

Title: PSI 2019/2020 - Quantum Field Theory I (Wohns/Xu) - Lecture 14

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Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

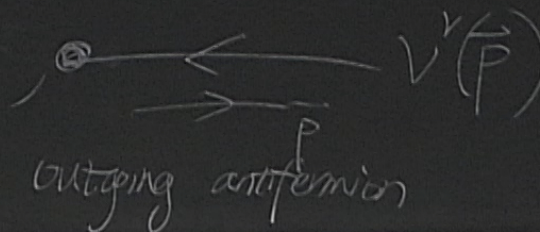
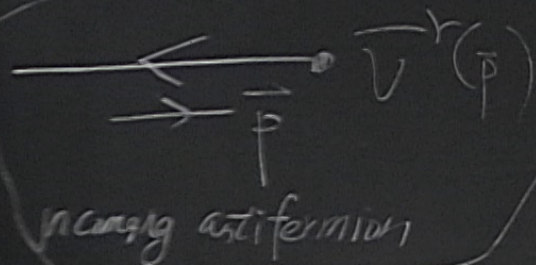
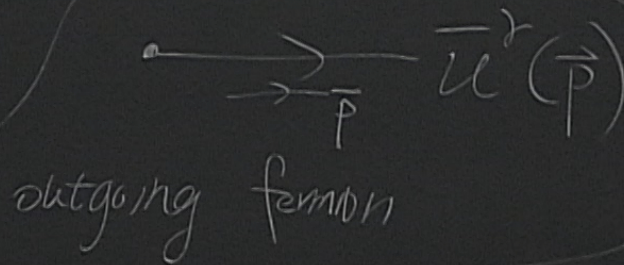
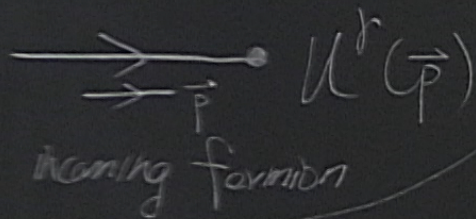
Date: October 31, 2019 - 9:00 AM

URL: <http://pirsa.org/19100071>

Yukawa theory

$$\mathcal{L}_{\text{int}} = -\lambda \phi \bar{\psi} \psi$$

fermion arrow
going away from
vertex barred.



fermion arrow
going away from
vertex barred.

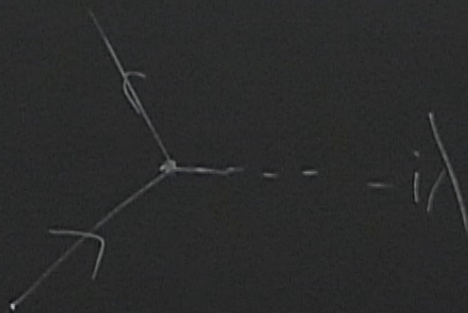
in the spinor space
row vector: $\bar{u}, \bar{v}$
column vector: u, v .

going with fermion
arrow in the
reverse order

fermion arrow
going away from
vertex barred.

going with fermion
arrow in the
reverse order

in the spinor space
row vector: $\bar{u}, \bar{v}$
column vector: u, v .



or space

$$\longrightarrow \frac{i(\not{p} - m)}{p^2 - m^2 + i\epsilon}$$

minus signs:

if a diagram looks
identical to another
diagram after odd

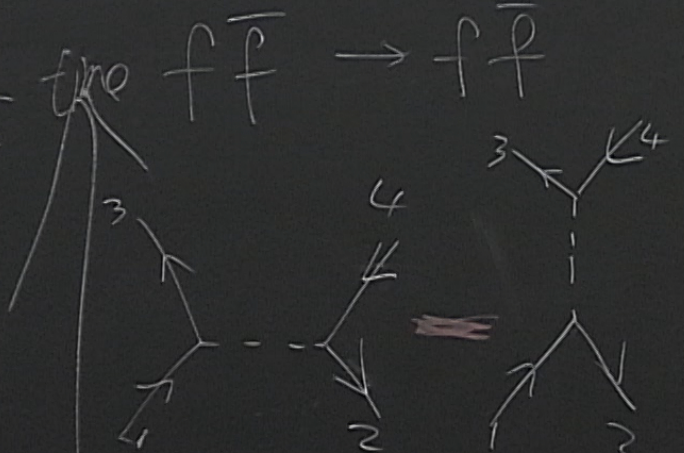
permutation of
external legs (minus sign)

