Title: PSI 2019/2020 - Quantum Field Theory I (Wohns/Xu) - Lecture 13

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Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Date: October 29, 2019 - 9:00 AM

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The Glory (or Gory) detail of the hunt for fermionic Fe

1. recipe:

① pick a theory

Feynman diagram } { ② pick a process
+ Feynman rules

③ Feynman diagrams

=: iM

④ use LSZ to co

⑤ compare and

⑥ repeat

The Glory (or Gory) detail of the hunt for fermionic

1. recipe: ① pick a theory

Feynman diagram } ② pick a process

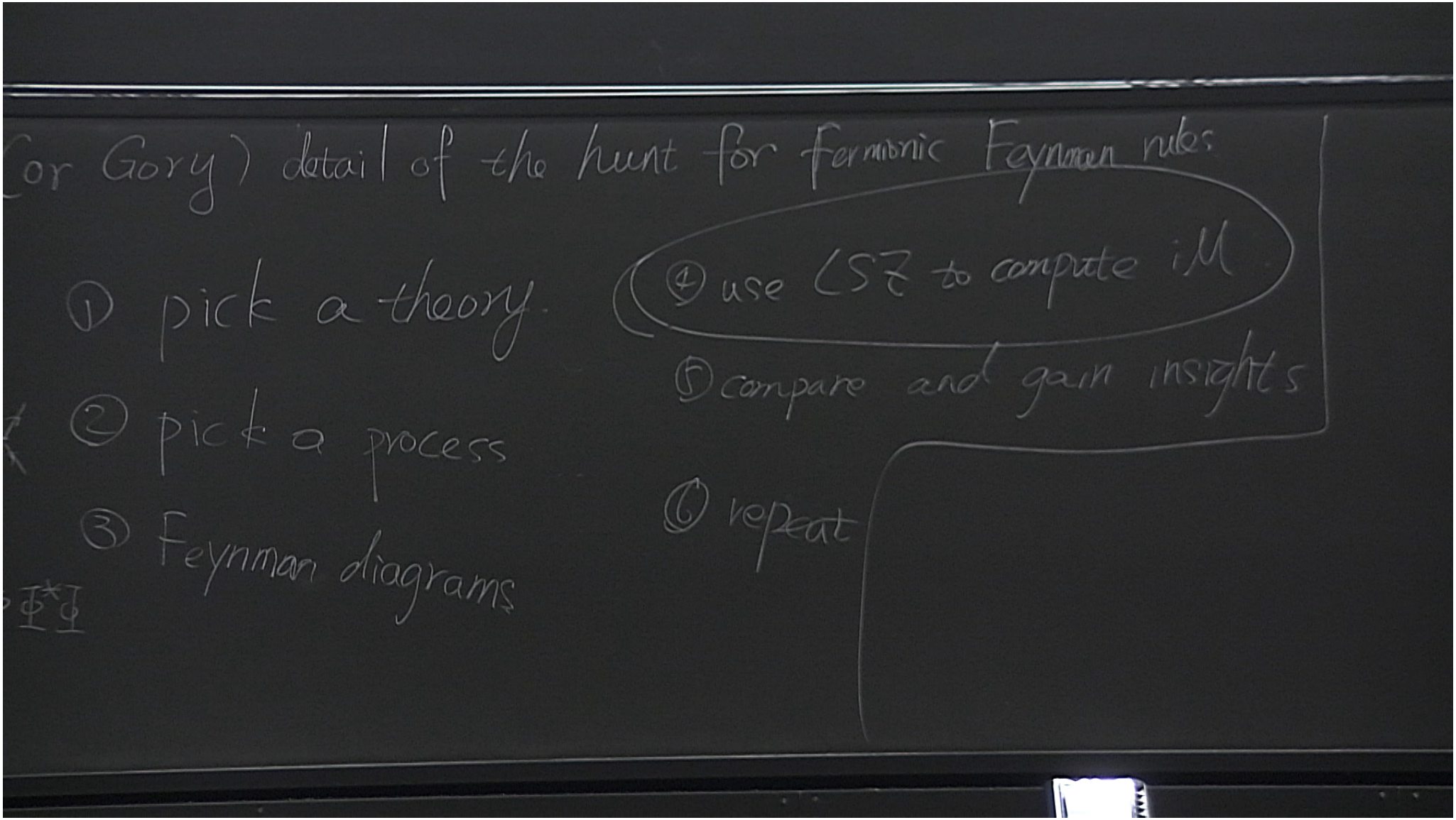
+ feynmen rules ③ Feynman diagrams

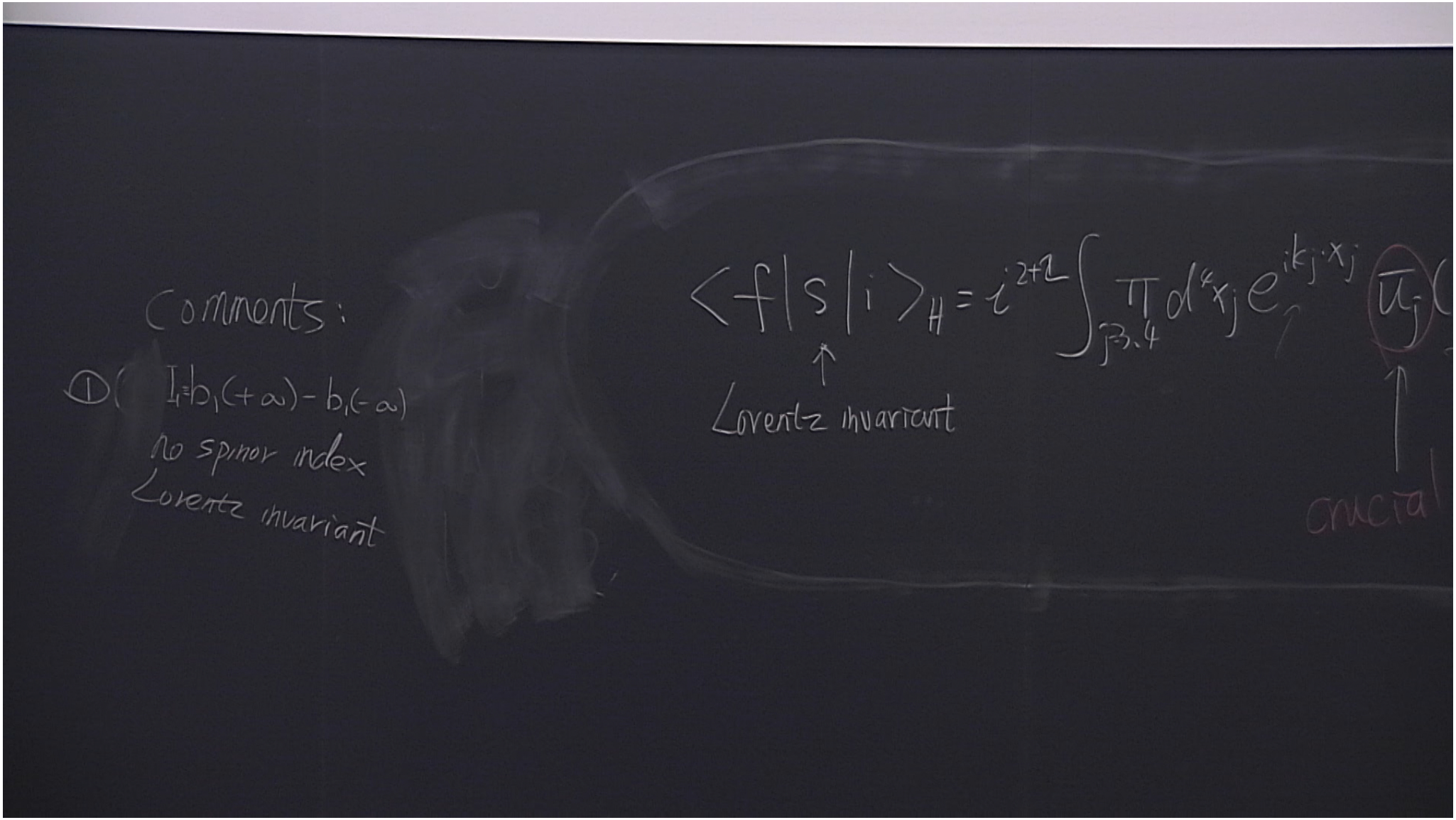
$=: iM$ $\varphi \Phi^* \Phi$

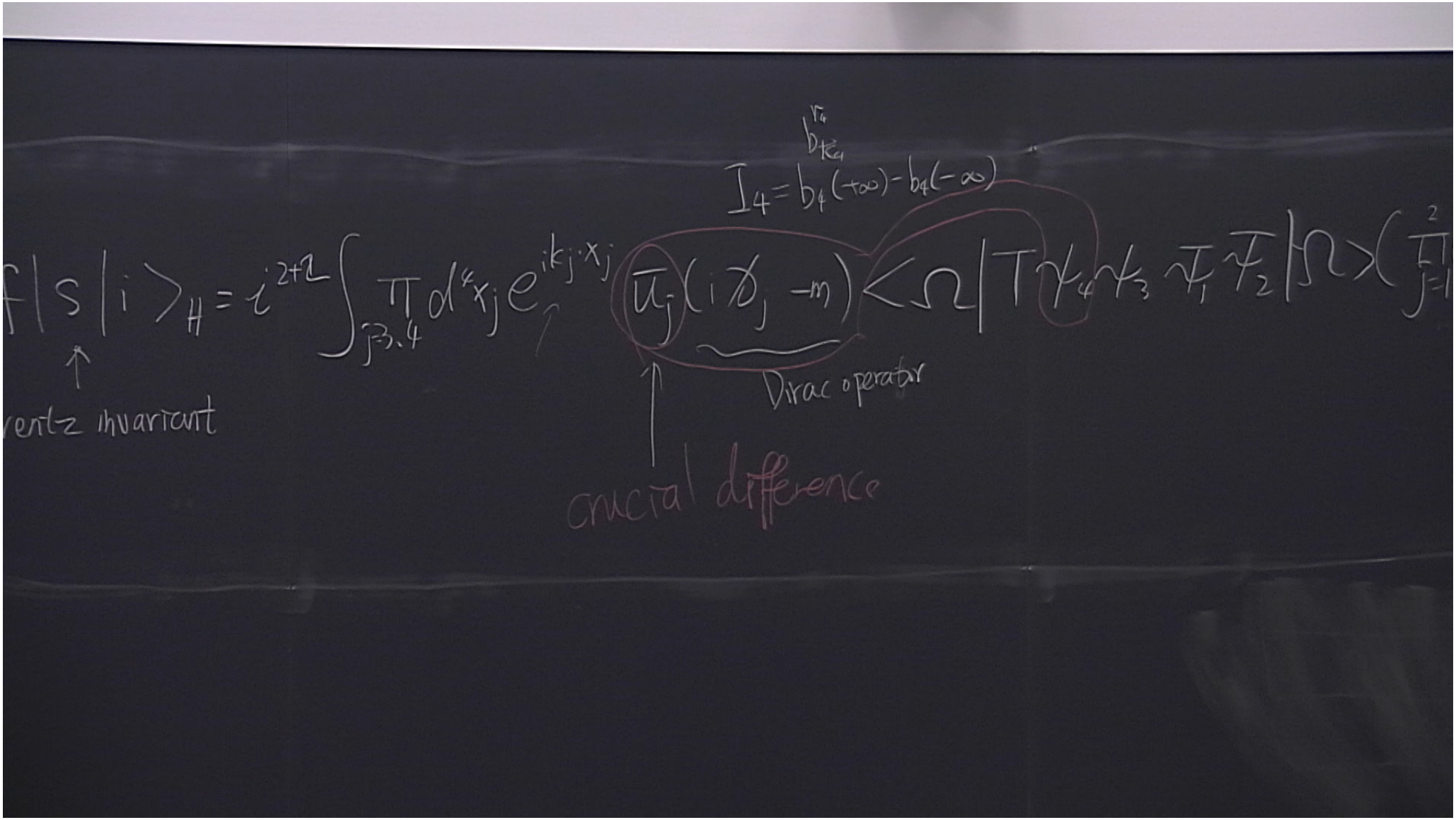
④ use LSZ to

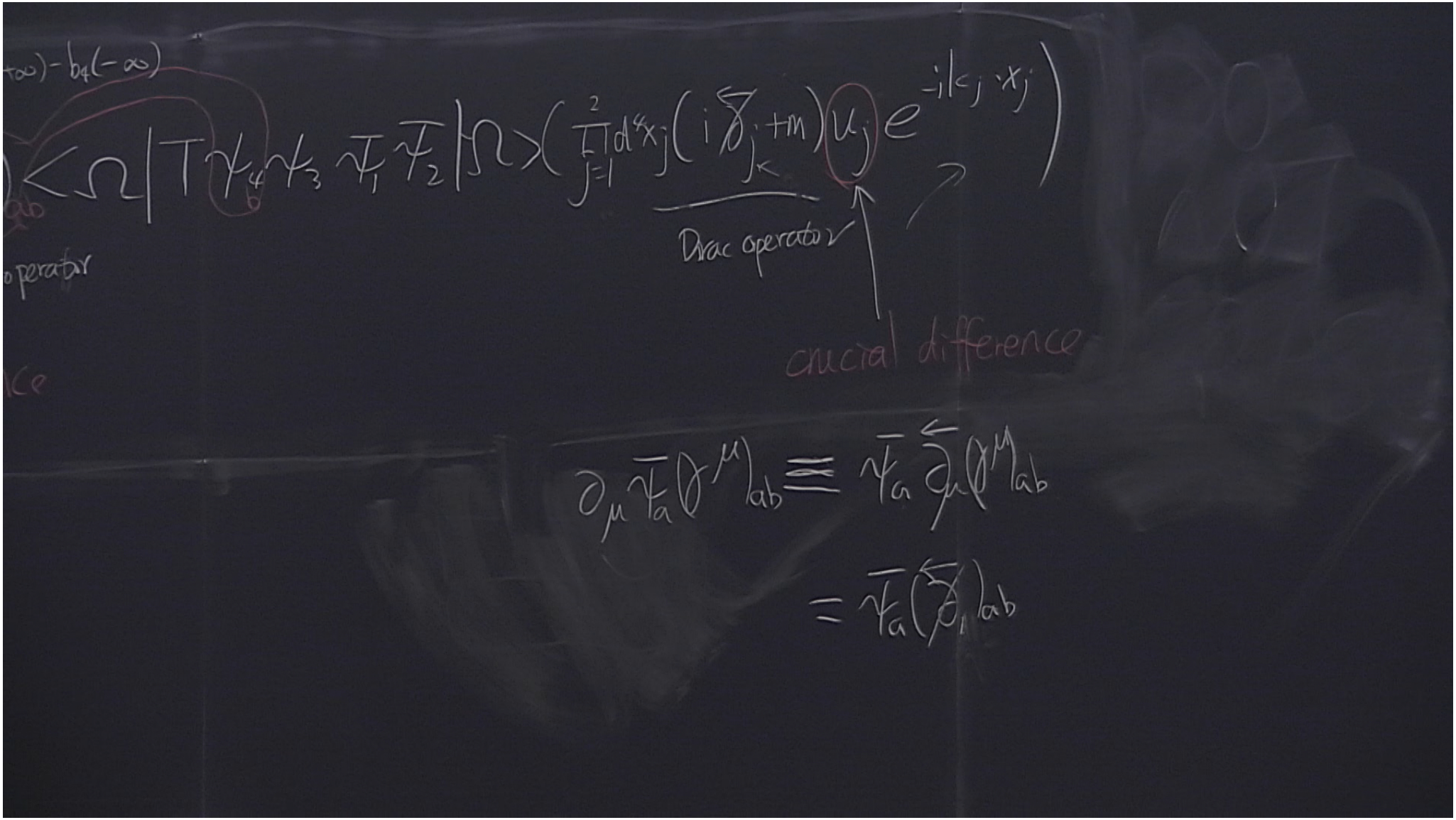
⑤ compare and

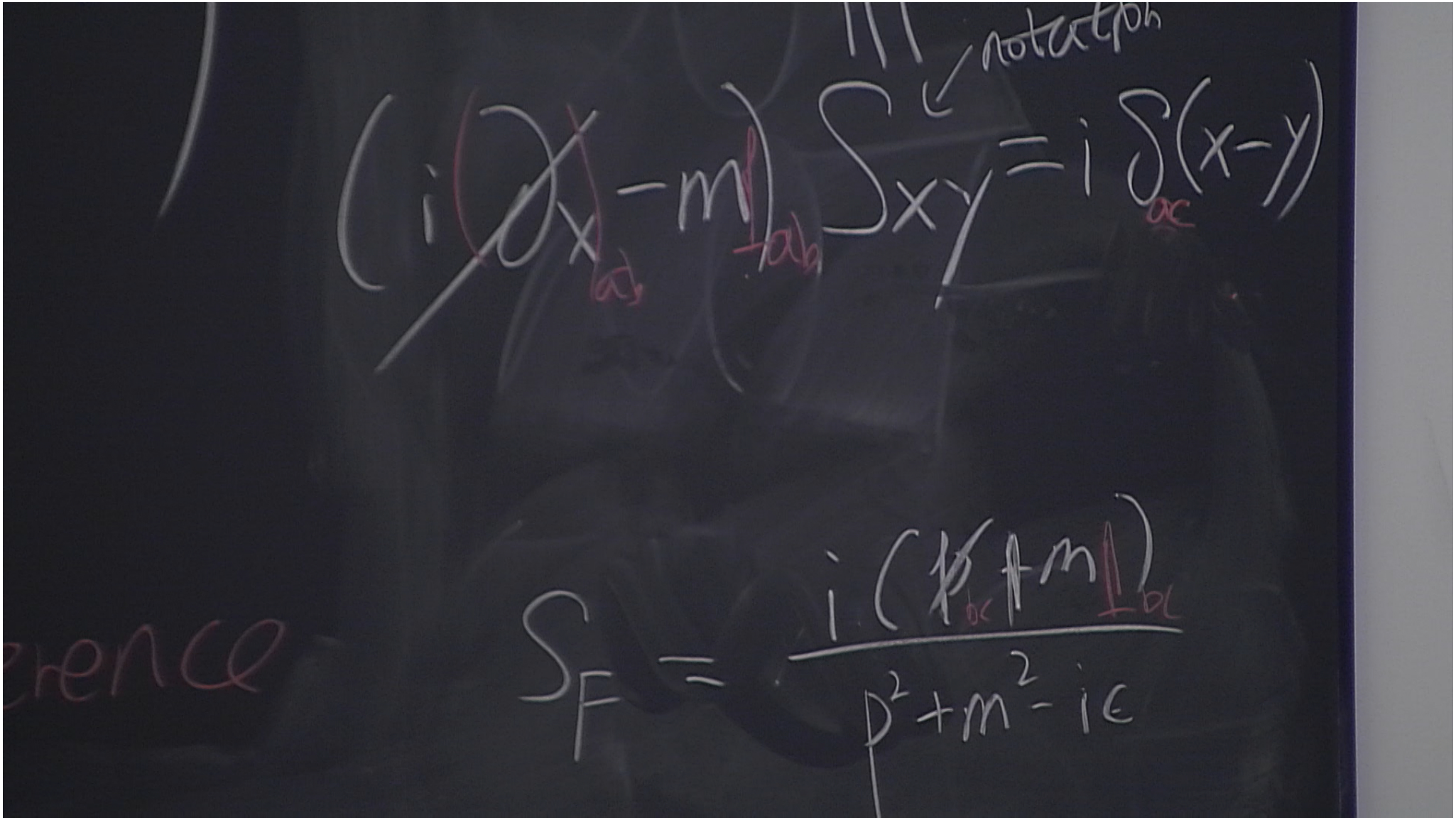
⑥ repeat


(or Gory) detail of the hunt for fermionic Feynman rules
① pick a theory
② pick a process
③ Feynman diagrams
④ use LSZ to compute iM
⑤ compare and gain insights
⑥ repeat


Comments:
① $I_1 = b_1(+\omega) - b_1(-\omega)$
no spinor index
Lorentz invariant
$\langle f|S|i\rangle_H = i^{2+2} \int \prod_{j=3,4} d^4x_j e^{ik_j \cdot x_j}$
Lorentz invariant
crucial


$\vec{b}_{k_4}^{r_4}$
$I_4 = b_4(+\infty) - b_4(-\infty)$
$\langle f|S|i\rangle_H = i^{2+2L} \int \prod_{j=3,4} d^4x_j e^{ik_j \cdot x_j}$
$\bar{u}_j(i\not\partial_j - m)\langle\Omega|T\gamma_4\gamma_3\bar{\psi}_1\bar{\psi}_2|\Omega\rangle(\prod_{j=1}^{2}$
Lorentz invariant
Dirac operator
crucial difference


