Title: Long-lived interacting phases of matter protected by multiple time-translation symmetries in quasiperiodically driven systems

Speakers: Dominic Else

Series: Condensed Matter

Date: October 08, 2019 - 3:30 PM

URL: http://pirsa.org/19100059

Abstract: The discrete time-translation symmetry of a periodically-driven (Floquet) system allows for the existence of novel, nonequilibrium interacting phases of matter. A well-known example is the discrete time crystal, a phase characterized by the spontaneous breaking of this time-translation symmetry. In this talk, I will show that the presence of *multiple* time-translational symmetries, realized by quasiperiodically driving a system with two or more incommensurate frequencies, leads to a panoply of novel non-equilibrium phases of matter, both spontaneous symmetry breaking ("discrete time quasi-crystals") and topological. In order to stabilize such phases, I will outline rigorous mathematical results establishing slow heating of systems driven quasiperiodically at high frequencies. As a byproduct, I will introduce the notion of many-body localization (MBL) in quasiperiodically driven systems.

Long-lived interacting phases of matter protected by multiple time-translation symmetries in quasiperiodically driven systems

arXiv:1910.xxxxx

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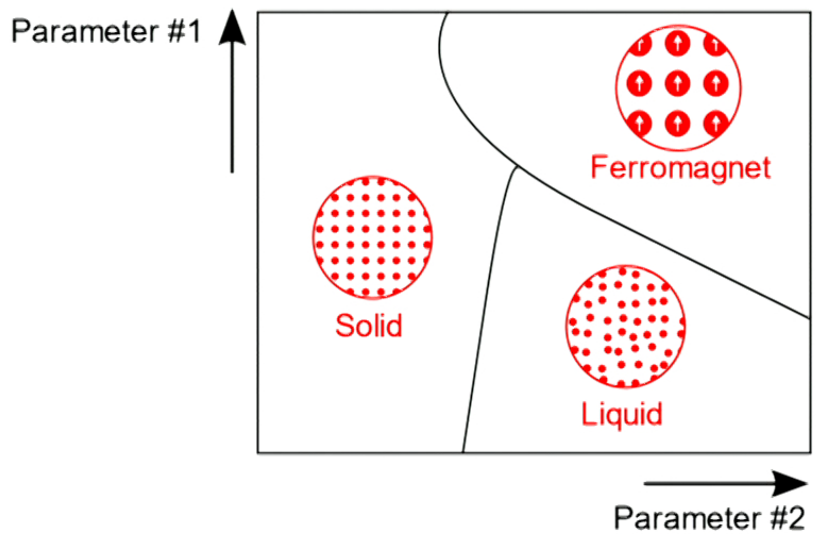
Phases of matter



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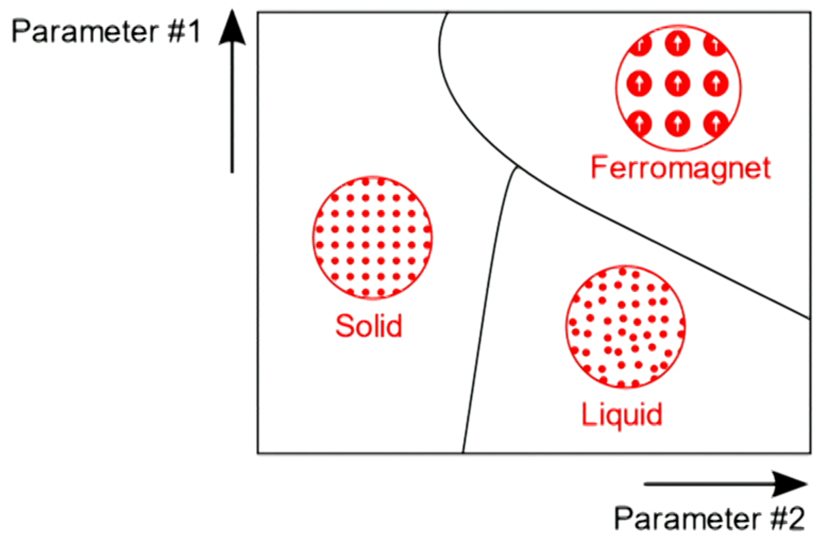
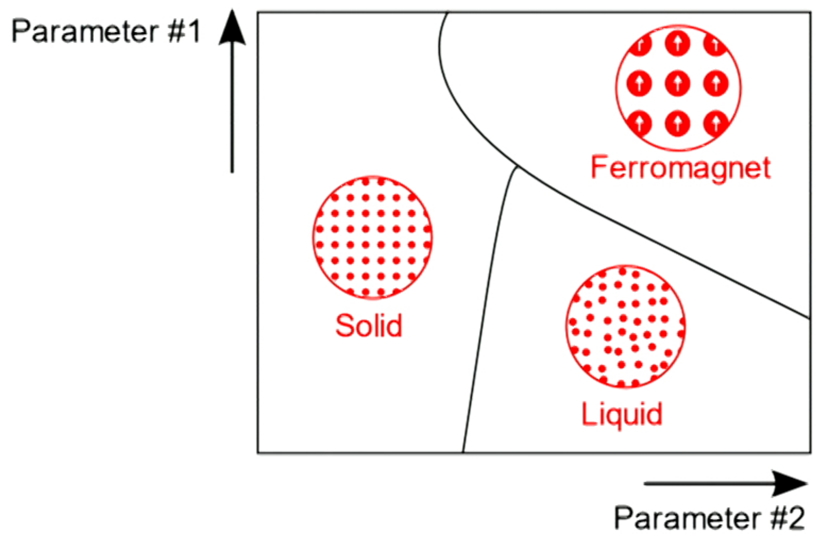


Symmetry →

Spontaneous symmetry breaking phases
e.g. crystal, ferromagnet, ...

Symmetry-protected topological (SPT)
and **symmetry-enriched topological (SET)**
e.g. topological insulators, spin liquids, ...

Phases of matter



Non-equilibrium phases of matter?

Symmetry →

- **Spontaneous symmetry breaking phases**
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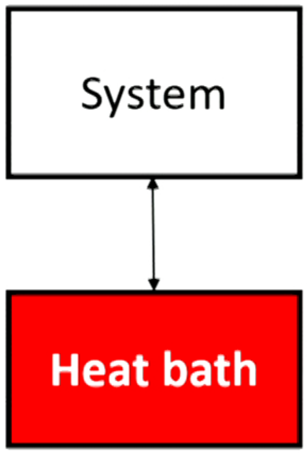
Non-equilibrium phases of matter



Equilibrium

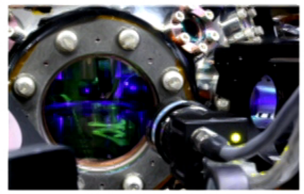
Non-equilibrium phases of matter

System

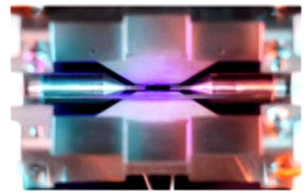
Non-equilibrium?

Unitary time evolution $i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$

Cold Atoms

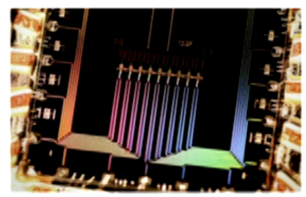


Trapped Ions



NV Centers

SC Qubits



Non-equilibrium phases of matter

System

Non-equilibrium?

Unitary time evolution $i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$

Thermalization

$$|\psi(t)\rangle \longrightarrow \frac{1}{Z}e^{-\beta H}$$ (in terms of local observables)

When does thermalization fail?

- Integrable systems (fine-tuned)
- Many-body localization

$$|\Psi\rangle = |\tau_1^z \tau_2^z \cdots \tau_L^z\rangle$$

Non-equilibrium phases of matter

System

Non-equilibrium?

Unitary time evolution $i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$

e.g. Floquet (periodic driving) $H(t+T) = H(t)$

Thermalization

$|\psi(t)\rangle \longrightarrow \frac{1}{Z}\mathbb{I}$ (in terms of local observables)

"Heating"

When does thermalization fail?

