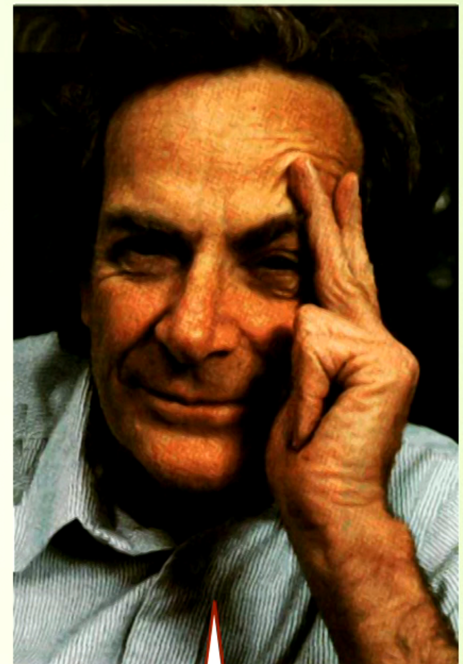
- Bits: 01010001010101.....
- Qubits $|\Psi\rangle = \sum_{i_1,i_2\cdots} a_{i_1,i_2,\cdots}|i_1, i_2, \cdots\rangle$
- N Qubits require 2^N complex coefficients
- Logic gates == Unitary operations acts on both states at the same time <= Quantum parallelism



Quantum Simulator



Picture courtesy Prof. Xinhua Peng



with it, with quantum-mechanical rules). For example, the spin waves in a spin lattice imitating Bose-particles in the field theory. I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. But I don't know

Quantum Simulator

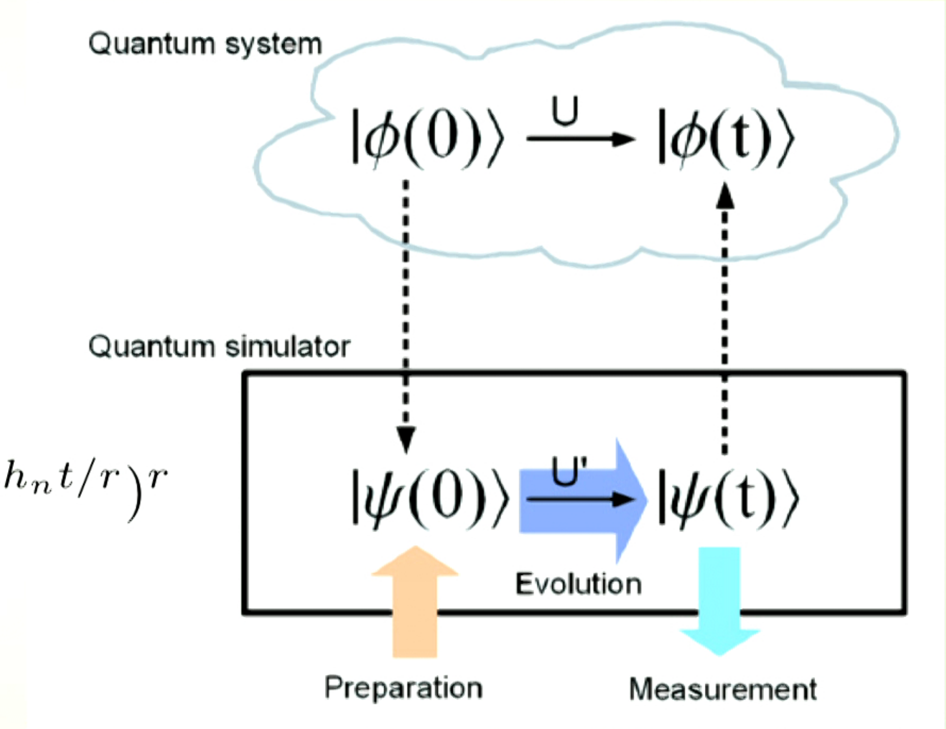
- 1) mapping
- 2) Hamiltonian engineering
- Lloyd's method :
- Quantum gates implemented

by sequence of Hamiltonian

(Average Hamiltonian theory)

