Title: Dimensionally Restricted Causal Sets

Speakers: William Cunningham

Series: Quantum Gravity

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Abstract: We study dimensionally restricted non-perturbative causal set quantum dynamics in two and three spacetime dimensions with non-trivial global spatial topology. The causal set sample space is generated from causal embeddings into latticisations of flat background spacetimes with global spatial topology and in two and three dimensions, respectively. The quantum gravity partition function over these sample spaces is studied using Markov Chain Monte Carlo (MCMC) simulations via an analytic continuation of a parameter analogous to an inverse temperature. In both two and three dimensions we find a phase transition that separates the dominance of the action from that of the entropy. The action dominated phase is characterised by "layered" posets with a high degree of connectivity, while the causal sets in the entropy dominated phase are manifold-like. These results are similar in character to those obtained for topologically trivial causal set dynamics over the sample space of 2-orders. The current simulations use a newly developed framework for causal set MCMC calculations, and provide the first implementation of a three-dimensional causal set dynamics.

arXiv paper: https://arxiv.org/abs/1908.11647

Dimensionally Restricted Causal Sets

Will Cunningham
Perimeter Institute for Theoretical Physics

arXiv:1908.11647
In collaboration with S. Surya

31 October 2019
Quantum Gravity Seminar

from DISCRETUUM to CONTINUUM

compute canada

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Overview

Causal Set Theory: What, Why, How?

- Definition of the CST model
- Where it fits in to other QG approaches
- Statistical physics of causal sets

Restricted Sample Spaces

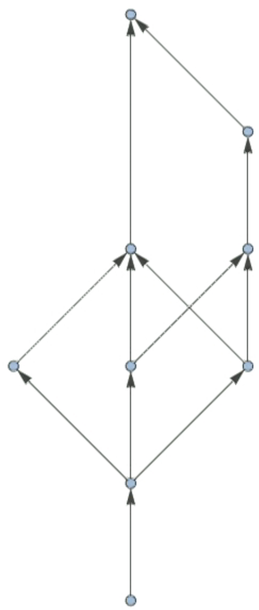
- 2D and 3D lattices
- Monte Carlo moves
- Typical configurations

Dynamics in 2D and 3D

- Phase structure
- Observables
- Future work

What is a causal set?

A causal set $\mathcal{C}_n$ is a set of n unlabeled elements $\{e_1, \ldots, e_n\}$ endowed with an irreflexive partial order relation $\prec$.



Structures are *partial orders* which are...

Acyclic: $e_i \prec e_j \Rightarrow e_j \nprec e_i$

Transitive: $e_i \prec e_k \wedge e_k \prec e_j \Rightarrow e_i \prec e_j$

Locally Finite: $|I(e_i, e_j)| \equiv |\mathrm{Fut}(e_i) \cap \mathrm{Past}(e_j)| < \infty$

Causal Sets: "Order + Number = Geometry"

Order $\sim$ Causality: $e_i \prec e_j$

Number $\sim$ Volume: $|\mathcal{C}_n| = n$

Fundamental Discreteness Scale: $\ell \sim n^{-1/d}$

Bombelli et al. '87
Surya '19


Reebok

Why Causal Sets?

Key concepts in CST are...

1. Minimalist QG: causality, discreteness
2. Spacetime is fundamentally discrete at the Planck scale
3. Diffeomorphism symmetry is realized via invariance under automorphisms

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- Key concepts in CST are...
  1. Minimalist QG: causality, discreteness
  2. Spacetime is fundamentally discrete at the Planck scale
  3. Diffeomorphism symmetry is realized via invariance under automorphisms
  4. Topology can be recovered from causal structure alone
  5. Kinematic observables recoverable for large n: dimension, distance, spatial homology, d'Alembertian, scalar curvature, boundary geometry
     a. Myrheim '78, Meyer '88, Reid '03, Glaser et al. '13, Aghili et al. '18
     b. Myrheim '78, Brightwell et al. '91, Rideout et al. '09, Roy '13, Eichhorn et al. '18
     c. Major et al. '07, '09
     d. Sorkin '07, Henson '10, Benincasa et al. '10, Dowker et al. '13, Aslanbeigi et al. '14
     e. Benincasa et al. '10, Dowker et al. '13, Belenchia et al. '16, Cunningham et al. '18
     f. Benincasa et al. '11, Buck et al. '15, Jubb et al. '17, Cunningham '18, Cunningham et al. w.i.p.

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  5. Kinematic observables recoverable for large n: dimension, distance, spatial homology, d'Alembertian, scalar curvature, boundary geometry
  6. Dynamics are determined by the measure in a discretized path integral formulation, similar to EDT/CDT

$$\int \mu(g_{\mu\nu})\mathcal{D}[g_{\mu\nu}] \xrightarrow{\mathcal{M}\to\Omega_n} \sum_{\mathcal{C}\in\Omega_n} \mu(\mathcal{C})$$

Brightwell et al. '08, Surya '12, Glaser et al. '18, Cunningham et al. w.i.p.

Statistical Physics of CST

Where is continuum physics? We must choose a suitable $\mu(\mathcal{C})$ and possibly restrict Ω_n.

$$\int \mu(g_{\mu\nu})\mathcal{D}[g_{\mu\nu}] \xrightarrow{\mathcal{M}\to\Omega_n} \sum_{\mathcal{C}\in\Omega_n} \mu(\mathcal{C})$$

We analytically continue a quantum partition function:

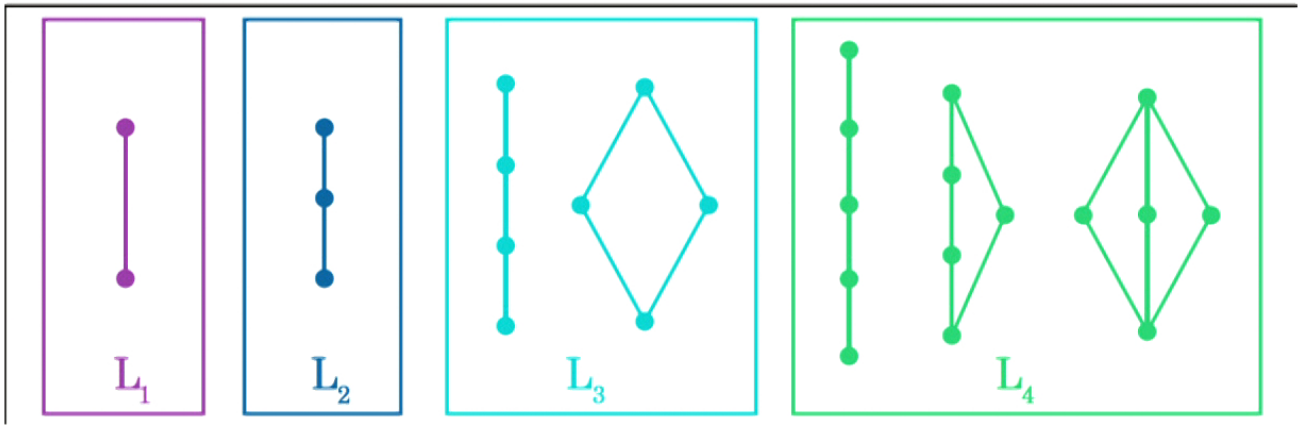
$$Z_n(\beta) = \sum_{\mathcal{C}\in\Omega_n} e^{i\beta S(\mathcal{C})/\hbar} \to \mathcal{Z}_n(\beta) = \sum_{\mathcal{C}\in\Omega_n} e^{-\beta S(\mathcal{C})}$$

We now have an additional *inverse temperature* parameter β, and we need to choose the *action* $S(\mathcal{C})$.