- Integrable systems (fine-tuned)
- Many-body localization

$|\Psi\rangle = |\tau_1^z \tau_2^z \cdots \tau_L^z\rangle$

- Floquet prethermalization

Floquet phases of matter

- Floquet ⟶ fundamentally new phases
  (can't occur without periodic driving)

Floquet phases of matter

- Floquet $\longrightarrow$ fundamentally new phases
  (can't occur without periodic driving)

- Why? Floquet systems have
  discrete time-translation symmetry $H(t) = H(t+T)$

Other symmetries are just operators that commute with the Hamiltonian $[\hat{g}, H] = 0$

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- New symmetry ⟶ new phases of matter

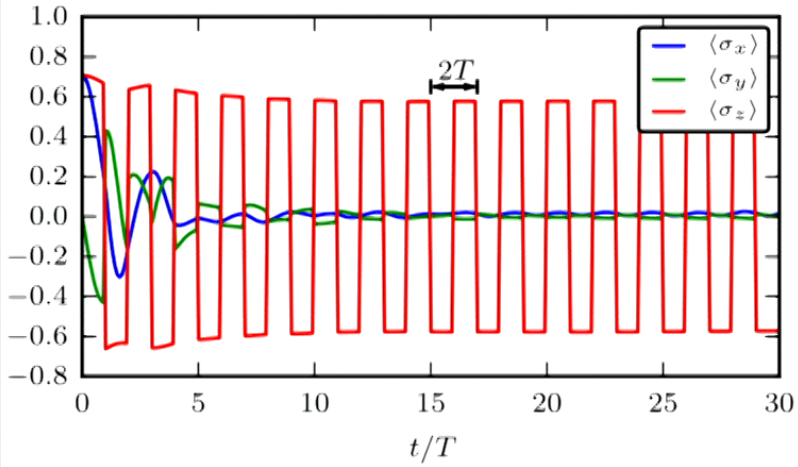
Spontaneous symmetry-breaking phases

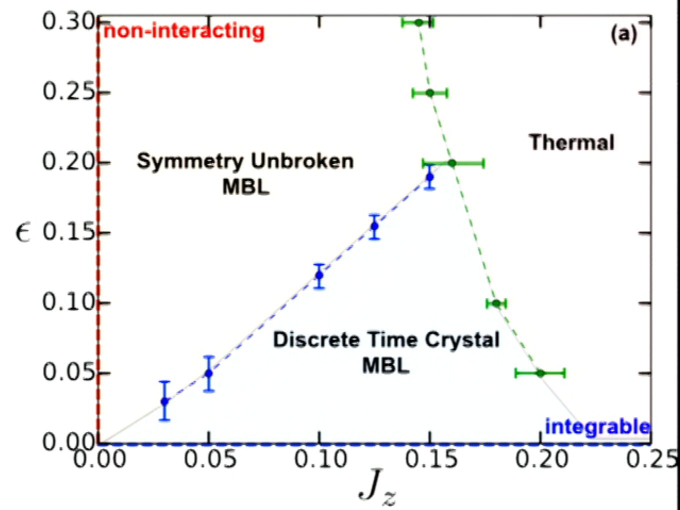
Topological phases

Discrete time crystal

- A discrete time crystal is a *spontaneous symmetry breaking phase* for the discrete time-translation symmetry

[Khemani et al, PRL 2016; DVE et al, PRL 2016; Yao et al, PRL 2017]





$T \to 2T$ Subharmonic response

Multiple time-translation symmetries?

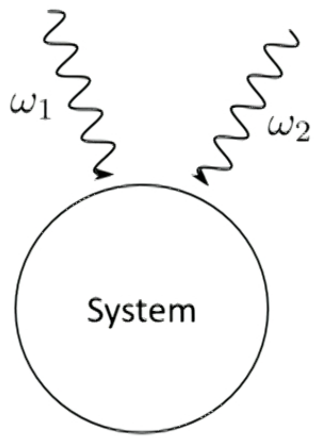
Gauged duality, conformal symmetry, and spacetime with two times

I. Bars, C. Deliduman, and O. Andreev
Phys. Rev. D **58**, 066004 – Published 11 August 1998

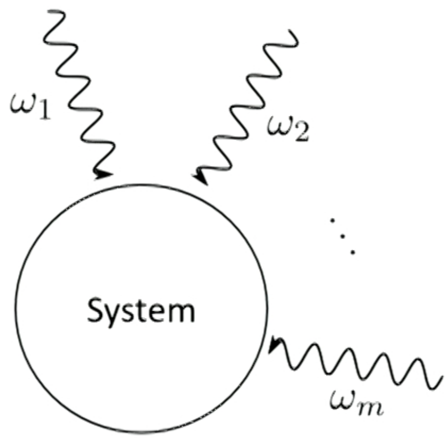
Outline

- Introduction to non-equilibrium phases of matter
- Quasiperiodic driving
- Slow heating and dynamics in quasiperiodically driven systems
- New phases: Discrete time quasicrystals
- New phases: Quasiperiodic topological phase

Quasiperiodic driving



Quasiperiodic driving



$$\boldsymbol{\omega} = (\omega_1, \cdots, \omega_m)$$

Frequencies should be **rationally independent**,

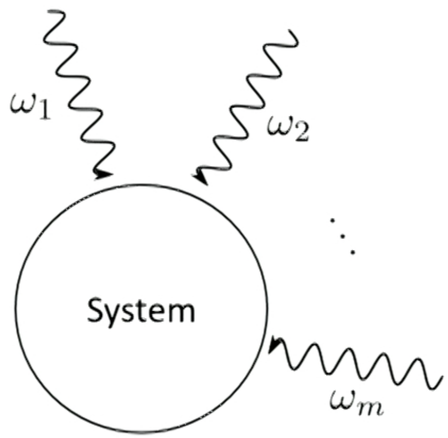
i.e. $\boldsymbol{n} \cdot \boldsymbol{\omega} \neq \mathbf{0}$ for any integer vector $\boldsymbol{n} \neq \mathbf{0}$

Example:

$\boldsymbol{\omega} = (\omega_1, \omega_2)$ with

ω_1 / ω_2 an irrational number

Quasiperiodic driving



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Example:
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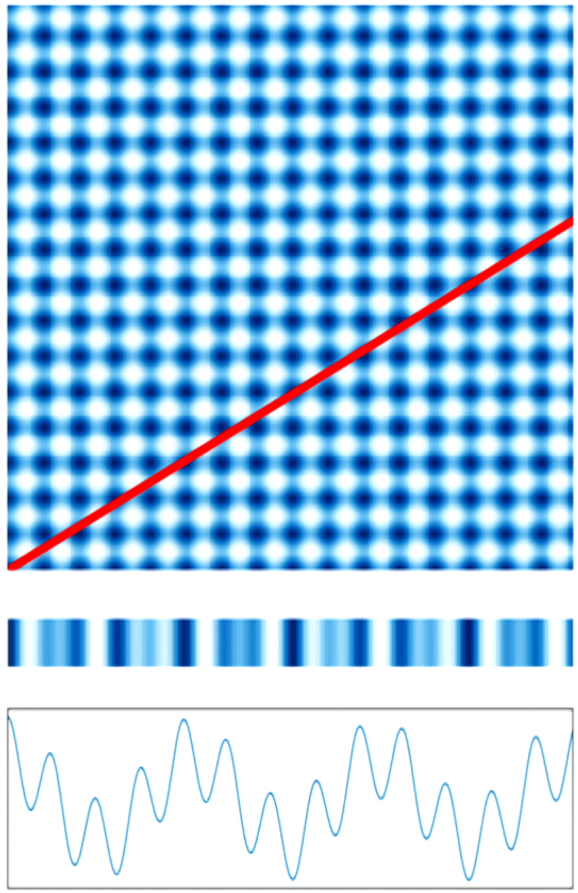
A function $f(t)$ is **quasiperiodic** with frequency vector $\boldsymbol{\omega}$ if