$$H = \sum_i^n h_i \qquad e^{iHt} = (e^{ih_1 t/r} \cdots e^{ih_n t/r})^r$$

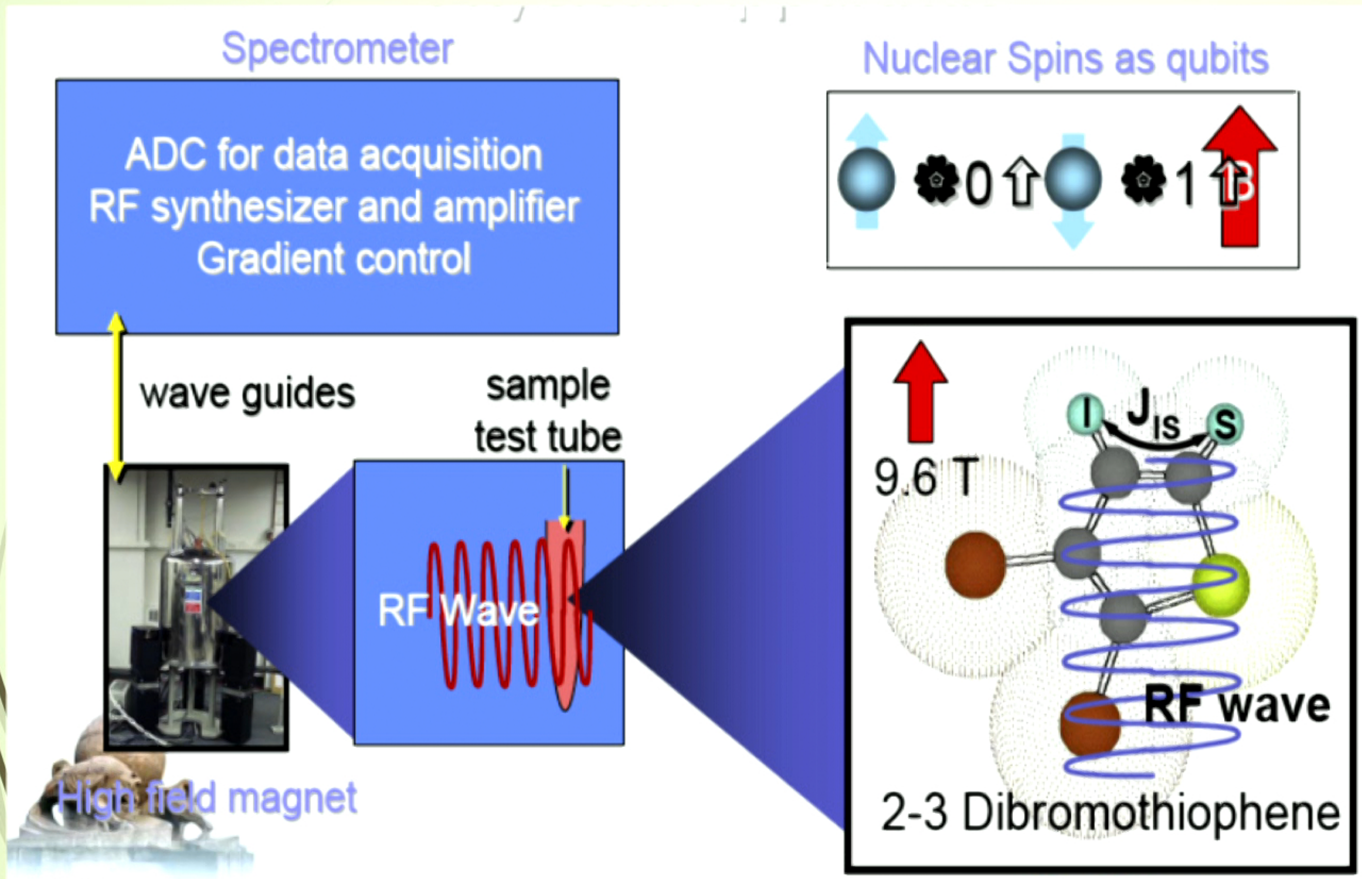
- Measurement



Picture courtesy Prof. Xinhua Peng

- I. M. Georgescu et al., Rev. Mod. Phys., Vol. 86, No. 1, January–March 2014

NMR Quantum Simulator



Picture courtesy Prof. Xinhua Peng

How (liquid)NMR Quantum computation works

- Operates at room temperature and pressure. Long coherence time ~ seconds
- (upto ~ 1000 pulses in an experiment).
- different molecules have slightly different energy levels and so allow for suitable choice of pulses to control them individually
- The system operating at room temperature is in Pseudo pure state
  Cory, Fahmy, Havel
  Proc Natl Acad Sci U S A. 1997 Mar 4; 94(5): 1634–1639.

