Title: Quantum complexity, irreversibility, learnability and fluctuations.

Speakers: Alioscia Hamma

Series: Perimeter Institute Quantum Discussions

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Abstract: Quantum complexity is a notion characterizing the universality of the entanglement arising from a quantum evolution. A universal evolution will result in a complex entanglement. At the same time, this also corresponds to small fluctuations and to unlearnability from the point of view of machine learning. All these aspects are connected to the different features of k-designs, which are under-samplings of the Hilbert space. 

We study the transition in complexity due to the doping of a quantum circuit by universal gates and show that the transition to complex entanglement can be obtained by just a single gate. 
These results are relevant for the notions of scrambling, quantum chaos, OTOCs and operator spreading.

We conjecture that the transition to 4âˆ'design, W-D and unlearnability are one and the same.

# Work based on

**Quantum Adiabatic Brachistochrone**
A.T. Rezakhani, W.-J. Kuo, A. Hamma, D.A. Lidar, P. Zanardi
**Phys. Rev. Lett. 103, 080502 (2009)**

**Emergent irreversibility and entanglement spectrum statistics**
Claudio Chamon, A.H., Eduardo R. Mucciolo
**Phys. Rev. Lett. 112, 240501 (2014)**

**Irreversibility and Entanglement Spectrum Statistics in Quantum Circuits**
Daniel Shaffer, Claudio Chamon, A.H. Eduardo R. Mucciolo
**J. Stat. Mech. (2014) P12007**

**Two-component Structure in the Entanglement Spectrum of Highly Excited States**
Zhi-Cheng Yang, Claudio Chamon, A.H. , Eduardo R. Mucciolo
**Phys. Rev. Lett. 115, 267206 (2015)**

**Entanglement Complexity, Thermalization, and Many-Body Localization**
Zhi-Cheng Yang, A. H., Salvatore M. Giampaolo, Eduardo R. Mucciolo, Claudio Chamon
**Phys. Rev. B 96, 020408 (2017)**

**Single T gate in a Clifford circuit drives transition to universal entanglement spectrum statistics**
Shiyu Zhou, Zhi-Cheng Yang, Alioscia Hamma, Claudio Chamon
**arXiv:1906:01079**

- Unitarity = Time Reversal
- Entropy is constant with unitary evolution
- Entanglement Entropy is not!
- Does increase in entropy mean irreversibility?
- Where does irreversibility come from?
- Irreversibility is a feature of Quantum Complexity

## IRREVERSIBILITY IN QUANTUM PHYSICS

- There are two ways of approaching the notion of quantum complexity
- 1. (Kolomogorov, Susskind) complexity relative to a fiducial state as the least number of elementary operations needed to make that state.
- If a quantum evolution is ergodic, it can eventually reach complex states: Gibbs POV
- 2. (this talk) The state is complex if it belongs to a large equivalence class, i.e., it is hard to tell it apart from many other states: Boltzmann POV

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## WHAT IS QUANTUM COMPLEXITY?

- States are complex if their entanglement is complex
- This means that it is very difficult to distinguish them from many other entangled states
- The landscape of entanglement would have many local minima
- Trying to disentangle a state would be hard

## QUANTUM COMPLEXITY AND ENTANGLEMENT

# RANDOM QUANTUM CIRCUIT (RQC)

- CONSIDER A SYSTEM OF N SPINS 1/2 INITIALIZED IN A PRODUCT STATE
- WE DRAW SOME RANDOM GATES BETWEEN RANDOM QUBITS AND MONITOR THE ENTANGLEMENT (HEATING)
- AT SOME POINT THE ENTANGLEMENT BECOMES MAXIMAL AND EQUILIBRATES
- THEN WE TRY TO REVERSE THE EVOLUTION AND DISENTANGLE THE STATE (COOLING)

## QUANTUM GATES

- **NOT** gate: flips the state of the qubit

$|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$

- **H** (Hadamard) gate:

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

- Phase gates $\mathbf{P}_\delta$: gives a state dependent phase

$|0\rangle \rightarrow |0\rangle$ and $|1\rangle \rightarrow e^{i\delta}|1\rangle$

The phase gate with $\delta = \pi/4$ is called **T**

The phase gate with $\delta = \pi/2$ is called **S**.

