Title: Higher spin symmetry in gravity and string

Speakers: Wei Li

Series: Colloquium

Date: October 09, 2019 - 2:00 PM

URL: http://pirsa.org/19100051

Abstract: Higher spin symmetries are gauge symmetries sourced by massless particles with spin greater than two. When coupled with diffeomorphism, they give rise to higher spin gravity. After a review on higher spin gravity, I will discuss its holography and its embedding in the string theory. Finally I will talk about some applications of higher spin symmetry, both in string theory and in QFT.

Higher spin symmetry in gravity and string

Wei Li

Institute of Theoretical Physics, Chinese Academy of Sciences

Perimeter, October 9, 2019

Based on work with *Datta, Eberhardt, Gaberdiel, Lin, Longhi, Gopakumar, Peng, Theisen, Wang, and Zhang*

Higher spin symmetry

Spin	Particles	symmetry
1	photon	$\mathfrak{u}(1)$
$3/2$	gravitino	supersymmetry
2	graviton	diffeomorphism
> 2	higher spin particles	higher spin gauge symmetry

Massless higher spin particles $\Longrightarrow$ higher spin gauge symmetry

1. Can couple to gravity $\Longrightarrow$ higher spin gravity
   $\Longrightarrow$ higher spin holography
   (simple, weak-weak duality, no need for susy...)
2. Subsector of stringy symmetry
   $\Longrightarrow$ help understand string theory
3. Symmetry of interesting QFT systems (from string theory)
   topological string, AGT correspondence, BPS algebras, $\cdots$

Brief history of Higher-spin theories

Massless higher spin particles in flat space

- Free spin-s field: *Fronsdal '78*

  totally-symmetric, double traceless, rank-s tensor $\phi_{\mu_1\mu_2\ldots\mu_s}$
  with gauge symmetry $\delta\phi_{\mu_1\mu_2\ldots\mu_s} = \partial_{\{\mu_1}\zeta_{\mu_2\ldots\mu_s\}}$ (ζ: traceless)

$$\mathrm{DOF} = \frac{(D-4+2s)(D-5+s)!}{(D-4)!s!} = \begin{cases} D-2 & \text{for } s=1 \\ \frac{D(D-3)}{2} & \text{for } s=2 \\ 0 & \text{for } D=3 \\ 2 & \text{for } D=4 \end{cases}$$

- Interacting, non-minimal coupling to gravity

  Bengtsson Bengtsson Brink '83, Metsaev '91

No-go against coupling HS to gravity (minimally) in flat space

Theorem (Coleman-Mandula '67)

S-matrix of interacting theory in flat space cannot have higher-spin symmetry

Vasiliev's higher-spin theory

Vasiliev: Go to AdS or dS *Fradkin Vasiliev '87*

- No S-matrix in (A)dS
- $\ell_{(\mathrm{A})\mathrm{dS}}$ serves as expansion parameter to control higher derivatives expansions. (non-local $\longrightarrow$ quasi-local.)

Vasiliev system

1. lives in AdS_d or dS_d
2. one field per spin from $s = 2, 3, \ldots, \infty$
   Vasiliev '91,Sundborg '01, Witten '01, Mikhailov '02, Klebanov-Polyakov '02
3. Extension of Einstein gravity
   - graviton couples to an infinite tower of massless higher-spin particles
   - diffeomorphism coupled with higher-spin gauge symmetry
   - Extension of Riemannian geometry
4. No lagrangian, a system of equation of motions.

Simplest higher spin gravity

3D Einstein gravity + negative c.c. $\Leftrightarrow$ $\mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$ Chern-Simon

From pure Einstein gravity to pure higher spin gravity:

$$\boxed{\mathfrak{sl}(2) \longrightarrow \mathfrak{sl}(N)}$$

Action: $S = S_{\mathrm{CS}}[A] - S_{\mathrm{CS}}[\tilde{A}]$

$$S_{\mathrm{CS}}[A] = \frac{k}{4\pi} \int_{\mathcal{M}} \mathrm{Tr}[A dA + \frac{2}{3} A^3] \qquad \text{with} \quad A, \tilde{A} \in \mathfrak{sl}(N, \mathbb{R})$$

Translate to metric-like formalism

1. Dreibein $e = \frac{A-\tilde{A}}{2}$ and spin connection $\omega = \frac{A+\tilde{A}}{2}$
2. metric and spin-3 field

$$G_{\mu\nu} = \mathrm{Tr}[e_\mu e_\nu] \qquad \varphi^{(3)}_{\mu\nu\rho} = \mathrm{Tr}[e_{\{\mu} e_\nu e_{\rho\}}] \qquad \ldots$$