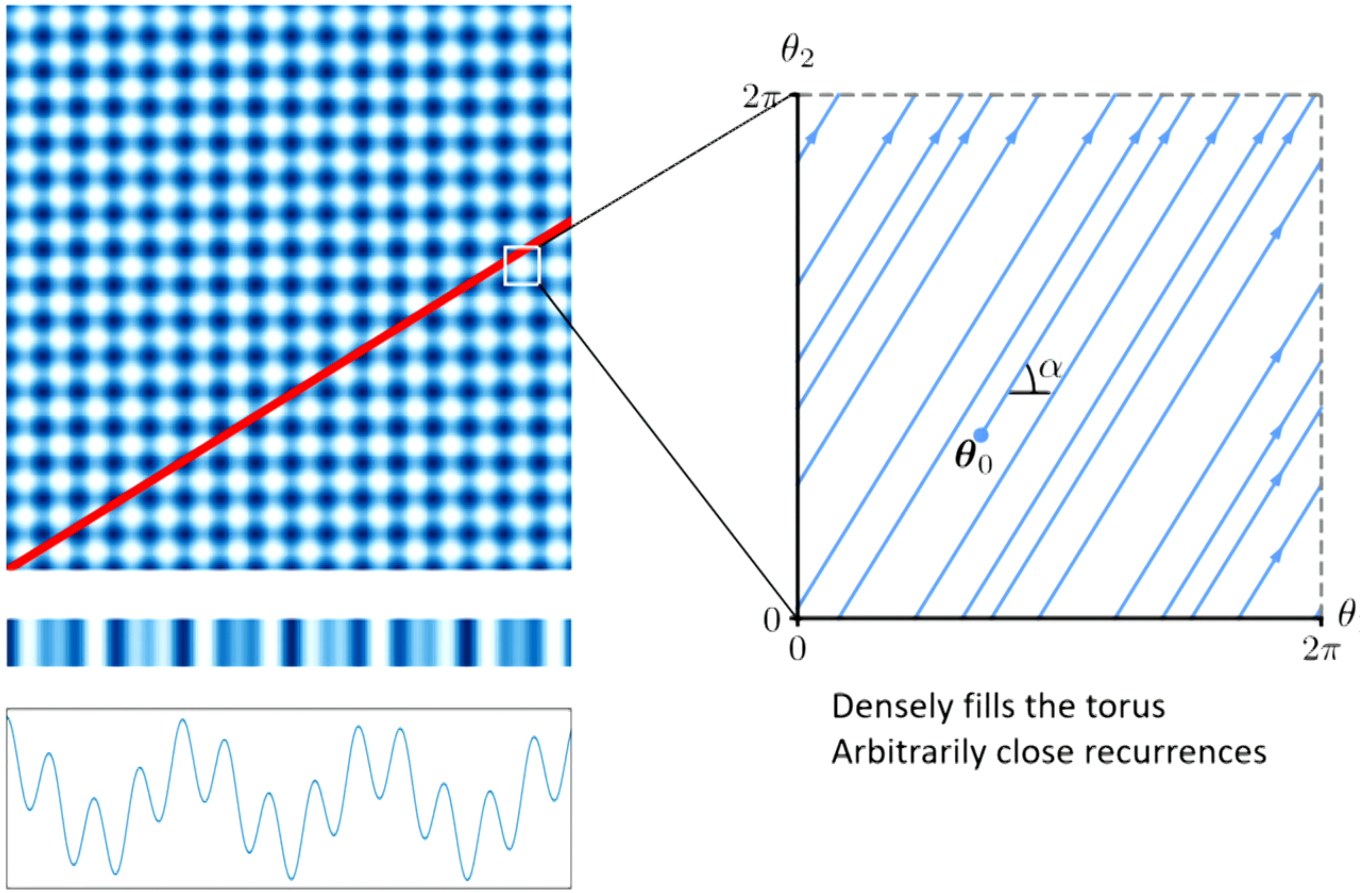
$$f(t) = f(\boldsymbol{\omega} t)$$

where $f(\boldsymbol{\theta}) = f(\theta_1, \cdots, \theta_m)$ is a piecewise continuous function that is 2π periodic in each argument.

Visualizing quasiperiodic driving



Visualizing quasiperiodic driving



Densely fills the torus
Arbitrarily close recurrences

Band-structure invariants

Topological Frequency Conversion in Strongly Driven Quantum Systems

Ivar Martin, Gil Refael, and Bertrand Halperin
Phys. Rev. X **7**, 041008 – Published 16 October 2017

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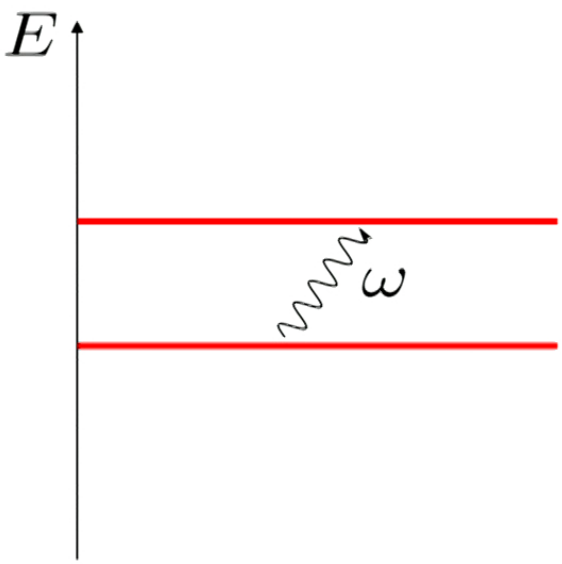
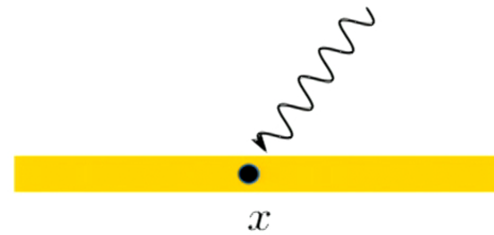
Heating in Floquet systems

[Abanin, de Roeck, Huveneers, PRL 2015]

$$H(t) = \overline{H} + V(t)$$





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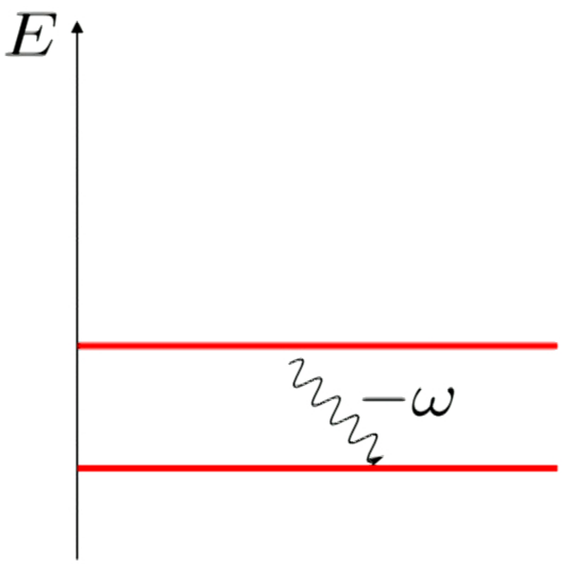
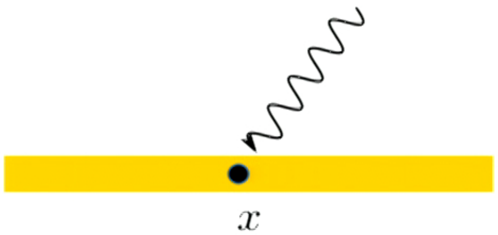
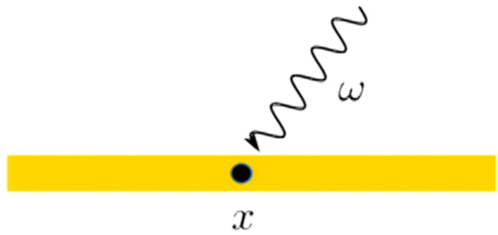
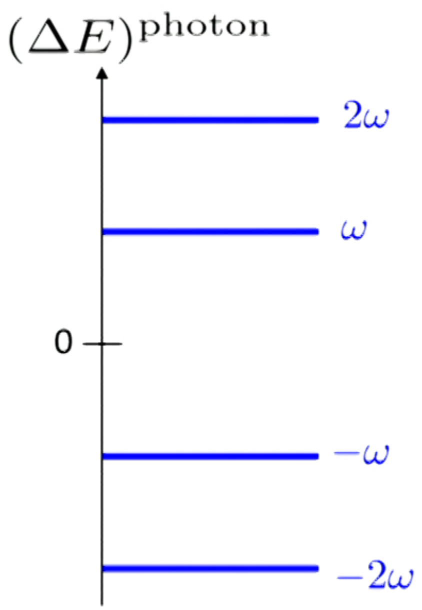




Resonances and how to avoid them

[Abanin, de Roeck, Huveneers, PRL 2015]