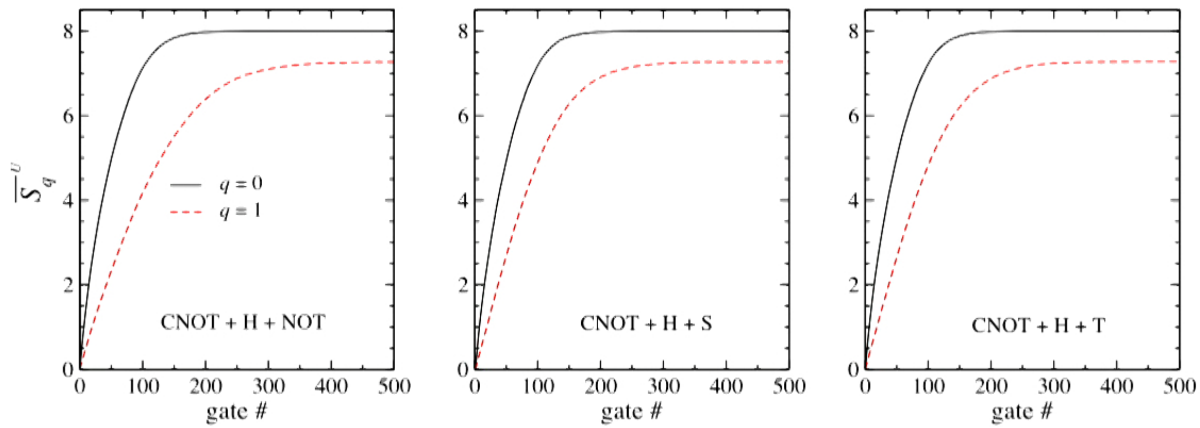
- **CNOT** gate or controlled-NOT gate:

$|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow |01\rangle$

$|10\rangle \rightarrow |11\rangle$, $|11\rangle \rightarrow |10\rangle$

## Universal and not sets of Gates

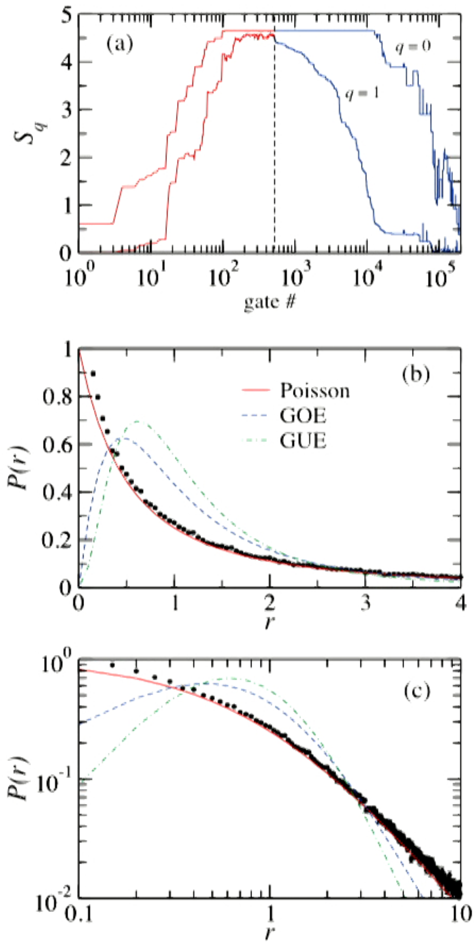
- (H,CNOT,S) dense in Clifford Group, **NOT UNIVERSAL**
- (H,CNOT,NOT) = **NOT UNIVERSAL**
- (H,CNOT,T) = **UNIVERSAL**
- Entanglement Heating by RQC



