Title: PSI 2019/2020 - Quantum Field Theory I (Wohns/Xu) - Lecture 12

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Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

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Last week

We quantize Dirac field

with anti-commutator relationship.

Last week

We quantize Dirac field

with anti-commutator relationship.

Today + tomorrow

today ill general theory

tomorrow Yukawa: $\boxed{\varphi \bar{\psi} \psi}$

c field

tor relationship.

aw

l theory

ory 4

Goal: cross section → experimentalist.

$$\sigma \longrightarrow |\mathcal{M}|^2 \rightarrow \underline{i\mathcal{M}}$$

current goal $i\mathcal{M}$: scattering amplitude

$$\Big\Downarrow \text{LSZ}$$

toolkit so far   calculate correlation function in free field vacuum

$$\langle 0 | \text{field} \ \overline{\text{fields}} | 0 \rangle$$

...tion → experimentalist.

→ $|\mathcal{M}|^2 \rightarrow \underline{i\mathcal{M}}$

$i\mathcal{M}$: scattering amplitude

LSZ

calculate correlation function in free field vacuum

$\langle 0 | \text{field}\ \text{field} | 0 \rangle$

scalar case.

$$\langle f | S | i \rangle_{\text{Heisenberg}} \qquad a_1^\dagger \equiv a_{k_1}^\dagger$$

$$S|i\rangle = a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)\, |\Omega\rangle$$

$$\left(|f\rangle = a_3^\dagger(+\infty)\, a_4^\dagger(+\infty)\, |\Omega\rangle \right)^\dagger$$

$$\langle f | S | i \rangle_H = \langle \Omega | a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty) | \Omega \rangle$$

case.

$\rangle_{\text{Heisenberg}} \quad a_1^\dagger = a_{k_1}^\dagger$

$= a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)|\Omega\rangle$

$= \left(a_3^\dagger(+\infty)\, a_4^\dagger(+\infty)|\Omega\rangle\right)^\dagger$

$\rangle_H = \langle \Omega | a_4(+\infty) a_3(+\infty) a_1^\dagger(-\infty) a_2^\dagger(-\infty) | \Omega \rangle \overset{\text{identity}}{=} \langle \Omega | T\, a_4(+\infty) a_3(+\infty) a_1^\dagger(-\infty) a_2^\dagger(-\infty) | \Omega \rangle$

why time order things?

$$a_1^\dagger(-\infty) = \boxed{a_1^\dagger(-\infty) - a_1^\dagger(+\infty)} + a_1^\dagger(+\infty)$$

$$\equiv -I_1^\dagger + a_1^\dagger(+\infty)$$

$$\boxed{f(+\infty) - f(-\infty) = \int_{-\infty}^{\infty} \partial_0 f \, dt}$$

naively 16 terms

$\langle f|S|i\rangle_H$

only true of time ordering imposed!

$= \langle\Omega| I_4 I_3 I_1^{\dagger} I_2 |\Omega\rangle$

$I_1^{\dagger} = -i\int d^4x\, e^{-ik_1\cdot x} \underbrace{(\partial^2+m^2)}_{\text{KG operator}} \varphi(x)$

interacting field

It does NOT satisfy KG equation

$$(\partial_j^2 + m^2)\,\underbrace{\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle}_{\text{correlation function. all the propagators.}}$$

amputation: chop off external propagators.

...momentum conservation.