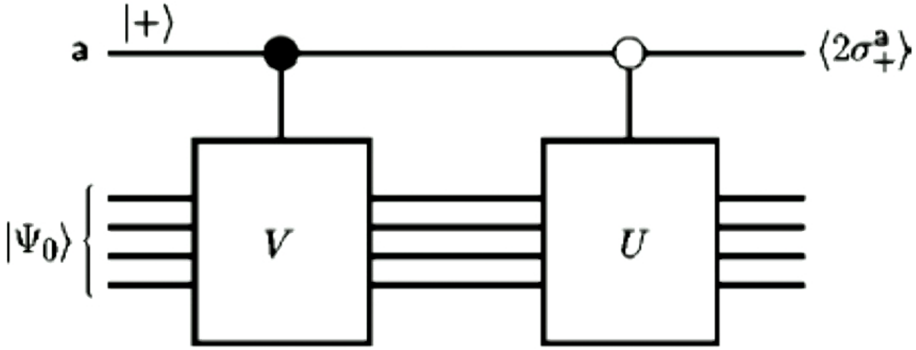
$$\Psi = \frac{(1-\alpha)\mathbf{1} + 2\alpha|\psi\rangle\langle\psi|}{(1-\alpha)2^n + 2\alpha} \quad (-1 \leq \alpha \leq 1),$$

- Ensemble computing: measure small magnetization can detect occupation and allows one to measure

$$\mathrm{Tr}(K\Psi) = (1-\alpha)\mathrm{Tr}(K) + 2\alpha\langle\psi|K|\psi\rangle$$

- Parallel computation without wavefunction collapse.

Measurement



$$2\sigma_+^{a} = \sigma_x^{a} + i\sigma_y^{a}$$

$$\tilde{V} = |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes V$$

$$\tilde{U} = |0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes \mathbb{1}.$$

$$\langle 2\sigma_+^{a}\rangle = \langle \Psi_0 | U^\dagger V | \Psi_0\rangle$$

|+> = 1/Sqrt[2](|0> + |1>)