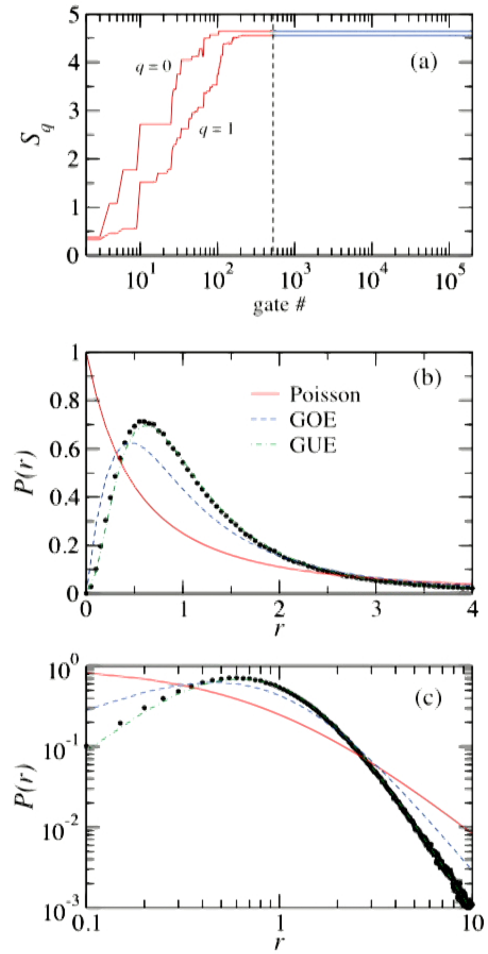
## Reversing the evolution: *Disentangling the state*

Metropolis algorithm:

- (1) Compute the entanglement entropies (all bipartitions)
- (2) Choose a gate randomly
- (3) Apply the gate
- (4) Recompute the entanglement entropies
    - If entropies are reduced, add gate to reverse list
    - If entropies increase, keep gate with Boltzmann probability or discard it and go back to (2)
- (5) Stop when entanglement entropies are zero




# CNOT+H+S




# CNOT+H+T

# Irreversibility and Complexity

- Irreversibility arises not just because there are many ways of entangling vs few ways of disentangling.
- Even with active disentangling (Metropolis) it may be difficult
- Complex entanglement means that searching for a disentangler requires exhaustive search
- Complex entanglement is a property of the statistics of the entanglement gaps.
- Simple entanglement means that a disentangler can be found by some principle (annealing)
- This also characterizes universal computation as ergodic and non-universal as non complex

# Entanglement in Hamiltonian systems

- Entanglement is expressed by a single number (VN entropy), but both integrable systems and non-integrable systems evolve towards very entangled states. The latter thermalize, the former do not
- MBL states do not obey ETH and they do not evolve towards very entangled states although these systems are not integrable
- If we look only at a single number for entanglement we cannot make sense of how entanglement could explain all these situations

# Heisenberg Spin chain

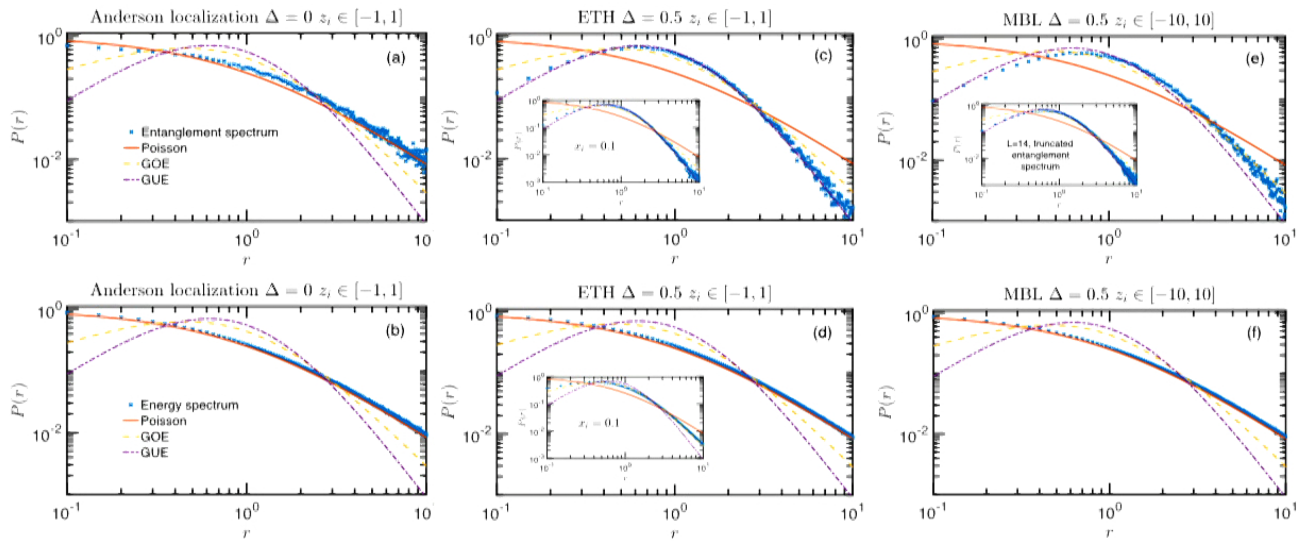
- What happens for a spin system described by a Hamiltonian?
- Time evolution is given by exp(-iHt)
- We can also study the entanglement complexity of the eigenstates