$$\rightarrow \frac{i(\not{p}-m)}{p^2-m^2+i\epsilon} \text{ tree } f\bar{f} \rightarrow f\bar{f}$$

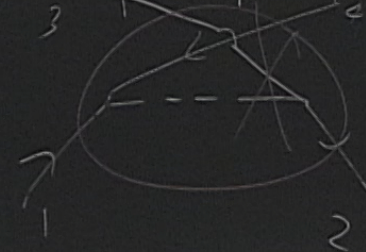
minus signs:

if a diagram looks identical to another diagram after odd permutation of

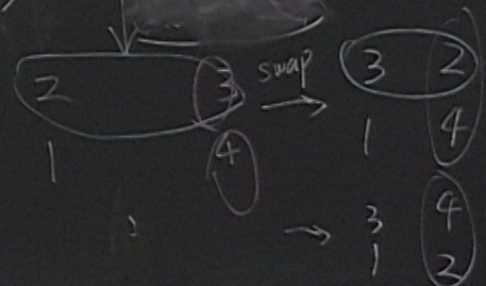
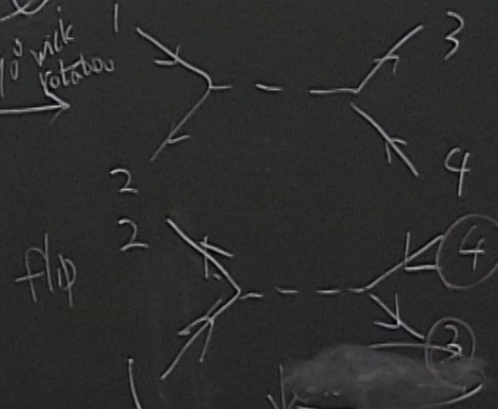
external legs relative minus sign