R. Somma et al.,Phys. Rev. A, 65, 042323, (2002)

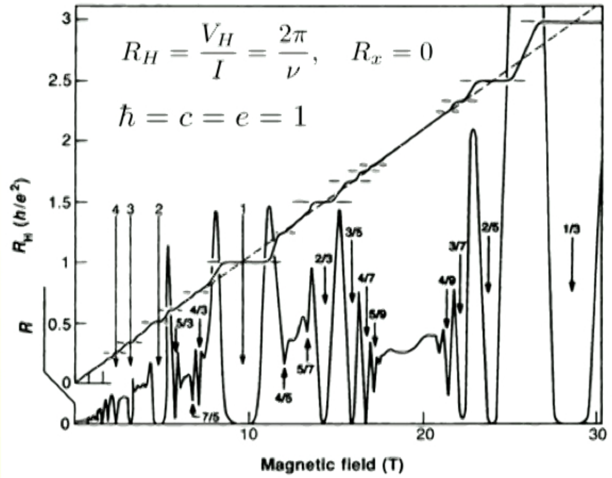
Topological Phases of Matter

Gapped phases of matter beyond the Landau Paradigm

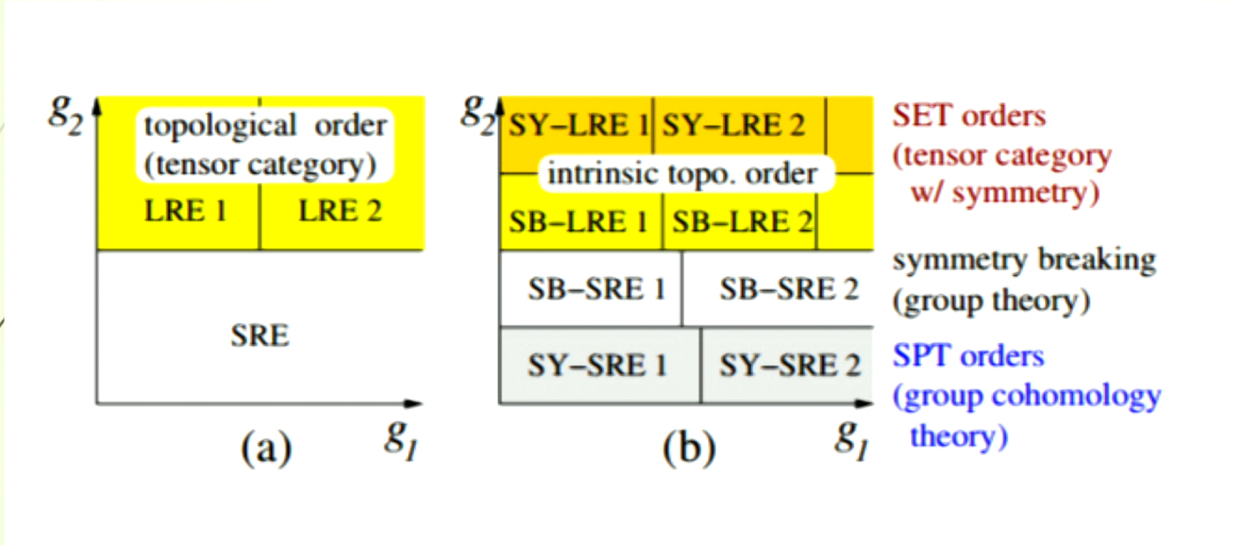
Well known example: Fractional quantum hall states