$$H = J \sum_{i=1}^{L-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z + z_i \sigma_i^z + x_i \sigma_i^x)$$

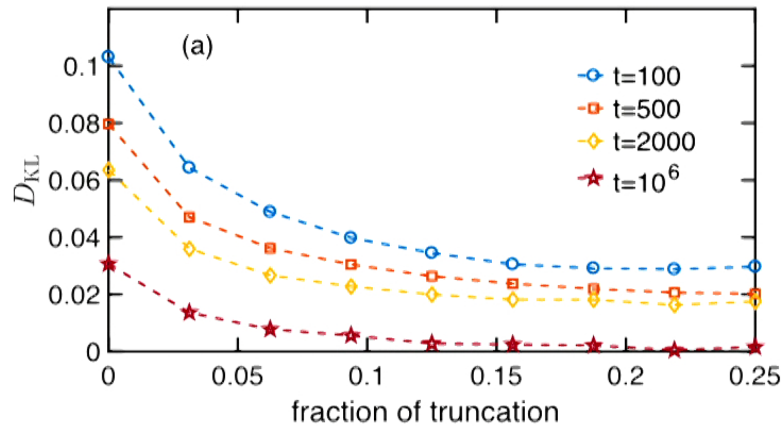
$$\begin{array}{ll} \Delta = x_i = 0, z_i \in [-1, 1] & \text{XX model (AA localization)} \\ \Delta = 0.5, z_i \in [-1, 1] & \text{ETH} \\ \Delta = 0.5, z_i \in [-10, 10] & \text{MBL} \end{array}$$

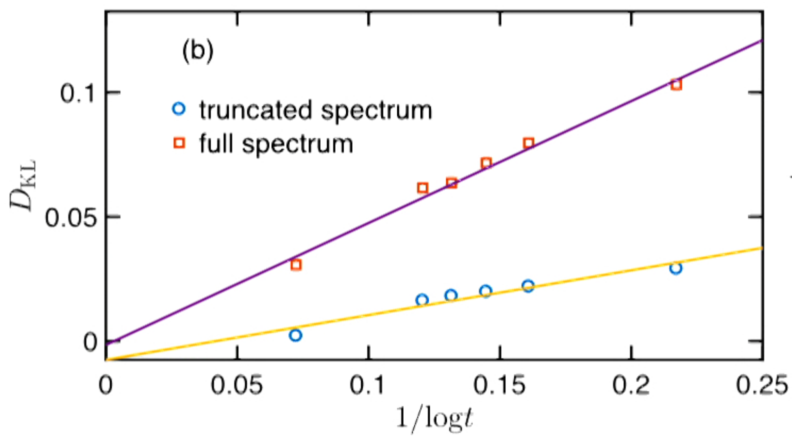
- XX is integrable and localized
- ETH thermalizes in subsystems
- MBL has quasi-local integrals of the motion and slow entanglement production

