Title: ETH, EE & OTOC

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Series: Perimeter Institute Quantum Discussions

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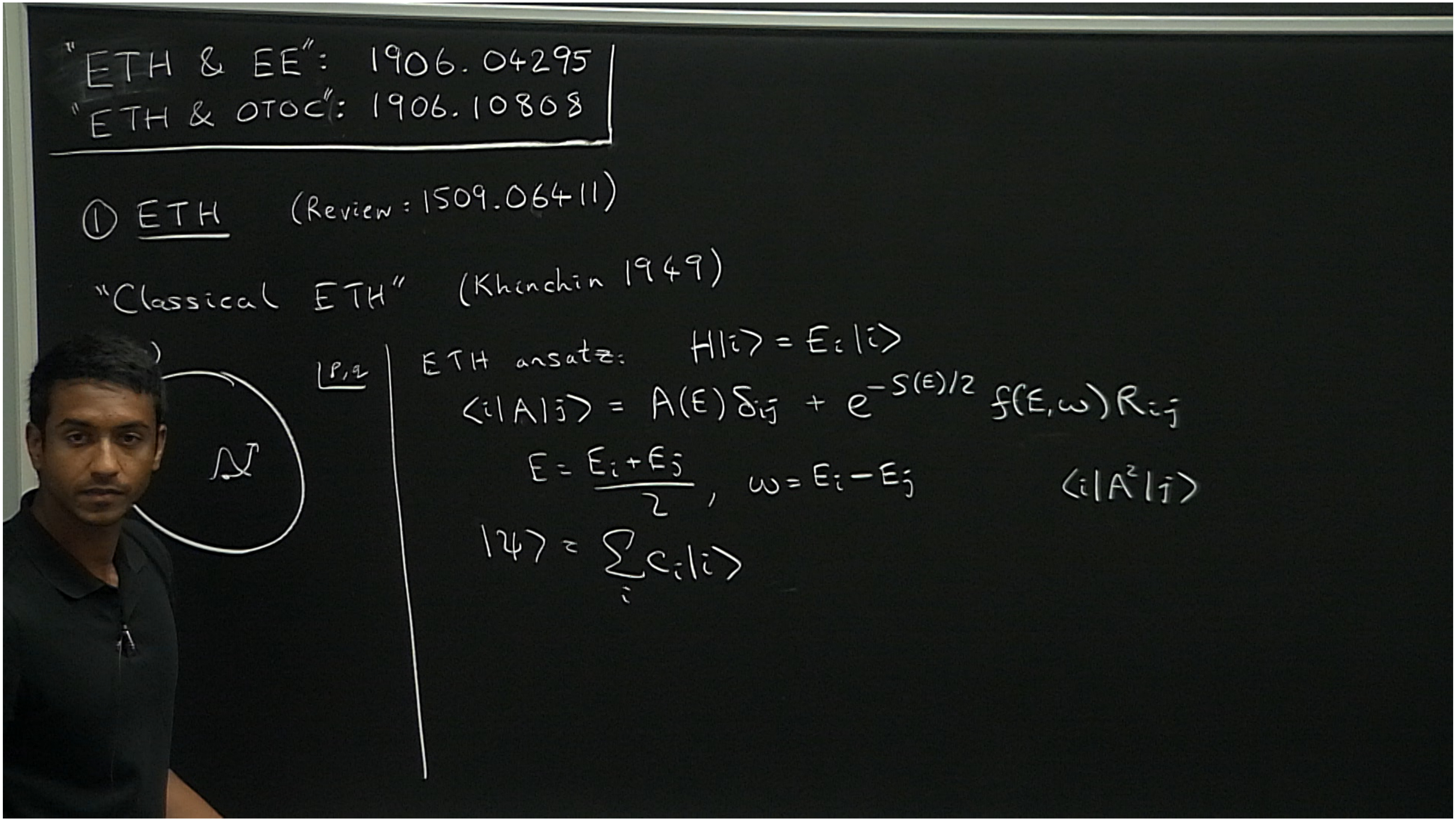
Abstract: I will describe some connections between the Eigenstate Thermalization Hypothesis (ETH), the entanglement structure of generic excited eigenstates of chaotic quantum systems ("EE", arXiv:1906.04295), and the "bound on chaos" limiting the growth rate of the out-of-time-order four-point correlator in such systems ("OTOC", arXiv:1906.10808).