N.B.: We can consider a fully quantum treatment by using the quantum measure via the decoherence functional w.i.p.

The Benincasa-Dowker Action

- The Benincasa-Dowker action for CST is a function of the abundance of primitive subsets called *order intervals*.



$$S^2_{BD}(\mathcal{C}) = 2(n - 2n_1 + 4n_2 - 2n_3)$$

$$S^3_{BD}(\mathcal{C}) = \frac{1}{\Gamma(5/3)}\left(\frac{\pi}{3\sqrt{2}}\right)^{2/3}\left(n - n_1 + \frac{27}{8}n_2 - \frac{9}{4}n_3\right)$$

We use a variant called the *smeared action*, which introduces a mesoscale ε

Benincasa et al. '10, Dowker et al. '13, Glaser '14

Restricted Sample Spaces

- Motivations to restrict dimension of $\mathcal{C} \in \Omega_n$
  1. The Benincasa-Dowker action makes us pick a dimension.
  2. The full sample space Ω_n is *super-exponential*: $|\Omega_n| = 2^{n^2/4}$

  The ensemble Ω_n is entropically dominated by *Kleitman-Rothschild* tri-layered orders and other crystalline orders

  Kleitman et al. '75, Dhar '78, '80

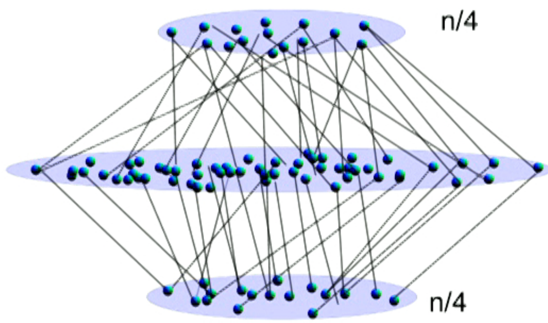


Image credit: Surya '19

Restricting to causal sets embeddable in $(\mathbb{M}^d, \eta_{\mu\nu})$ greatly reduces the sample space.

Can we estimate the size of the subspace $\Omega_n(\mathbb{M}^d, \eta_{\mu\nu})$?

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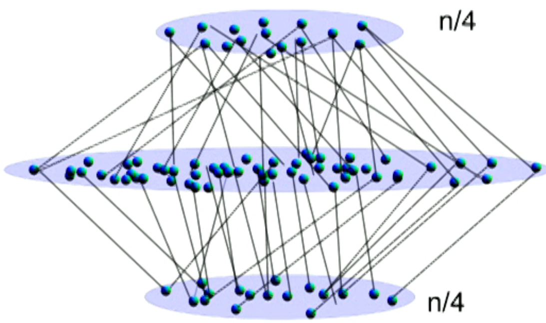

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Can we estimate the size of the subspace $\Omega_n(\mathbb{M}^d, \eta_{\mu\nu})$?

2d-Orders

- The 2d-orders correspond to topologically trivial 2d CST...
  $U = \{u_1, \ldots, u_n\}$ and $V = \{v_1, \ldots, v_n\}$ are *total orders*,
  $i \prec j$ iff $u_i < u_j \wedge v_i < v_j$ Brightwell et al. '08, Surya '12