# ENTANGLEMENT COMPLEXITY AND DYNAMICAL PHASES




- XX model: free fermions/ integrable, Anderson localized
- ETH: thermalizes
- MBL: Hybrid behavior

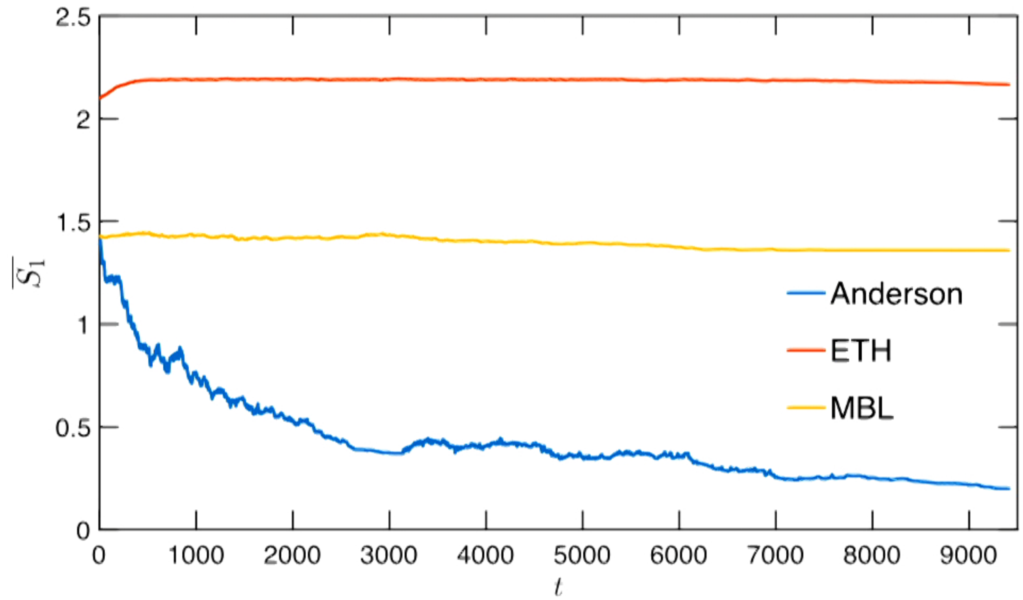



- MBL spectrum approaches WD if we truncate heavy weights
- At long times, it approaches WD with vanishing truncation
- Entanglement in MBL is universal at large times

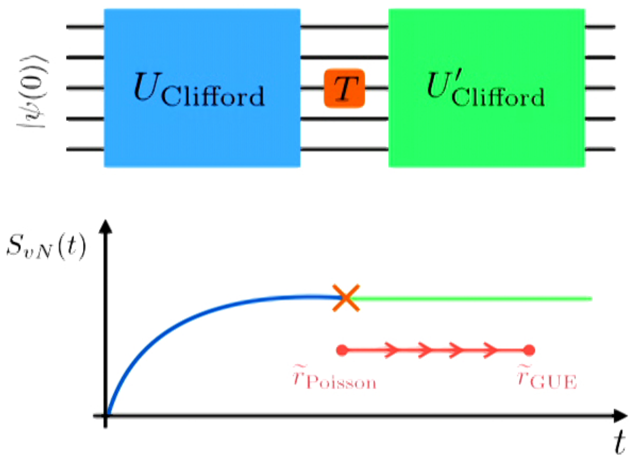
$$D_{KL}(p\|q) = \sum_i p_i \log(p_i/q_i)$$

# Disentangling and Complexity




- XX can be disentangled
- ETH is complex and cannot be disentangled
- MBL is hybrid in its disentangling behavior. In spite of low entanglement its pattern is complex