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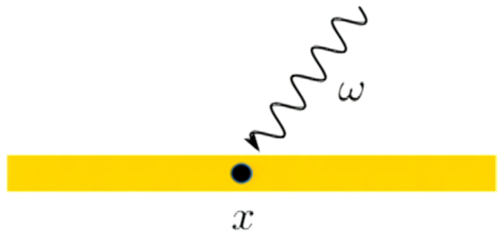
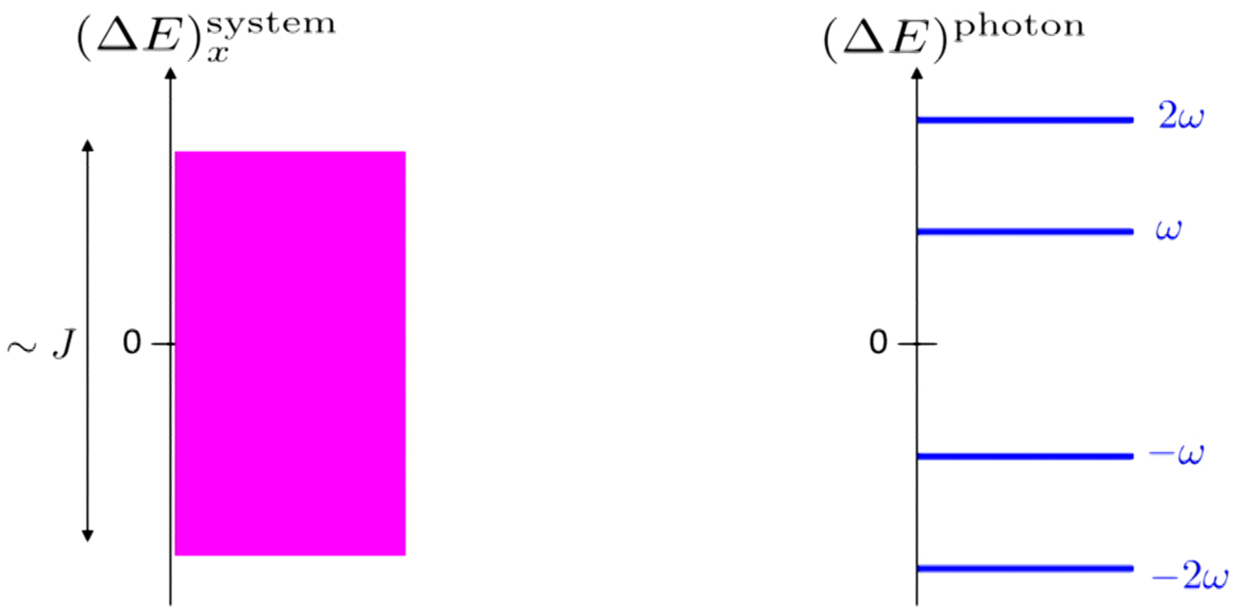


J is the local coupling strength of the Hamiltonian



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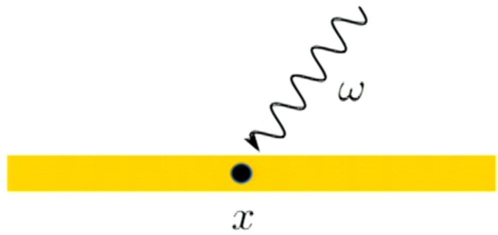

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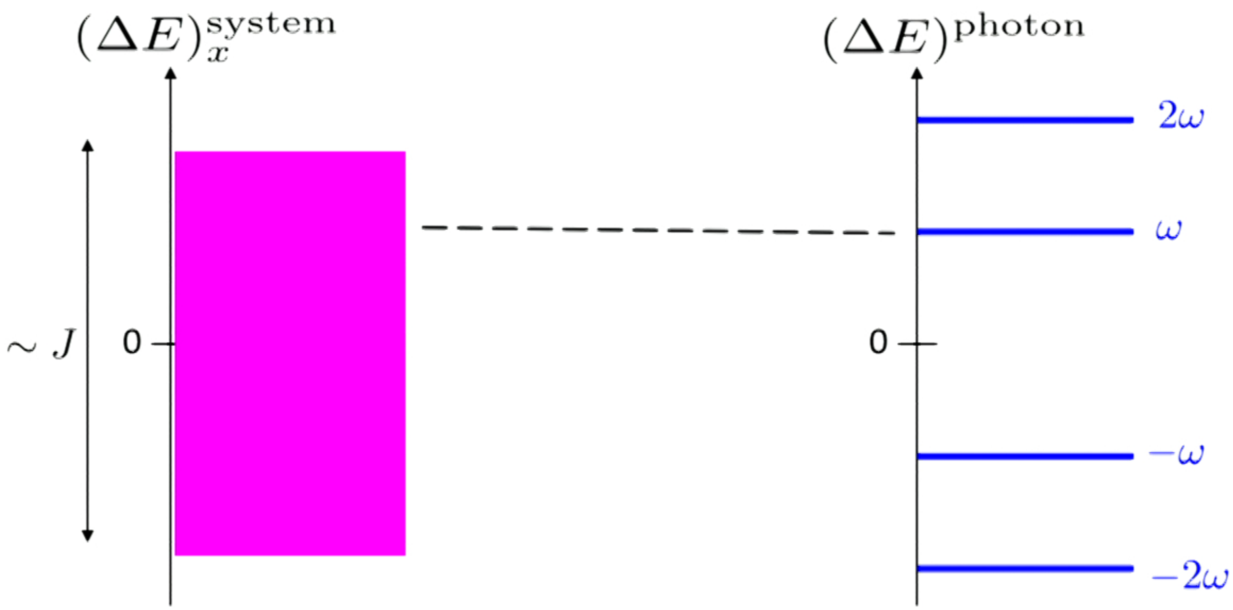

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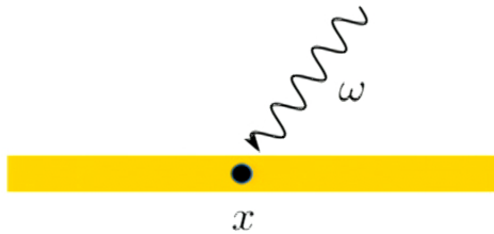

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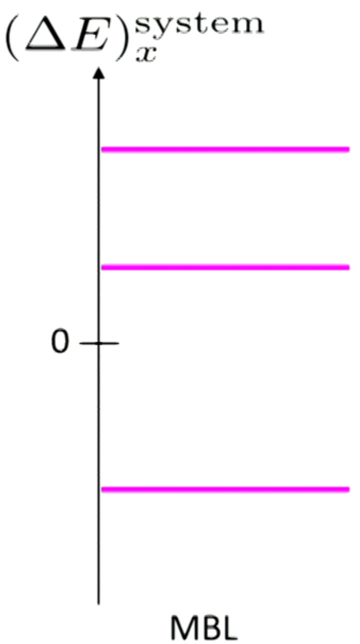

