Title: Central extension in gravity

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Series: Quantum Gravity

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Abstract: The asymptotic symmetry charge algebra of certain class of spacetimes could have a nontrivial central extension, which measures the non-equivariance of the charges of the large gauge transformations. The Cardy formula of the dual CFT has been famously used to derive black hole entropy. However, it remains obscure from the point of view of gravity why such a constant on the gravitational phase space could encode the information about the density of black hole micro-states, and what the degrees of freedom accounting for the black hole entropy truly are. I will discuss the ongoing efforts of understanding these questions in the covariant phase space formalism.

Central Extension in Gravity

with Slava Lysov

- There has been surprising effectiveness of assuming the validity of Cardy formula for computing BH entropy

? How to understand the appearance of c and the d.o.f from the phase space of gravity?

Using Cardy formula to compute BH entropy.

- Strominger 97'

Near horizon geometry locally AdS $\Lambda = -\frac{1}{\ell^2}$

Semiclassical $\ell \gg G$ $\underbrace{\text{large}}$ c

$$S_{cardy} = \ln \rho = 2\pi\left(\sqrt{\frac{c_R \Delta}{6}} + \sqrt{\frac{c_L \bar{\Delta}}{6}}\right) \longrightarrow S = \frac{2\pi r_+}{4G}$$

Δ $\bar{\Delta}$ eigenvalue L_0. $\bar{L}_0$

$$\begin{cases} M = \frac{1}{\ell}(L_0 + \bar{L}_0) = \frac{r_+^2 + r_-^2}{8G\ell} \\ J = L_0 - \bar{L}_0 = \frac{r_+ r_-}{4G\ell} \end{cases}$$

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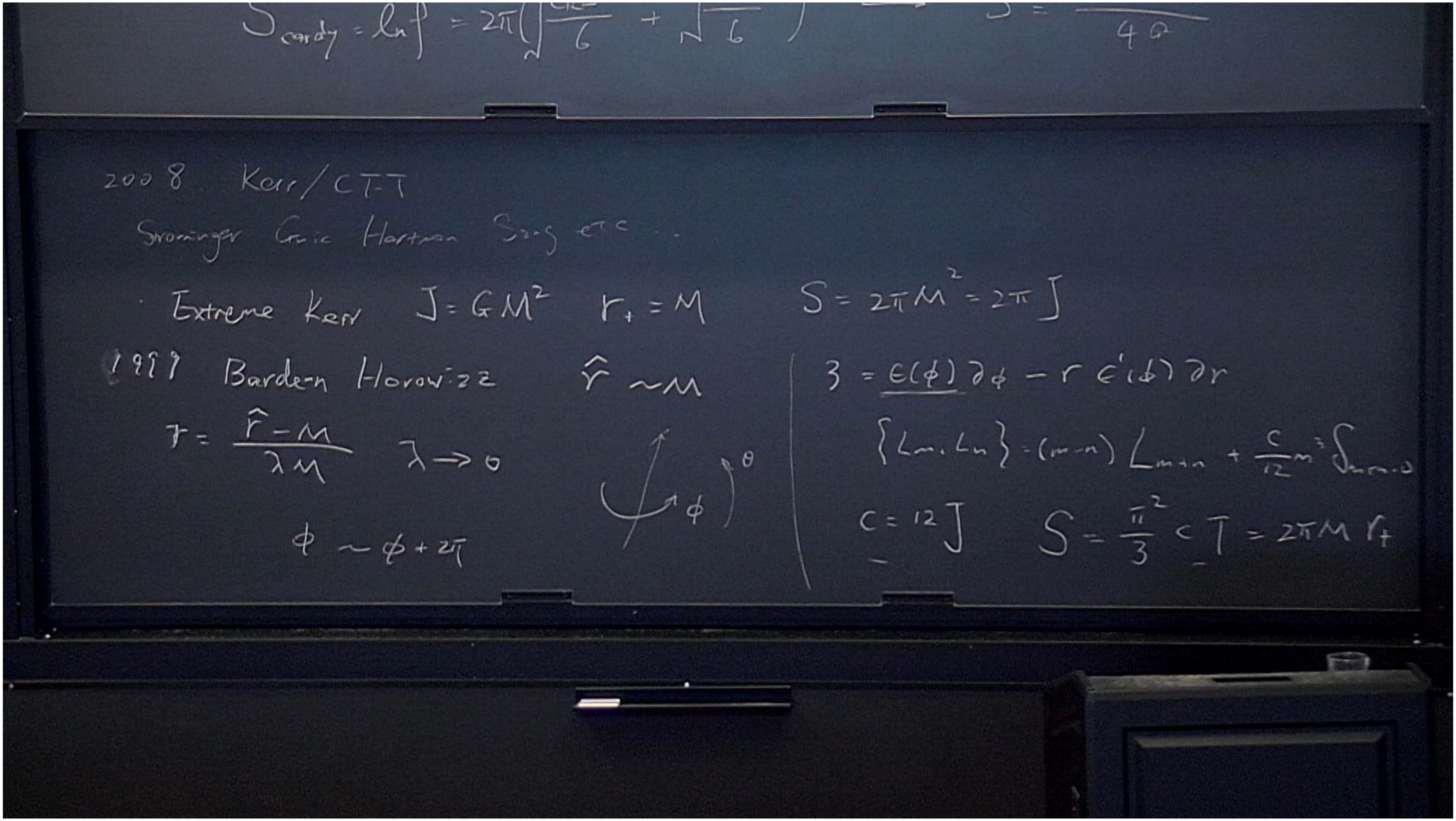