$+\infty) - b_4(-\infty)$
$\langle \Omega | T \bar{\psi}_4 \psi_3 \bar{\psi}_1 \psi_2 | \Omega \rangle \left(\prod_{j=1}^{2} d^4x_j \left(i \overleftarrow{\not\partial}_j + m \right) u_j \, e^{-i k_j \cdot x_j} \right)$
Dirac operator
crucial difference
$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab} = \bar{\psi}_a \overleftarrow{\partial}_\mu (\gamma^\mu)_{ab}$
$= \bar{\psi}_a (\overleftarrow{\not\partial})_{ab}$


notation
$(i\partial\!\!\!/_x - m)_{ab} S_{xy} = i\,\delta_{ac}(x-y)$
$S_F = \frac{i(\not{p}_{ac} + m\mathbb{1}_{ac})}{p^2 + m^2 - i\epsilon}$

$$\left(\prod_{j=1}^{2} d^4x_j \left(i\overleftarrow{\not\partial}_j + m\right) u_j e^{-ik_j \cdot x_j}\right)$$

Dirac operator

crucial difference

$$S_F(x-y)$$

notation

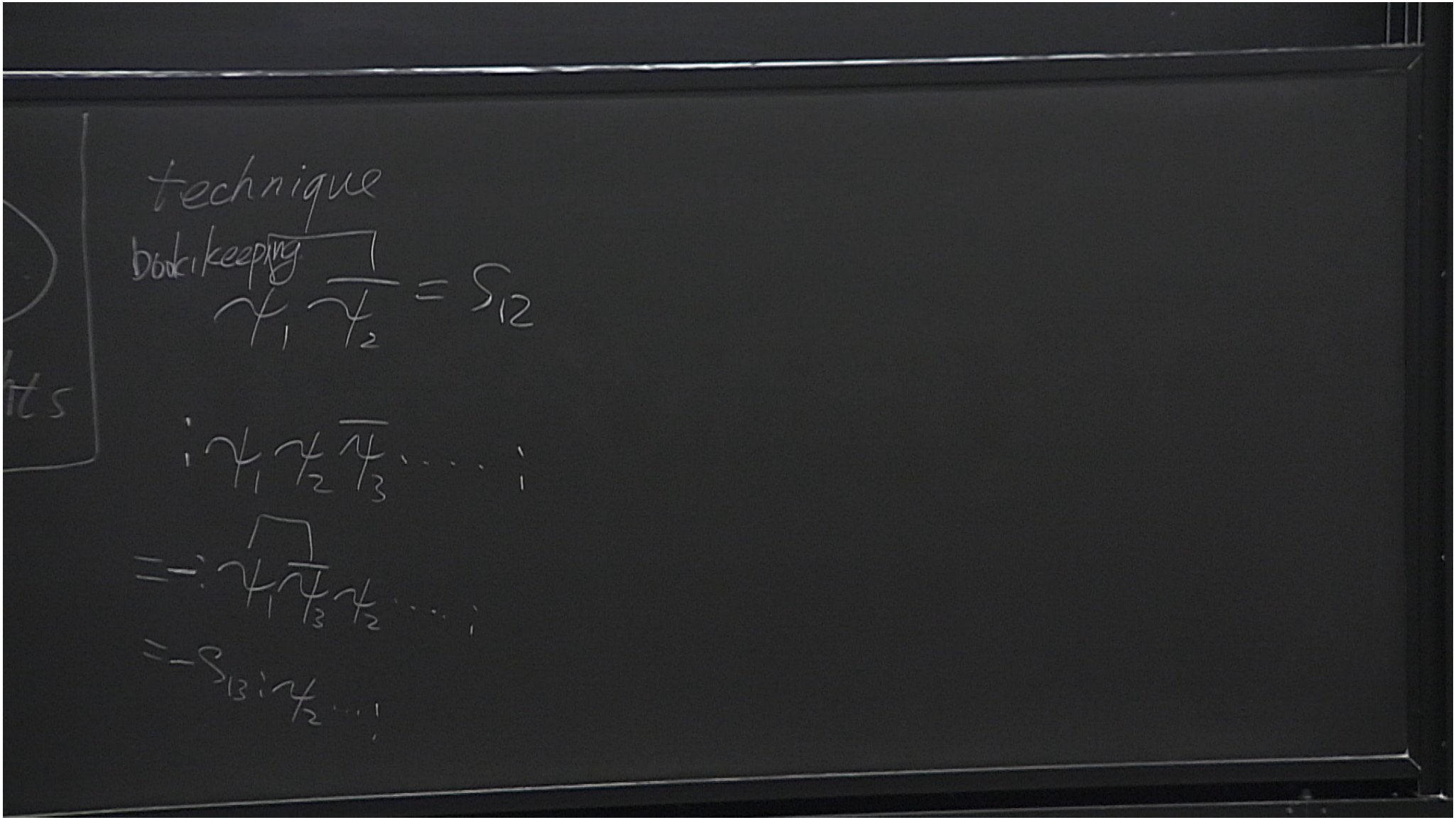
$$(i\not\partial_x - m)_{ab} S_{xy} = i\delta(x-y)$$

$$S_F = \frac{i(\not p + m)_{ab}}{p^2 + m^2 - i\epsilon}$$

$$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab} \equiv \bar{\psi}_a \overleftarrow{\partial}_\mu (\gamma^\mu)_{ab}$$

$$S_{xy}(i\overleftarrow{\not\partial}_y + m) = -i\delta(x-y)$$

$$= \bar{\psi}_a (\overleftarrow{\not\partial})_{ab}$$

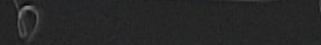

technique
bookkeeping
$\tau_1 \overline{\tau_2} = s_{12}$
$; \tau_1 \tau_2 \overline{\tau_3} \cdots ;$
$= -; \tau_1 \overline{\tau_3} \tau_2 \cdots ;$
$= -s_{13} ; \tau_2 \cdots ;$

$\psi_1 \psi_2 = 0$

$= S_{12}$ $\bar{\psi}_1 \bar{\psi}_2 = 0$

different species always give 0

$\psi \bar{\psi}_1 = 0$



① pick a theory.

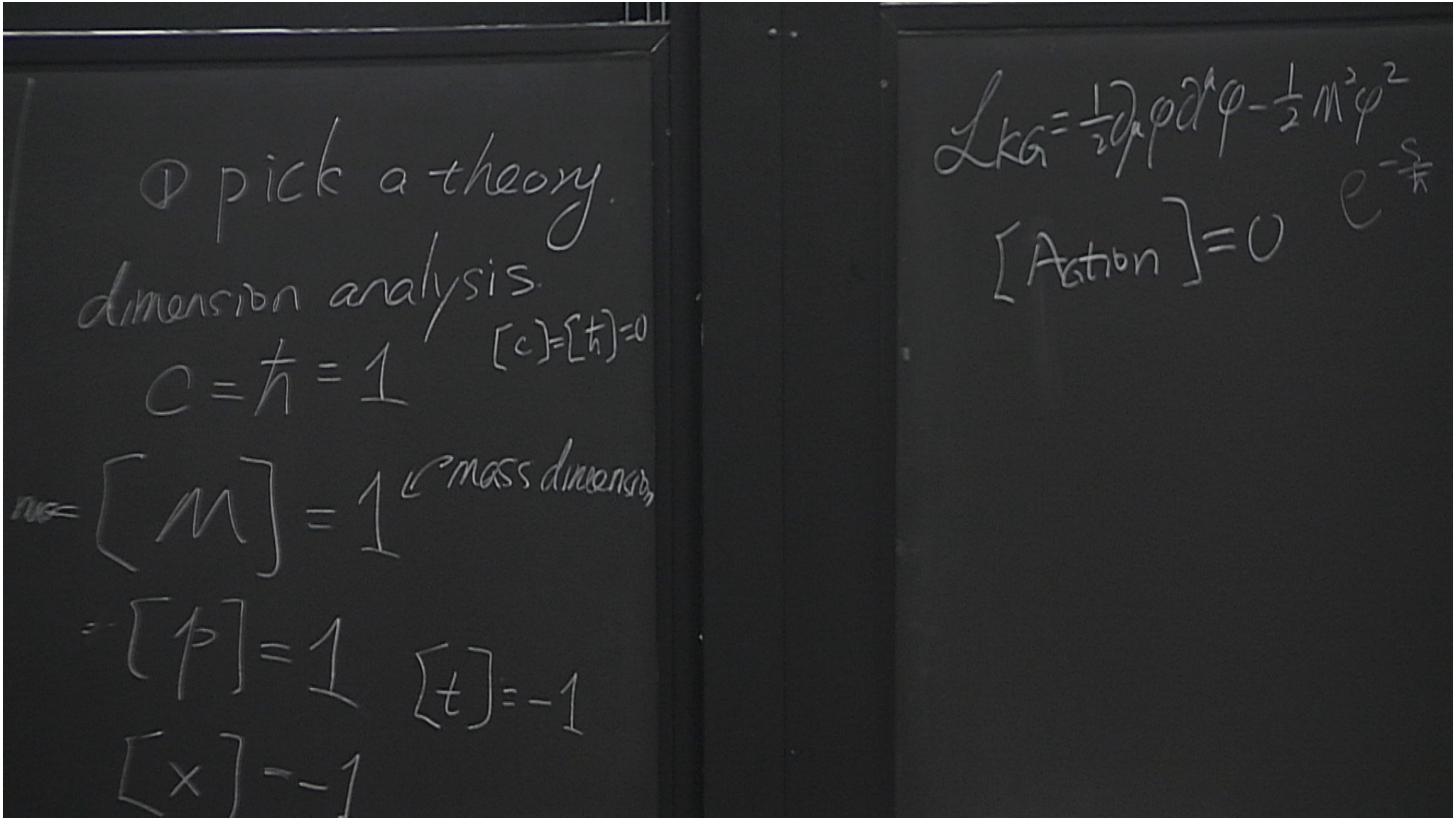
dimension analysis.