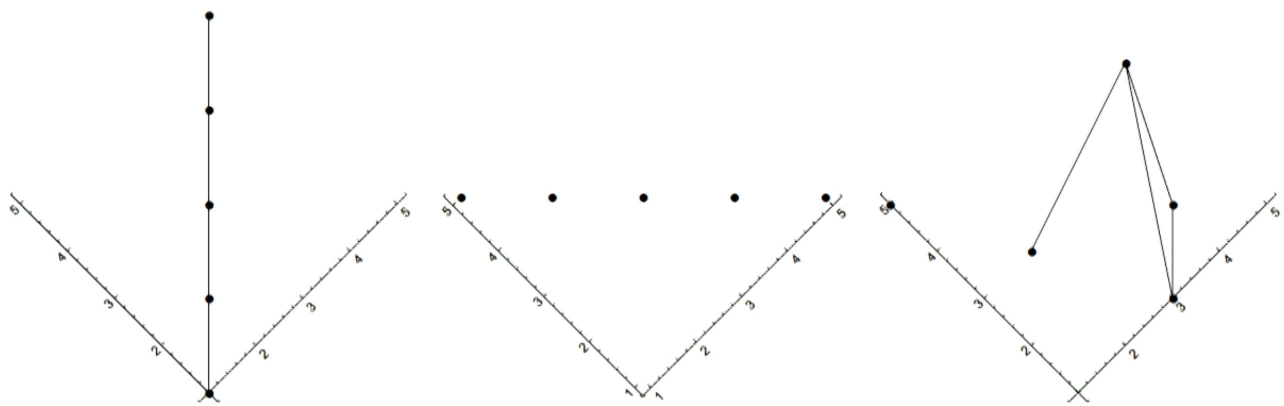


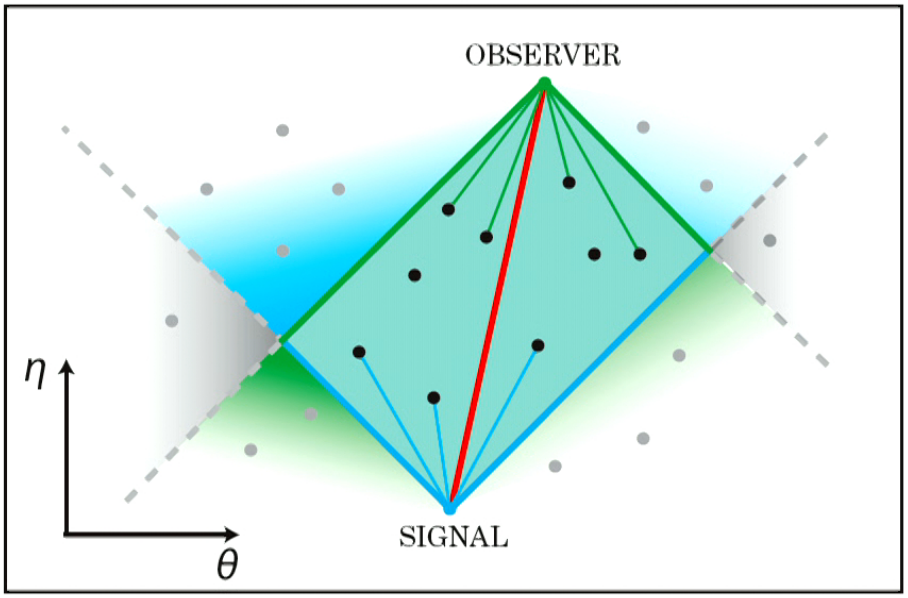
Image Credit: Brightwell et al. '08

$\Omega_n(D_2, \eta)$: All permutations of U and V

Entropically dominated by causal sets which approximate the 2D Minkowski diamond El-Zahar et al. '88, Winkler '91

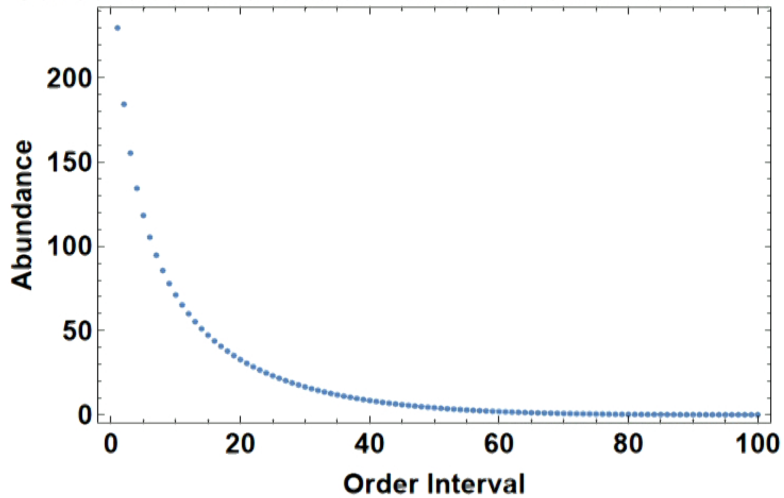
Continuum Approximations

- How do we know what a "manifold-like" causal set looks like?
  1. Poisson sprinkling into a compact spacetime region, use coordinates to identify timelike relations



Continuum Approximations

- How do we know what a "manifold-like" causal set looks like?
  1. Poisson sprinkling into a compact spacetime region, use coordinates to identify timelike relations
  2. Calculate the *order interval abundances*



Glaser et al. '13, '18

Claim: These curves are not unique to a particular manifold, but the form of them indicates manifold-likeness.

Higher Dimensions and Other Restrictions

The d-orders cannot be extended to $d > 2$

- The 3-orders form a cubic lattice
- The associated "light-cones" are cubic, with square rather than circular cross sections
- We may also be interested in $d = 2$ with non-trivial topology...

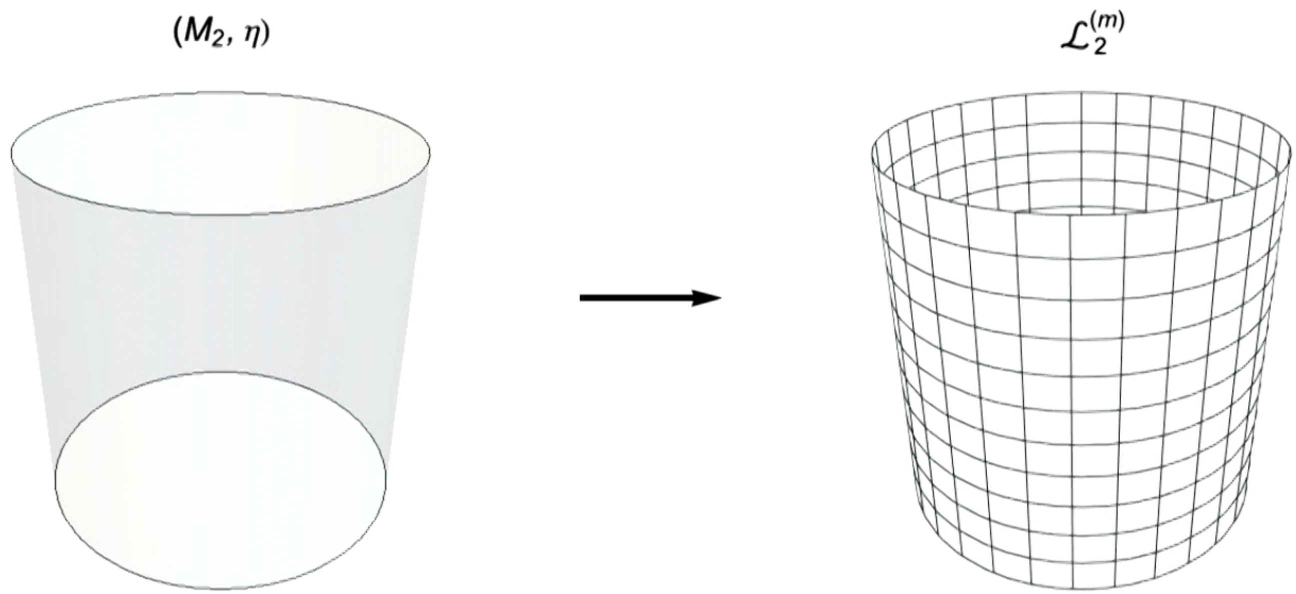
Is there another way to impose dimensional restriction for Minkowski spacetime?