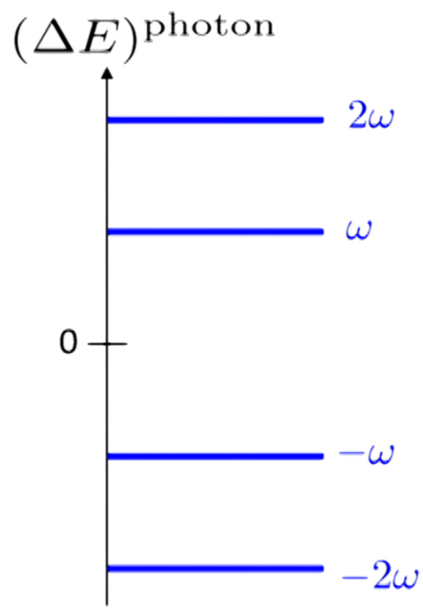

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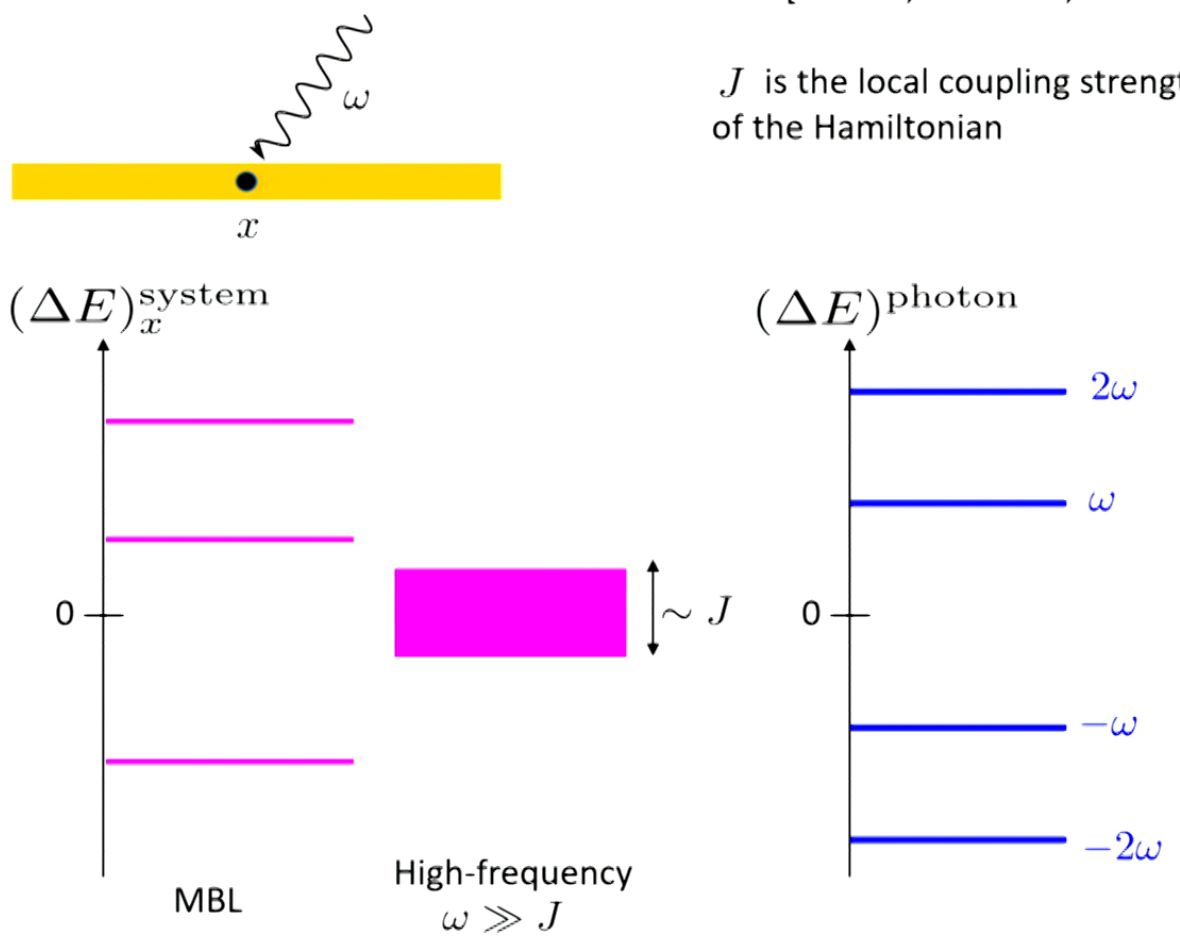


Summary of Floquet heating

- Heating can be suppressed in Floquet systems:
  - By high-frequency driving:
    - Heating time $t_* \sim e^{C\omega/J}$

    Rigorously proven in

    [Kuwahara et al, Ann. Phys 2016; Abanin et al, Comm. Math.Phys. 2017]

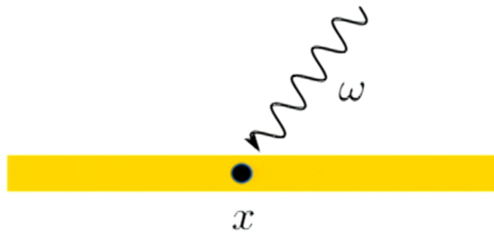
  - By strong disorder (MBL)

    $t_* = \infty$?

    [Ponte et al, Ann. Phys. 2015; Ponte et al, PRL 2015; Lazarides et al, PRL 2015; Abanin et al, Ann. Phys. 2016]

Resonances and how to avoid them

[Abanin, de Roeck, Huveneers, PRL 2015]



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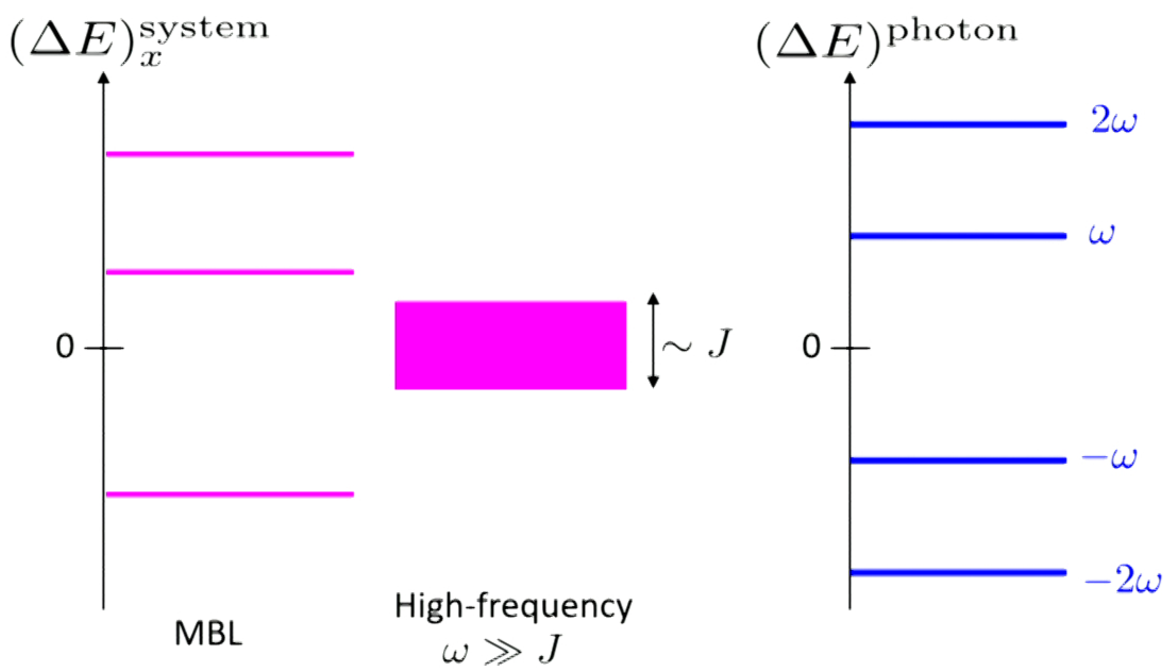

Smooth drives

$$H(t) = \overline{H} + V(t)$$

$V(t) = V(\boldsymbol{\omega} t)$ for some function $V(\boldsymbol{\theta})$

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Expand in a Fourier series

$$V(\boldsymbol{\theta}) = \sum_{\boldsymbol{n}} e^{i\boldsymbol{n}\cdot\boldsymbol{\theta}} V_{\boldsymbol{n}}$$

$$\implies V(t) = \sum_{\boldsymbol{n}} e^{i(\boldsymbol{n}\cdot\boldsymbol{\omega})t} V_{\boldsymbol{n}}$$

Smooth drives

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Expand in a Fourier series

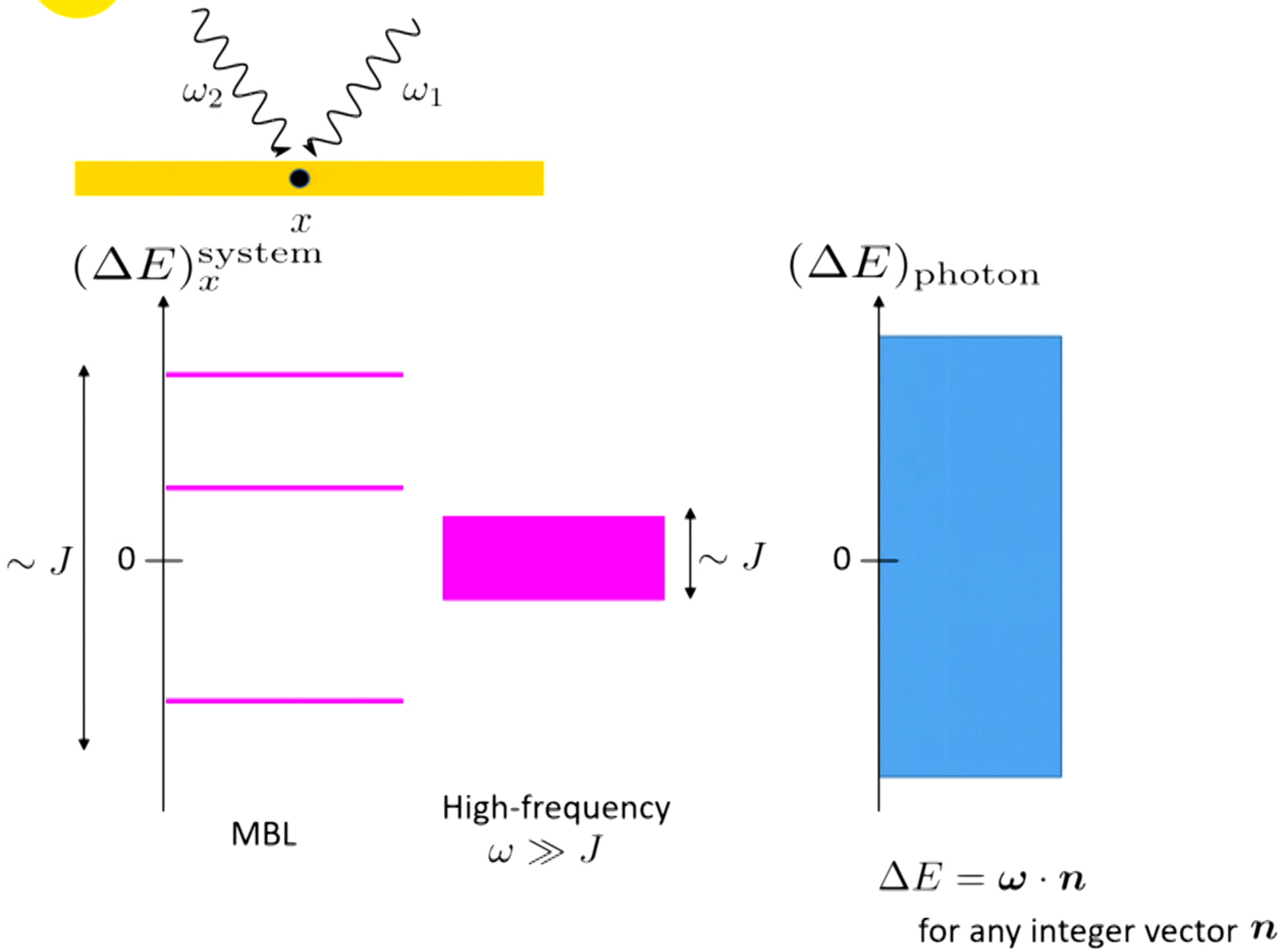
$$V(\boldsymbol{\theta}) = \sum_{\boldsymbol{n}} e^{i\boldsymbol{n}\cdot\boldsymbol{\theta}} V_{\boldsymbol{n}}$$