$c = \hbar = 1$

$m = [m] = 1$ ← mass dimension

$\therefore [p] = 1$ $[t] = -1$

$[x] = -1$


① pick a theory.
dimension analysis.
$c = \hbar = 1$
$[c] = [\hbar] = 0$
$[m] = [M] = 1$ ← mass dimension
$[p] = 1$
$[t] = -1$
$[x] = -1$
$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$
$[\text{Action}] = 0$
$e^{-\frac{S}{\hbar}}$

$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$

$e^{-\frac{S}{\hbar}}$

$\mathcal{L}_{DIRAC} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$

$[\text{Action}] = 0$

$[\mathcal{L}] = +4$

$\text{Action} = \int d^4x \mathcal{L}_{KG}$

-4

$1 \times 1 + 2[\psi] = 4$

$[\psi] = \frac{3}{2}$

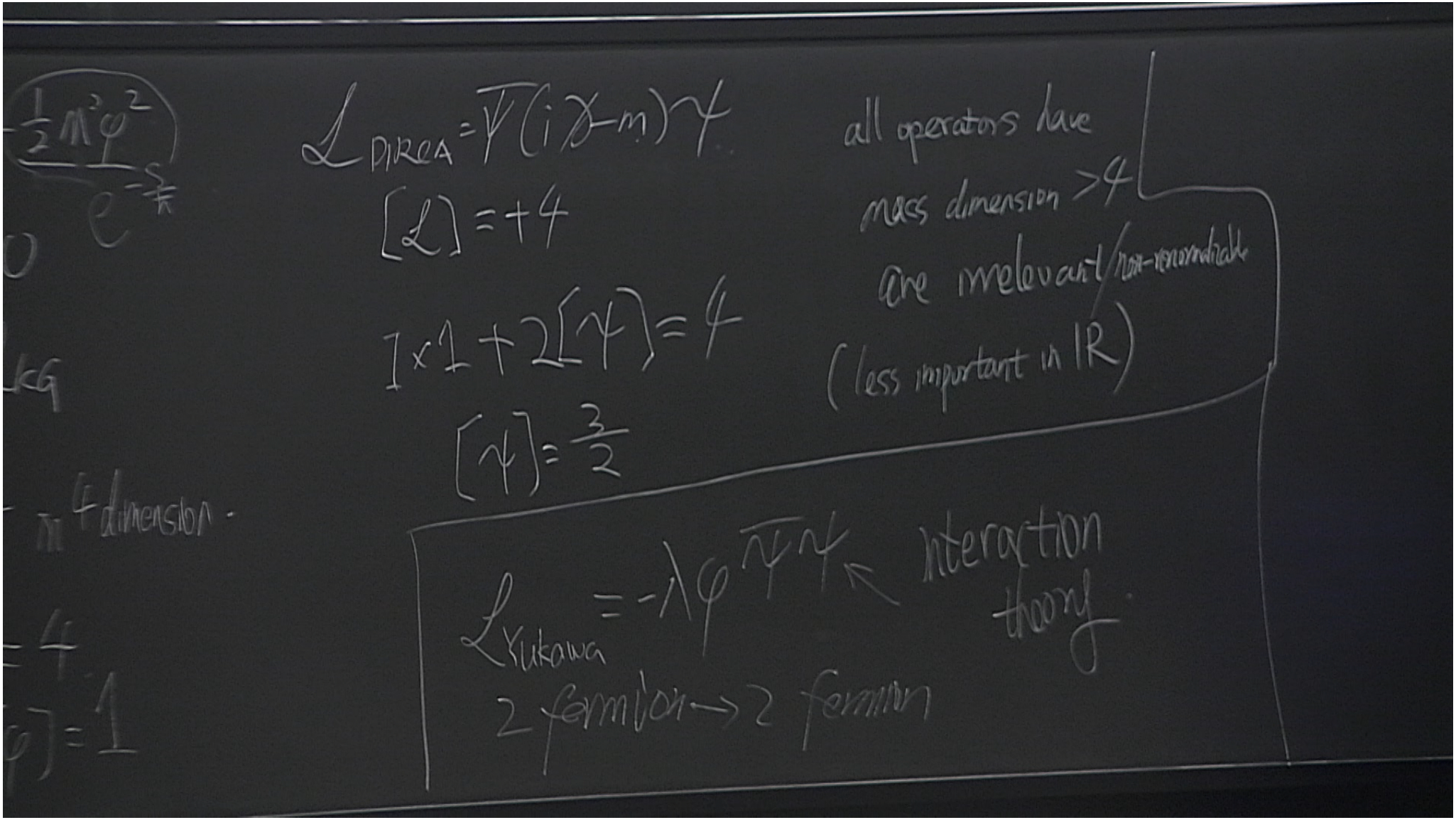
$[\mathcal{L}] = +4$ in 4 dimension.

$2 \times 1 + 2[\varphi] = 4$

$[\varphi] = 1$

power

each thing contribute


$\frac{1}{2}m^2\varphi^2$
$\mathcal{L}_{DIRAC} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$
$[\mathcal{L}] = +4$
$1 \times 1 + 2[\psi] = 4$
$[\psi] = \frac{3}{2}$
all operators have mass dimension > 4 are irrelevant/non-renormalizable (less important in IR)
$\mathcal{L}_{Yukawa} = -\lambda\varphi\bar{\psi}\psi$ ← interaction theory
2 fermion → 2 fermion
4 dimension.
$= 4$
$\varphi] = 1$

$$\langle \Omega | T \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 | \Omega \rangle = \frac{\langle 0 | T \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 e^{\int dx\, i\mathcal{L}_{INT}} | 0 \rangle}{\langle 0 | T e^{\int dx\, i\mathcal{L}_{INT}} | 0 \rangle}$$

→ not worry today

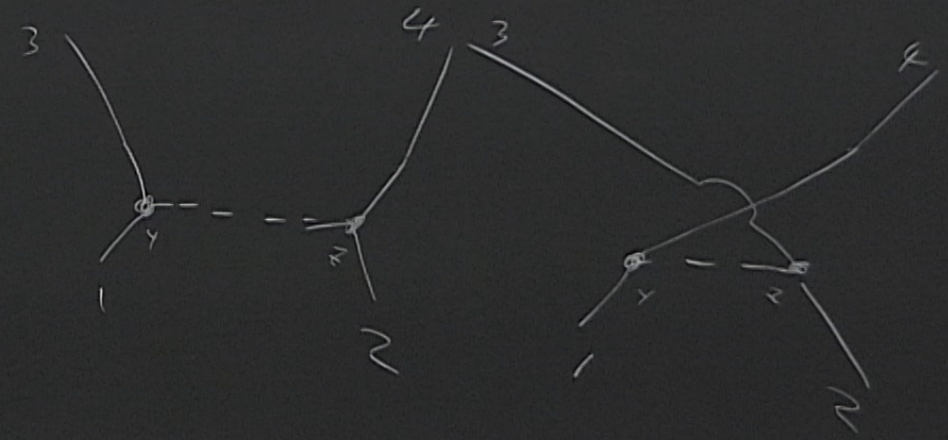
perturbation theory order 0 elastic scattering

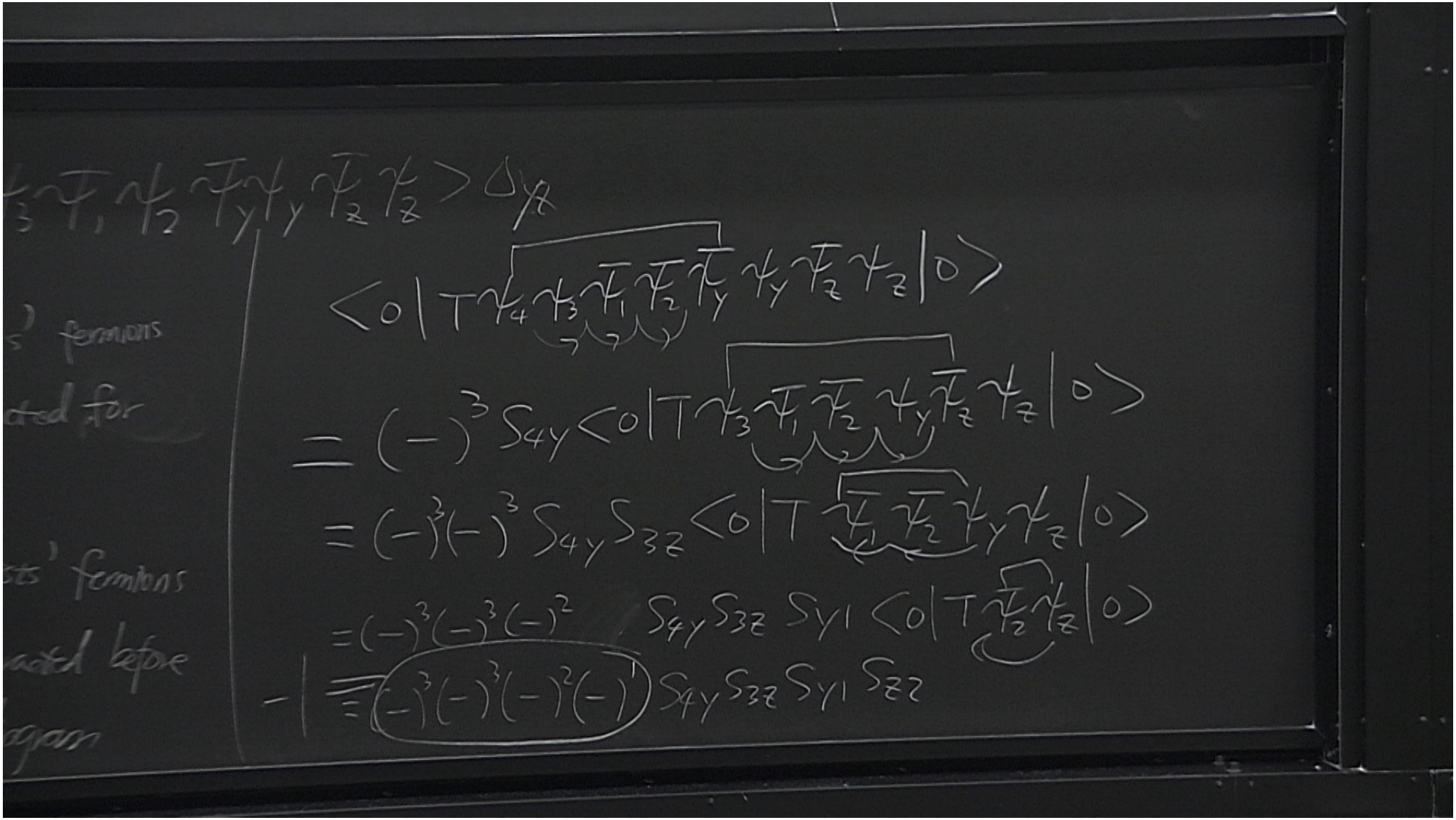
order 1 $\int dy (-i\lambda \varphi_y \bar{\psi}_y \psi_y)$ 0 do not have full contraction

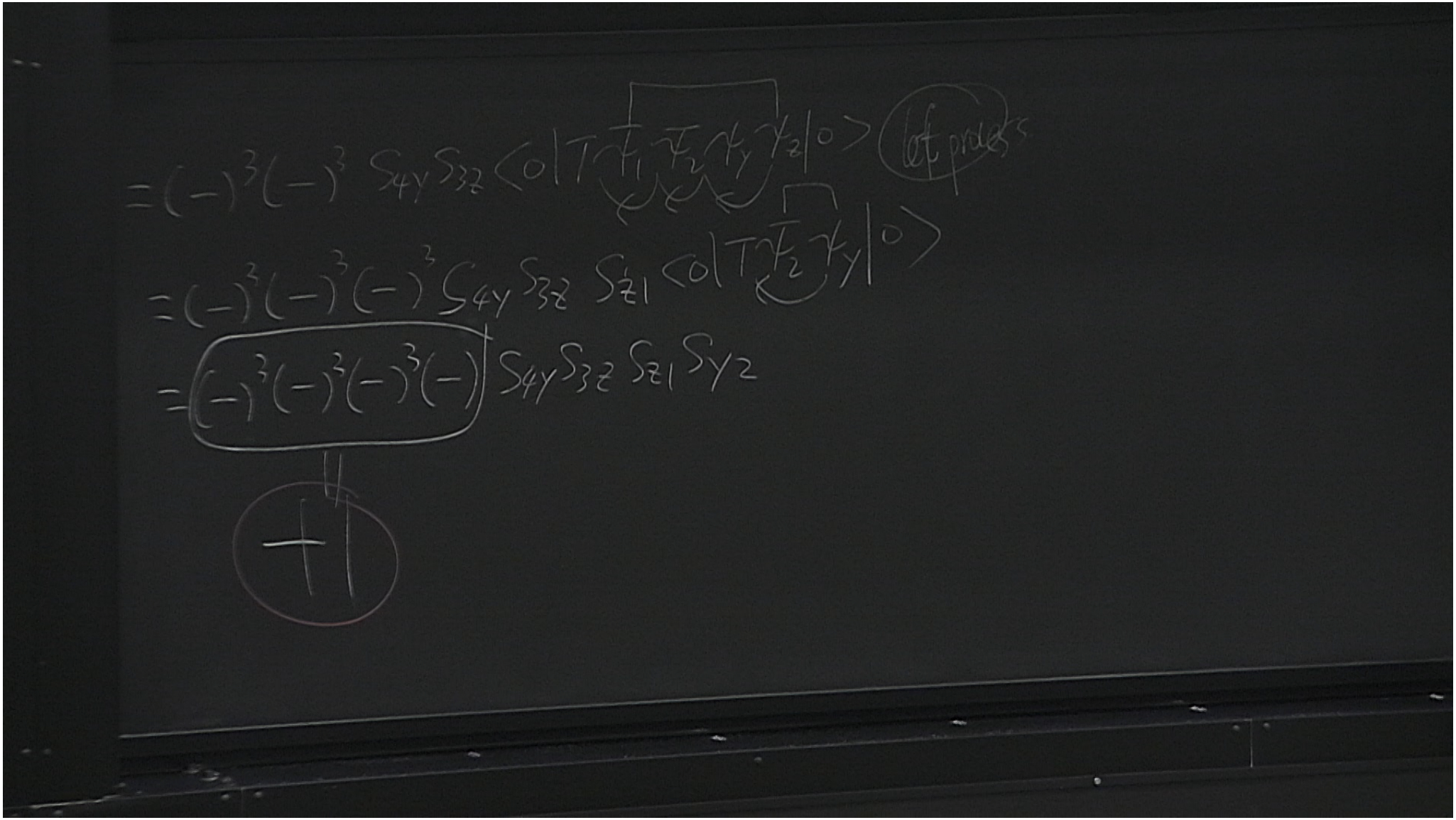
2nd order (leading order) $\frac{(-i\lambda)^2}{2}\langle 0|T\psi_4\psi_3\bar{\psi}_1\bar{\psi}_2\varphi_{y'}\bar{\psi}_{y'}\psi_{y'}\varphi_{z'}\bar{\psi}_{z'}\psi_{z'}|0\rangle$

4 externals

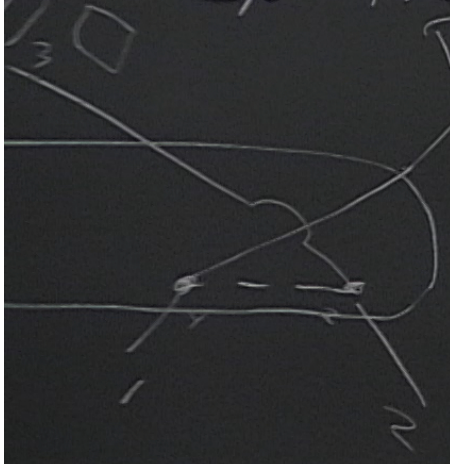
Δ_{yz} 4 internals




fermions
fermions
fermions


$= (-)^3(-)^3 S_{4Y} S_{3Z} \langle 0|T(\bar{T}_1 T_2 \gamma_Y \gamma_Z)|0\rangle$
let pride
$= (-)^3(-)^3(-)^3 S_{4Y} S_{3Z} S_{Z1} \langle 0|T(\bar{T}_2 \gamma_Y)|0\rangle$
$= (-)^3(-)^3(-)^3(-) S_{4Y} S_{3Z} S_{Z1} S_{Y2}$
$+1$

Δ_{yz} 4 internals



fix your external legs

$T(\psi_1, \psi_2) = -T(\psi_2, \psi_1)$

the left scientists' fermions have interacted for left diagram.