2008 Kerr/CFT
Strominger Guica Hartman Song etc ...
Extreme Kerr J = GM² r+ = M
S = 2πM² = 2πJ
1999 Bardeen Horowitz r̂ ~ M
r = (r̂ − M)/(λM) λ → 0
φ ~ φ + 2π
θ
φ
ξ = ε(φ)∂φ − r ε'(φ)∂r
{Lm, Ln} = (m−n) Lm+n + c/12 m³ δm+n,0
c = 12J S = π²/3 c T = 2πM r+

Covariant Phase Space Formalism

$$S = \int_M L(\phi, \partial_\mu \phi, \dots) + \int_{\partial M} \ell$$

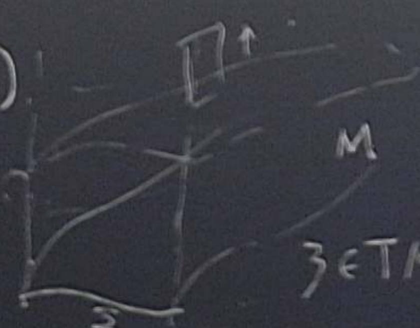
d-dim

Solution space

$$\downarrow \; \delta\phi\big|_{\partial M} = 0$$

$$\text{E.O.M} + \int_M d\theta(\phi, \delta\phi)$$

$\delta\phi$ $\quad S$



$$\omega = \delta(\phi, \delta\phi) = \delta_{[1}\theta(\phi, \delta_{2]}\phi)_{\partial\Sigma}$$

e.g.: QED: $\omega = \delta A \wedge * \delta F$

$$\xi \in TM \longrightarrow \hat{\xi} = \int d^dx\, \mathcal{L}_\xi \phi(x) \frac{\delta}{\delta\phi}$$

from the phase space of gravity!

Semiclassical $\ell \gg G$ large c $\quad 4G\ell$

$$S_{Cardy} = \ln \rho = 2\pi\left(\sqrt{\frac{c_L \Delta}{6}} + \sqrt{\frac{c_R \bar{\Delta}}{6}}\right) \longrightarrow S = \frac{2\pi r_+}{4G}$$

$$j_\xi = \theta(\phi, \mathcal{L}_\xi \phi) - \iota_\xi L$$

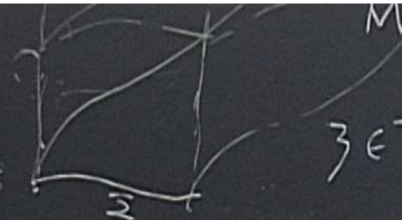
$$\downarrow \quad \delta_\xi L = \mathcal{L}_\xi L$$

$$d j_\xi = 0$$

$$\Downarrow$$

$$\exists\, Q_\xi^N \quad j_\xi = dQ_\xi^N$$

$$\omega = \delta(\phi, \delta\phi) = \delta_{[1}\Theta(\phi, \delta_{2]}\phi)$$



$$\xi \in TM \longrightarrow \hat{\xi} = \int d^d x \, \xi\phi(x) \frac{\delta}{\delta\phi}$$

eg: QED: $\omega = \delta A \wedge * \delta F$

$$\not\delta Q_\xi = \int_\Sigma \iota_{\hat{\xi}}\,\omega \hat{=} \int_\Sigma d\big(\underbrace{\delta Q^N_\xi - \iota_\xi \theta(\phi, \delta\phi)}_{k_\xi[\delta\phi,\, \phi]}\big)$$

$$\exists Q_\xi : \not\delta Q_\xi = \delta Q_\xi$$

$N_\xi[\bar{\phi}]$

$\bar{\phi}$ ϕ

$$Q_\xi[\phi; \bar{\phi}] = \int_{\bar{\phi}}^{\phi} \int_{\partial\Sigma} k_\xi[\delta\phi, \phi] + N_\xi[\bar{\phi}]$$