...ertex

$$\langle f | S | i \rangle_{H} = i^{2+2} \int \prod_{j=1}^{2+2} d^4 x_j \, e^{-i\lambda_j k_j \cdot x_j} \left(\partial_j^2 + m^2\right) \langle \Omega |$$

Scattering amplitude

$\lambda_1, \lambda_2 = 1$ initial

$\lambda_3, \lambda_4 = -1$ final

4-momentum conservation

vertex

amputation: chop off external propagator scalar case.

step 1

$\langle f | S | i \rangle_{\text{Heisenberg}}$ $a_1^\dagger \equiv a_{k_1}^\dagger$

$S|i\rangle = a_1^\dagger(-\infty) a_2^\dagger(-\infty) |\Omega\rangle$

$\left(|f\rangle = a_3^\dagger(+\infty) a_4^\dagger(+\infty) |\Omega\rangle \right)^\dagger$

$\langle f | S | i \rangle_H = \langle \Omega | a_4(+\infty) a_3(+\infty) a_1^\dagger(-\infty) a_2^\dagger(-\infty) | \Omega \rangle \overset{\text{identity}}{=} \langle \Omega | a_4(+\infty) a_3(+\infty) a_1^\dagger(-\infty) a_2^\dagger(-\infty) | \Omega$

why time order things?

step 2

$a_1^\dagger(-\infty) = \boxed{a_1^\dagger(-\infty) - a_1^\dagger(+\infty)} + a_1^\dagger(+\infty)$

$\equiv -I_1^\dagger + a_1^\dagger(+\infty)$

$\boxed{f(+\infty) - f(-\infty) = \int_{-\infty}^{\infty} \partial_0 f \, dt}$

& guys, ...

$$\langle f|S|i\rangle_{I} = i^{2+2}\int \prod_{j=1}^{2+2} d^4x_j \left(e^{-i\lambda_j k_j\cdot x_j} (\partial_j^2+m^2) \right) \langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle$$

↓ scattering amplitude

$\lambda_1, \lambda_2 = 1$ initial

$\lambda_3, \lambda_4 = -1$ final

4-momentum conservation

vertex

amputation: chop off external propagators.

correlation: all the prop

step 3.

$$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \frac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp\left(i\int d^4x\,\mathcal{L}_{int}(\varphi_0)\right)|0\rangle}{\langle 0|T\exp\left(i\int d^4x\,\mathcal{L}_{int}(\varphi_0)\right)|0\rangle}$$

$\varphi_2|\Omega\rangle$

lation function

ll the propagators

Step 4: Wick's theorem.

$$T\varphi_1\cdots\varphi_n = :\varphi_1\cdots\varphi_n: + :\text{all possible contractions}:$$

$$\varphi(x)\varphi(y) = \Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\frac{i}{(p^2-m^2)+i\epsilon}e^{-ip(x-y)}$$

Fermion modification?: 2 fermion to 2 fermion scattering

$$S|i\rangle = b_1^{\dagger}(-\infty)\, b_2^{\dagger}(-\infty)|\Omega\rangle$$

$$b_1^{\dagger} \equiv b^{s_1 \dagger}_{\vec{k}_1}$$

fermion ones

$$\langle f|S|i\rangle_H = \langle\Omega|T\, \tilde{I}_4\, \tilde{I}_3\, \tilde{I}_1^{\dagger}\, \tilde{I}_2^{\dagger}|\Omega\rangle$$

formion
scattering

new integrals:

$$b^r_{\vec{k}} = \int d^3x\, e^{ik\cdot x} (u^r)^\dagger(\vec{k})\, \psi(x)$$

$$\tilde{I}_1 = \int_{-\infty}^{\infty} dt\, \partial_0 b^r_{\vec{k}}$$

$$= \int d^4x\, (ik_0)\left(e^{ikx} (u^r)^\dagger(\vec{k})\psi(x)\right.$$

$$\left. + e^{ikx} u^{r\dagger}(\vec{k})\, \partial_0 \psi(x)\right)$$

$\gamma^0\gamma^0 = 1$

$\psi(x)$

$\psi(x))$

$\psi(x))$

$\gamma^0 = 1$

I am expecting $(i\gamma_0\partial^0 + i\gamma_i\partial^i - m)\psi(x)$

first trick e.o.m for u

$$(i\gamma^\mu\partial_\mu - m)u(\vec{k})e^{-ik\cdot x} = 0$$

$$(\gamma^\mu k_\mu - m)u(\vec{k}) = 0$$

$$\gamma^0 k_0 + \gamma^i k_i$$

take dagger

am expecting $(i\gamma_0\partial^0 + i\gamma_i\partial^i - m)\psi(x)$

first trick e.o.m for u

$$(i\gamma^\mu\partial_\mu - m)u(\vec{k})e^{-ik\cdot x} = 0$$

$$(\gamma^\mu k_\mu - m)u(\vec{k}) = 0$$

$\gamma^0 k_0 + \gamma^i k_i$

take dagger

use e.o.m

$\gamma^i k_i e^{ik\cdot x}(u^r)^\dagger(k)\psi(x)$

write $k_i e^{ik\cdot x} = -\partial_i e^{ik\cdot x}$

trick 2: integrate by parts

move derivative from $e^{ik\cdot x}$

to $\psi(x)$

$$\tilde{I}_1 = -i\int d^4x\, e^{ik_1\cdot x_1}\, \bar{u}_1(k_1)(i\not\partial - m)\psi(x)$$

some spinor

we need something to carry the info

it is a spinor.

Dirac operator

the field

Can show

$$\bar\psi(i\overleftarrow{\not\partial}+m)=0$$

if EL $\mathcal{L}=\bar\psi(i\not\partial - m)\psi$

on ψ.

$$\tilde{I}_1^\dagger = -i\int d^4x\, e^{-ik_1\cdot x_1}(i\partial_\mu\bar\psi\gamma^\mu u_1 + m\bar\psi u_1)$$

$$= -i\int d^4x\, \bar\psi(i\overleftarrow{\partial}_\mu\gamma^\mu + m)\, e^{-ik_1\cdot x_1}u_1$$

Dirac operator

take derivative from right operator.

$\psi \in \mathcal{L}$

$+m) u_1$

on ψ.

tor

$-ik_1 \cdot x_1 \; u_1$

from right operator.