Start with fixed geometry and topology, then allow it to vary

Cunningham & Surya w.i.p

Lattices without Spatial Boundary

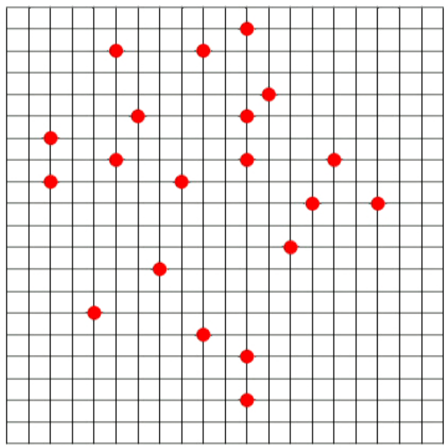
- Consider the latticisation of the 2D flat cylinder

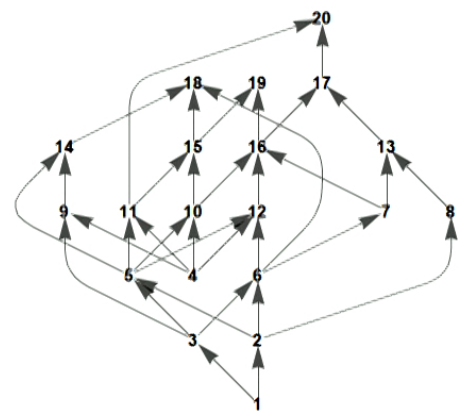


The lattice is parametrized by the total number of sites, $m = h \times w$, and the aspect ratio $\alpha = h/w$

A Lattice-Gas Model for CST

Lattice sites are either *occupied* or *unoccupied* by causal set elements, and coordinates determine causal relations

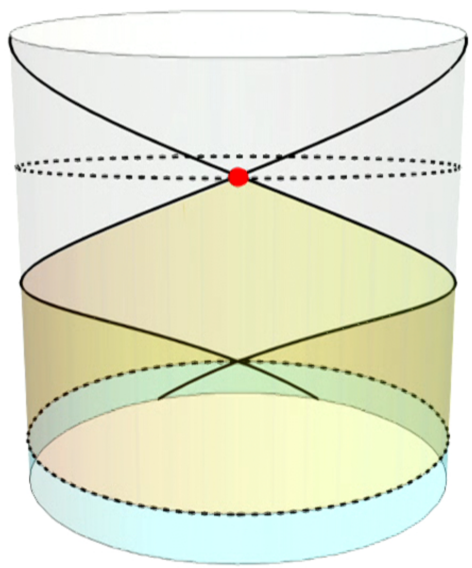




This forms the set of (m, n)-*orders* when we consider n elements on m sites: all $\mathcal{C}_n$ embeddable in M_2.

Non-Trivial Topology

- We consider $\alpha = 4$ to ensure light cones can wrap around the entire cylinder



Taking $\alpha < 1$ restricts us to trivial topology, and $\alpha \gg 1$ will too greatly constrain the allowed configurations

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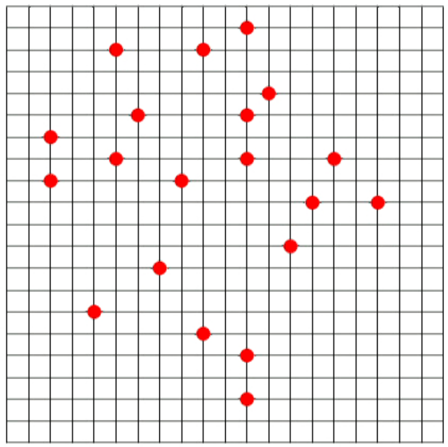

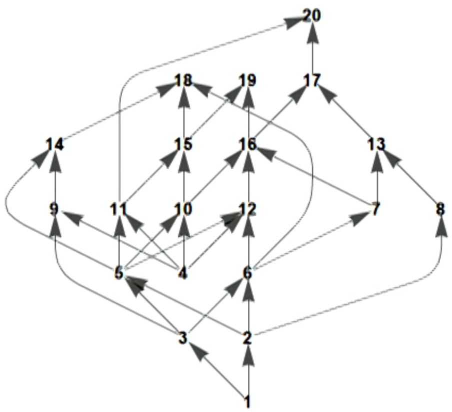

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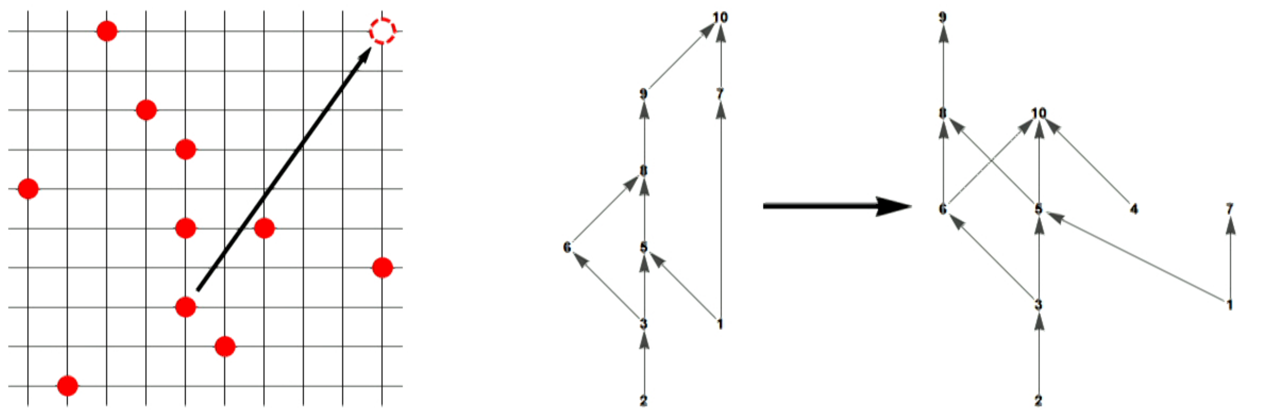
Monte Carlo Moves

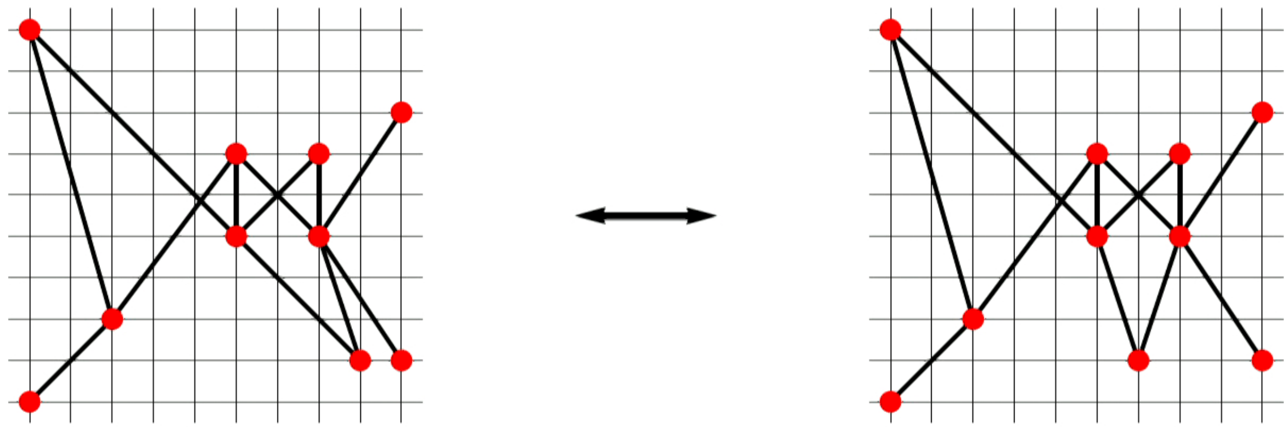
We can define a Markov chain using the *lattice-gas move*



Is this an efficient move?