Locally indistinguishable degenerate ground state depending only on topology

iStockphoto.com/buyit



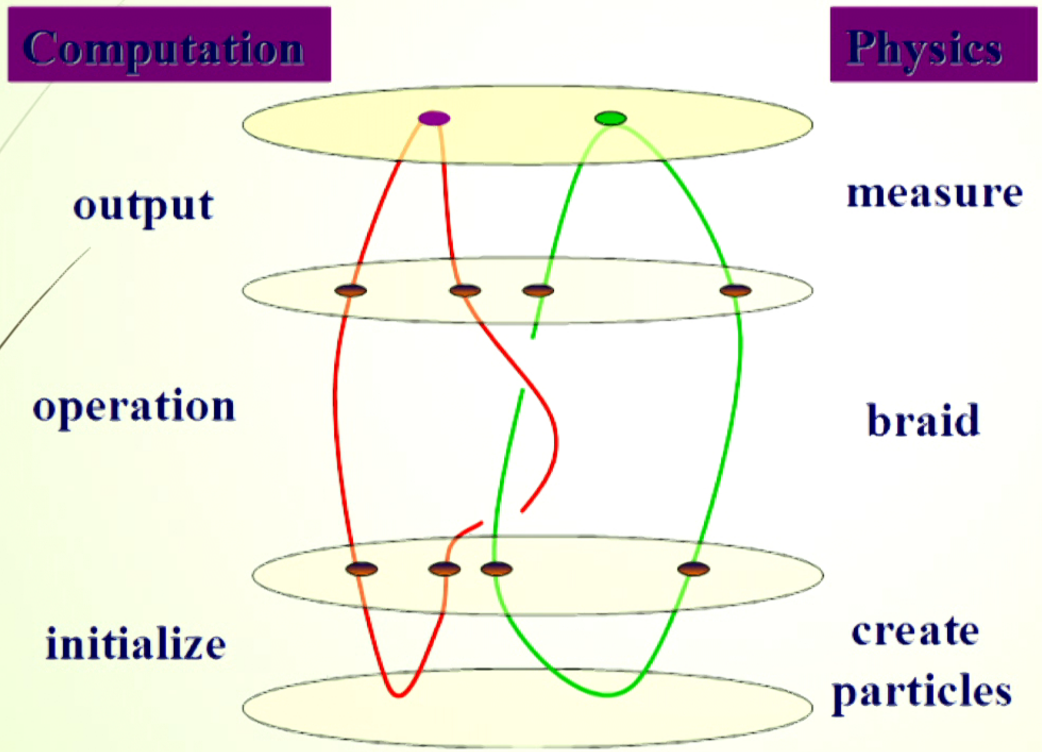
Phases of matter



Applications of topological phases

- Topological order involves Long Range Entanglement
- Degeneracy protected against Local perturbations
  (energy separation ~ Exp(-L))
- In 2+1 d there are anyons with non-trivial statistics
- Use it for an error correcting code! GS subspace = code subspace
- Message encoded in topological charge which cannot be corrupted by local interactions => errors
- (in 2+1 d e.g.) Braiding of anyons to realize logic gates – robust against decoherence since they arises from local interactions with the environment