$$\Longrightarrow V(t) = \sum_{\boldsymbol{n}} e^{i(\boldsymbol{n}\cdot\boldsymbol{\omega})t} V_{\boldsymbol{n}}$$

In linear response, $V_{\boldsymbol{n}}$ generates transitions with $\Delta E = \boldsymbol{n}\cdot\boldsymbol{\omega}$

If $V(\boldsymbol{\theta})$ is smooth, then $V_{\boldsymbol{n}} \sim e^{-\kappa|\boldsymbol{n}|}$!

Resonances and how to avoid them



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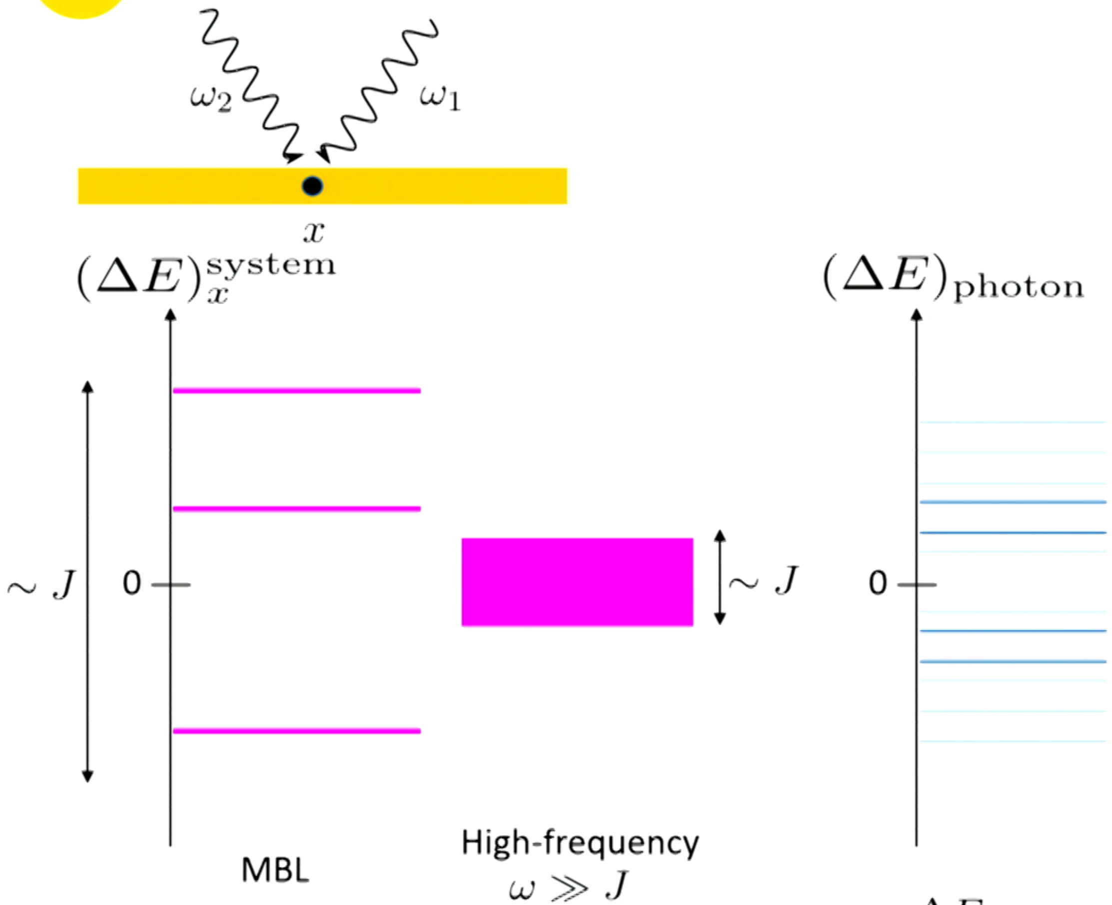

$\Delta E = \boldsymbol{\omega} \cdot \boldsymbol{n}$

for any integer vector $\boldsymbol{n}$

Diophantine approximation

- Set $\boldsymbol{\omega} = |\boldsymbol{\omega}|\boldsymbol{\alpha}$
- Theorem. *Almost all m*-dimensional vectors $\boldsymbol{\alpha}$ satisfy

$$|\boldsymbol{\alpha} \cdot \boldsymbol{n}| \geq c|\boldsymbol{n}|^{-(m-1+\epsilon)}$$

for some constant *c* depending on $\boldsymbol{\alpha}$

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$$|\boldsymbol{\alpha} \cdot \boldsymbol{n}| \geq c|\boldsymbol{n}|^{-(m-1+\epsilon)}$$

for some constant *c* depending on $\boldsymbol{\alpha}$

- Gives an estimate of heating time (for smooth drives)

$$t_* \sim \exp\left(C\left[\frac{|\boldsymbol{\omega}|}{J}\right]^{1/(m+\epsilon)}\right)$$

Our theorem

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- In the preheating regime $(t < t_*)$, the dynamics is well approximated by

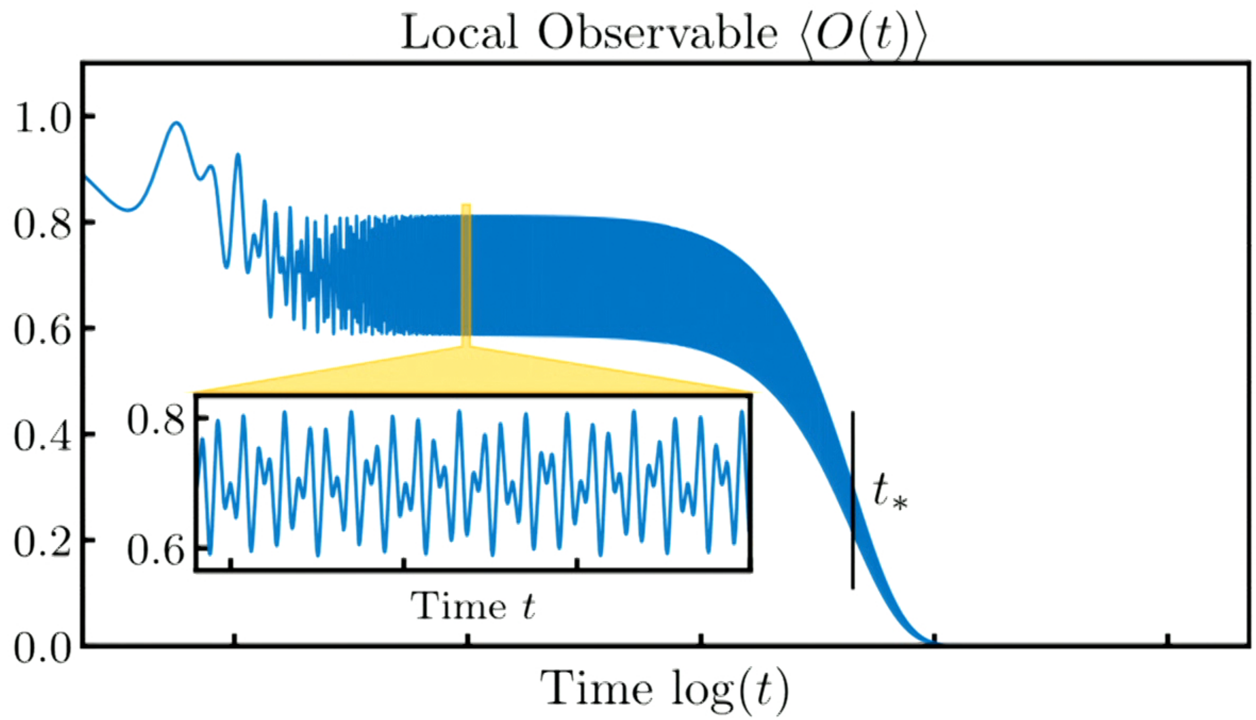
$$U(t) \approx P(t) \exp(-iDt) P^\dagger(0)$$

where:

  - D is a quasi-local static Hamiltonian
  - $P(t)$ is a local unitary that is *quasi-periodic* in time

If D is ergodic

$$U(t) \approx P(t)\exp(-iDt)P^{\dagger}(0)$$



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Many-body localization (MBL)

- MBL in a static system:

$$[\tau_i^z, H] = 0$$

Complete set of local integrals of motion ("l-bits")

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Complete set of local integrals of motion ("l-bits")

- Floquet-MBL (periodically driven system)

$$[\tau_i^z, U_F] = 0 \qquad U_F = \mathcal{T}\exp\left(-i\int_0^T H(t)dt\right)$$

- NEW Quasiperiodically driven MBL

  Reverse Heisenberg evolution $\tau_i^z(t) = U(t)\tau_i^z U^\dagger(t)$

  is *quasiperiodic* in time

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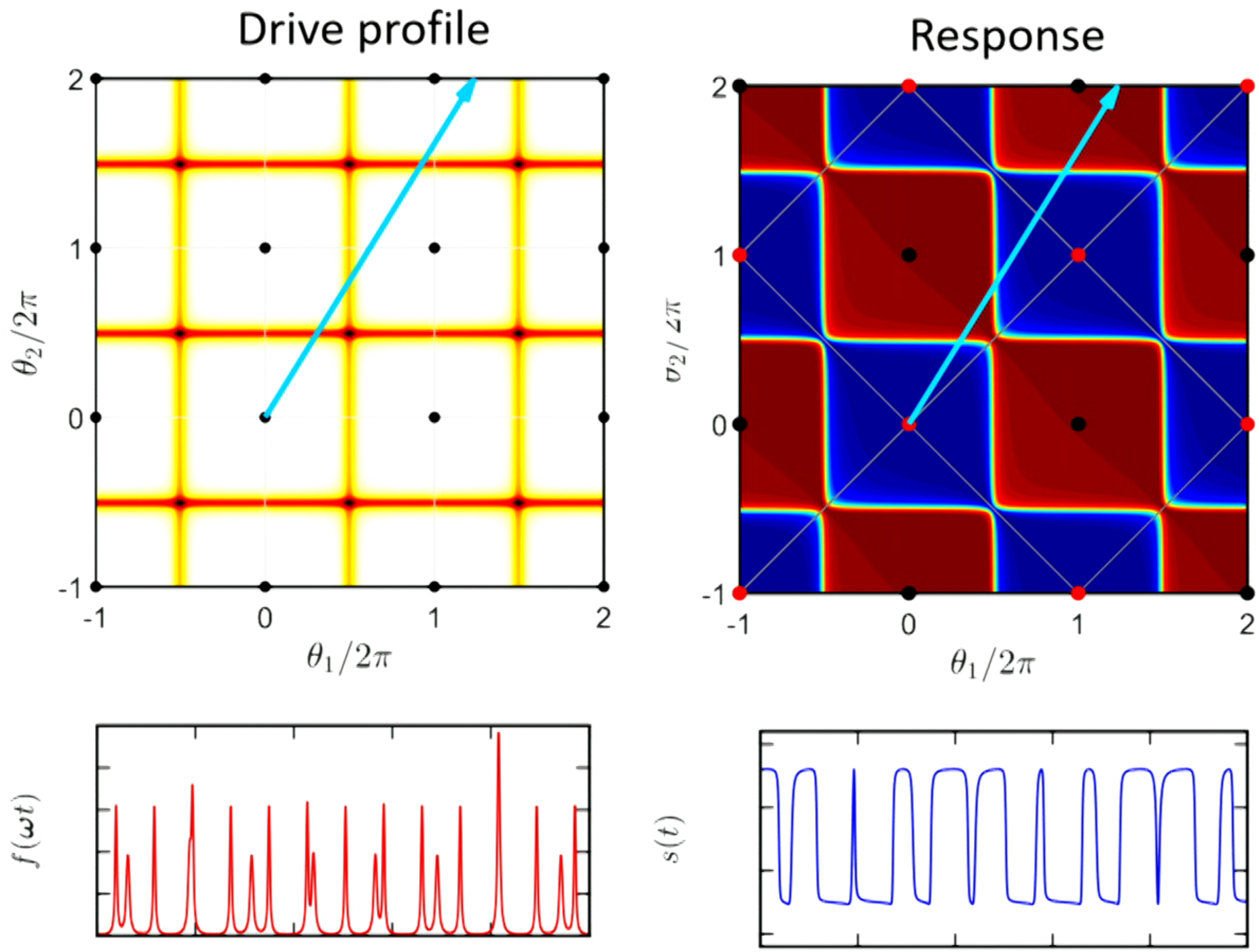
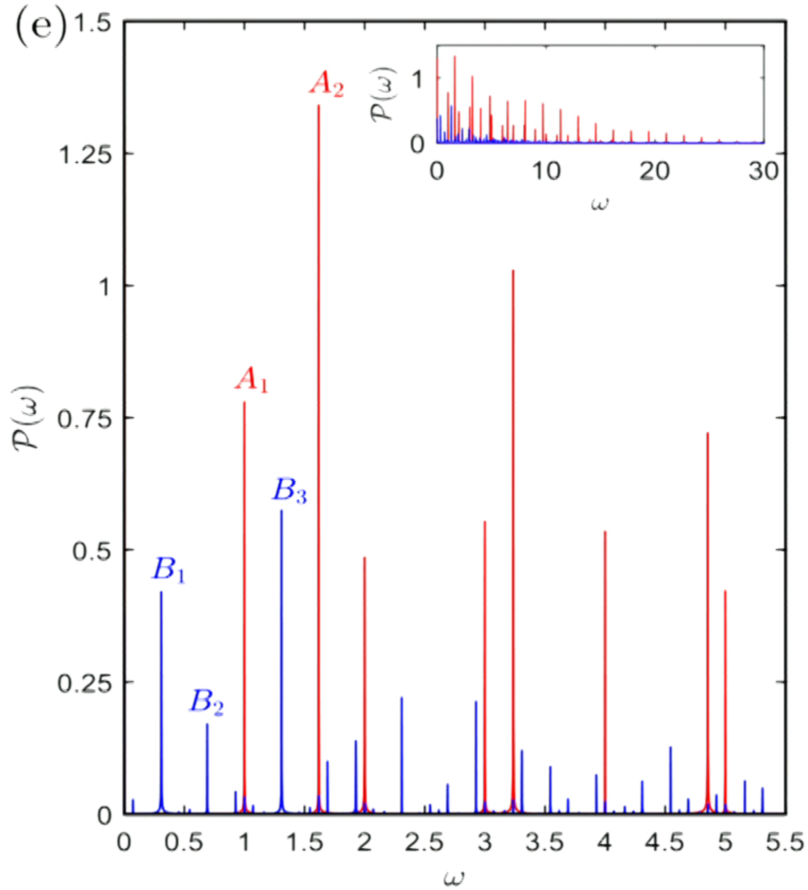
- New phases: Quasiperiodic topological phases

Discrete time quasicrystal

[See also: Potter et al, PRL 2018]