Move Efficiency

Each causal set can be represented multiple times in $\Omega^{(m,n)}$:



This is less likely as $n \to m$, so we let $n = w$, or $m = \alpha n^{d/(d-1)}$ in experiments to follow

Observables

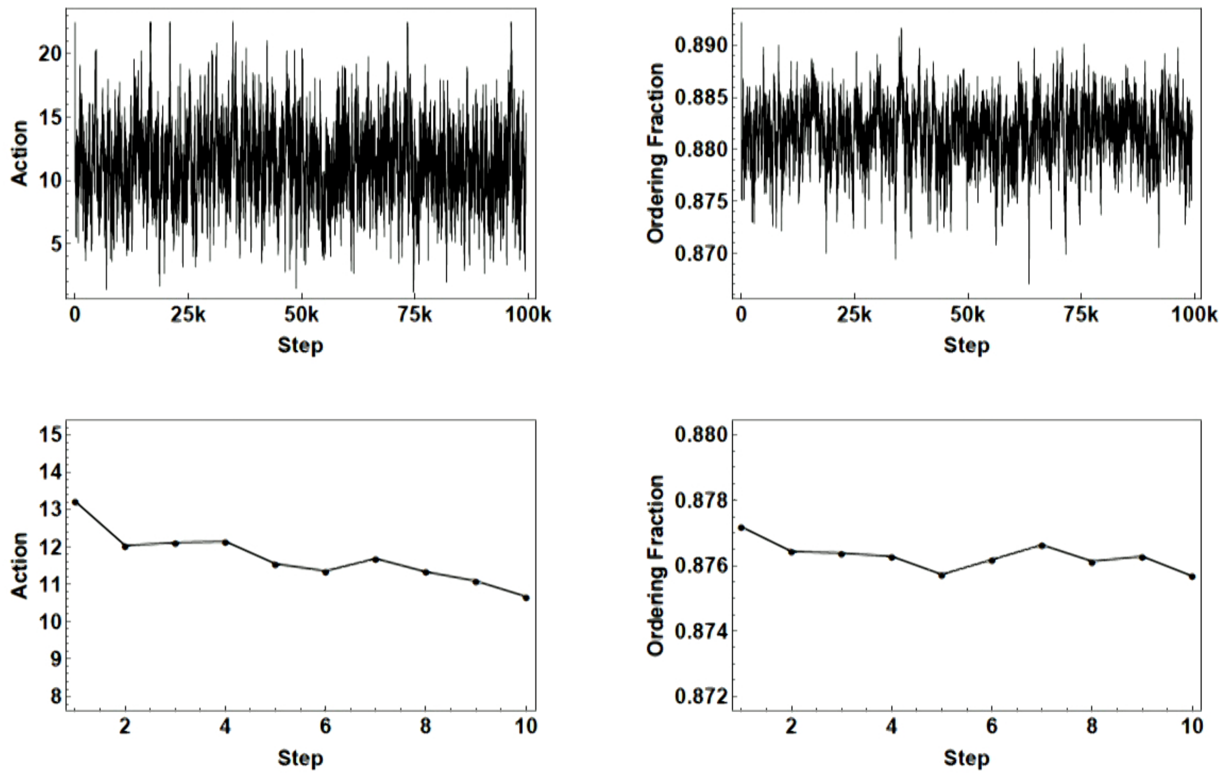
We can define several structural observables to measure:

1. **Action**: the Benincasa-Dowker action with smearing $\varepsilon = 0.1$
2. **Ordering Fraction**: (number of relations) / (total possible relations); $r_2 \approx 0.88$ and $r_3 \approx 0.815$ for $\alpha = 4$ sprinklings
3. **Height**: longest future-directed path in the causal set
4. **Number of Links**: irreducible relations, first term in the action

If the move is efficient, a new causal set is generated at each step, and the observables *should* change.

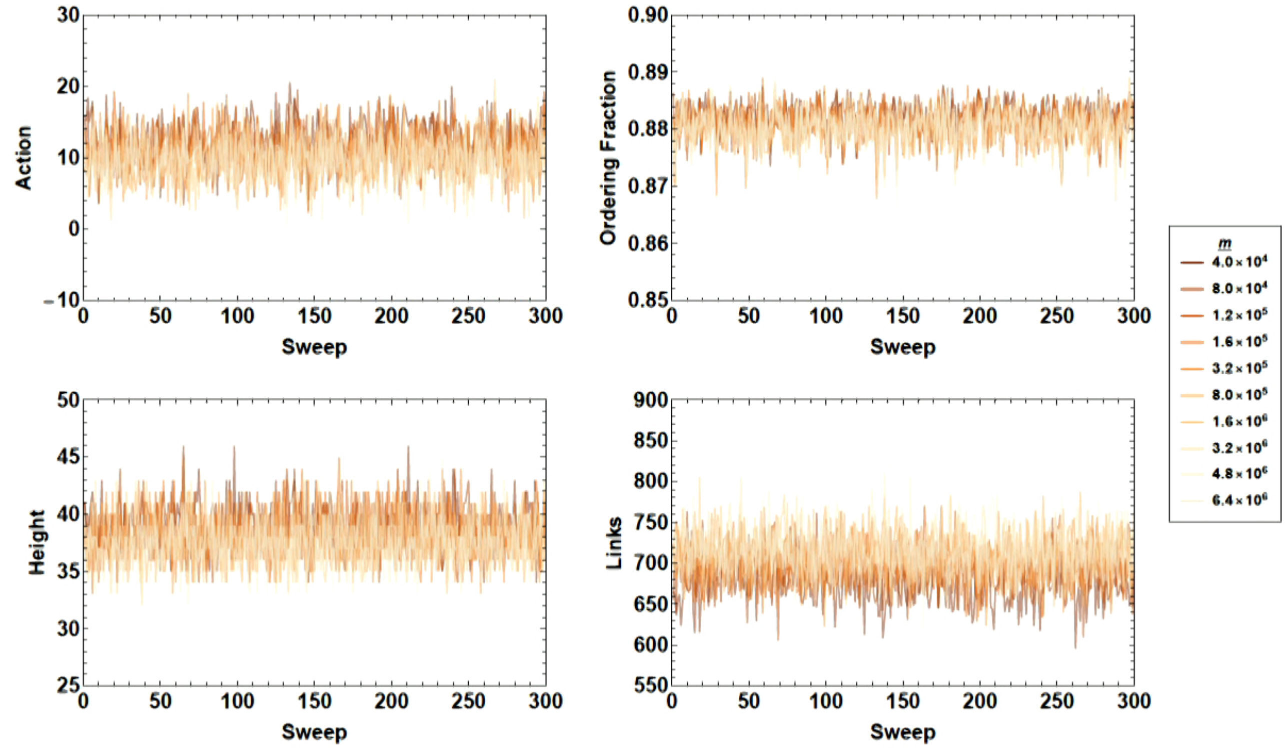
Observables change $\rightarrow$ causal set has changed