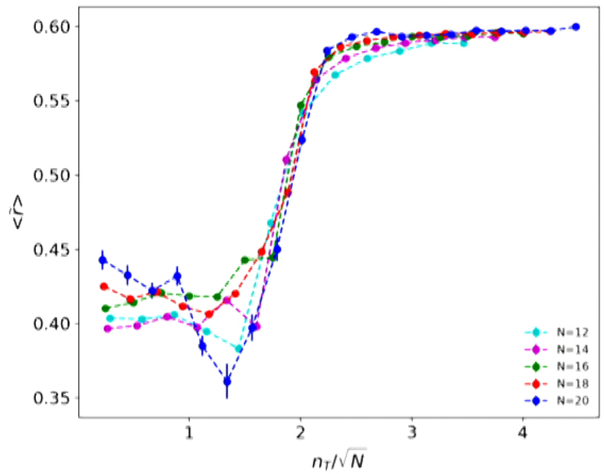
# Transition to Quantum Complexity




- We place n (possibly just one) T gates sandwiched by Clifford circuits
- n=0 will be just a Clifford circuit: ESS will be Poisson
- Can we drive a transition to Universal (GUE) ESS?
- Is integrability immediately destroyed?
- What does Universal GUE correspond to?

## A Quantum KAM, the onset of quantum chaos

$$(T^\dagger)U_{\mathrm{Clifford}}\,T\,U^\dagger_{\mathrm{Clifford}}$$

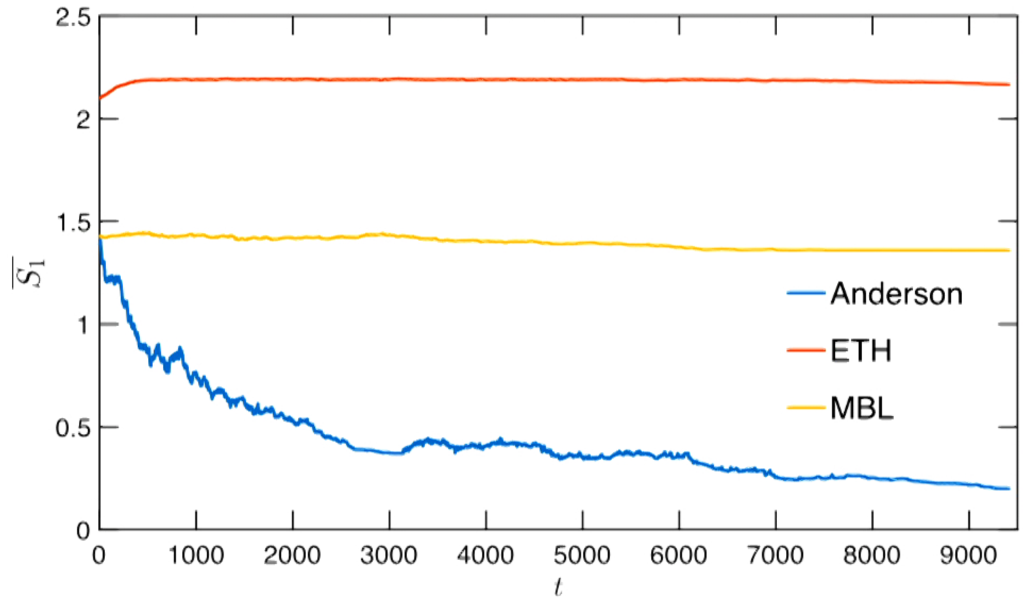



- There is a threshold ~sqrt{N} before Clifford goes into chaos
- This quantity is like a OTOC

## Open Problems

- When does the onset of complexity happen by universal gates? Is it sqrt{N}?
- What is the role of time fluctuations of the entropy in the effectiveness of cooling?
- When does the unlearnability transition happen?
- When does the transition to 4-designs happen?
- Can you dope anything to a 4-design?
- Can you dope Clifford to Universal by sqrt{N}?
- What is the relationship between conserved quantities and entanglement complexity?
- Quantum complexity, OTOC, and scrambling

# Disentangling and Complexity




- XX can be disentangled
- ETH is complex and cannot be disentangled
- MBL is hybrid in its disentangling behavior. In spite of low entanglement its pattern is complex

- FIRST LAW OF QUANTUM COMPLEXITY: The average entanglement complexity across all the TPS is constant.
- SECOND LAW OF QUANTUM COMPLEXITY: In every TPS, universal quantum evolution always entangles the state in the most possible complex way (with small fluctuations).

## THE LAWS OF QUANTUM COMPLEXITY