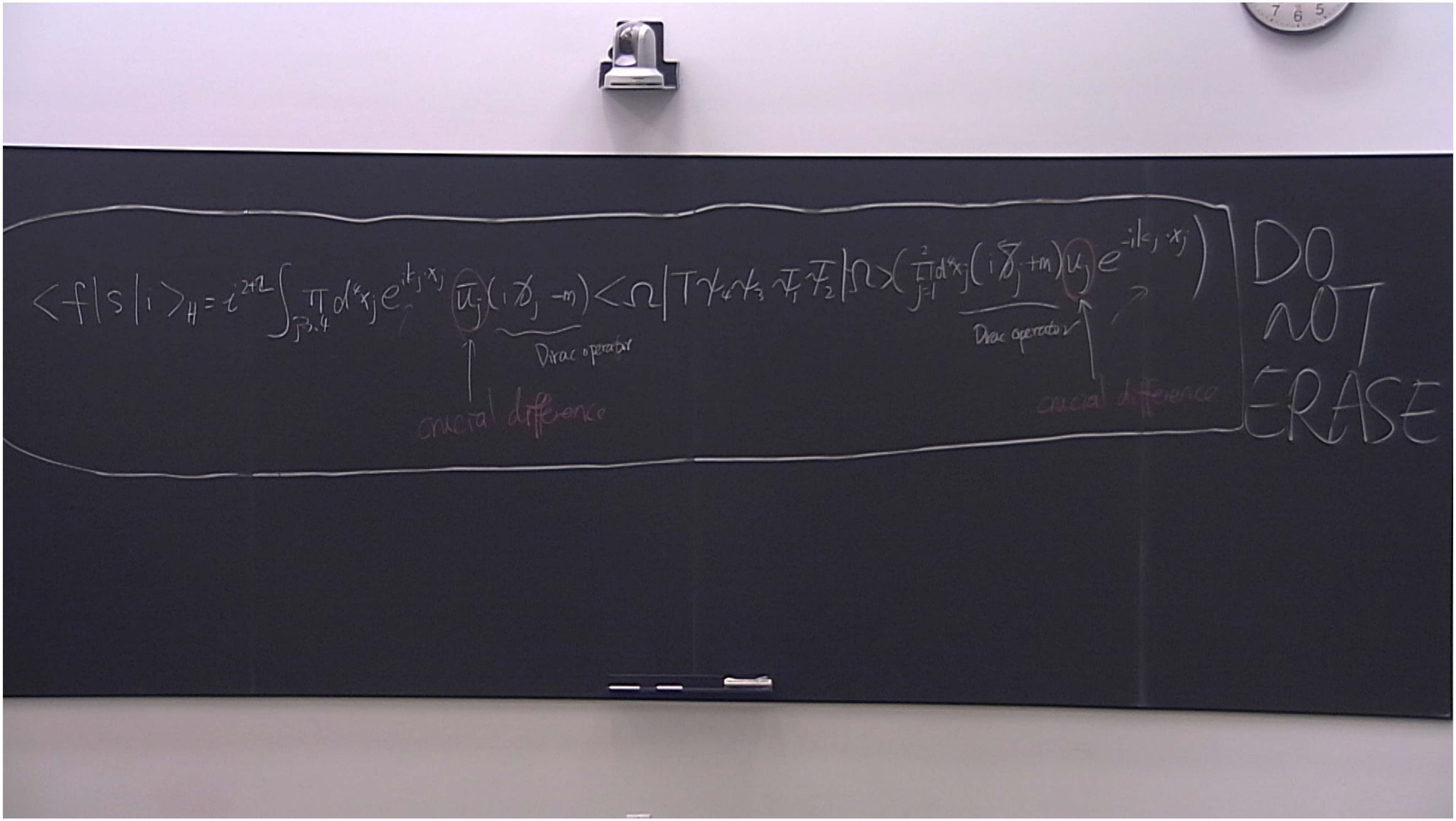
carries momentum.
spinor label
info about the fermion.

$$\cdots{}^{j\cdot x_j}\ \bar{u}_j \underbrace{(i\overleftarrow{\partial\!\!\!/}_j - m)}_{\text{Dirac operator}} \langle \Omega | T \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 | \Omega \rangle \left(\prod_{j=1}^{2} \int d^4 x_j \underbrace{(i\overrightarrow{\partial\!\!\!/}_j + m)}_{\text{Dirac operator}} u_j \, e^{-ik_j \cdot x_j} \right)$$

$$\bar{u}_j \underbrace{(i\not{\partial}_j - m)}_{\text{Dirac operator}} \langle \Omega | T \psi_4 \psi_3 \bar{\psi}_1 \bar{\psi}_2 | \Omega \rangle \left(\prod_{j=1}^{2} d^4x_j \underbrace{(i\overleftarrow{\not{\partial}}_j + m)}_{\text{Dirac operator}} u_j e^{-ik_j \cdot x_j} \right)$$

cial difference

crucial difference


$\langle f|S|i\rangle_H = i^{2+2}\int \prod_{j=3,4} d^4x_j\, e^{ik_j\cdot x_j}\,\bar{u}_j\,\underbrace{(i\not{\partial}_j - m)}_{\text{Dirac operator}}\langle\Omega|T\psi_4\psi_3\bar{\psi}_1\bar{\psi}_2|\Omega\rangle\Big(\prod_{j=1}^{2} d^4x_j\,\underbrace{(i\not{\partial}_j + m)}_{\text{Dirac operator}}\,u_j\,e^{-ik_j\cdot x_j}\Big)$
crucial difference
crucial difference
DO NOT ERASE

Learn some technique.

$$T\,\psi_a(x)\bar{\psi}_b(y) = :\psi_a(x)\bar{\psi}_b(y): + \psi_a(x)\bar{\psi}_b(x)$$

what is this thing?