Move Efficiency ($\beta = 0$)



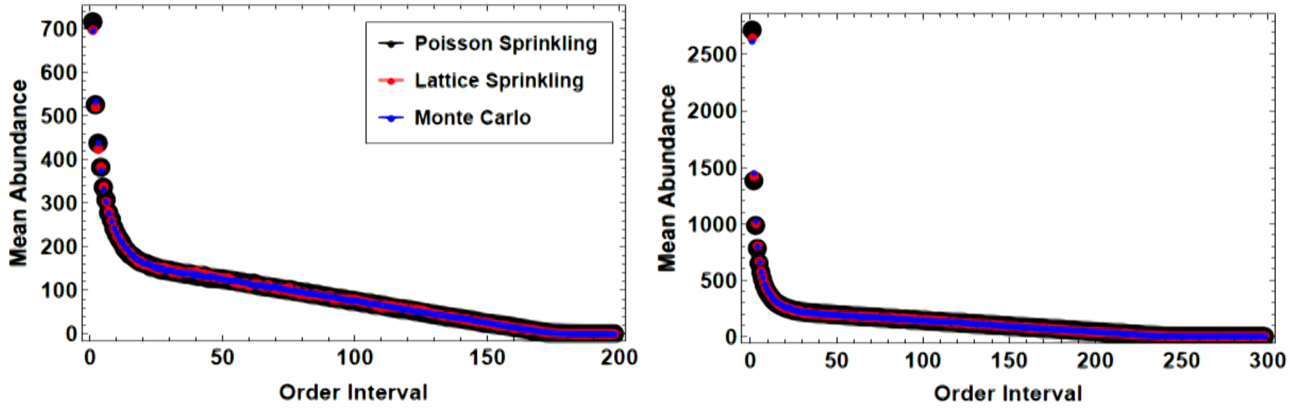
Dependence on the Lattice ($\beta = 0$)

- For large m and $n \ll m$, observables are independent of m



Comparison to Sprinklings

- A typical element of $\Omega^{(m,n)}$ has the same properties as a sprinkling into the continuum:



This gives strong evidence the ensemble is dominated by continuum-like causal sets, similar to the 2-orders. By restricting to a lattice, we can possibly analyze the relative sizes of the sample spaces using combinatorics Cunningham et al. w.i.p.

Simulation Details

General setup...

- We select $n = 200$ for $d = 2$ and $n = 300$ for $d = 3$
- Lattices: $\mathcal{L}_2 = 200 \times 800$ and $\mathcal{L}_3 = 17 \times 17 \times 70$
- Monte Carlo importance sampling is used to sample $\Omega^{(m,n)}$
- Generate a Markov chain from a random initial state:
  1. Move a random element to a random unoccupied site
  2. Topologically sort the causal matrix ($e_i \prec e_j \Rightarrow i < j$)
  3. Construct the causal matrix
  4. Calculate the action
  5. Accept/reject: Metropolis condition accepts an increase in action with probability $e^{-\beta \Delta S_{BD}}$
- Let $\varepsilon = 0.1$ in the action and measure observables while varying β to identify a phase transition