And the left scientists' fermions have not interacted before for the right diagram

$\langle 0 | T \psi_4 \psi_3$

$= (-)^3 S_{4y}$

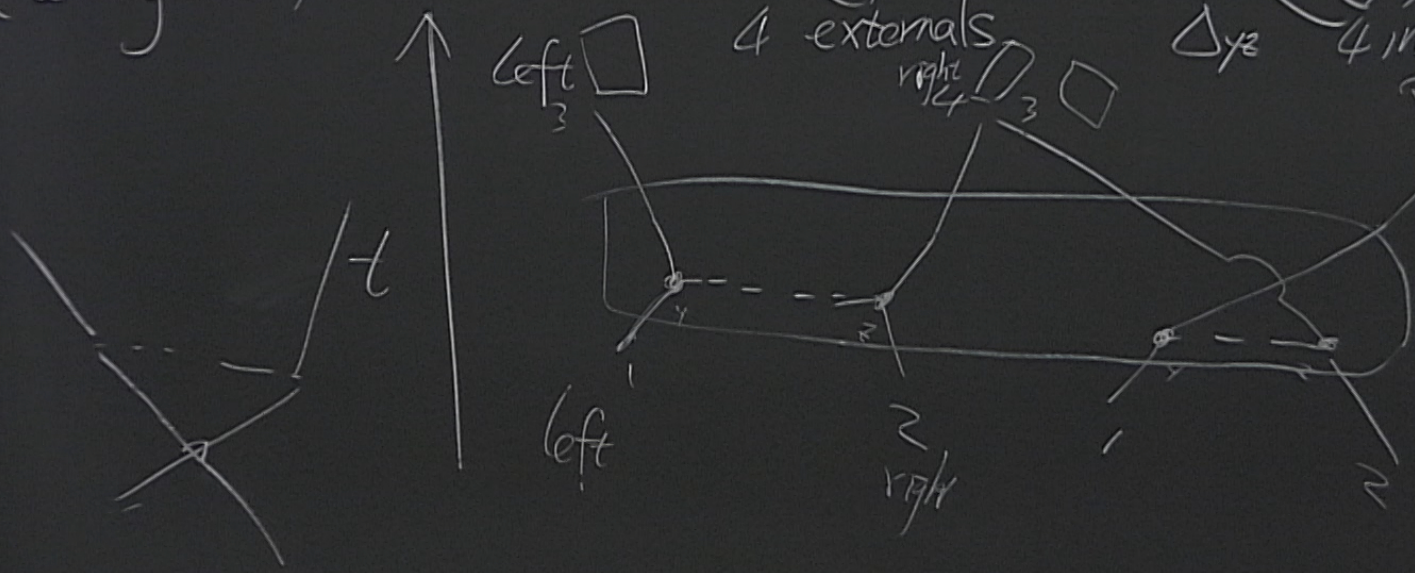
$= (-)^3 (-)^3 S_{4y}$

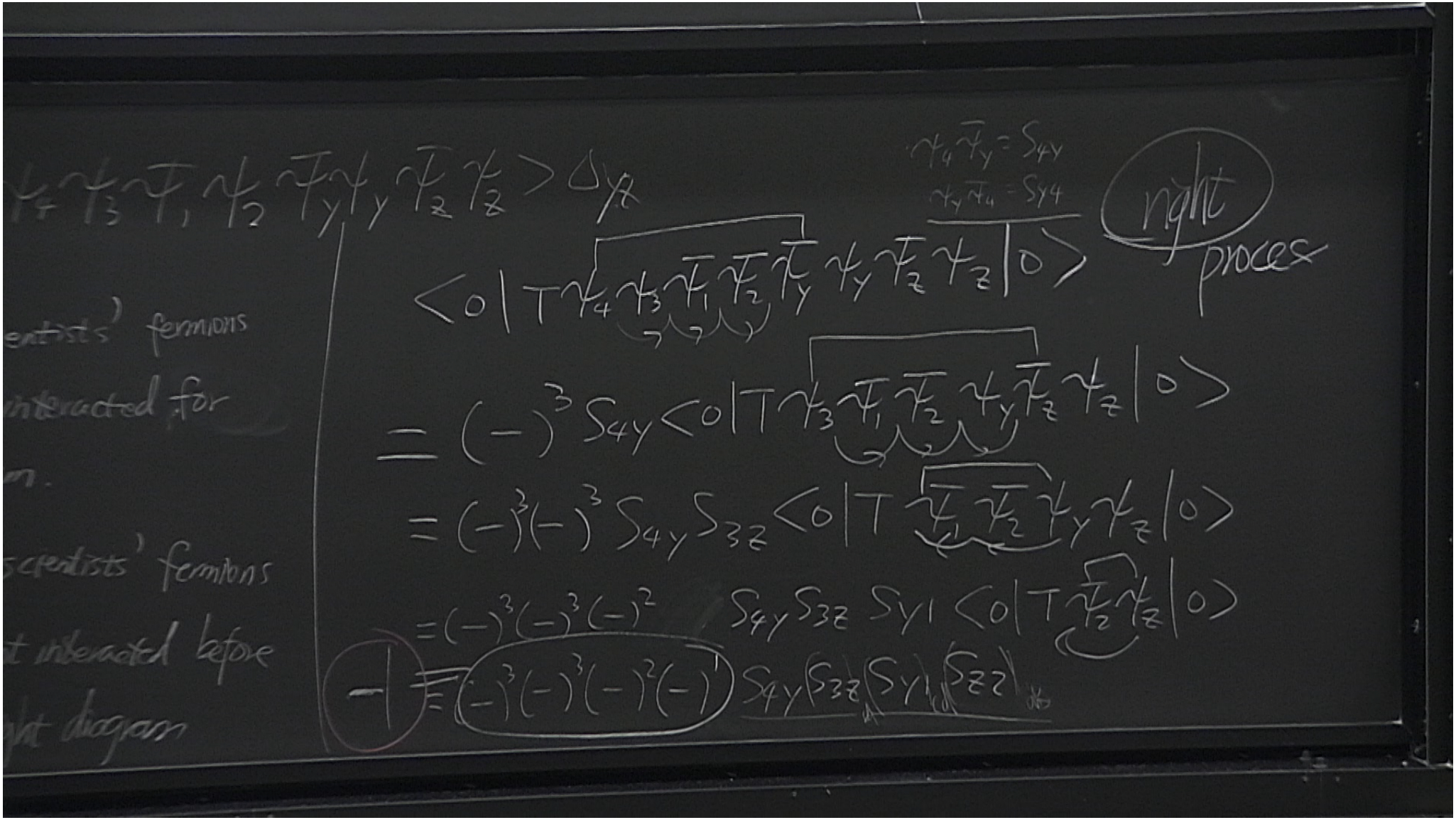
$= (-)^3 (-)^3 (-)^2$

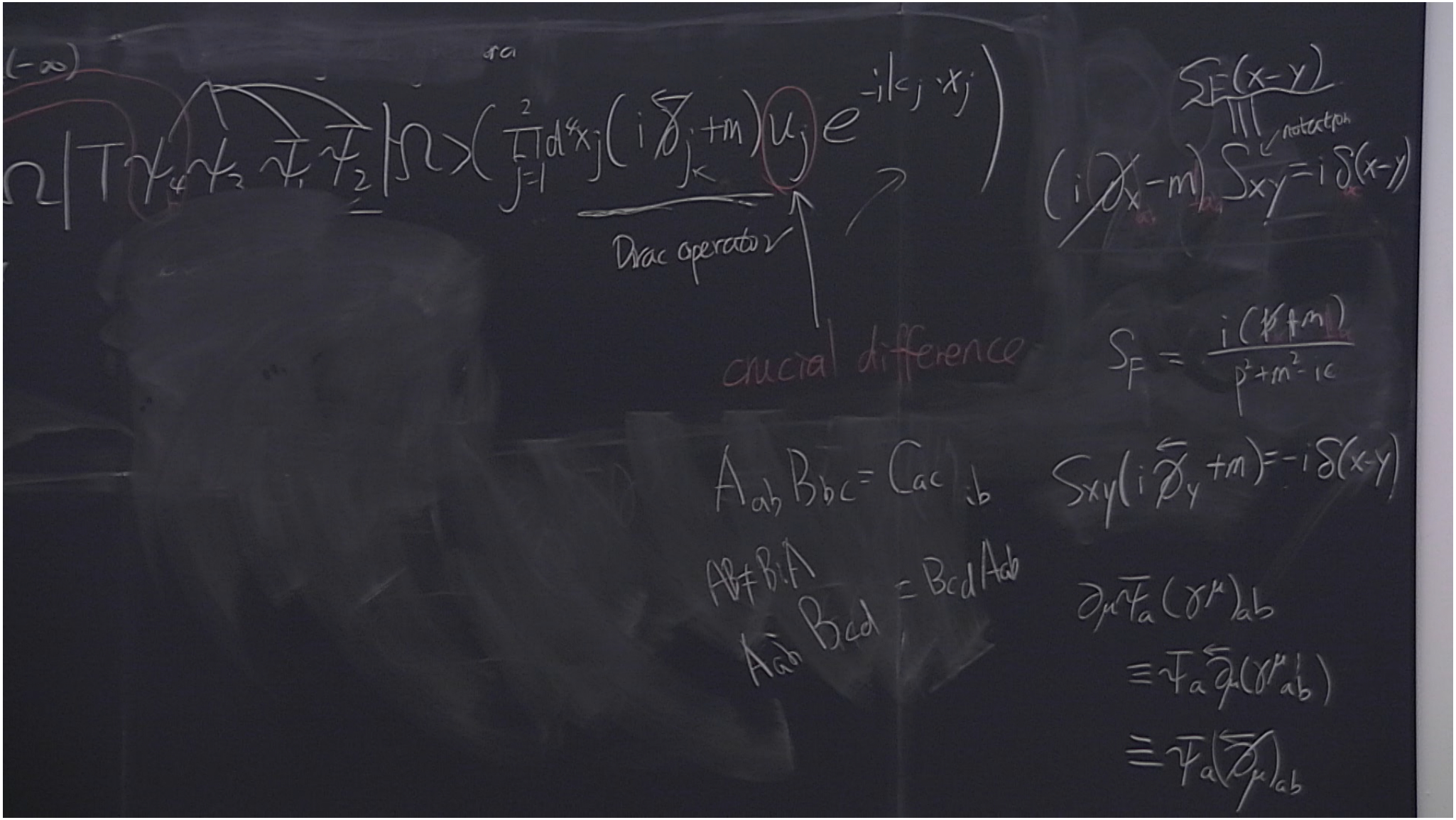
$-1 = (-)^3 (-)^3 (-)^2 (-$

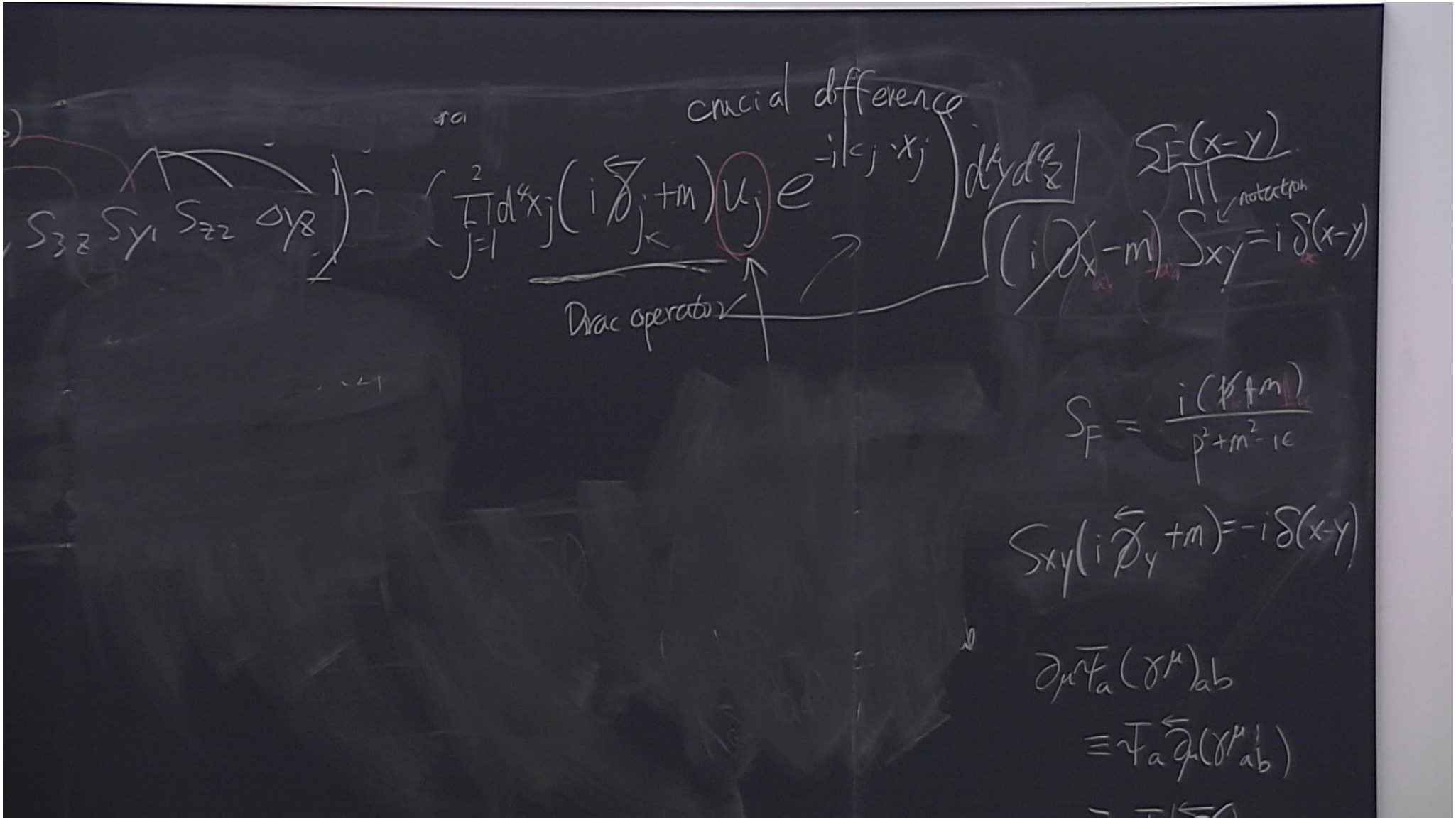
2nd order $\frac{(-i\lambda)^2}{2}\langle 0|T\,\varphi_4\varphi_3\bar{\psi}_1\bar{\psi}_2\,\varphi_y\bar{\psi}_y\psi_y\,\varphi_z\bar{\psi}_z\psi_z|0\rangle =$

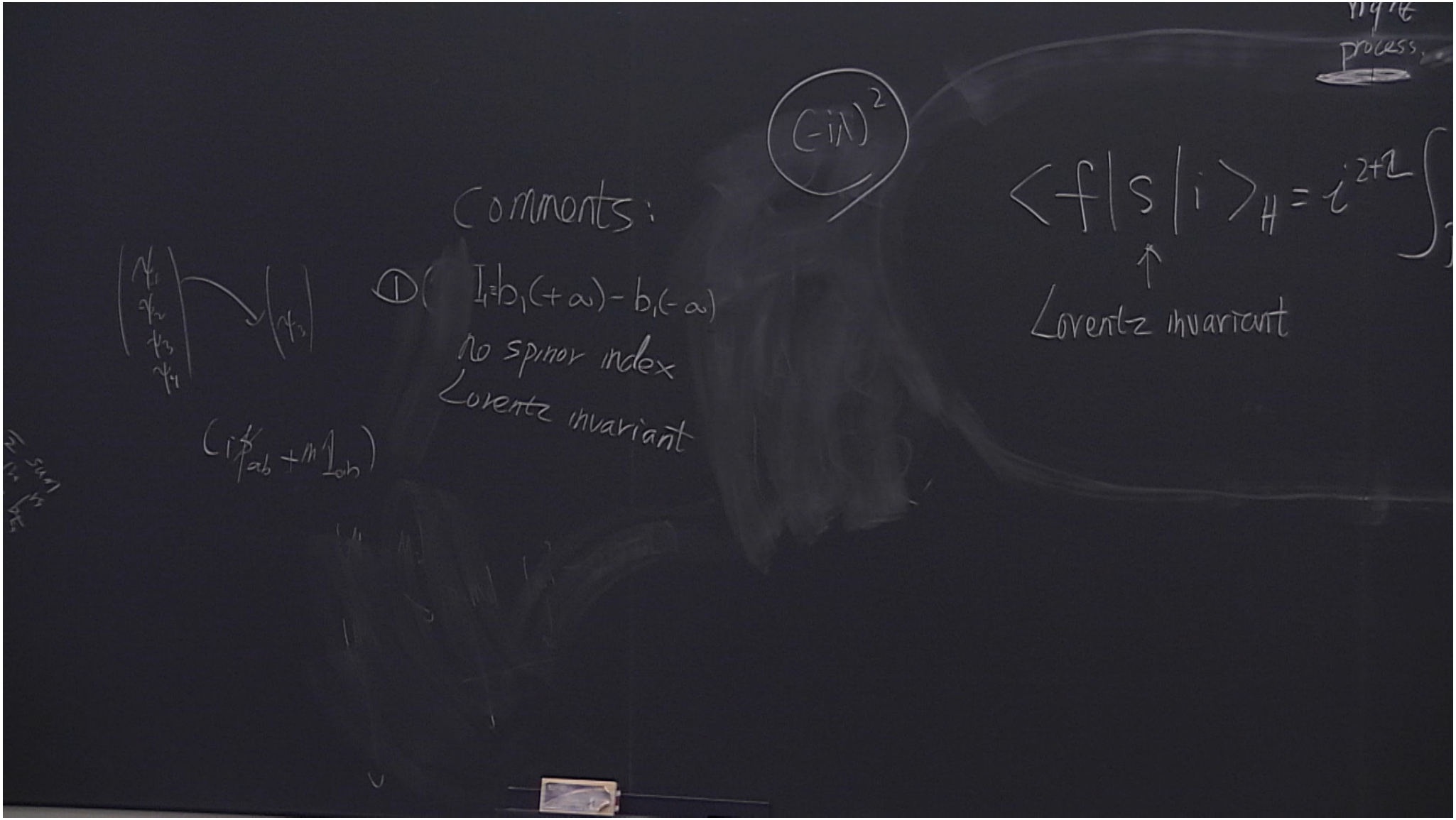
(leading order)