recall in scalar case $T\left(\varphi(x)\varphi(y)\right) = \begin{cases} \varphi(x)\varphi(y) \\ \varphi(y)\varphi(x) \end{cases}$

close our eyes define the time ordering the same

$\psi(x)\psi(y)$

$) \text{ if } x^0 > y^0$

$x) \text{ if } y^0 > x^0$

se our eyes define the time ordering the same

$$T(\psi(x)\psi(y)) = -T(\psi(y)\psi(x))$$

minus sign b.c

$$\{c_{\vec p}, c^\dagger_{\vec k}\}$$

$$\{b_{\vec p}, b^\dagger_{\vec k}\} = \delta(\vec p - \vec k)$$

$$\{b, b\} = \{c, c\} = 0$$

Time order it $x^0 > y^0$

time ordered

$$\psi(x)\psi(y) = -\psi(x)\psi(y)$$

$$\psi(x)\psi(y) = 0 \; ???$$

$$T\left(\psi(x)\psi(y)\right) \equiv \begin{cases} \psi(x)\psi(y) & x^0 > y^0 \\ -\psi(y)\psi(x) & y^0 > x^0 \end{cases}$$

minus sign

$$\psi(x)_+ = \int dV_p\, c^\dagger v\, e^{ip\cdot x}$$

$$\psi(x)_- = \int dV_p\, b\, u\, e^{-ip\cdot x}$$

$$\bar{\psi}(x)_+ = \int dV_p\, b^\dagger \bar{u}\, e^{ip\cdot x}$$

$$\bar{\psi}(x)_- = \int dV_p\, c\, \bar{v}\, e^{-ip\cdot x}$$

$x^0 > y^0$

$$T\left(\psi(x)\bar{\psi}(y)\right) = \psi(x)\bar{\psi}(y)$$

$$= \left(\psi_+(x) + \psi_-(x)\right)\left(\bar{\psi}_+(x) + \bar{\psi}_-(y)\right)$$

$$= (++) + (+-) + (--) + \psi_-(x)\bar{\psi}_+(y)$$

$$= 0 + 0 + 0 + \boxed{\{\psi_-(x), \bar{\psi}_+(y)\}} - \bar{\psi}_+(y)\psi_-(x)$$

wick contraction

$+ :\psi_-(x)\bar{\psi}_+:$ ← 4th normal order

$$= :\psi(x)\bar{\psi}(y): +$$

$$\psi(x)\bar{\psi}(y) \equiv \{\psi_-(x), \bar{\psi}_+(y)\} \quad (x^0 > y^0)$$

ex $\quad y^0 > x^0 \quad -\{\bar{\psi}_-(y), \psi_+(x)\}$

minus sign

$$\psi(x)\psi(y) = 0$$

$$\bar{\psi}(x)\bar{\psi}(y) = 0$$

$$\{\psi_a(x)_-, \bar{\psi}_b(y)_+\} = \left\{\int dV_{\vec{p}}\, b^r_{\vec{p}}\, u^r_a(\vec{p})\, e^{-ip\cdot x},\ \int dV_{\vec{q}}\, b^{\dagger s}_{\vec{q}}\, \bar{u}^s_b(\vec{q})\, e^{iq\cdot y}\right\}$$

$$= \int dV_{\vec{p}}\, dV_{\vec{q}}\, \{b^r_{\vec{p}}, b^{\dagger s}_{\vec{q}}\}\, u^r_a(\vec{p})\, \bar{u}^s_b(\vec{q})\, e^{-ip\cdot x + iq\cdot y}$$

$$= \int dV_{\vec{p}}\, u^r_a(\vec{p})\, \bar{u}^r_b(\vec{p})\, e^{-ip(x-y)}$$

$$\{\psi_a(x)_-, \bar{\psi}_b(y)_+\} = \int dV_{\vec{p}}\, (\not{p}_{ab} + m\mathbb{1}_{ab})\, e^{-ip(x-y)}$$