"ETH & EE": 1906.04295
"ETH & OTOC": 1906.10808
① ETH (Review: 1509.06411)
"Classical ETH" (Khinchin 1949)
p,q
ETH ansatz: $H|i\rangle = E_i|i\rangle$
$\langle i|A|j\rangle = A(E)\delta_{ij} + e^{-S(E)/2} f(E,\omega) R_{ij}$
$E = \frac{E_i+E_j}{2}, \quad \omega = E_i - E_j$
$\langle i|A^2|i\rangle$
$|\psi\rangle = \sum_i c_i|i\rangle$

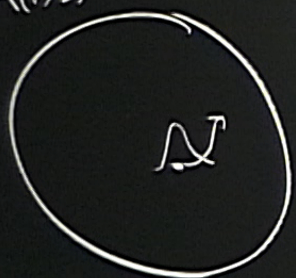
E : 1906.0…

oc": 1906.10808

① ETH (Review: 1509.06411)

"Classical ETH" (Khinchin 1949)

$A(p,q)$



p,q

ETH ansatz: $H|i\rangle = E_i|i\rangle$

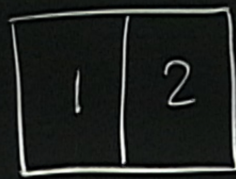
$$\boxed{\langle i|A|j\rangle = A(E)\,\delta_{ij} + e^{-S(E)/2} f(E,\omega) R_{ij}}$$

$$E = \frac{E_i + E_j}{2}, \quad \omega = E_i - E_j \qquad \langle i|A^2|j\rangle$$

$$|\psi\rangle = \sum_i c_i |i\rangle$$

$$\langle\psi|A(t)|\psi\rangle = \sum_i |c_i|^2 A(E_i) + \sum_{i\neq j} c_i^* c_j e^{i\omega t} e^{-S(E)/2} f(E,\omega) R_{ij}$$

② "EE":



$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$

$|\psi\rangle \in \mathcal{H}$

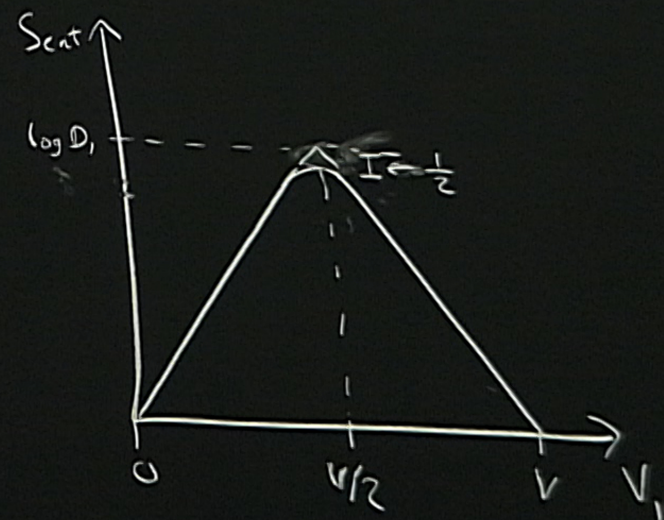
$$|\psi\rangle = \sum_{ij} M_{ij} |i\rangle_1 \otimes |j\rangle_2$$

$$\rho_1 = MM^\dagger$$

$$\rho_2 = M^\dagger M$$

$$\overline{\rho}_1 = \frac{1}{D_1} I$$

$$\overline{S}_{ent,1} = \log D_1 - \frac{D_1}{2D_2}$$

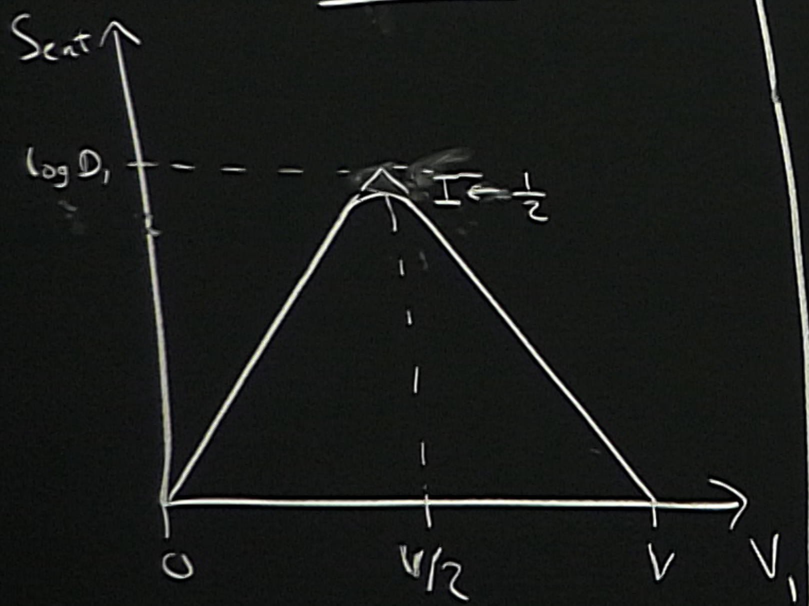


$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$|\psi\rangle \in \mathcal{H}$$