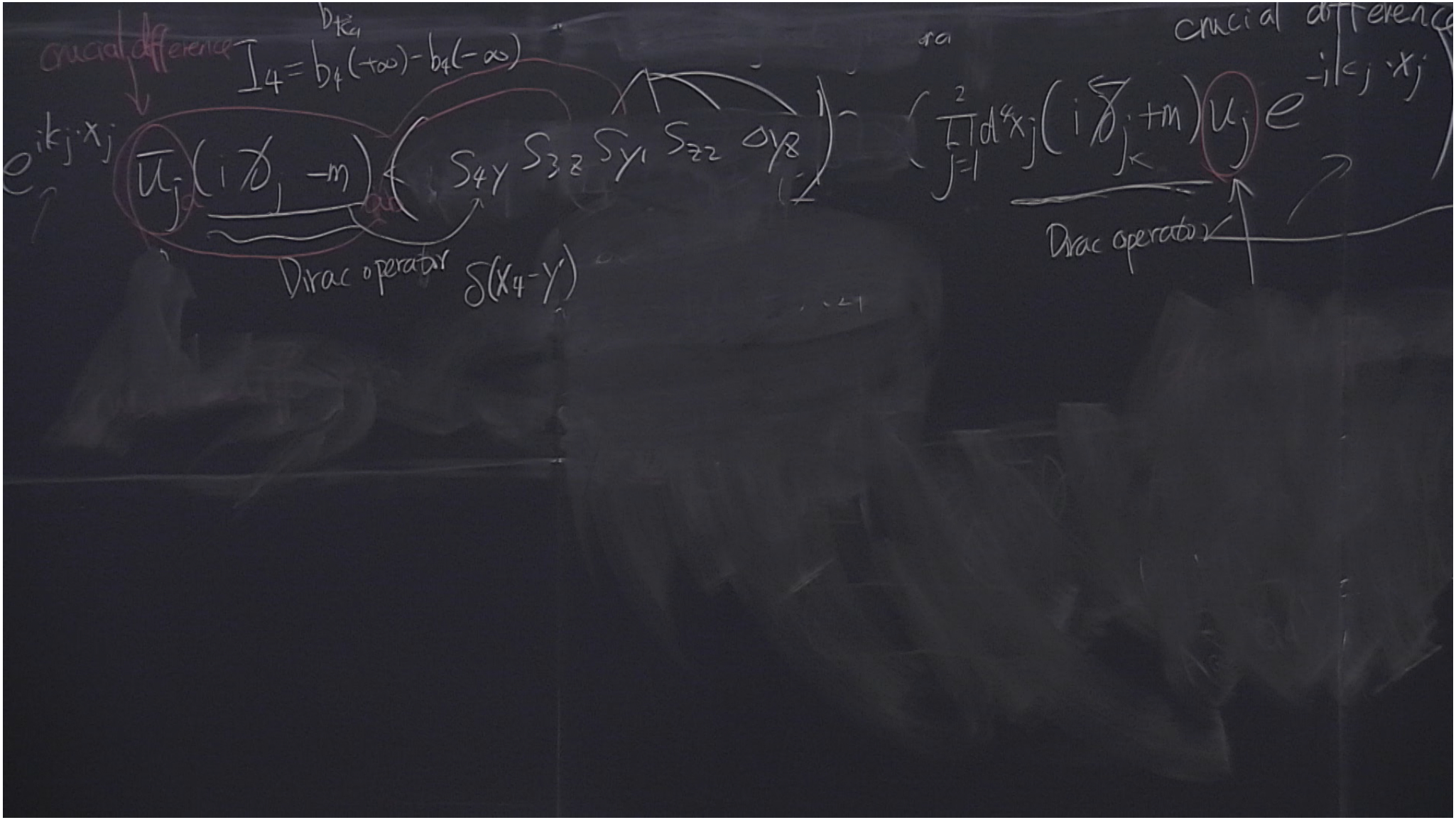


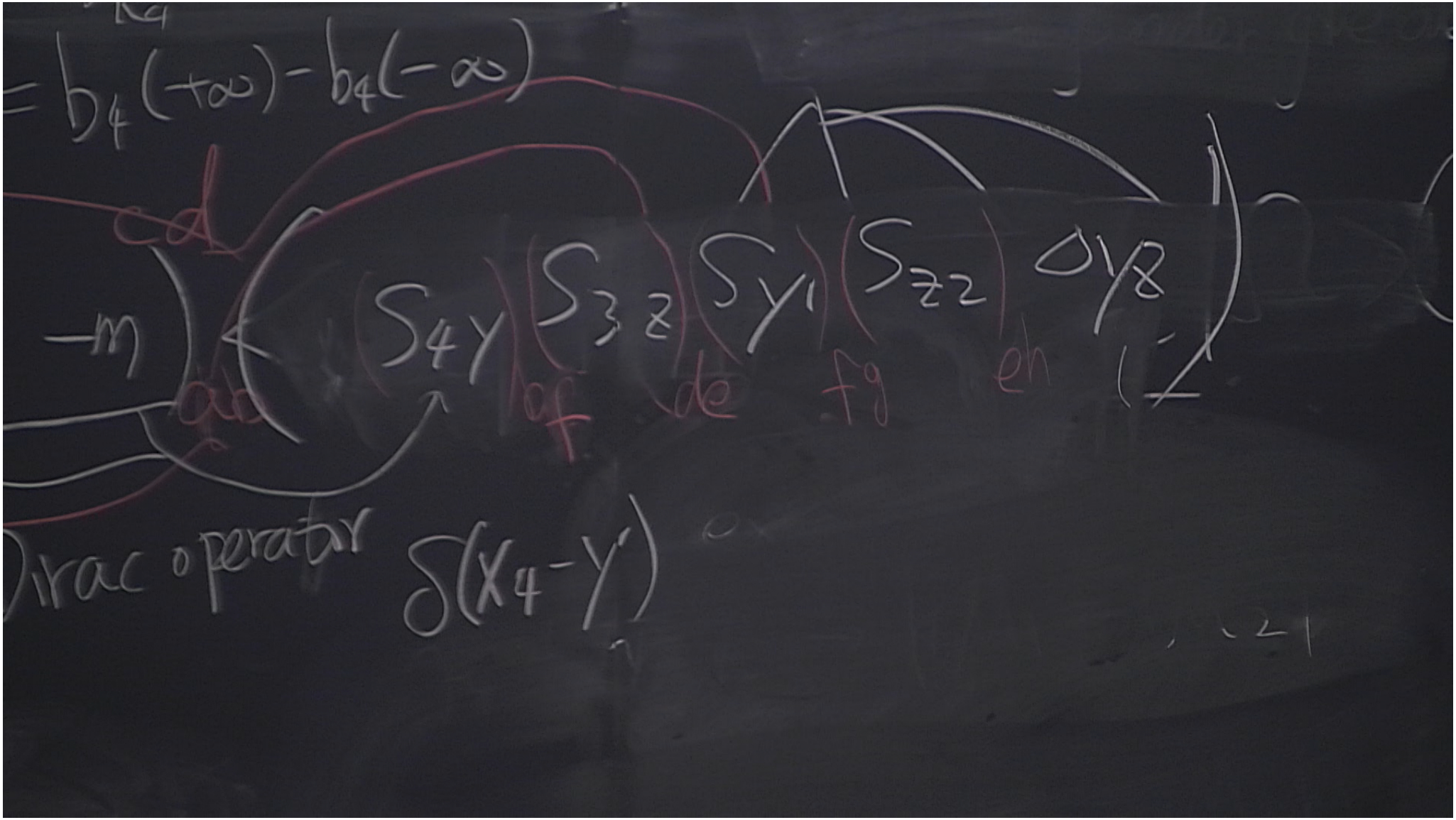

$\psi_4\bar{\psi}_y = S_{4y}$
$\psi_y\bar{\psi}_4 = S_{y4}$
right
proces
entists' fermions
nteracted for
n.
scientists' fermions
t interacted before
ht diagram

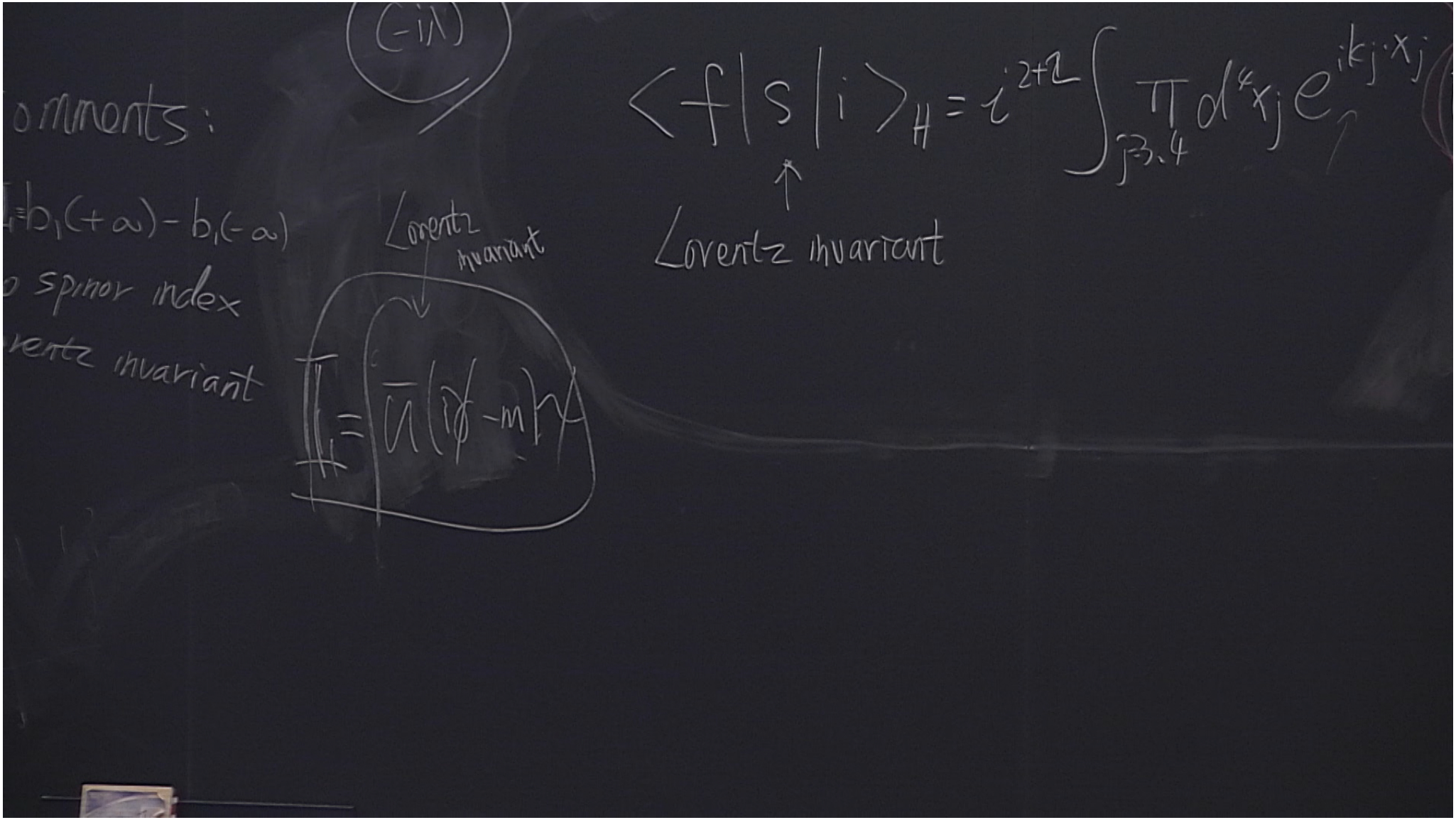

$(-\infty)$
$\Omega | T \psi_4 \psi_3 \psi_2 \psi_1 \bar{\psi}_2 | \Omega \rangle \left(\prod_{j=1}^{2} d^4x_j \left(i \overleftarrow{\not\partial}_j + m \right) u_j e^{-i k_j \cdot x_j} \right)$
Dirac operator
crucial difference
$\Sigma_F = (x-y)$
propagator
$(i \not\partial_x - m) S_{xy} = i \delta(x-y)$
$S_F = \frac{i(\not p + m)}{p^2 + m^2 - i\epsilon}$
$S_{xy} (i \overleftarrow{\not\partial}_y + m) = -i \delta(x-y)$
$A_{ab} B_{bc} = C_{ac}$
$AB \neq BA$
$A_{ab} B_{cd} = B_{cd} A_{ab}$
$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a \overleftarrow{\partial}_\mu (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a (\overleftarrow{\not\partial})_{ab}$