$\mathfrak{sl}(N) \oplus \mathfrak{sl}(N)$ Chern-Simons theory — Spectrum

Spectrum of $\mathfrak{sl}(N)$ Chern-Simons theory

1. Choose an $\mathfrak{sl}(2)$ subalgebra that corresponds to spin-2:

$$\text{spin-2}: \qquad \{L_1, \quad L_0, \quad L_{-1}\}$$

2. Decompose $\mathfrak{sl}(N)$ in terms of irreps of the gravitonal $\mathfrak{sl}(2)$

$$\text{spin-s} \ : \qquad \{W_m^{(s)}\} \qquad m = -s+1, \ldots, s-1$$

Principal embedding: 1 spin-s field for each $s = 2, ..., N$

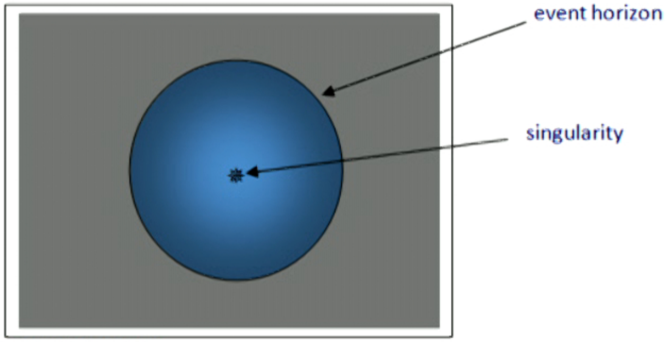
$$\begin{array}{ccccccc} W_3^{(4)} & W_2^{(4)} & W_1^{(4)} & W_0^{(4)} & W_{-1}^{(4)} & W_{-2}^{(4)} & W_{-3}^{(4)} \\ & W_2^{(3)} & W_1^{(3)} & W_0^{(3)} & W_{-1}^{(3)} & W_{-2}^{(3)} & \\ & & L_1 & L_0 & L_{-1} & & \end{array}$$

$\mathfrak{sl}(N) \oplus \mathfrak{sl}(N)$ Chern-Simons theory — Classical solutions

1. No propagating degrees of freedom
2. Non-trivial classical solutions such as black holes

Black hole in Einstein gravity:

singlularity behind horizon
(cosmic censorship)



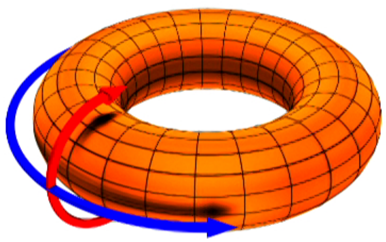
Model of a black hole

3. Problem of defining black holes in higher spin gravity:
   Singularity and horizon are not gauge-invariant concepts.
   (metric is coupled to higher-spin fields)
4. Question: how to define black hole in higher spin gravity"

Higher-spin black hole and conical surplus

Cosmic censorship = smoothness condition

gauge theory: $\boxed{\mathrm{Hol}_{\mathcal{C}}(A) \equiv \mathcal{P}e^{\oint_{\mathcal{C}} A} \text{ is trivial around a shrinkable cycle.}}$



- Shrinkable cycle is time $\Longrightarrow$ higher spin black holes

Gutperle Kraus '11

- Shrinkable cycle is space $\Longrightarrow$ Conical surplus *Castro et.al. '11*
  (higer spin version of thermal AdS_3)

- $SL(2,\mathbb{Z})$-family of solutions

WL Lin Wang '13

higher-spin holography

Understand quantum higher spin gravity via its holographic dual

$$\text{gravity with higher spin gauge symmetry in } \mathrm{AdS}_{d+1}$$
$$\Updownarrow$$
$$\mathrm{CFT}_d \text{ with high spin currents}$$

Examples of higher spin holography

- Vasiliev in AdS_4 = O(N) model — *Klebanov Polyakov '02*, *Sezgin Sundell '02*
- Vasiliev in AdS_3 = $\mathcal{W}_{N,k}$ minimal model $\left(= \frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}\right)$ — *Gaberdiel Gopakumar '10*

Features of higher-spin holography

1. Weak/Weak duality

$$\left(\frac{R}{\ell_p}\right)^4 \sim N \qquad \left(\frac{R}{\ell_s}\right)^4 \sim \lambda = (g_{YM})^2 N$$