$$|i\rangle_1 \otimes |J\rangle_2$$

Random



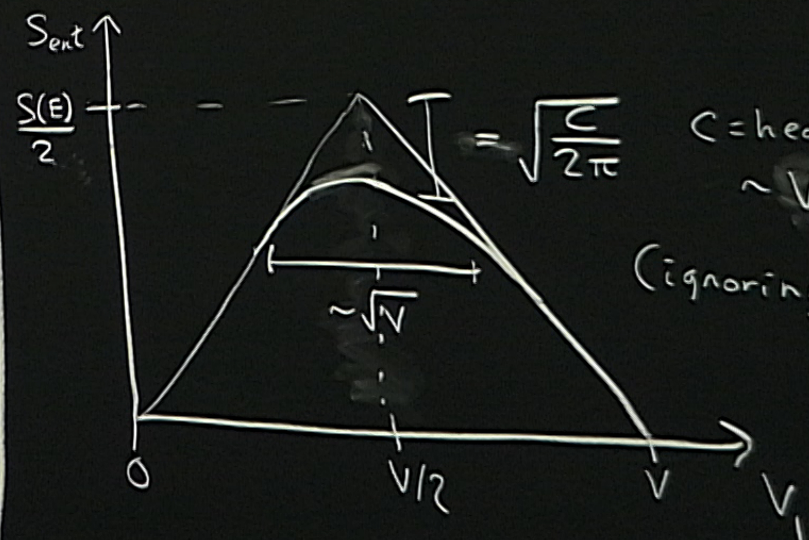
$$H = H_1 + H_2 + H_{12}$$

$$H_1 |i\rangle_1 = E_{1i} |i\rangle_1$$

$$H_2 |J\rangle_2 = E_{2J} |J\rangle_2$$

$$H|E\rangle = E|E\rangle$$

$$|E\rangle = \sum_{iJ} M_{iJ} |i\rangle_1 \otimes |J\rangle_2$$



C = heat capacity $\sim V$

(ignoring $O(A)$ terms)

capacity

O(A) terms)

$$M_{iJ} = e^{-S(E_{1i}+E_{2J})/2}\, F(E_{1i}+E_{2J}-E)^{1/2}\, C_{iJ}, \qquad \langle E|H_{12}|E\rangle = 0$$

$F(\varepsilon)$ center $\varepsilon = 0$, width $\sim \Delta = \sqrt{\langle E|H_{12}^2|E\rangle} \sim \sqrt{A}$

$\overline{C_{iJ}} = 0$, $\overline{C^*_{iJ}\, C_{i'J'}} = \delta_{ii'}\,\delta_{JJ'}$

$$d \geq 2: \quad F(\varepsilon) = \frac{e^{-\varepsilon^2/2\Delta^2}}{\sqrt{2\pi}\,\Delta}$$

$$\langle E|E\rangle = \sum_{iJ} |M_{iJ}|^2 = \sum_{iJ} e^{-S(E_{1i}+E_{2J})}\, F(E_{1i}+E_{2J}-E)\, \overset{1}{\cancel{|C_{iJ}|^2}}$$

$$\approx \int dE_1 \int dE_2\, e^{S_1(E_1)+S_2(E_2)-S(E_1+E_2)}\, F(E_1+E_2-E)$$

$\Delta \to 0: \quad F(\varepsilon) = \delta(\varepsilon)$

$$\Rightarrow \langle E|E\rangle = \int dE_1\, e^{S_1(E_1)+S_2(E-E_1)-S(E)} = 1$$

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$$\langle E|(H_1+H_2-E)^n|E\rangle = \int d\varepsilon\, F(\varepsilon)\, \varepsilon^n$$