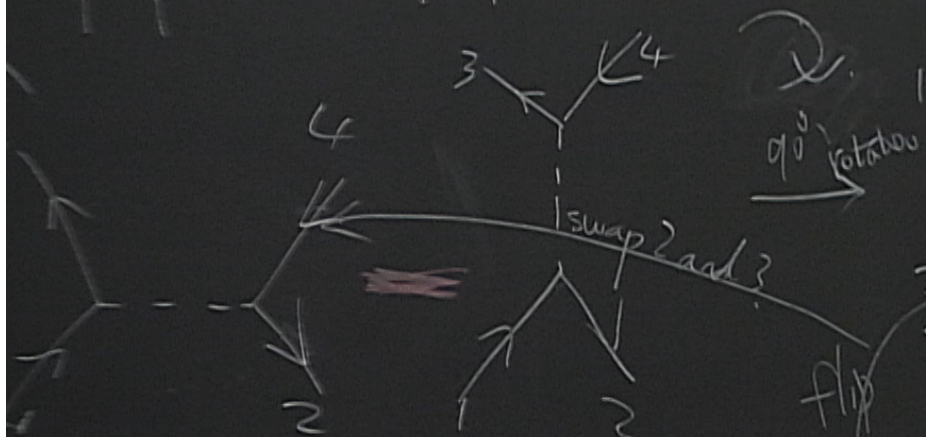
different from scalars



90° Wick rotation

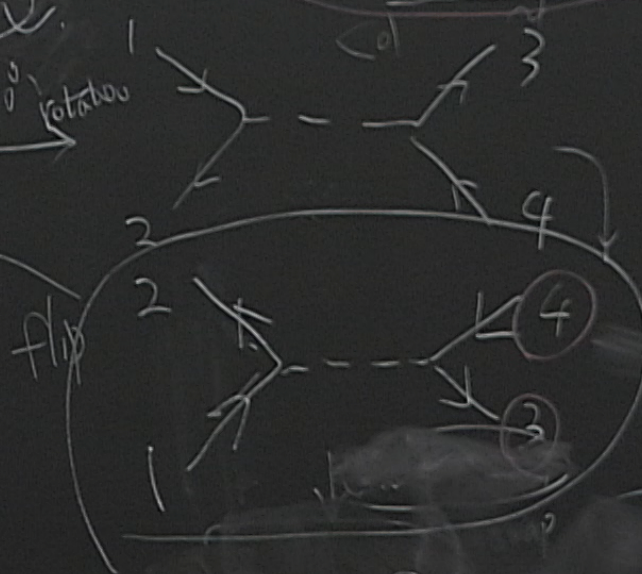


$$f\bar{f} \rightarrow f\bar{f}$$



different from scalars

$$\langle \psi_1 \bar{\psi}_2 \psi_3 \bar{\psi}_4 \rangle$$



$$\text{fermion loop} = \int \frac{d^4 p}{(2\pi)^4} \text{Tr}$$

Maxwell theory.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x) \quad F_{\mu\nu} \text{ invariant}$$

antisymmetric

$$\partial^\mu F_{\mu\nu} = 0 \quad \downarrow \text{vacuum}$$

$$\partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) = 0$$

$$\square A_\nu - \partial^\mu \partial_\nu A_\mu = 0$$

gauge condition

$$\partial^\mu A_\mu = 0$$

$$: \partial^\mu A_\mu = \tilde{\mathcal{L}}(x)$$

$$\partial^\mu (\tilde{A}_\mu + \partial_\mu \alpha(x)) = \tilde{\mathcal{L}}(x)$$

$$\partial^\mu \tilde{A}_\mu + \square \alpha(x) = 0$$

Seiberg
19100053

$\tilde{Z}(x)$

$$\partial_\mu \tilde{A}_\mu = 0 \quad \checkmark$$

$$\partial_\mu \alpha(x) = \tilde{Z}(x)$$

$$+ \square \alpha(x) = \tilde{Z}(x)$$

solvable

undance

$$\square \alpha(x) = 0$$

solution is

Coulomb $\vec{\nabla} \cdot \vec{A} = 0$ $\rightarrow A_0 = 0$ 2 dof

What about A^0 $\partial_0 A_0$ is not is Lagrangian

$$\partial_\mu A^\mu = 0 \quad \text{Lorenz}$$

$$\square A^\nu = 0$$

$$\partial_\mu \partial^\mu A^\nu = 0$$

massless KG equation

$$\partial_\nu A^\nu = 0$$

$$\sum_{\lambda=0}^3 (\epsilon^\nu)^\lambda \cdot k_\nu = 0$$

$$A^\nu = \int d^4k \sum_{\lambda=0}^3 (\epsilon^\nu)^\lambda e^{-ikx} + (\epsilon^\nu)^\lambda e^{+ikx}$$

polarization vector $\swarrow$
 solution index $0 \rightarrow 3$ $\nwarrow$

$$\epsilon^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \epsilon^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\partial_\nu A^\nu = 0$$

KG equation, for any $(\epsilon^\nu) \cdot k_\nu = 0$

$$\epsilon^0 k + \epsilon^3 \cdot k = 0$$

for any solution

pick a wave
moves in z-direction
 $k^\mu = (k, 0, 0, k)$

$x \rightarrow 3$

$$(\epsilon^\nu)^\lambda e^{+ikx}$$

$$\epsilon^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\partial_\nu A^\nu = 0$$

$$\sum_{\lambda=0}^3 (\epsilon^\nu)^\lambda \cdot k_\nu = 0$$

$$\sum_{\lambda=0}^3 (\epsilon^\nu)^\lambda \cdot k_\nu = 0$$

$$+ \epsilon^\nu_0 k^\nu + \epsilon^\nu_1 k^\nu + \epsilon^\nu_2 k^\nu + \epsilon^\nu_3 k^\nu = 0$$