Traditional Gauge/Gravity correspondence: Strong/Weak duality

$$R \gg \ell_s \gg \ell_p \qquad \Longrightarrow \qquad \boxed{N \to \infty \qquad \lambda \gg 1}$$

Higher-Spin theory: Weak/Weak duality

$$\ell_s \gg R \gg \ell_p \qquad \Longrightarrow \qquad \boxed{N \to \infty \qquad \lambda \ll 1}$$

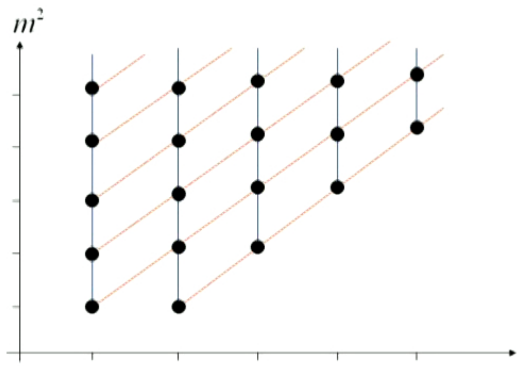
2. Supersymmetry is not required.

3. CFT dual is always free *Maldacena-Zhiboedov '11, Stanev '13*
   - Except for AdS_3/CFT_2
   - AdS version of Coleman-Mandula

tensionless limit and stringy symmetry

String theory has infinite number of massive higher spin particles

$$m \sim \frac{1}{\ell_s}$$



Tensionless limit: $\frac{\ell_s}{\ell_{\text{AdS}}} \to \infty$ (quantum: 1)

massive higher spin particle $\Longrightarrow$ massless $\Longrightarrow$ stringy symmetry

- subalgebra: Vasiliev higher spin symmetry (one per spin)

  (from Leading Regge trajectory)

Vasiliev '91, Sundborg '01, Witten '01, Mikhailov '02, Klebanov-Polyakov '02

Tensionless string in AdS$_3$

- **Symmetry enhancement** from bulk AdS$_3$ to boundary CFT$_2$

	Bulk Isometry		Bndy global symmetry
Einstein	$\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$	$\xrightarrow{\text{Brown Henneaux '86}}$	Virasoro $\oplus$ Virasoro
	$\downarrow$		$\downarrow$
Higher-Spin	$\mathfrak{hs}[\lambda] \oplus \mathfrak{hs}[\lambda]$	$\xrightarrow[\text{Henneaux Rey '10}]{\text{Campoleoni et.al. '10}}$	$\mathcal{W}_\infty[\lambda] \oplus \mathcal{W}_\infty[\lambda]$

$$\mathfrak{hs}[\lambda]$$

⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
spin-5		X_{-4}	X_{-3}	X_{-2}	X_{-1}	X_0	X_1	X_2	X_3	X_4
spin-4			U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3	
spin-3				W_{-2}	W_{-1}	W_0	W_1	W_2		
spin-2					L_{-1}	L_0	L_1			

Tensionless string in AdS_3

- Symmetry enhancement from bulk AdS_3 to boundary CFT_2

	Bulk Isometry		Bndy global symmetry
Einstein	$\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$	$\xrightarrow{\text{Brown Henneaux '86}}$	Virasoro $\oplus$ Virasoro
	$\downarrow$		$\downarrow$
Higher-Spin	$\mathfrak{hs}[\lambda] \oplus \mathfrak{hs}[\lambda]$	$\xrightarrow[\text{Henneaux Rey '10}]{\text{Campoleoni et.al. '10}}$	$\mathcal{W}_\infty[\lambda] \oplus \mathcal{W}_\infty[\lambda]$

- In tensionless limit, the worldsheet theory of string in $\mathrm{AdS}_3 \times S^3 \times T^4$ is a symmetric orbifold of T^4.

  Gaberdiel Gopakumar '18, Eberhardt Gaberdiel Gopakumar '19

- Stringy symmetry at $\mathrm{AdS}_3 \quad \sim \quad$ chiral algebra of $\mathrm{Sym}^N(T^4)$
  - What is the mathematical description?