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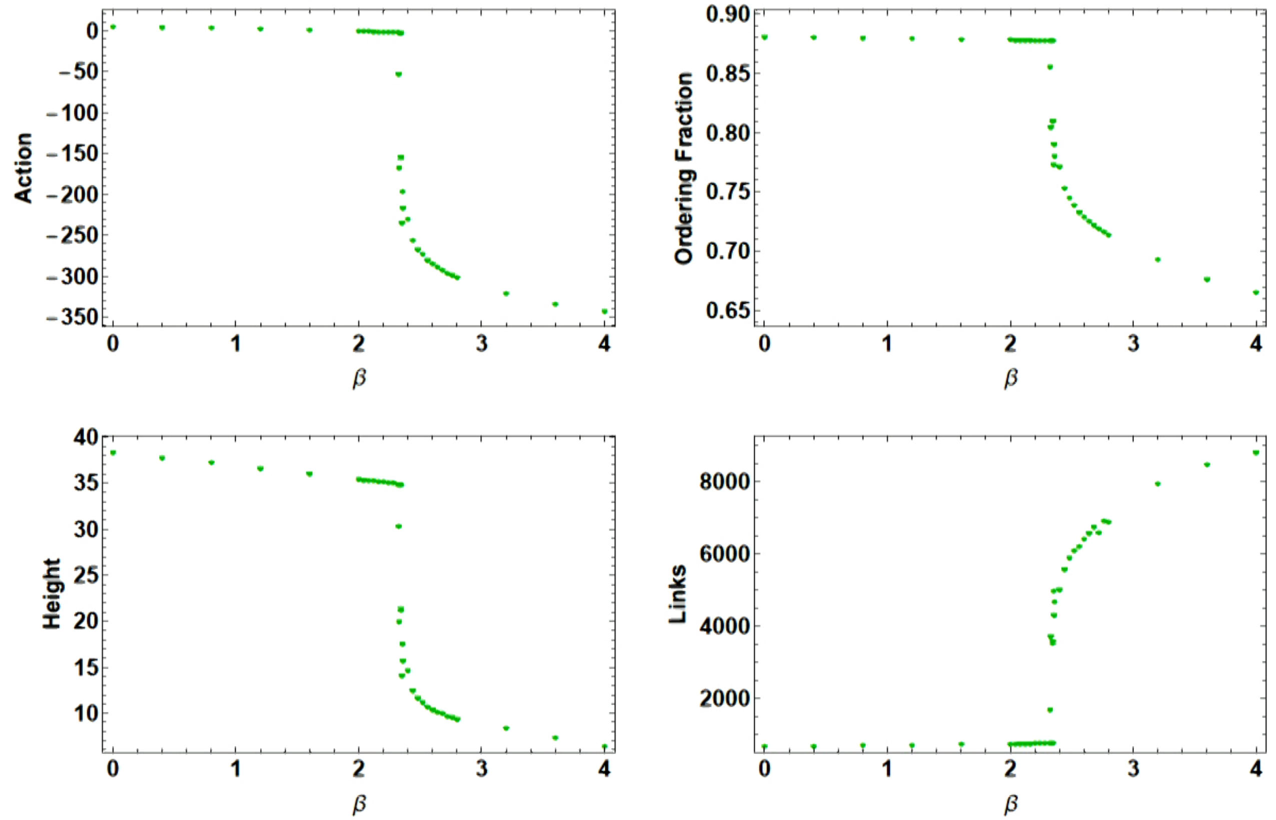
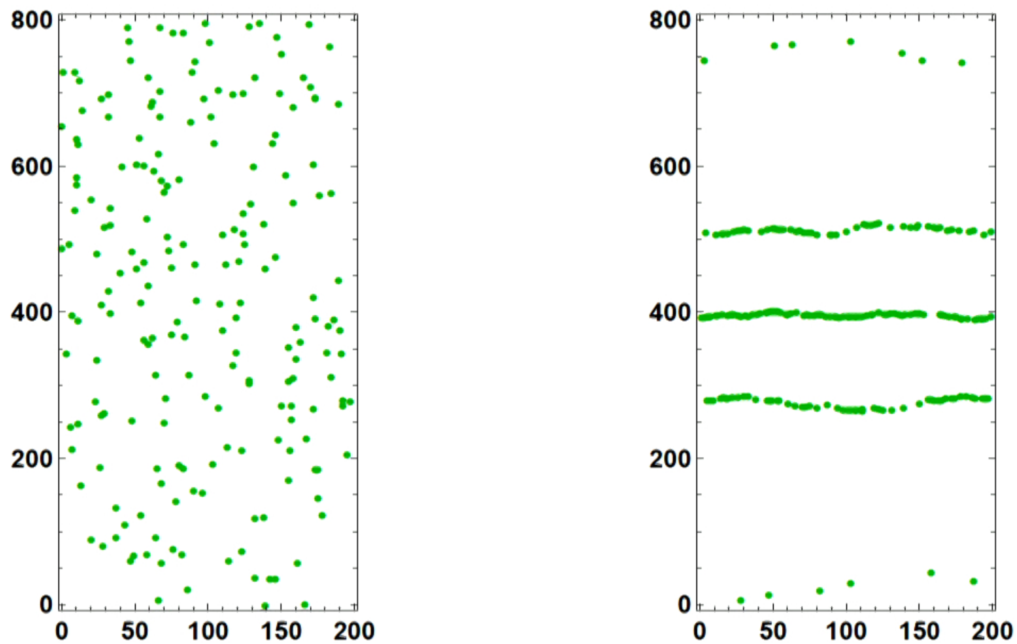
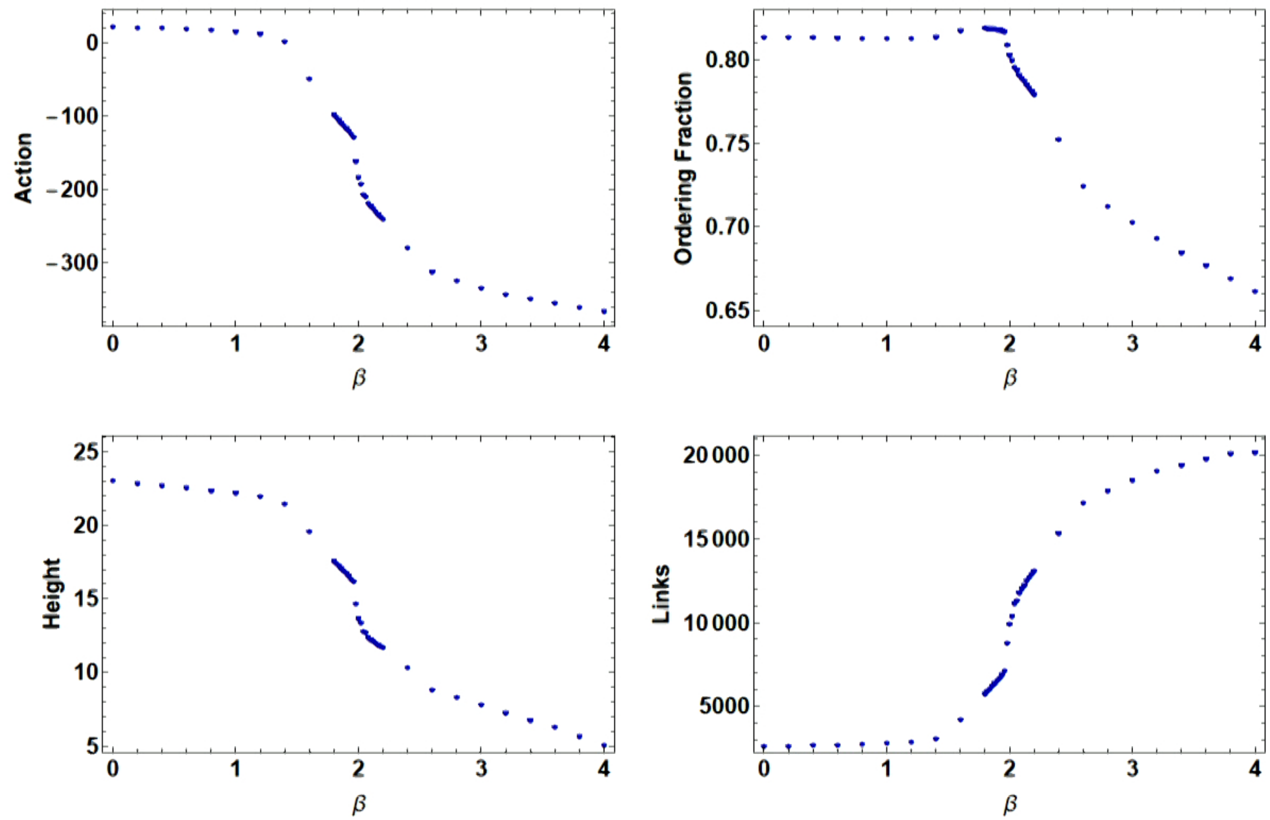
Phase Transitions ($d = 2$)



Disordered and Ordered Phases ($d = 2$)



Phase Transitions ($d = 3$)