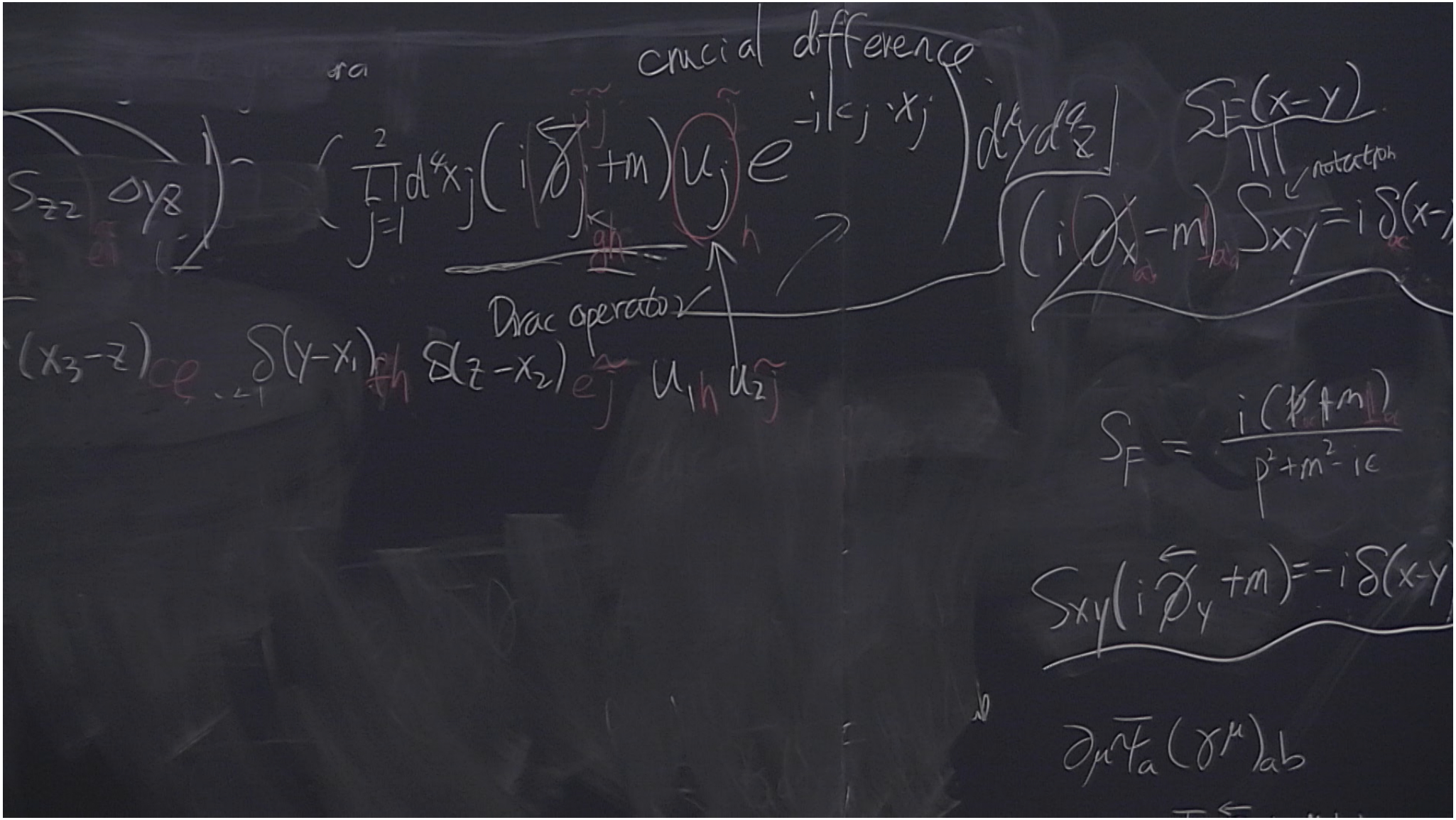

crucial difference
Dirac operator
notation
$S_F = \frac{i(\not{p}+m)}{p^2+m^2-i\epsilon}$
$S_{xy}(i\overleftarrow{\not\partial}_y + m) = -i\delta(x-y)$
$(i\not\partial_x - m)S_{xy} = i\delta(x-y)$
$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a \overleftarrow{\partial}_\mu (\gamma^\mu)_{ab}$

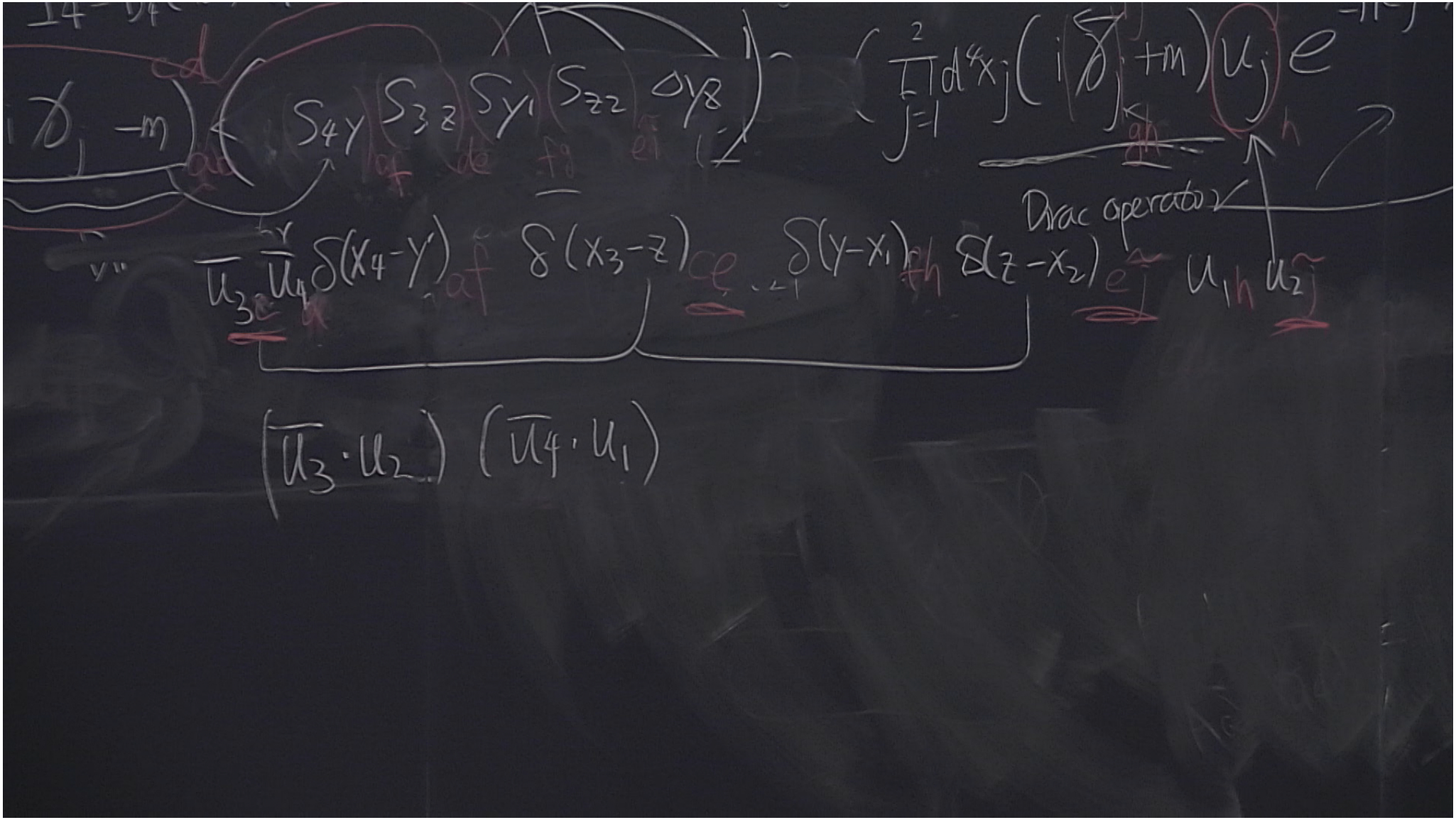

Comments:
$(-iN)^2$
$\langle f|S|i\rangle_H = i^{2+2}\int$
Lorentz invariant
$I_1 = b_1(+\omega) - b_1(-\omega)$
no spinor index
Lorentz invariant
$(i\not{p}_{ab} + m\mathbb{1}_{ab})$


crucial difference
$I_4 = b_4(+\infty) - b_4(-\infty)$
$e^{ik_j \cdot x_j}$
$\bar{u}_j(i\not{\partial}_1 - m)$
$S_{4Y} S_{3Z} S_{Y1} S_{Z2} \Theta/8$
Dirac operator
$\delta(x_4 - y')$
crucial difference
$\prod_{j=1}^{2} d^4x_j (i\not{\partial}_j + m) u_j e^{-i k_j \cdot x_j}$
Dirac operator


$= b_4(+\infty) - b_4(-\infty)$
$-m$
S_{4Y} S_{3Z} S_{Y1} S_{Z2} O_{18}
cd
ab
af
de
fg
eh
Dirac operator
$\delta(X_4 - Y')$


(-iN)
omments:
$\langle f|S|i\rangle_H = i^{2+2}\int_{j=3,4}\Pi\, d^4x_j\, e^{ik_j\cdot x_j}$
$=b_1(+\omega)-b_1(-\omega)$
spinor index
Lorentz invariant
Lorentz invariant
Lorentz invariant
$\bar{u}(p-m)v$


crucial difference
Dirac operator
notation
$S_F = \frac{i(\not{p}+m)}{p^2+m^2-i\epsilon}$
$(i\not{\partial}_x - m)S_{xy} = i\delta(x-y)$
$S_{xy}(i\not{\partial}_y + m) = -i\delta(x-y)$
$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab}$


Dirac operator

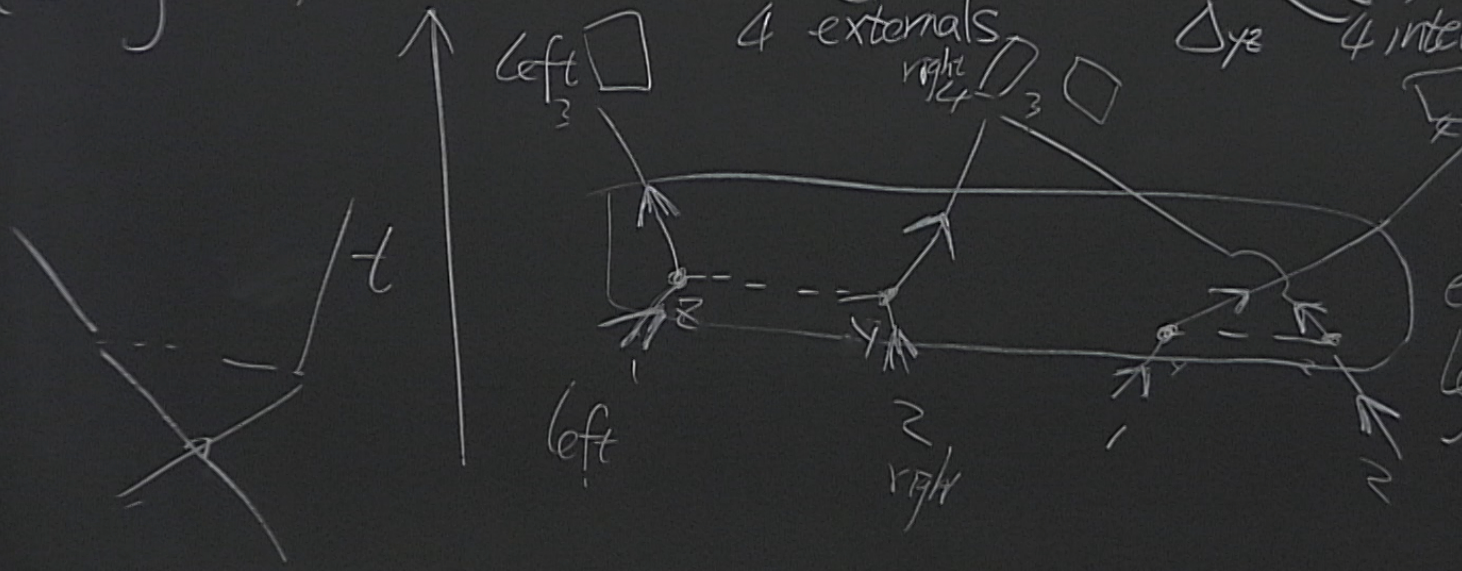
2nd order (leading order)

$$\frac{(-i\lambda)^2}{2}\langle 0|T\phi_{x_4}\phi_{x_3}\phi_{x_1}\phi_{x_2}\phi_{y_1}\phi_{y_2}\phi_{y_3}\phi_{y_4}\phi_{z_1}\phi_{z_2}|0\rangle = \langle$$