Stringy symmetry $\gg$ higher-spin symmetry $\gg$ conformal symmetry

Size of stringy symmetry

Virasoro ($\sim$ partition) *Euler*

$$\prod_{k=1}^{\infty} \frac{1}{1-q^k} = \sum_{n=0} p(n) q^n = 1 + q + 2\,q^2 + 3\,q^3 + 5\,q^4 + 7\,q^5 + 11\,q^6 + \cdots$$

$$p(n) \sim \frac{1}{n} \cdot \exp\left(\sqrt{\tfrac{2}{3}}\,\pi\,\sqrt{n}\right)$$ *Hardy Ramanujan '18*

higher spin symmetry ($\sim$ plane partition) *MacMahon*

$$\prod_{k=1}^{\infty} \frac{1}{(1-q^k)^k} = \sum_{n=0} M(n) q^n = 1 + q + 3\,q^2 + 6\,q^3 + 13\,q^4 + 24\,q^5 + 48\,q^6 + \cdots$$

$$M(n) \sim n^{-\frac{25}{36}} \cdot \exp\left(\frac{3\zeta(3)^{\frac{1}{3}}}{2^{\frac{2}{3}}} n^{\frac{2}{3}}\right)$$ *Wright '31*

stringy symmetry ($\sim$ double partition)

$$\prod_{k=1}^{\infty} \frac{1}{(1-q^k)^{p(k)}} = \sum_{n=0} D(n) q^n = 1 + q + 3\,q^2 + 6\,q^3 + 14\,q^4 + 27\,q^5 + 58\,q^6 + \cdots$$

$$D(n) \sim n \cdot \exp\left(\frac{\pi^2}{6} n\right)$$ *Kaneiwa '79*

plane partition

plane partition *MacMahon '12*

$$\sum_{n=0} M(n) q^n = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^k} = 1 + q + 3\,q^2 + 6\,q^3 + 13\,q^4 + 24\,q^5 + 48\,q^6 + \cdots$$

$$M(n) \sim n^{-\frac{25}{36}} \cdot \exp\left(\frac{3\zeta(3)^{\frac{1}{3}}}{2^{\frac{2}{3}}} n^{\frac{2}{3}} \right)$$ *Wright '31*

MacMahon's conjecture on d-dim partition

$$\sum_{\Lambda_d} q^{|\Lambda_d|} = \sum_{n=0} P^{(d)}(n) q^n = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^{\binom{k+d-3}{d-2}}}$$

partition $(d=2)$: $\sum_{n=0} p(n) q^n = \prod_{k=1}^{\infty} \frac{1}{1-q^k}$ ✓

plane partition $(d=3)$: $\sum_{n=0} M(n) q^n = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^k}$ ✓

solid partition $(d=4)$: $\sum_{n=0} S(n) q^n = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^{\frac{k(k+1)}{2}}}$