$$\psi(x)\,\bar{\psi}(y) \equiv \{\psi_-(x), \bar{\psi}_+(y)\} \quad (x^0 > y^0) \qquad \{\psi_a(x)_-, \bar{\psi}_b$$

ex $\quad y^0 > x^0 \quad -\{\bar{\psi}_-(y), \psi_+(x)\}$

minus sign

ex

$$\psi(x)\psi(y) = 0$$

$$\bar{\psi}(x)\bar{\psi}(y) = 0$$

$$\{\psi_a(x)_-,$$

$$e^{-ip\cdot x},\ \int dV_q\, b^{\dagger s}_{\vec{q}}\, \bar{u}^s_b(\vec{q})\, e^{iq\cdot y} \Big\}$$

$$b^{\dagger s}_{\vec{q}} \Big\}\, u^r_a(\vec{p})\, \bar{u}^s_b(\vec{q})\, e^{-ip\cdot x + iq\cdot y}$$

$$b_{\vec{q}}$$

$$\psi(x)\bar{\psi}(y) = \int dV_{\vec{p}}\, (\not{p}+m)\, e^{-ip(x-y)}\, \Theta(x^0 - y^0)$$

$$+ \text{ similar thing } \Theta(y^0 - x^0)$$

$$\vec{p})\, e^{-ip(x-y)}$$

$$m\,](\, e^{-ip(x-y)}$$

$$= \int \frac{d^4p}{(2\pi)^4}\, \frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon}\, e^{-ip(x-y)}$$

$$\int dV_{\vec{q}}\, b^{\dagger s}_{\vec{q}}\, \bar{u}^{s}_{b}(\vec{q})\, e^{iq\cdot y}\Big\}$$

$$\boxed{\begin{aligned} &S_F(x-y) \\ &= (i\partial\!\!\!/_x + m)\,\Delta_F(x-y)\end{aligned}}$$

KG = 0. Dirac

$${}^{r}_{a}(\vec{p})\, \bar{u}^{s}_{b}(\vec{q})\, e^{-ip\cdot x + iq\cdot y}$$

$$\psi(x)\bar{\psi}(y) = \int dV_{\vec{p}}\,(p\!\!\!/ + m)\, e^{-ip(x-y)}\,\Theta(x^0 - y^0)$$