Braiding and Quantum Computation

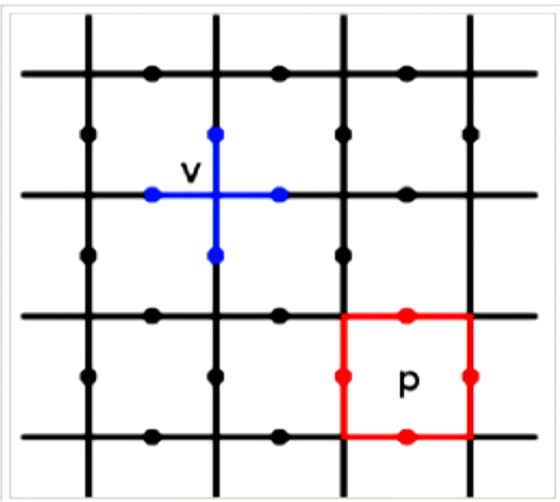


Prototypical Example: Toric code

$$H_{\text{toric}} = -\sum_s A_s - \sum_p B_p$$

$$A_s = \sigma^x_{sa}\sigma^x_{sb}\sigma^x_{sc}\sigma^x_{sl}$$

$$B_p = \sigma^z_{ij}\sigma^z_{jk}\sigma^z_{kl}\sigma^z_{li}$$

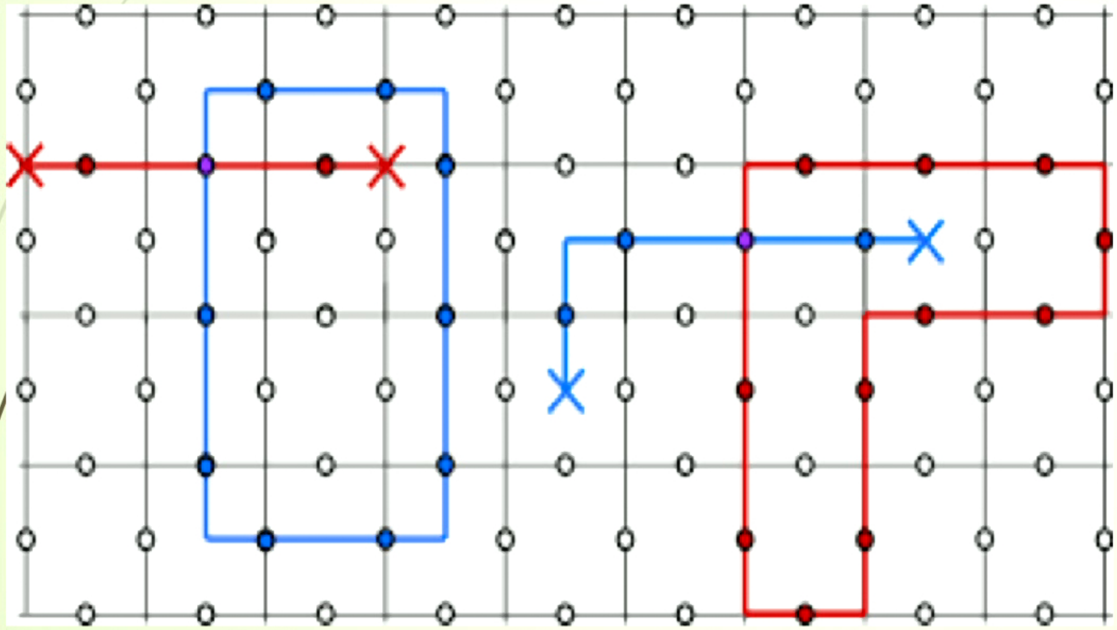


A. Kitaev, Ann. Phys. 303, 2 (2003) ; X. G. Wen, PRL. 90, 016803 (2003)

Excitations = anyons

4 types of anyons

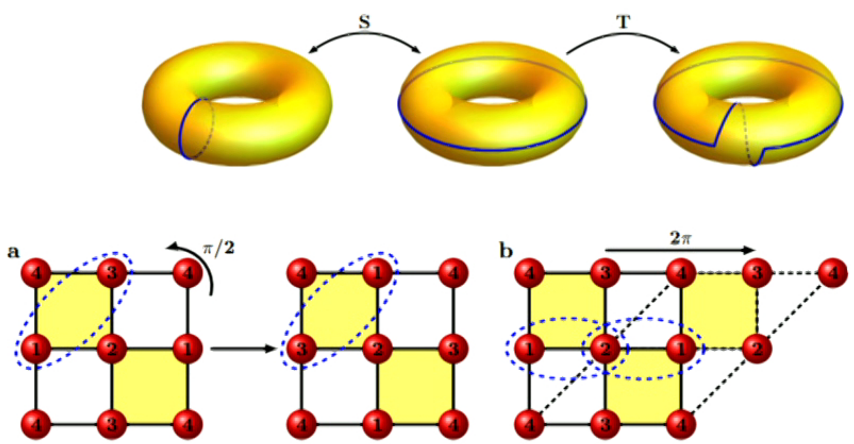
W_e=∏σz, W_m=∏σx.



Courtesy: topocondmat.org

Degenerate states on a torus characterized by global winding numbers of these Wilson loops

Modular Matrices



$$S_{\text{standard}} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}, \quad T_{\text{standard}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Modular matrices of the toric code
S and T recovers statistics of the anyons

A lot of numerical investigations:

[1] H.-C. Jiang et al., Identifying topological order by entanglement entropy. Nat Phys, 8, 902–905 (2012).
[2] Y. Zhang et al., Quasiparticle statistics and braiding from ground-state entanglement. Phys. Rev. B, 85:235151 (2012).
[3] M. P. Zaletel et al., Topological characterization of fractional quantum hall ground states from microscopic hamiltonians. Phys. Rev. Lett., 110:236801 (2013).
[4] P. Bonderson et al. Probing non-abelian statistics with quasiparticle interferometry. Phys. Rev. Lett., 97:016401 (2006).
[5] L. Cincio & G. Vidal. Characterizing topological order by studying the ground states on an infinite cylinder. Phys. Rev. Lett., 110:06720, (2013).
[6]H. He et al. Modular matrices as topological order parameter by a gauge-symmetry-preserved tensor renormalization approach. Phys. Rev. B, 90:205114 (2014).
[7] Fangzhou Liu et al., Modular transformations and topological orders in two dimensions. arXiv: 1303.0829v2 (2013)
[8] Jacob C. Bridgeman et al., Detecting Topological Order with Ribbon Operators. arXiv:1603.02275v3 (2016)
……

How much physics of the Z2 order can the simulator recover?

- Strategy: Hamiltonian: $H = H_{\text{toric}} + H_z + H_{\text{disorder}}$

$$H_z = h \sum_i \sigma_i^z \qquad H_{\text{disorder}} = \sum_i \epsilon_i \sigma_i^z$$

- Z2 order for small h.
- should display phase transition for sufficiently large h

- We would like to detect them via the experiment.