$$= 1 + q + 4\,q^2 + 10\,q^3 + 26\,q^4 + 59\,q^5 + 141\,q^6 + \cdots$$

compute by hand

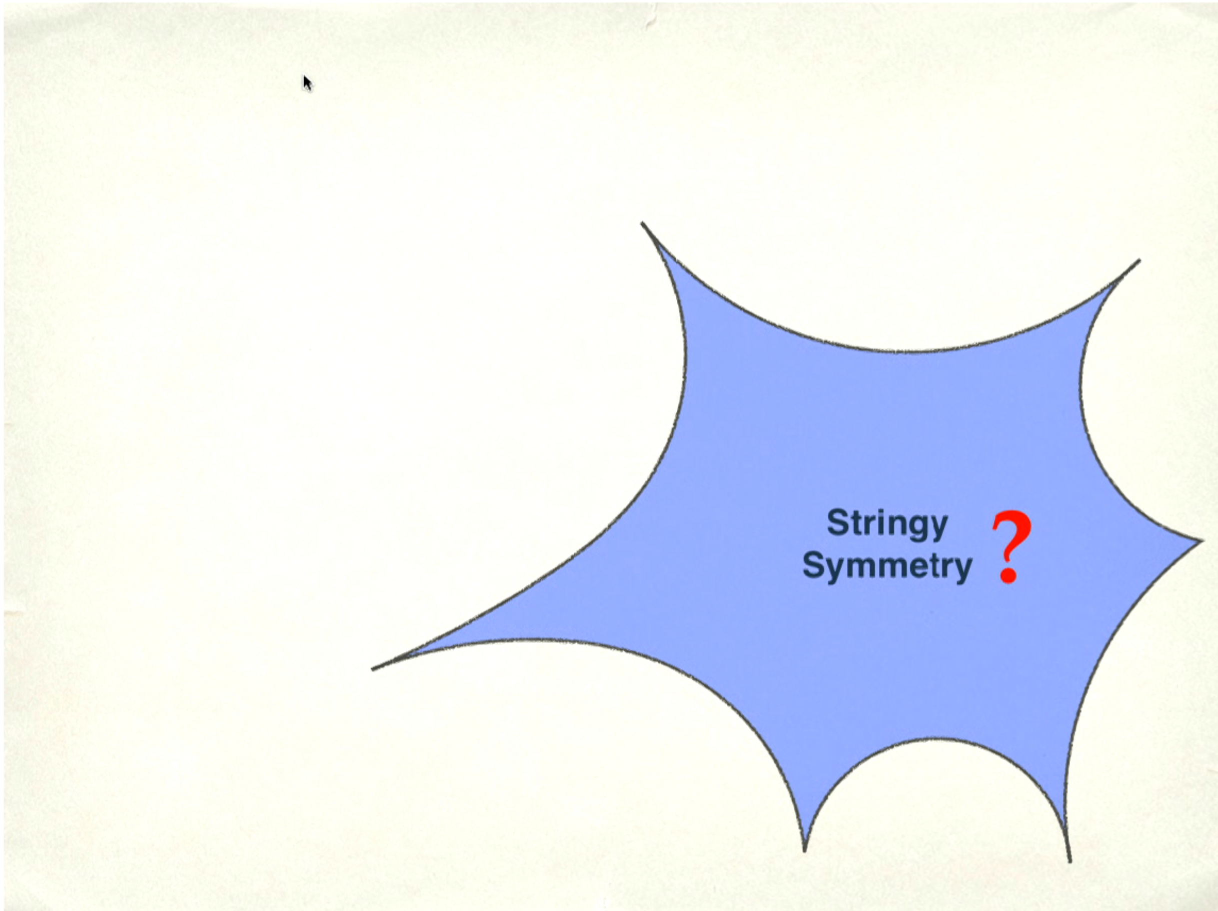
$$\sum_{n=0} S(n) q^n = 1 + q + 4\,q^2 + 10\,q^3 + 26\,q^4 + 59\,q^5 + 140\,q^6 + \cdots$$

Generating function for $d \geq 4$ dimensional partition is still unknown.

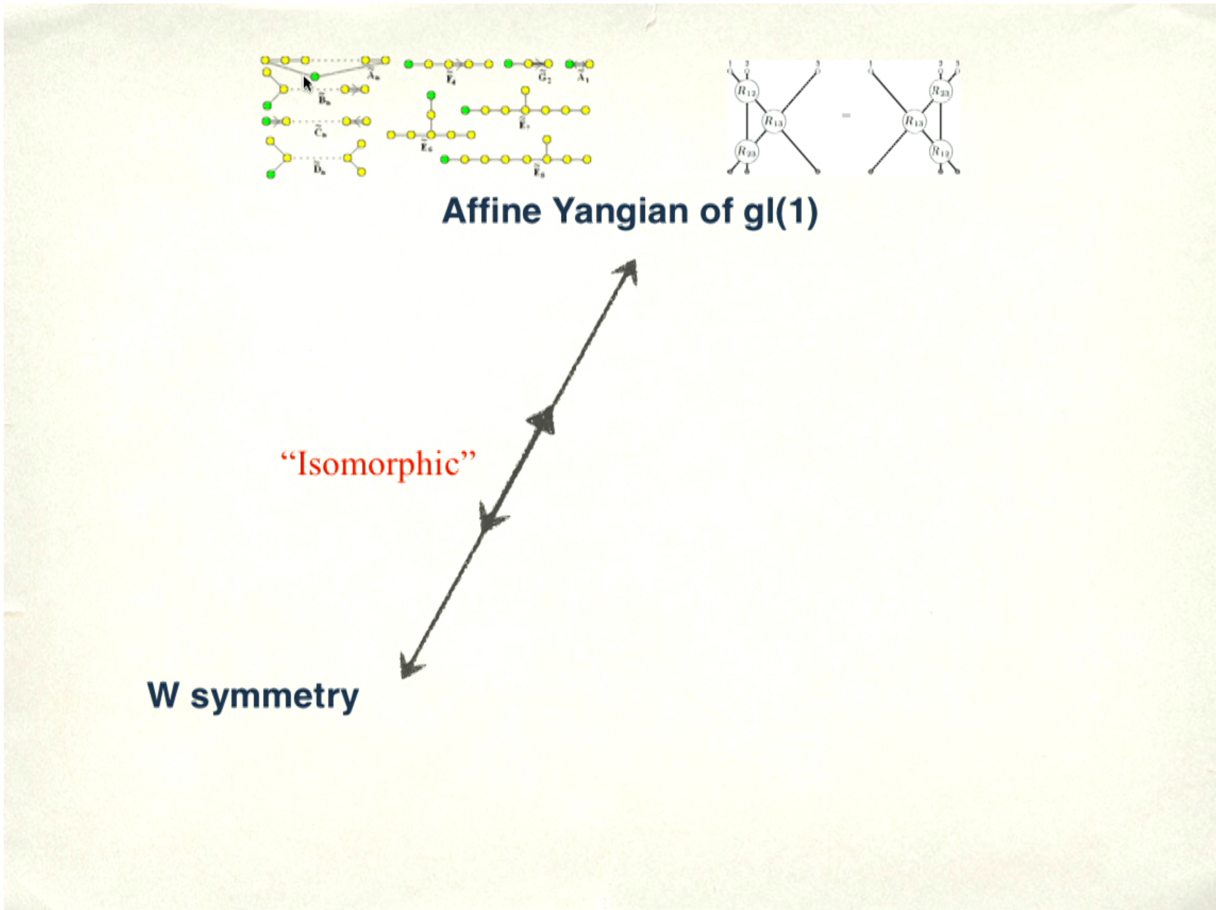
Embed higher-spin theory in string theory