Sufficient condition that $K_{\xi,\eta}$ trivial

- $\mathfrak{g}$ - semisimple

- $\Omega = -\delta \Theta$ & $\mathcal{L}_{\xi} \Theta = 0$

$$\{Q_{\xi}, Q_{\eta}\} = -\iota_{\hat{\xi}}\iota_{\hat{\eta}}\delta\Theta = -\iota_{\hat{\xi}}\underset{0}{\mathcal{L}_{\hat{\eta}}\Theta} + \iota_{\hat{\xi}}\delta\iota_{\hat{\eta}}\Theta$$

$$= -\mathcal{L}_{\hat{\xi}}\iota_{\hat{\eta}}\Theta + \iota_{\hat{\eta}}\mathcal{L}_{\hat{\xi}}\Theta = \iota_{[\hat{\eta},\hat{\xi}]}\Theta$$

tion that $K_{\xi\cdot\eta}$ trivial

& $\underline{\mathcal{L}_{\xi}}\,\Theta^{(1)} \underline{= 0}$

$= - I_{\hat\xi}\, I_{\hat\eta}\, \delta \Theta^{(1)} = - I_{\hat\xi}\, \mathcal{L}_{\hat\eta} \Theta^{(1)} + I_{\hat\xi}\, \delta\, I_{\hat\eta} \Theta^{(1)}$

$= - \mathcal{L}_{\hat\xi}\, I_{\hat\eta} \Theta^{(1)} + I_{\hat\eta}\, \mathcal{L}_{\hat\xi} \Theta^{(1)} = I_{[\hat\eta,\hat\xi]} \Theta^{(1)} = Q_{[\eta,\xi]}$

eg: QED: $\omega = \delta A \wedge * \delta F$

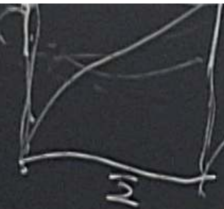
If $\Phi_g\left(\Theta(\phi, \delta\phi)\right) \neq \Theta\left(\Phi_g \phi, \ \delta \Phi_g \cdot \phi\right)$

$\xi \parallel \partial\Sigma \qquad \mathcal{L}_\xi\left(\Theta + \Theta_{\partial\Sigma}\right) = 0$

$\xi_\perp \neq 0 \qquad \xi_\perp = \varepsilon(\phi j \cdot r \partial_r$

$\omega = \delta(\phi, \delta\phi) = \delta_{[1]}\, \Theta(\phi, \delta_{[2]}\phi)\big|_{\partial\Sigma}$