1. State preparation

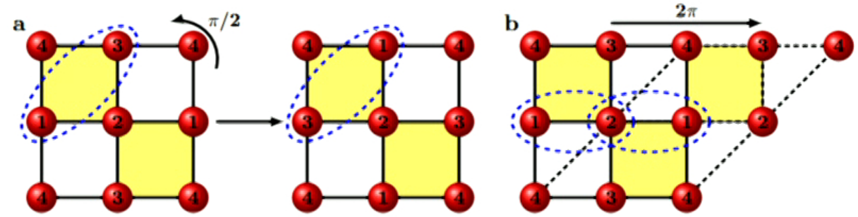
- 4- fold degenerate ground state: we apply the random adiabatic method

$$H_{\text{adiabatic}}(s) = sH + (1-s)H_{\text{random}}$$

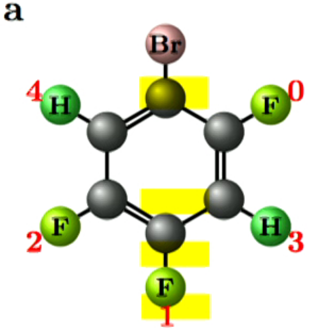
$$H_{\text{random}} = \sum \alpha_j^i \sigma_j^i$$

- $|\psi_i\rangle, \qquad i \in \{1,2,3,4\}$

2. Implement SWAP operation

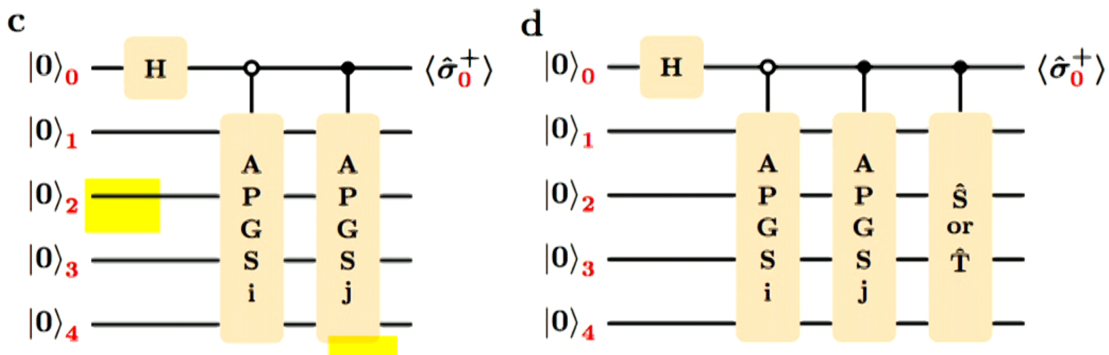


a



b

	0	1	2	3			
0	-37342						
1	334.8	-44793				40	0.8
2	114.0	-51.4	-47243			40	0.8
3	1875.9	1523.4	64.0	2701		100	1.5
4	62.8	127.4	1849.6	-11.8	2691	100	1.5



3. Recovery of the modular matrices

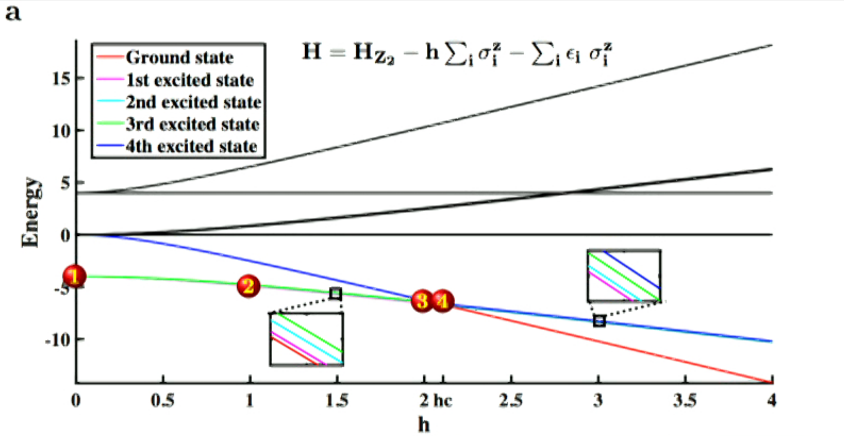
- First, obtain orthonormal basis $|\phi_i\rangle_n$ from $|\psi_i\rangle,\qquad i \in \{1,2,3,4\}$
- Then, look for standard anionic basis by writing