4 externals

Δ_{yz}

4 internals

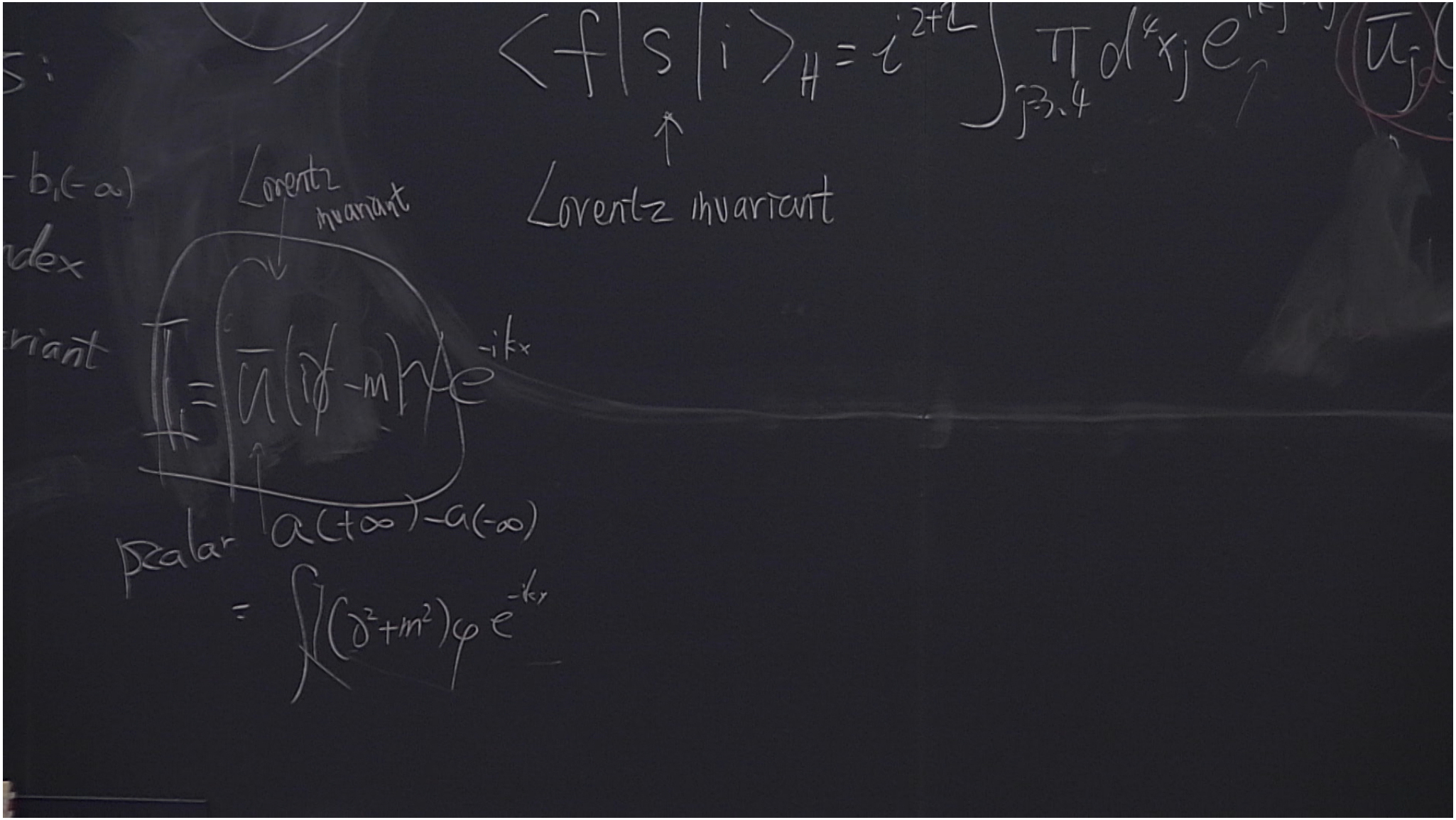


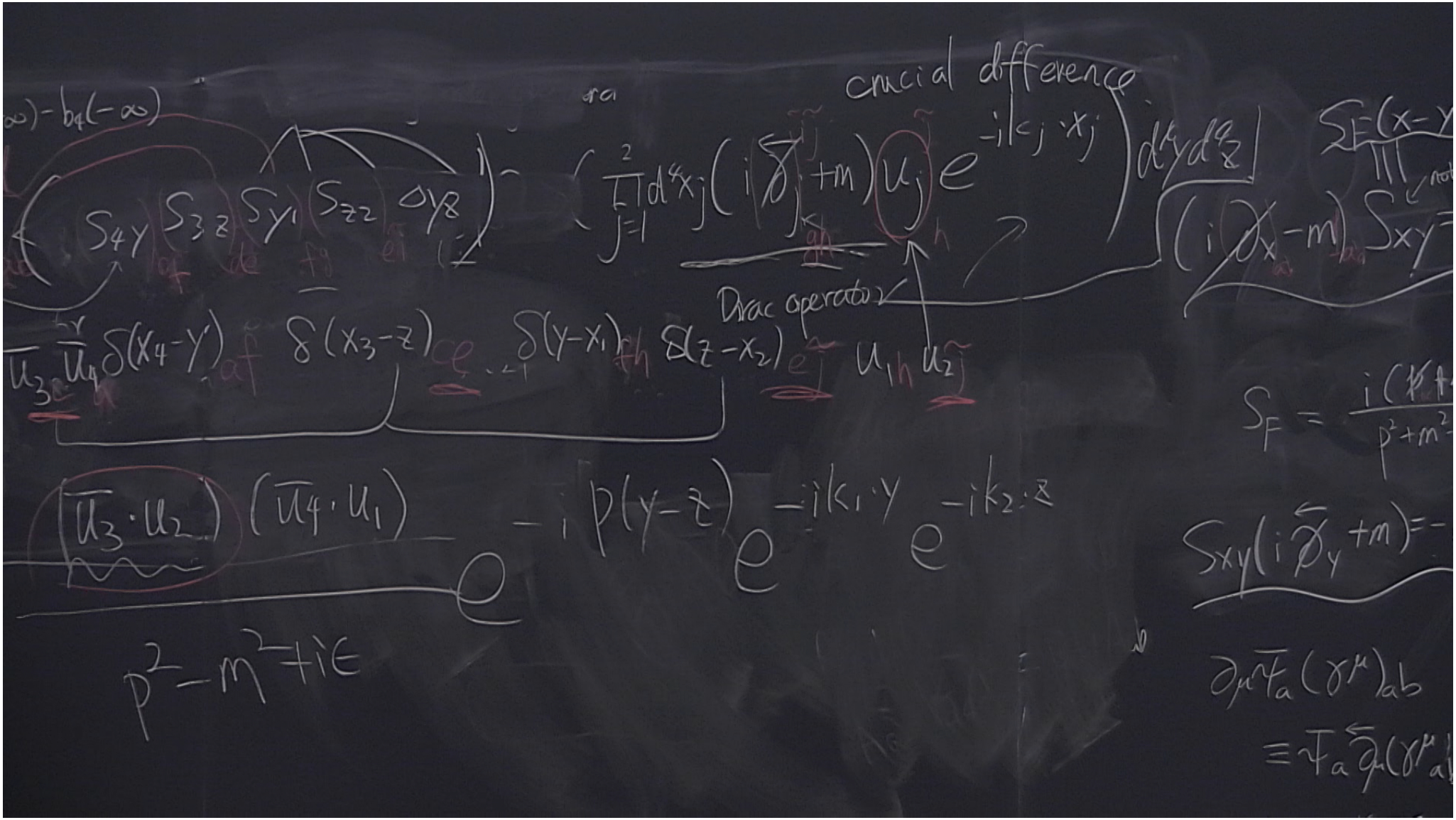
the

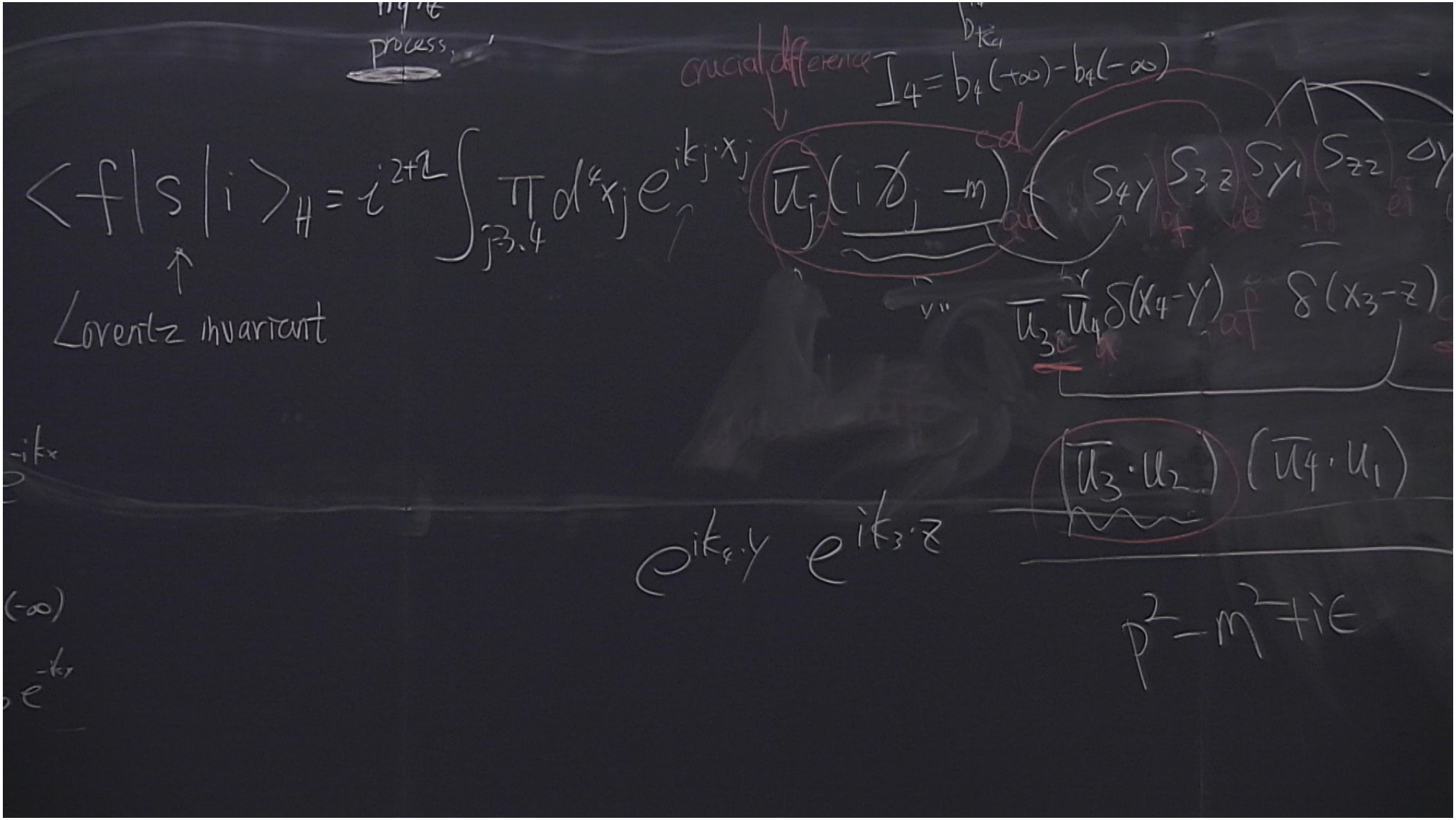
left

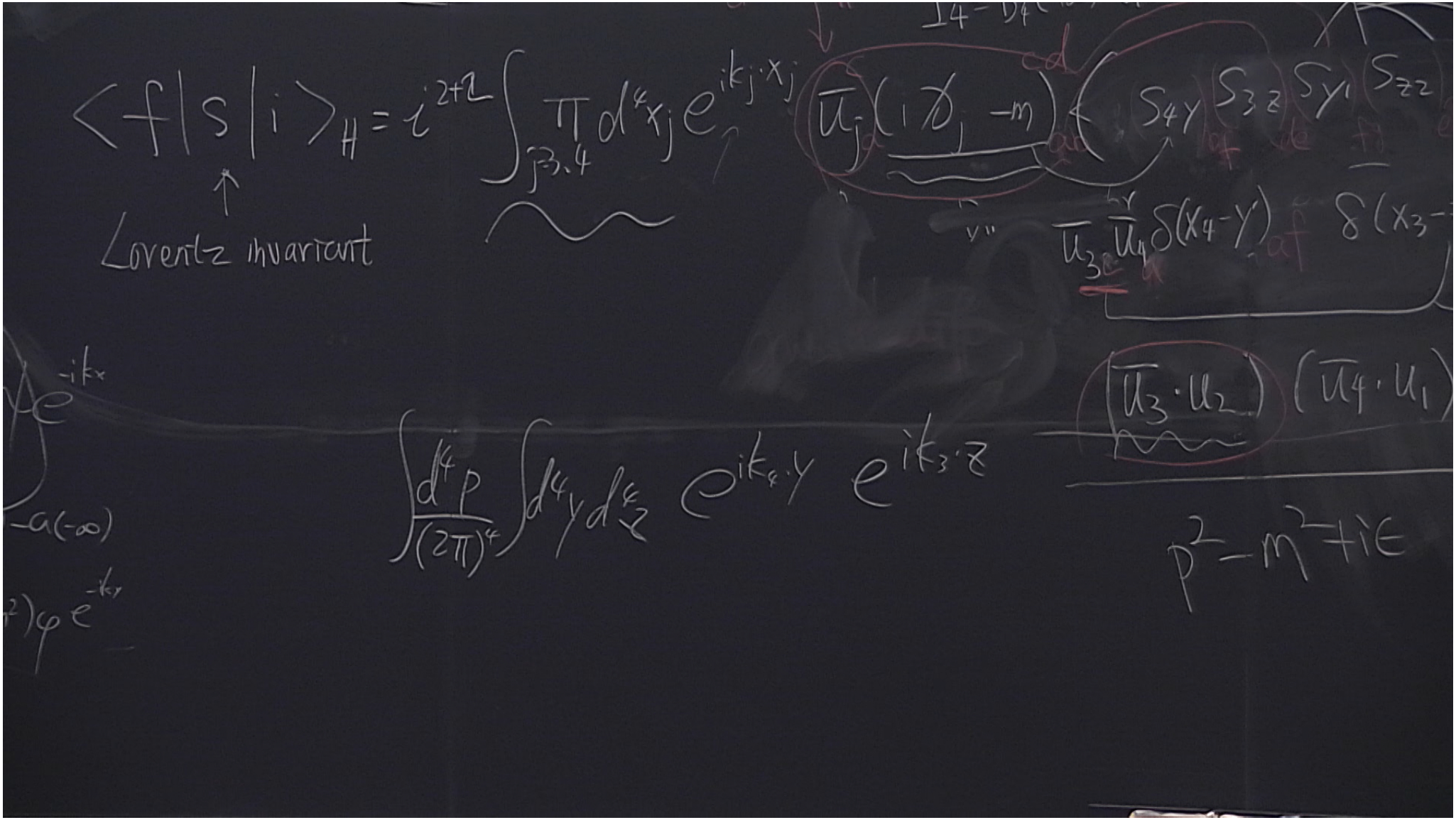
And

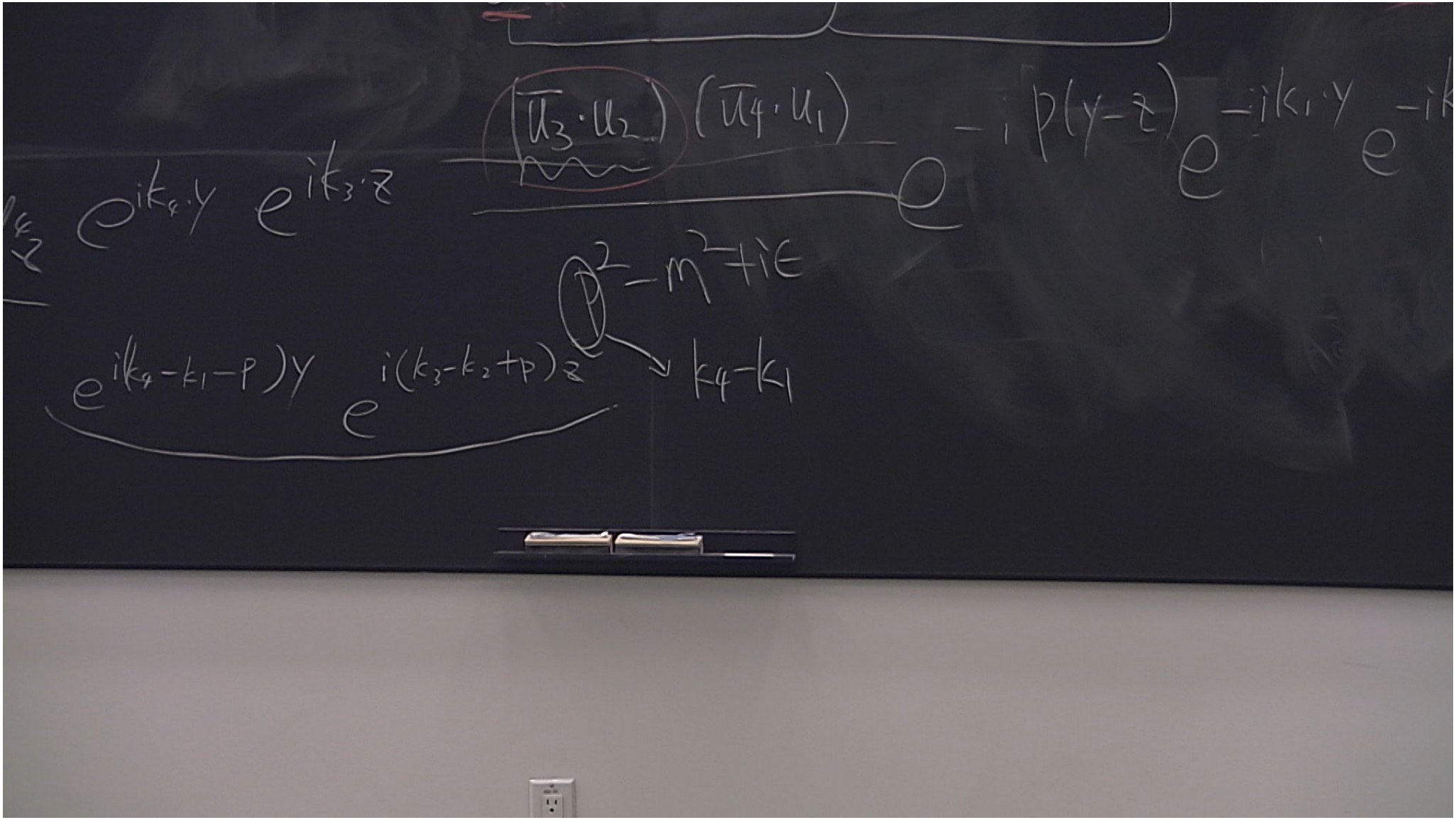
$T(\phi_{x_1}\phi_{x_2}) = T(\phi_{x_1}\phi_{x_3})$