eg: QED: $\omega = \delta A \wedge * \delta F$



$\xi \in TM \longrightarrow \hat{\xi} = \int d^dx\, \mathcal{L}_\xi \phi(x)$

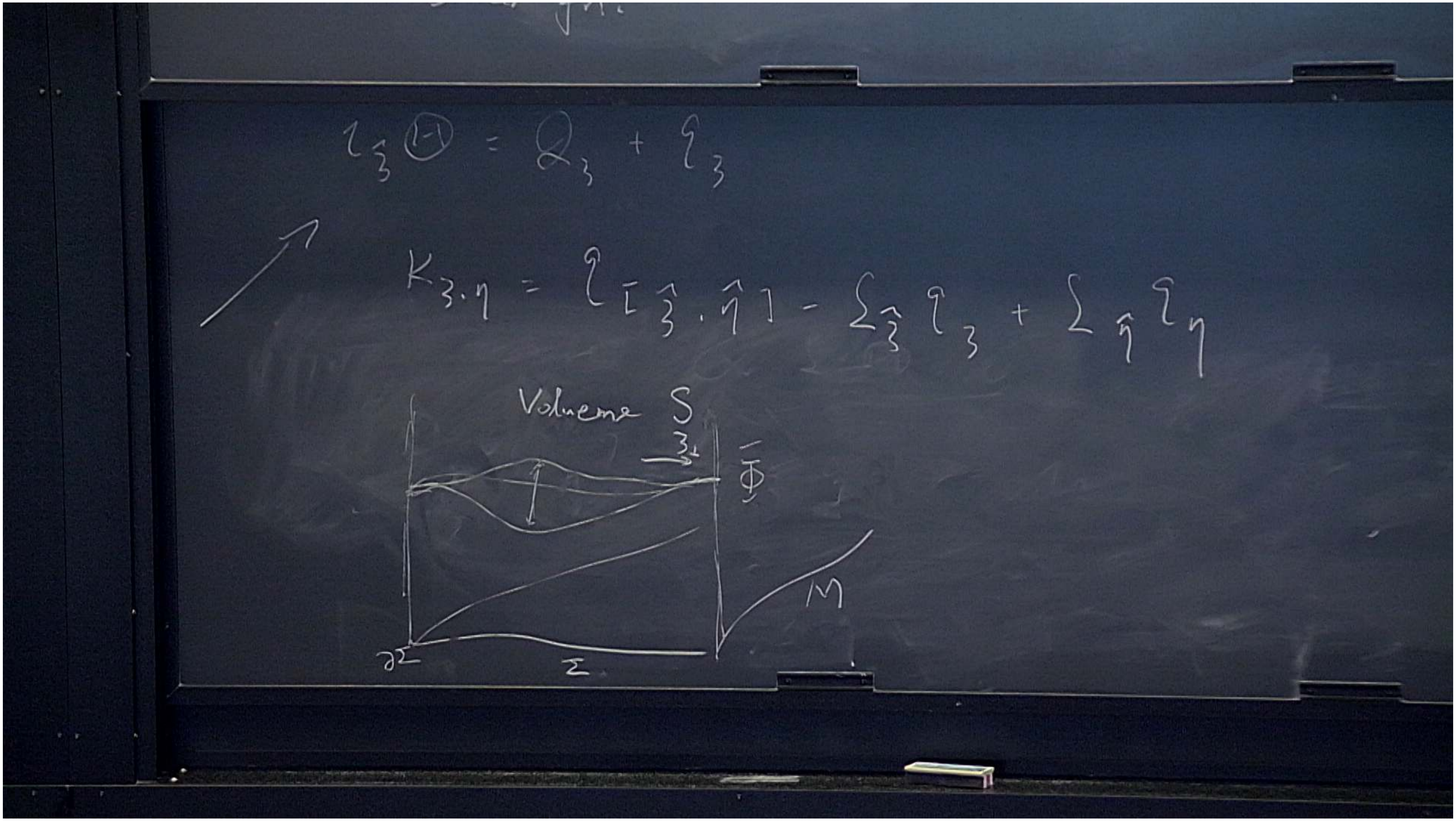
When $\mathcal{L}_{\hat{\xi}} \Theta \neq 0$ $\qquad \mathcal{L}_\xi \Omega = 0$

$\mathcal{L}_{\hat{\xi}} \Theta = g$ $\quad$ we can do a canonical trans. $\Theta \to \Theta'$ if

$\exists$ field space 1 form $\varphi = \delta \alpha$ $\qquad g = -\delta(\iota_\xi \delta\alpha)$

$\Theta \to \Theta + \delta\alpha$

$\mathcal{L}_{\hat{\xi}} \Theta = 0 \Rightarrow \delta\, \iota_{\hat{\xi}} \Theta = -\iota_{\hat{\xi}} \delta\Theta = -\iota_{\hat{\xi}} \Omega, \quad Q_\xi = \iota_{\hat{\xi}} \Theta$


Volume S
M
Σ

Non-equivariance

- Lie group G acts on P. Φ_g

Lie algebra element $\xi \in \mathfrak{g} \longrightarrow \xi_P$ on P

Under $\Phi_g^* \xi_P = (\mathrm{Ad}_{g^{-1}} \xi)_P$

$Q_\xi[\phi]$: $\hat{Q} \quad P \rightarrow \mathfrak{g}^*$ $\quad \left(\forall \xi \in \mathfrak{g}, \; -\delta \underbrace{\langle \hat{Q} \cdot \xi \rangle}_{} = \iota_\xi \Omega \right)$

$\langle \hat{Q} \cdot \xi \rangle = Q_\xi[\phi]$

· Casimir fn.

$\exists\ K_{\xi,\eta}$

$$\begin{array}{ccc} P & \xrightarrow{\hat{Q}} & \mathfrak{g}^* \\ \Phi_g \big\downarrow & & \big\downarrow \mathrm{Ad}^*_{g^{-1}} \\ P & \xrightarrow{\hat{Q}} & \mathfrak{g}^* \end{array}$$

$$Q_\xi[\Phi_g \cdot \phi] \neq Q_{\mathrm{Ad}_{g^{-1}}\xi}[\phi]$$

$$\underline{Q_\xi[\Phi_g \cdot \phi] - Q_\xi[\phi]} - \left(\underline{Q_{\mathrm{Ad}_{g^{-1}}\xi}[\phi] - Q_\xi[\phi]}\right)$$

$$\neq 0$$