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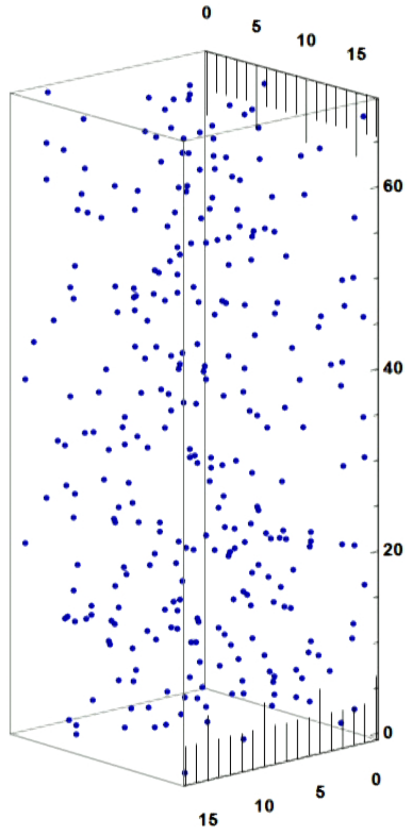
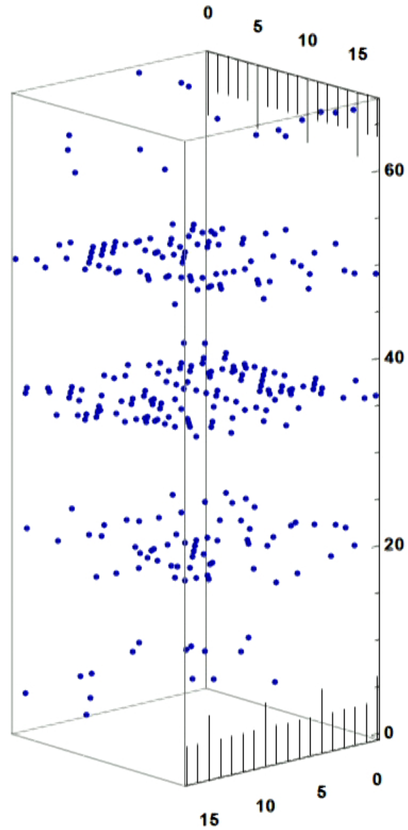
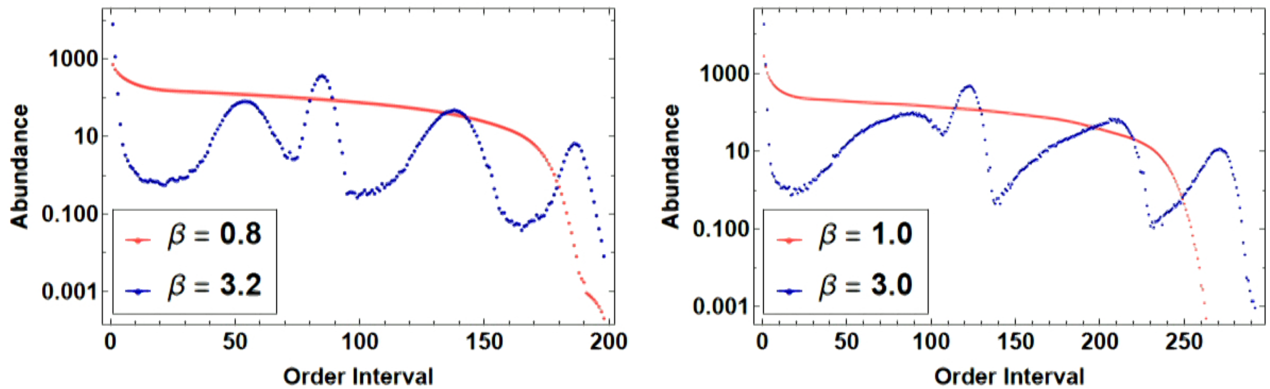


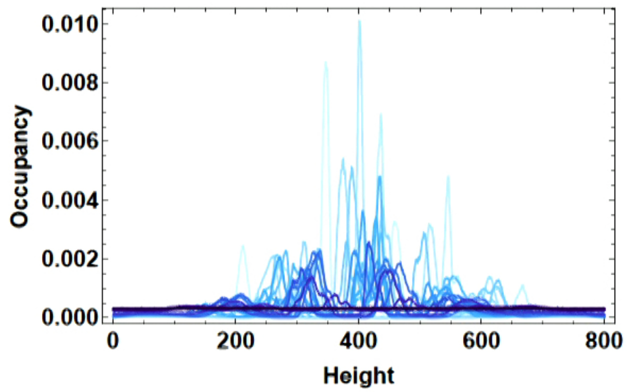


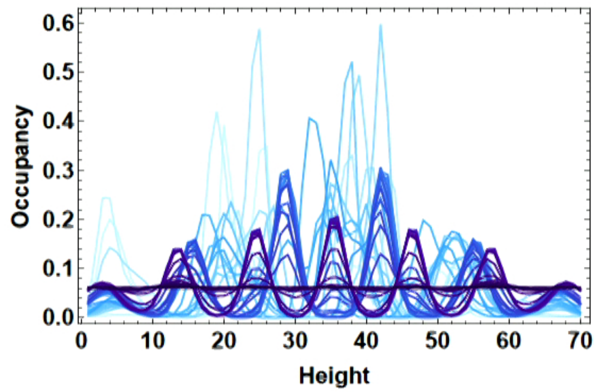
Order Interval Abundances



- For $0 < \beta < \beta_c$ we see an exponential cutoff
- For $\beta > \beta_c$, four peaks:
  - $\rightarrow$ first around $n/4$ (second/fourth layers)
  - $\rightarrow$ second just below $n/2$ (middle layer)
  - $\rightarrow$ third just below $3n/4$ (middle and adjacent layer)
  - $\rightarrow$ fourth close to n (middle three layers)

Layer Occupancy





Asymptotically we expect a bi-layer poset, as for the 2d-orders

Glaser et al. '18

Summary

- We have a dimensionally and topologically restricted partition function $\mathcal{Z}^{\mathbf{d}}_{m,n}(\beta)$ built on $\Omega_{m,n}$ and $S^{\mathbf{d}}_{BD}$
- For appropriate $n \ll m$ we achieve m-independence
- We observe what we believe to be a first order phase transition (pending further analysis)
- Two phases: entropy dominated (continuum-like) and action dominated (crystalline)
- Critical temperatures for these n: $\beta_c^{(2)} = 2.34(4)$ and $\beta_c^{(3)} = 1.98(0)$
- Much more to do...

Next Steps

Immediate:

- Finite size scaling analysis for this ensemble Cunningham, Glaser & Surya
- Variable geometry via non-uniform sampling Cunningham & Surya
- Variable topology - much easier on a lattice than in the continuum Cunningham & Surya
- Algorithmic improvements - replica exchange, histogram reweighting, GPU implementation

Some thoughts:

- Can we analytically calculate the sizes of these ensembles?
- Is it possible to study these ensembles using growth models?
- More natural choices for the action? Cunningham, Eichhorn & Surya

Thank you for listening!