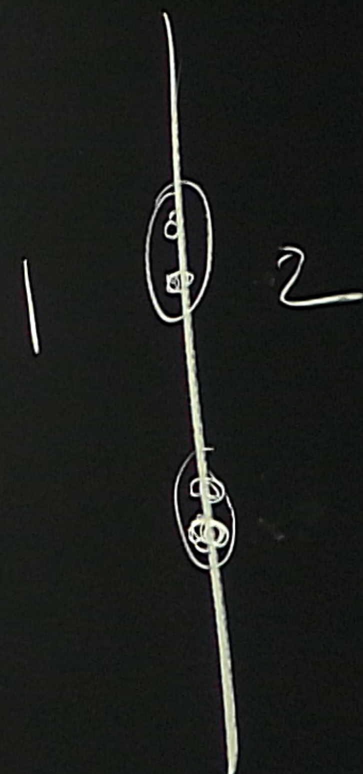
$$\langle H_{12}^n\rangle$$

$$\ldots J)\; F(E_{1i}+E_{2J}-E)\,|\overset{1}{\cancel{C_{iJ}}}|^2$$

$$\ldots(E_1+E_2)\; F(E_1+E_2-E)$$

$$\ldots -S(E) \;\; = 1$$

$$\mathcal{H}_{12} = \sum_{x\in B} h_x$$



acity

) terms)

$$\rho_1 = MM^\dagger$$

$$(\rho_1)_{ij} \approx e^{-S(E)+S_2(E-E_1)}\,\delta_{ij} + \cdots$$

In $[E_1, E_1 + dE_1]$, ρ_1 has

$e^{S_{min}(E_1)}$ nonzero eigenvalues

with value $\sim e^{-S(E)+S_{max}(E_1)}$

$$S_{\substack{min\\max}}(E_1) = \substack{\min\\\max}\left\{ S_1(E_1),\ S_2(E-E_1)\right\}$$

$$S_{ent} = -\mathrm{Tr}_1(\rho_1 \log \rho_1)$$

$$= \frac{\int dE_1\, e^{S_1(E_1)+S_2(E-E_1)}\left[S(E) - S_{max}(E_1)\right]}{\int dE_1\, e^{S_1(E_1)+S_2(E-E_1)}}$$

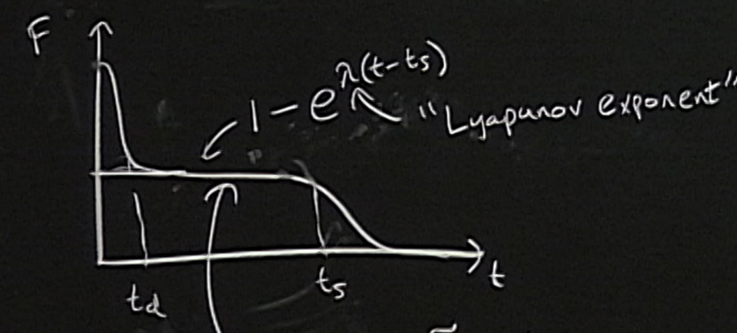
$$S'(E) = \beta$$

$$S''(E) = -\frac{\beta^2}{C} \qquad \int e^{-x^2/C}\,|x|$$

OTOC

$$F(t) = \mathrm{Tr}\left[\rho^{1/4} A(t)\, \rho^{1/4} A(0)\, \rho^{1/4} A(t)\, \rho^{1/4} A(0)\right], \quad \rho = \frac{1}{Z} e^{-\beta H}$$

$$\boxed{\lambda \leq \frac{2\pi}{\beta}}$$



$$F_{OTOC}(t) \to \tilde{F}_{OTOC}(\omega)$$

$$\mathrm{Tr}(\rho A^2) = \int d\omega\, e^{\beta\omega/2} |f(E,\omega)|^2$$

$$\Rightarrow f(E,\omega) \lesssim e^{-\beta|\omega|/4} \quad \text{as } |\omega| \to \infty$$

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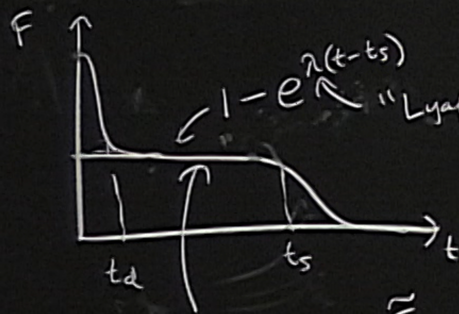

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$$F(t) = \sum_{ijkl} e^{i(\omega_1+\omega_3)t} A_{ij} A_{jk} A_{kl} A_{li}$$

$$\overline{R_{ij} R_{jk} R_{kl} R_{li}} = \delta_{ik} + \delta_{jl} + e^{-S(E)} g(\omega_1, \omega_2, \omega_3)$$

$$g(\omega_1, \omega_2, \omega_3) \approx e^{-\beta|\omega|/4} \text{ as } |\omega| \to \infty$$

$$\Rightarrow \tilde{F}_{OTOC}(\omega) \lesssim e^{-3\beta|\omega|/4}$$

$$F_{OTOC}(t) = \frac{1}{\left(1 + e^{\lambda(t-t_s)}\right)^n} \to \lambda \le \frac{3\pi}{4\beta}$$