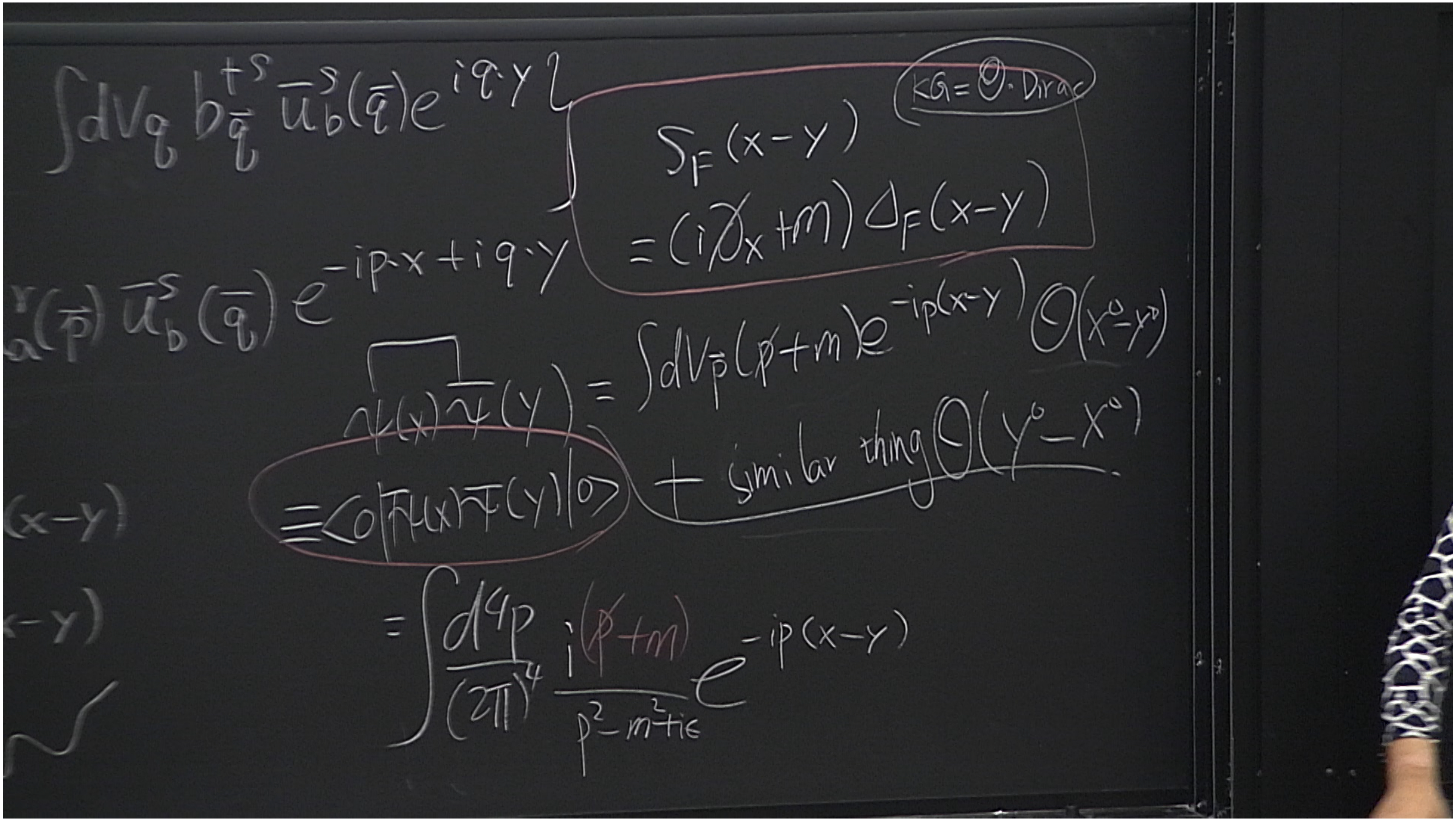
$$+ \text{similar thing}\,\Theta(y^0 - x^0)$$