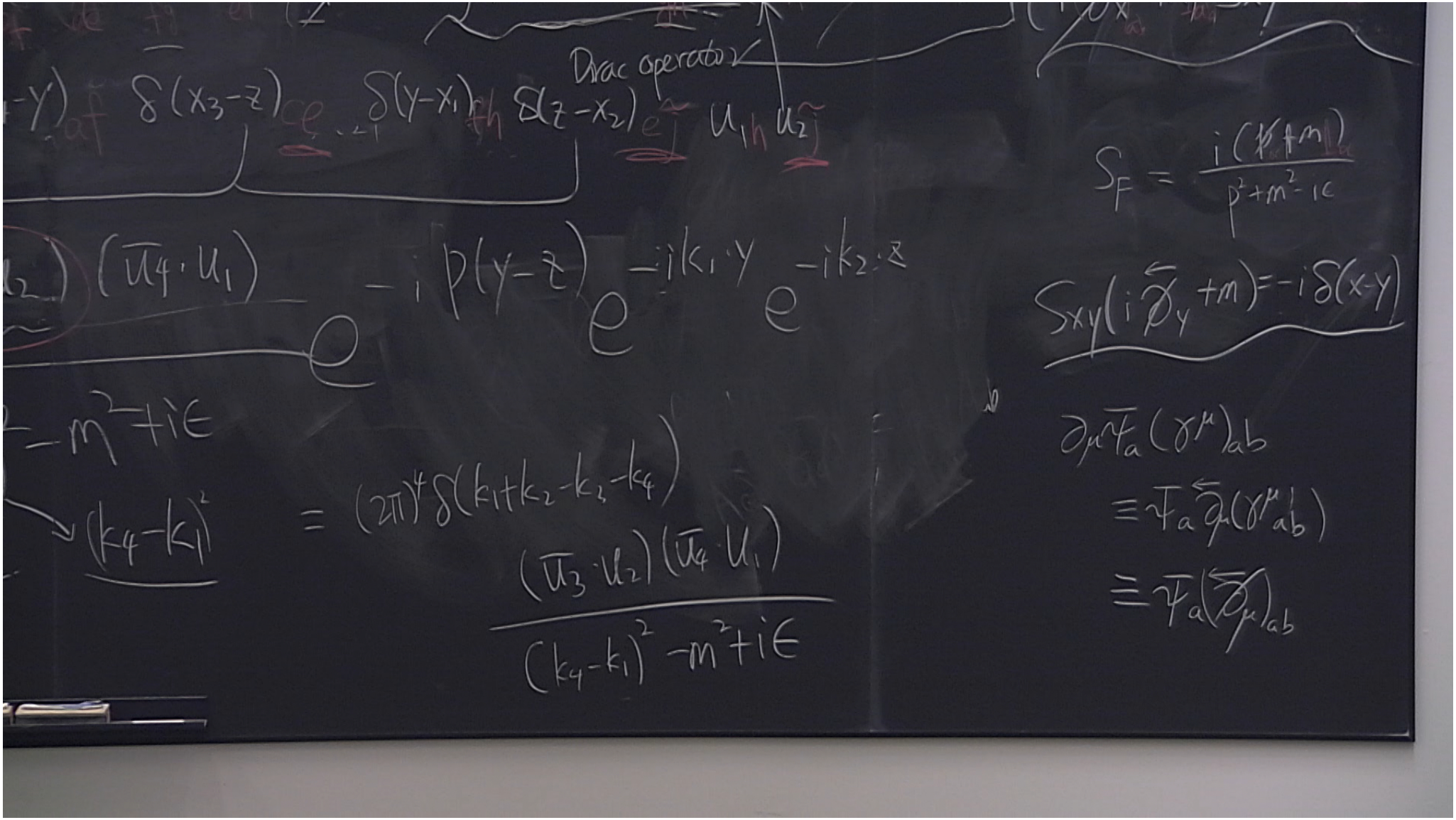

$\langle f|S|i\rangle_H = i^{2+L}\int \prod_{j=3,4} d^4x_j e^{}$
Lorentz invariant
Lorentz invariant
$b_i(-\infty)$
dex
riant
$\bar{u}(i\not\partial - m)$
e^{-ikx}
Scalar
$a(+\infty) - a(-\infty)$
$= \int (\partial^2 + m^2)\varphi\, e^{-ikx}$

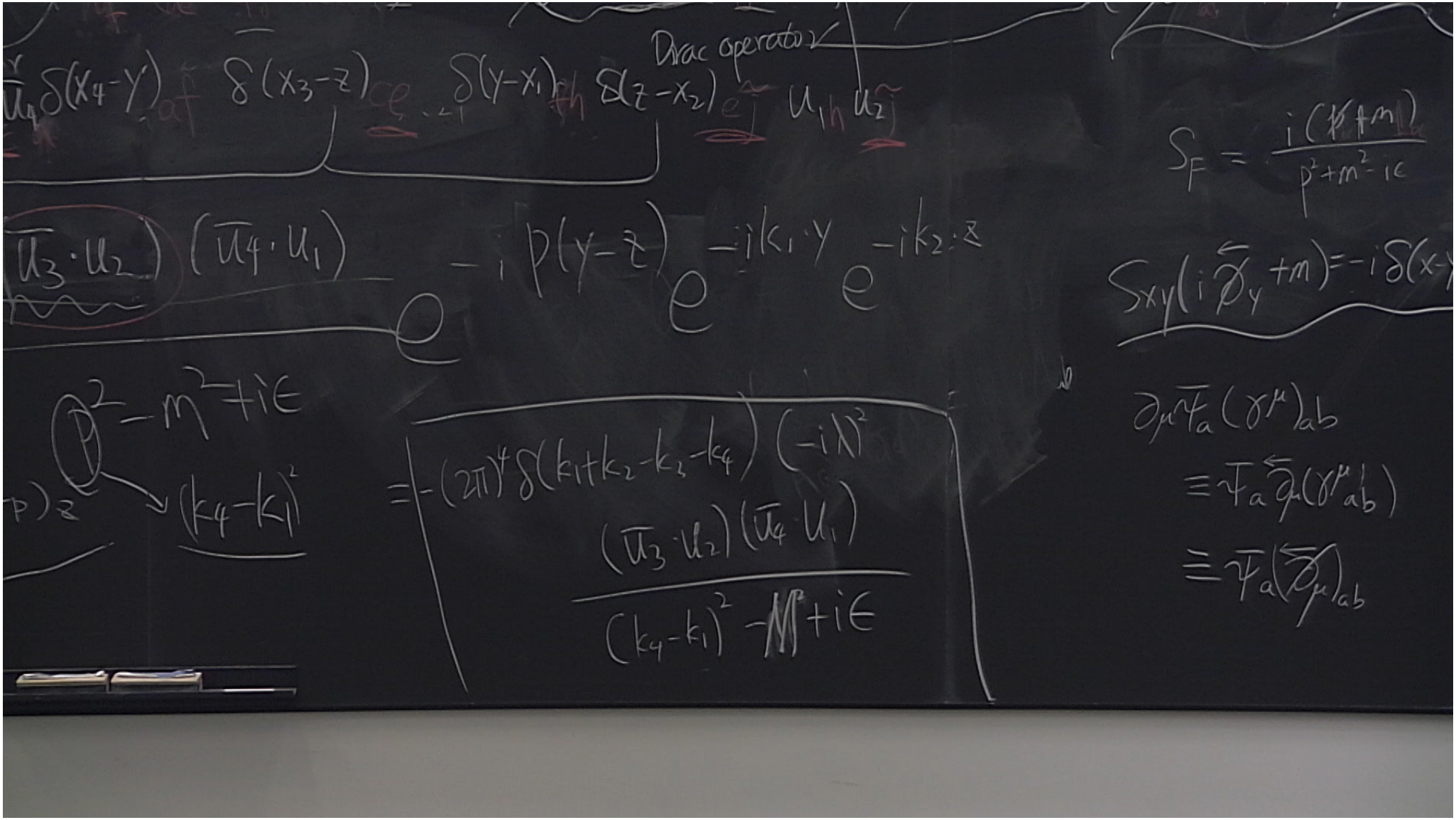

crucial difference
Dirac operator
$p^2 - m^2 + i\epsilon$


process
crucial difference
$I_4 = b_4(+\infty) - b_4(-\infty)$
$\langle f|S|i\rangle_H = i^{2+2} \int \prod_{j=3,4} d^4x_j e^{ik_j \cdot x_j}$
$\bar{u}_2(i\gamma_\mu - m)$
S_{4y} S_{3z} S_{y1} S_{z2}
Lorentz invariant
$\bar{u}_3 \bar{u}_4 \delta(x_4 - y)$ $\delta(x_3 - z)$
$e^{ik_4 \cdot y}$ $e^{ik_3 \cdot z}$
$(\bar{u}_3 \cdot u_2)(\bar{u}_4 \cdot u_1)$
$p^2 - m^2 + i\epsilon$


$\langle f|S|i\rangle_H = i^{2+2}\int \prod_{j=3,4} d^4x_j e^{ik_j\cdot x_j}$
Lorentz invariant
$\bar{u}_j(i\not{\partial}_j - m)$
$S_{4Y}\ S_{3Z}\ S_{Y1}\ S_{Z2}$
$\bar{u}_3\ \bar{u}_4\ \delta(x_4-y)\ \delta(x_3-$
$e^{-ik\cdot x}$
$-a(-\infty)$
$\varphi\, e^{-ik\cdot x}$
$\int \frac{d^4p}{(2\pi)^4}\int d^4y\, d^4z\ e^{ik_4\cdot y}\ e^{ik_3\cdot z}$
$(\bar{u}_3\cdot u_2)\ (\bar{u}_4\cdot u_1)$
$p^2 - m^2 + i\epsilon$


$(\bar{u}_3 \cdot u_2)(\bar{u}_4 \cdot u_1)$
$e^{ik_4 \cdot y}\, e^{ik_3 \cdot z}$
$e^{-ip(y-z)}$
$e^{-ik_1 \cdot y}$
$p^2 - m^2 + i\epsilon$
$e^{i(k_4 - k_1 - p)y}\, e^{i(k_3 - k_2 + p)z}$
$k_4 - k_1$


Dirac operator
$\delta(x_3-z)$
$\delta(y-x_1)$
$\delta(z-x_2)$
u_1 u_2
$S_F = \frac{i(\not{p}+m)}{p^2+m^2-i\epsilon}$
$S_{xy}(i\overleftarrow{\partial}_y+m) = -i\delta(x-y)$
$(\bar{u}_4 \cdot u_1)$
$e^{-ip(y-z)}$ $e^{-ik_1 \cdot y}$ $e^{-ik_2 \cdot z}$
$-m^2+i\epsilon$
$(k_4-k_1)^2$
$= (2\pi)^4 \delta(k_1+k_2-k_3-k_4)$
$\frac{(\bar{u}_3 \cdot u_2)(\bar{u}_4 \cdot u_1)}{(k_4-k_1)^2-m^2+i\epsilon}$
$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a \overleftarrow{\partial}_\mu (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a (\overleftarrow{\not{\partial}})_{ab}$


Dirac operator
$S_F = \frac{i(\not{p}+m)}{p^2+m^2-i\epsilon}$
$S_{xy}(i\overleftarrow{\not\partial}_y + m) = -i\delta(x-y)$
$\partial_\mu \bar{\psi}_a (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a \overleftarrow{\partial}_\mu (\gamma^\mu)_{ab}$
$\equiv \bar{\psi}_a (\overleftarrow{\not\partial})_{ab}$
$p^2 - m^2 + i\epsilon$
$(k_4 - k_1)^2$
$= \frac{-(2\pi)^4 \delta(k_1+k_2-k_3-k_4)(-i\lambda)^2 (\bar{u}_3 \cdot u_2)(\bar{u}_4 \cdot u_1)}{(k_4-k_1)^2 - M^2 + i\epsilon}$

2nd order (leading order)

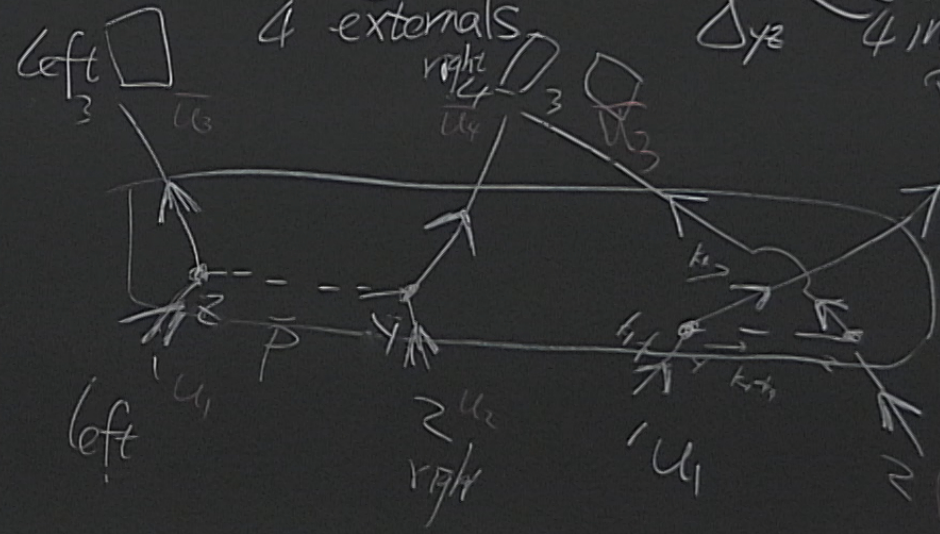
$$\frac{(-i\lambda)^2}{2}\langle 0|T\,\chi_4\chi_3\bar{\chi}_1\bar{\chi}_2\,\varphi_1\bar{\chi}_1\chi_1\,\varphi_2\bar{\chi}_2\chi_2|0\rangle =$$

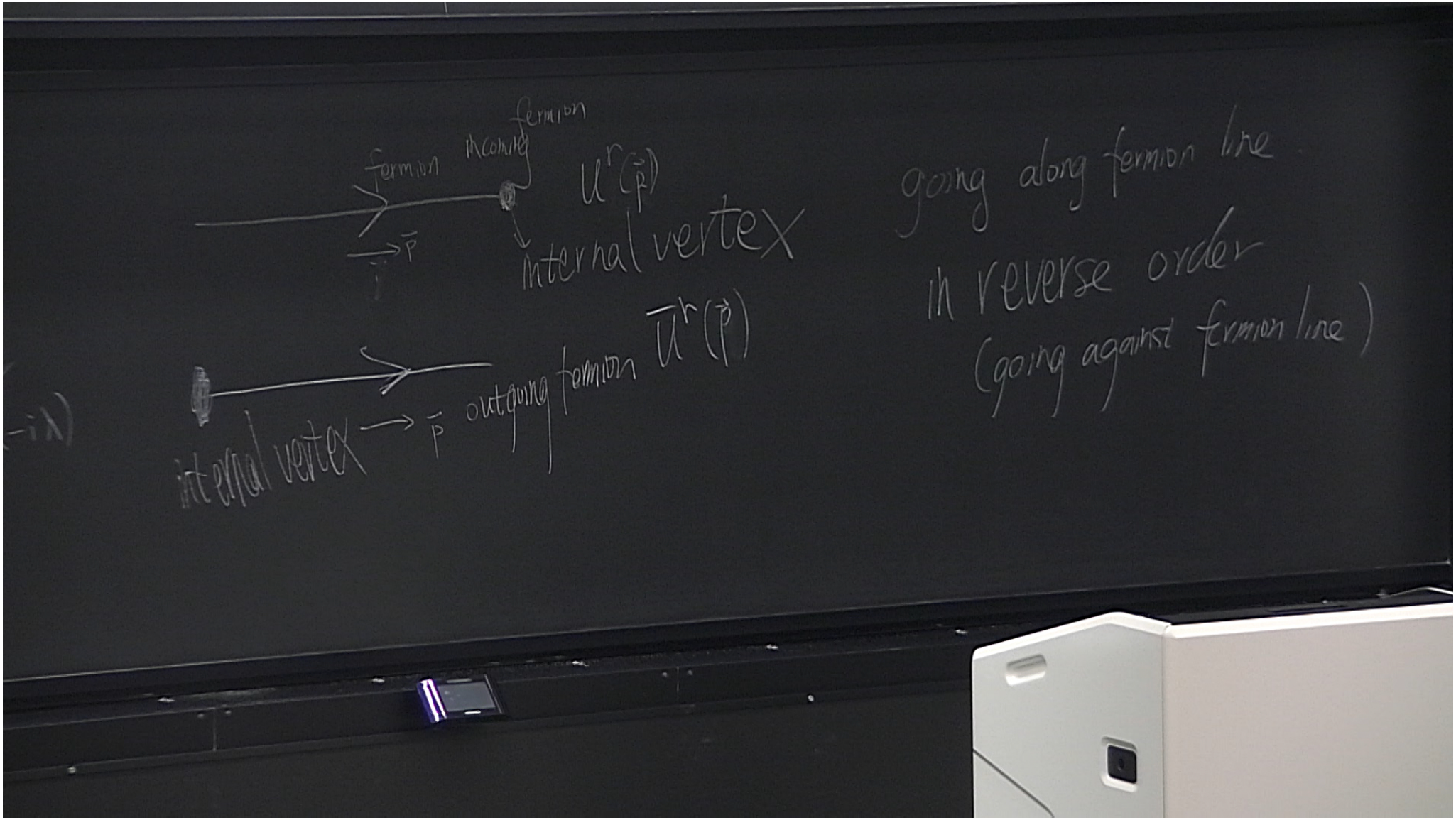
4 externals

Δy_3

4 internals

$$\frac{(\bar{u}_3\cdot u_1)(\bar{u}_4\cdot u_2)}{(k_3-k_1)^2-M^2+i\epsilon}$$




fermion
incoming fermion
$u^r(\vec{p})$
$\vec{p}$
internal vertex
going along fermion line
in reverse order
(going against fermion line)
internal vertex
$\vec{p}$ outgoing fermion $\bar{u}^r(\vec{p})$