$$|a\rangle = \sum_i \alpha_i^a |\phi_i\rangle_n$$

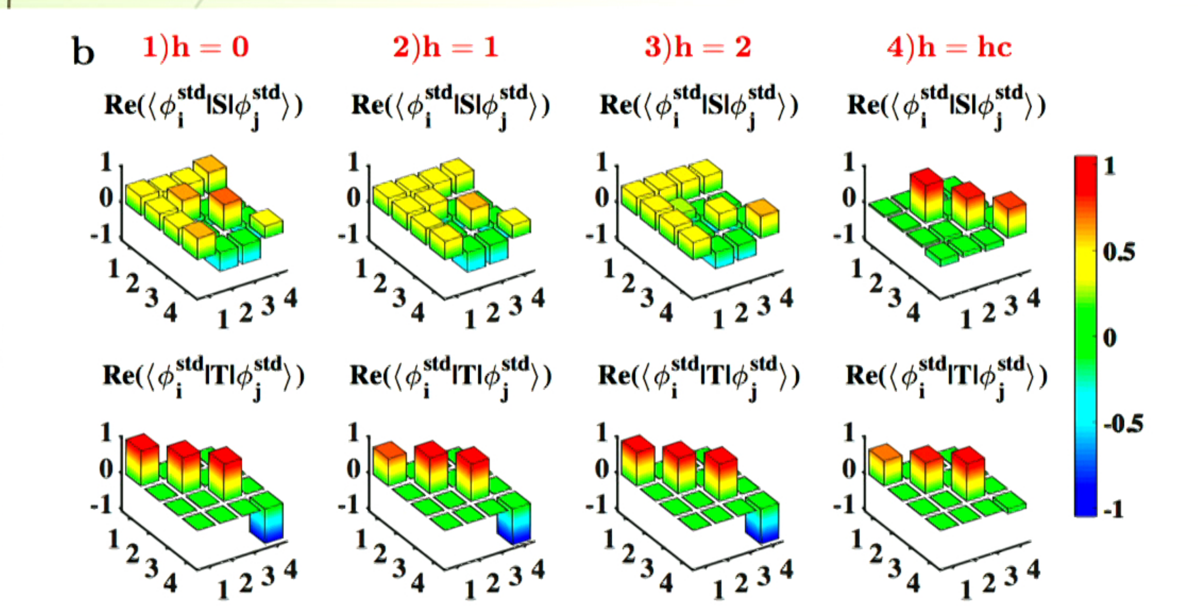
- Do optimisation of the constraints $\langle a|b\rangle = \delta_{ab}$
- Hermiticity of the modular matrices

$$T_{ab} = \exp(i\Theta_a)\delta_{ab} \qquad S_{a1} = 0$$

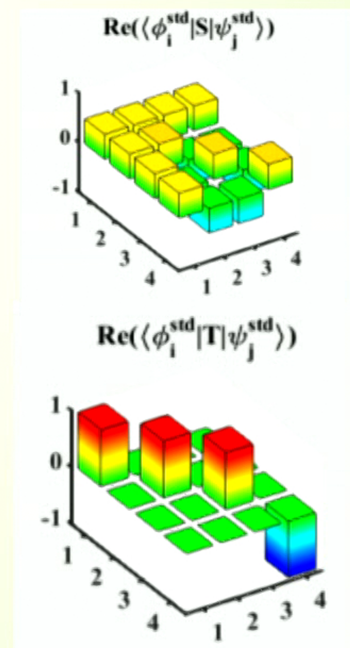
Results:



a
$H = H_{Z_2} - h\sum_i \sigma_i^z - \sum_i \epsilon_i \sigma_i^z$
Ground state
1st excited state
2nd excited state
3rd excited state
4th excited state
Energy
h



b
1)h = 0
2)h = 1
3)h = 2
4)h = hc
$Re(\langle \phi_i^{std}|S|\phi_j^{std}\rangle)$
$Re(\langle \phi_i^{std}|T|\phi_j^{std}\rangle)$



$Re(\langle \phi_i^{std}|S|\psi_j^{std}\rangle)$
$Re(\langle \phi_i^{std}|T|\psi_j^{std}\rangle)$

Summary

- We use minimal theoretical input to extract phase structure of the Z2 topological order via measurements of the modular matrices. It has mild requirements on symmetries.
- Our experiment is the first to measure modular matrices, also the first to get rid of knowledge of string operators, and the first to move away from the exactly solvable point.
- Bigger systems? (Complexity of our method only grows polynomially)
- More general topological orders? Braiding operations?

Thank you !