$$\equiv \langle 0 | T\,\psi(x)\bar{\psi}(y) | 0 \rangle$$

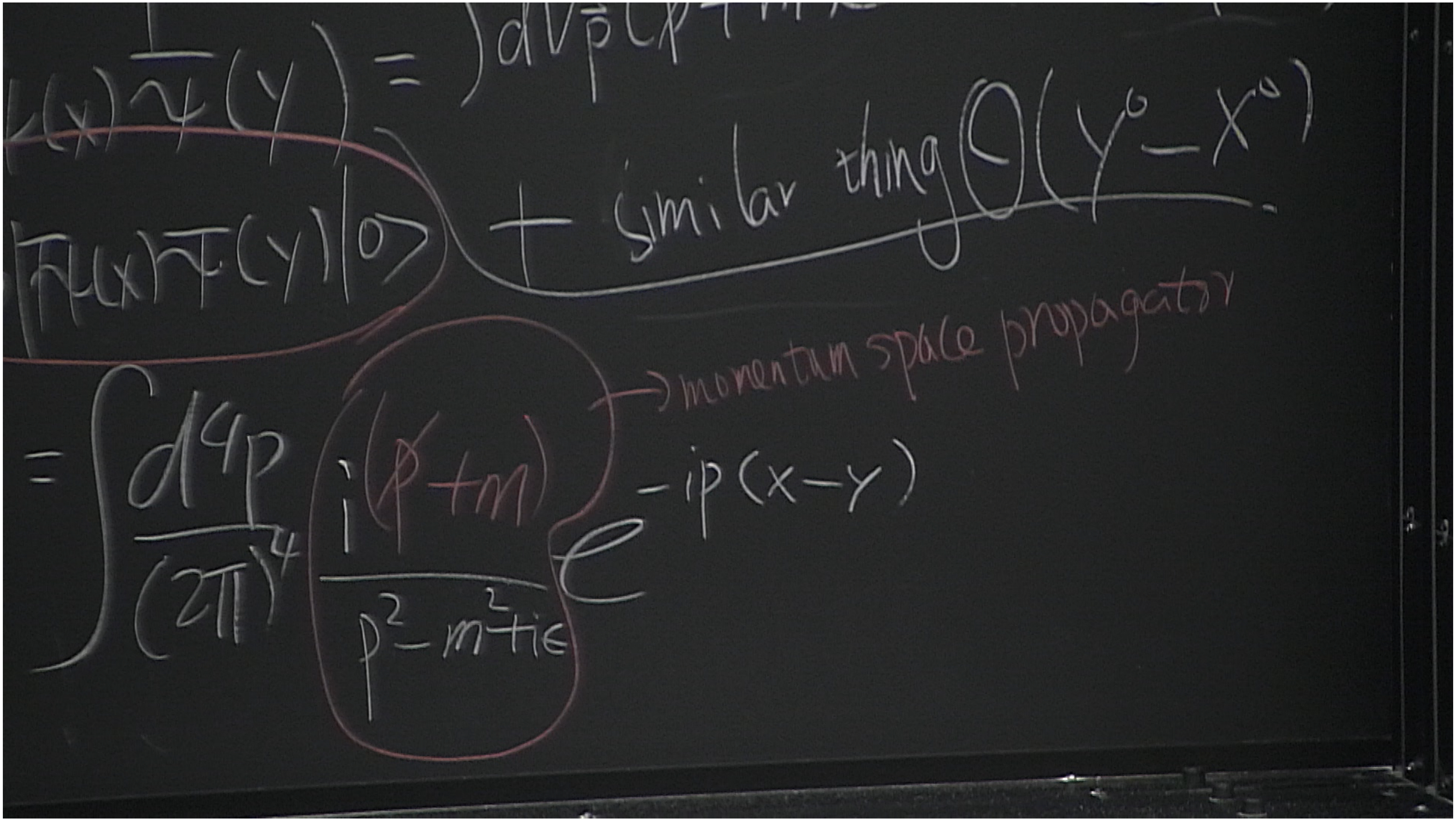
$$(x-y)$$

$$= \int \frac{d^4p}{(2\pi)^4}\, \frac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon}\, e^{-ip(x-y)}$$

$$x-y)$$



$\int dV_q \, b^{\dagger s}_{\vec{q}} \, \bar{u}^s_b(\vec{q}) e^{iq\cdot y}$
$KG = 0 \cdot Dirac$
$S_F(x-y)$
$= (i\partial\!\!\!/_x + m)\Delta_F(x-y)$
$e^{-ip\cdot x + iq\cdot y}$
$(\vec{p})\, \bar{u}^s_b(\vec{q})\, e$
$\psi(x)\bar{\psi}(y) = \int dV_{\vec{p}} (p\!\!\!/ + m) e^{-ip(x-y)} \Theta(x^0 - y^0)$
$+$ similar thing $\Theta(y^0 - x^0)$
$\equiv \langle 0 | T \psi(x) \bar{\psi}(y) | 0 \rangle$
$= \int \frac{d^4p}{(2\pi)^4} \frac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$
$(x-y)$
$(x-y)$


+ similar thing $\Theta(y^0 - x^0)$
$T\psi(x)\bar{\psi}(y)|0\rangle$
momentum space propagator
$= \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} e^{-ip(x-y)}$