Summing over polarizations

(we always pick $\epsilon^\nu_1 k^\nu = \epsilon^\nu_2 k^\nu = 0$)

$$\boxed{\epsilon^\nu_0 k^\nu + \epsilon^\nu_3 k^\nu = 0}$$

$$\epsilon^\lambda_\mu \epsilon^\lambda_\nu \eta^{\mu\nu} = \eta^{\lambda\lambda'}$$

$$\epsilon^\lambda_\mu \epsilon^{\lambda'}_\nu \eta_{\lambda\lambda'} = \eta_{\mu\nu}$$

Quantize.

① pick a Lagrangian

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

②

$$\pi^0 = \frac{\partial \mathcal{L}}{\partial (\partial_0 A^0)} = 0$$

$$[\varphi, \pi] = i\delta(x-y)$$

oops!!

we did: $\mathcal{L} \xrightarrow{\text{e.o.m. gauge condition}} KG$

we want $\mathcal{L}_{\text{new}} \xrightarrow{\text{e.o.m.}} KG$

||
 $\mathcal{L}_{\text{old}}$ if $(\partial_\mu A^\mu) = 0$

$$\mathcal{L}_{\text{new}} = -\frac{1}{4} \bar{F}_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu A^\mu)^2$$

check e.o.m. $\square \vec{A} = 0$

① pick a Lagrangian

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

②

$$\Pi^0 = \frac{\partial \mathcal{L}}{\partial(\partial_0 A^0)} = 0$$

A^0 completely determined by A^i

oops!!

$[\phi, \pi] = i\delta(x-y)$

we did: $\mathcal{L} \rightarrow \text{e.o.m} \xrightarrow{\text{gauge cond. 2nd}} \text{KG}$

we want $\mathcal{L}_{\text{new}} \xrightarrow{\text{e.o.m}} \text{KG}$

||

$\mathcal{L}_{\text{old}}$ if $(\partial_\mu A^\mu) = 0$

$$\mathcal{L}_{\text{new}} = -\frac{1}{4} \bar{F}_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu A^\mu)^2$$

check e.o.m $\square \vec{A} = 0$

$$\mathcal{L}_q = -\frac{1}{2} \partial_\mu A_\nu \partial^\mu A^\nu$$

(no gauge fixing)

$$\pi^\mu = -\dot{A}^\mu$$

$$[A^\mu(\vec{x}), \pi^\nu(\vec{y})] = i\delta(\vec{x}-\vec{y}) \eta^{\mu\nu}$$

$$[A^i, \pi^j] = [A^i, -\dot{A}^j]$$

$$= -\delta_{ij} \delta(\vec{x}-\vec{y})$$

$$[a^\dagger, (a^\dagger)^{k'}] =$$

$$[a_{\vec{k}}^\lambda, (a_{\vec{p}}^\dagger)^{\lambda'}] = -(2\pi)^3 2E_{\vec{p}} \delta(\vec{k}-\vec{p}) \eta^{\lambda\lambda'}$$

$$[a_{\vec{k}}^0, (a_{\vec{p}}^\dagger)^0] = -(2\pi)^3 2E_{\vec{p}} \delta(\vec{k}-\vec{p})$$

$$A_{\mu-}(x) = \sum_{\vec{k}} \int dV_{\vec{p}} (\epsilon_{\mu})^{\lambda} a_{\vec{p}}^{\lambda} e^{-ip \cdot x}$$

$$A_{\mu+}(x) = \sum_{\vec{k}} \int dV_{\vec{p}} (\epsilon_{\mu})^{\lambda} a_{\vec{p}}^{\lambda\dagger} e^{ip \cdot x}$$

bad states - negative norm
need to remove bad states!!

gauge condition

$\neq |\lambda, \vec{p}\rangle$ *not sum*
 $= \langle 0 | a_{\vec{p}}^0 a_{\vec{p}}^{0\dagger} | 0 \rangle$