Power spectrum of discrete time quasicrystal



Driving (red) has peaks at

$$\boldsymbol{n} \cdot \boldsymbol{\omega}$$

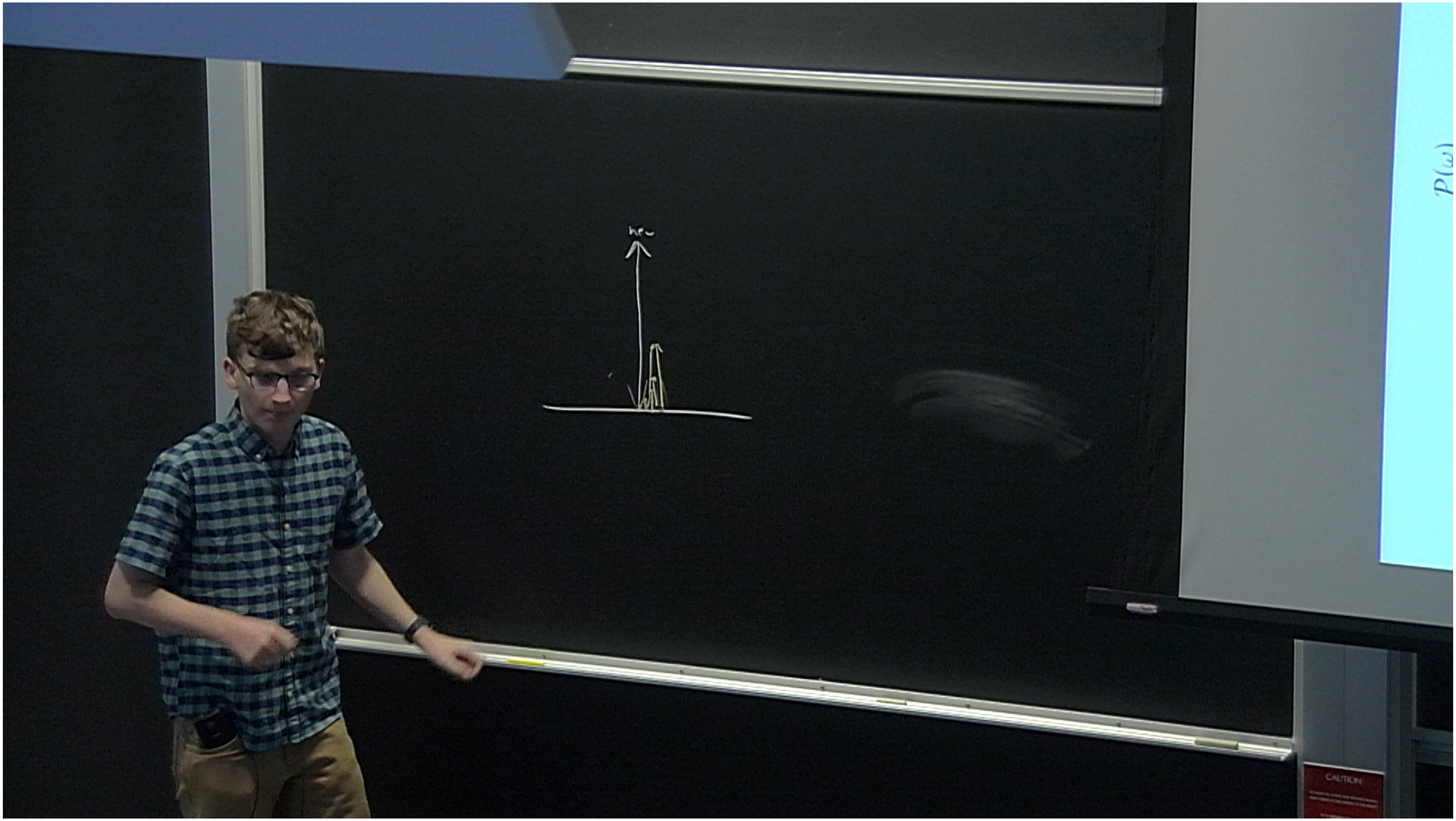
for integer vectors $\boldsymbol{n}$

Response (blue) has peaks at

$$\boldsymbol{\alpha} \cdot \boldsymbol{\omega}$$

where $\boldsymbol{\alpha}$ is a reciprocal lattice vector for the new lattice

All these patterns are *stable* (phase of matter) and sharply distinct from the "trivial" response



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Many-body localization (MBL)

- MBL in a static system:

$$[\tau_i^z, H] = 0$$

Complete set of local integrals of motion ("l-bits")

- Floquet-MBL (periodically driven system)

$$[\tau_i^z, U_F] = 0 \qquad U_F = \mathcal{T} \exp\left(-i \int_0^T H(t) dt\right)$$

- **NEW** Quasiperiodically driven MBL

  Reverse Heisenberg evolution $\tau_i^z(t) = U(t)\tau_i^z U^\dagger(t)$

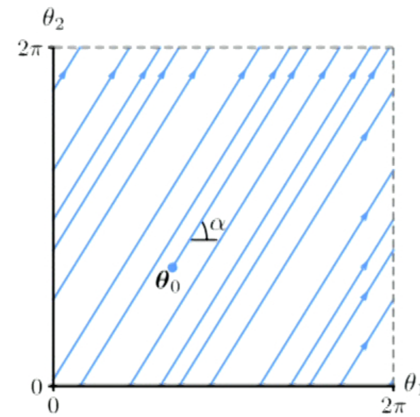
  is *quasiperiodic* in time

  Topological classification?

Micromotions with quasiperiodic driving

- One approach (not the whole story!)
  - Look at a single simultaneous eigenstate $|\psi\rangle$ of the τ_i^z's. It looks like a gapped ground state and its time evolution $|\psi(t)\rangle$ is quasiperiodic.
  - So we really have an *m*-dimensional function $|\psi(\boldsymbol{\theta})\rangle$
- So we really want to classify maps

$$\mathbb{T}^m \to \text{gapped ground states}$$



$m = 1$ (periodic driving):

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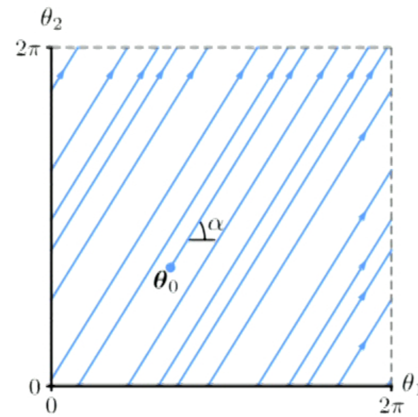

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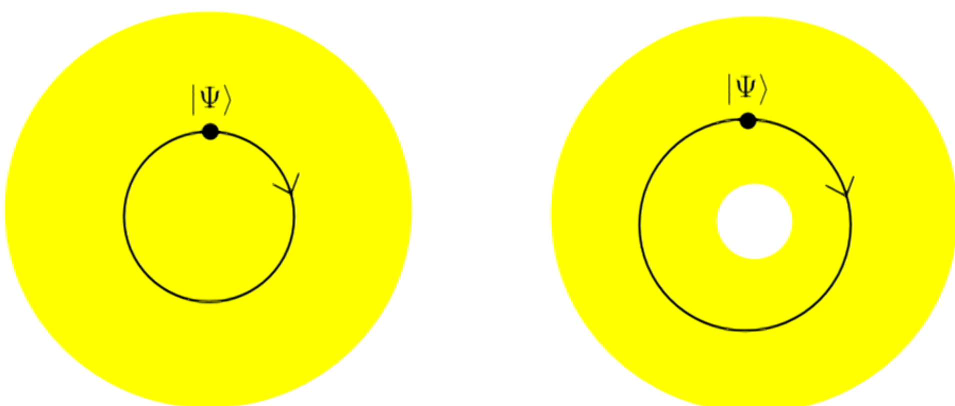
$$\mathbb{T}^m \to \text{gapped ground states}$$

$m = 1$ (periodic driving):



General classification results

- How do we classify maps

$$X \to \text{gapped ground states}$$

?

- The tentative answer is already contained within existing frameworks for topological phases

  [Turaev, math/0005291; Kitaev, unpublished;
  Thorngren & DVE, PRX '17; Cordova et al, 1905.09315]

- **Example:**
  - Bosonic SPTs w/ internal symmetry G classified by

    $$H^{d+2}(BG, \mathbb{Z})$$

  - Maps X -> gapped ground states (w/ internal symmetry G) are classified by

    $$H^{d+2}(X \times BG, \mathbb{Z})$$

Quasiperiodic equivalence principle

Classification of maps $\mathbb{T}^2 \to \text{gapped ground states}$
(with internal symmetry G)

$\longleftrightarrow$

Classification of phases with symmetry $G \times \mathbb{Z} \times \mathbb{Z}$

Topological phases protected by *two-dimensional* discrete time translation symmetry

Hamiltonian for the discrete time quasi-crystal

$$H(t) = f(t)\sum_i \sigma_i^x + \sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z + \cdots$$