= - something

impose $\partial_\mu A^\mu = 0$ as -operator equation

$$\partial_\mu \langle 0 | A^\mu A^\nu | 0 \rangle = \partial_\mu \text{propagator} \left(\int \frac{-i \eta^{\mu\nu}}{p^2 - m^2 + i\epsilon} e^{ip(x-x')} \right)$$

$$\partial_\mu A^\mu | \text{phys} \rangle = 0$$

also problematic

$$\partial_\mu A^\mu | 0 \rangle \neq 0$$

$|0\rangle$ is no longer physical

$$\partial_\mu A_-^\mu | \text{phys} \rangle = 0$$

$$\langle \text{phys} | \partial_\mu A_+^\mu$$

$$\langle \text{phys} | \partial_\mu A^\mu | \text{phys} \rangle = 0$$

$$\boxed{\partial_\mu A^\mu |phys\rangle = 0} \Rightarrow \sum_\lambda k_\mu (\epsilon^\mu)^\lambda a_{\vec{k}}^\lambda |phys\rangle = 0$$

$$\langle phys | \partial_\mu A^\mu$$

can pick $(k \cdot \epsilon^0 a_{\vec{k}}^0 + k \cdot \epsilon^3 a_{\vec{k}}^3) |phys\rangle = 0$
 $\epsilon_1 \cdot k = 0$
 $\epsilon_2 \cdot k = 0$

$$k \cdot \epsilon^0 (a_{\vec{k}}^0 - a_{\vec{k}}^3) |phys\rangle = 0$$

$$\langle phys | \partial_\mu A^\mu | phys \rangle = 0$$

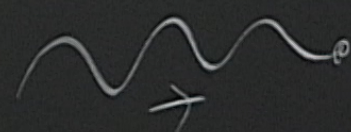
$$\epsilon_0 \cdot k + \epsilon_3 \cdot k = 0$$

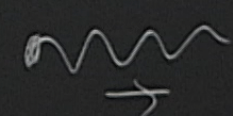
$$|a_{\vec{k}}^0 |phys\rangle|^2 = |a_{\vec{k}}^3 |phys\rangle|^2$$

$$N_0 = N_3$$

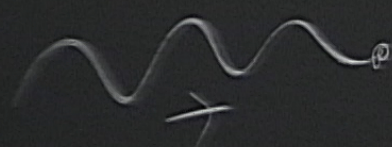
$$\mathcal{H} = \sum_{\vec{p}} \int dV_{\vec{p}} (-) \frac{E_{\vec{p}}}{2} a_{\vec{p}}^{+\lambda} a_{\vec{p}}^{\lambda'} \eta^{\lambda\lambda'}$$

$$= \int dV_{\vec{p}} \frac{E_{\vec{p}}}{2} (N_1 + N_2 - N_0 + N_3)$$

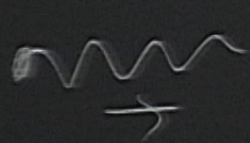


$$\epsilon_{\mu}^{in\kappa} (n=0,1,2,3)$$


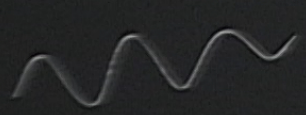
$$\epsilon_{\mu}^{out}$$



$$\epsilon_{\mu}^{in\kappa} (n=0,1,2,3)$$

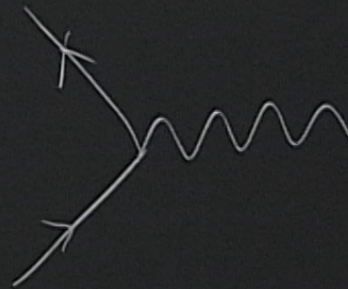


$$\epsilon_{\mu}^{out}$$



$$\frac{-i\eta^{\mu\nu}}{p^2 - m^2 + i\epsilon}$$

$$-Q\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$



$m=0,1,2,3$

$$-Q\bar{\psi}\gamma^\mu A_\mu\psi$$