	Bulk		Boundary
Higher spin	Vasiliev $\mathfrak{shs}_2[\lambda]$	$\longleftrightarrow$	Wolf coset
	$\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$		$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \otimes \mathfrak{u}(1)_k} \otimes \mathfrak{u}(1)$
	$\downarrow \lambda \to 0$		$\downarrow k \to \infty$
Free higher spin	Vasiliev $\mathfrak{shs}_2[0]$	$\longleftrightarrow$	$(T^4)^{\otimes(N+1)}/U(N)$
	$\mathrm{AdS}_3 \times S^3 \times T^4$		
	$\downarrow$		$\downarrow$
String	tensionless string theory	$\longleftrightarrow$	$(T^4)^{\otimes(N+1)}/S_{N+1}$
	$\mathrm{AdS}_3 \times S^3 \times T^4$		

How to characterize stringy symmetry mathematically?



A concrete relation between HS and integrability



Modes of $\mathcal{W}_{1+\infty}$

$$W^{(s)}(z) = \sum_{n\in\mathbb{Z}} \frac{W_n^{(s)}}{z^{n+s}} \qquad s = 1, 2, 3, \dots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
spin-5	$\dots$	X_{-4}	X_{-3}	X_{-2}	X_{-1}	X_0	X_1	X_2	X_3	X_4	$\dots$
spin-4	$\dots$	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3	U_4	$\dots$
spin-3	$\dots$	W_{-4}	W_{-3}	W_{-2}	W_{-1}	W_0	W_1	W_2	W_3	W_4	$\dots$
spin-2	$\dots$	L_{-4}	L_{-3}	L_{-2}	L_{-1}	L_0	L_1	L_2	L_3	L_4	$\dots$
spin-1	$\dots$	J_{-4}	J_{-3}	J_{-2}	J_{-1}	J_0	J_1	J_2	J_3	J_4	$\dots$

Regrouping the modes

$$W^{(s)}(z) = \sum_{n\in\mathbb{Z}} \frac{W_n^{(s)}}{z^{n+s}} \qquad s = 1, 2, 3, \ldots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
spin-5	$\ldots$	X_{-3}	X_{-2}	$X_{-1} \sim e_4$	$X_0 \sim \psi_5$	$X_1 \sim f_4$	X_2	X_3	X_4
spin-4	$\ldots$	U_{-3}	U_{-2}	$U_{-1} \sim e_3$	$U_0 \sim \psi_4$	$U_1 \sim f_3$	U_2	U_3	U_4
spin-3	$\ldots$	W_{-3}	W_{-2}	$W_{-1} \sim e_2$	$W_0 \sim \psi_3$	$W_1 \sim f_2$	W_2	W_3	W_4
spin-2	$\ldots$	L_{-3}	L_{-2}	$L_{-1} \sim e_1$	$L_0 \sim \psi_2$	$L_1 \sim f_1$	L_2	L_3	L_4
spin-1	$\ldots$	J_{-3}	J_{-2}	$J_{-1} \sim e_0$	$J_0 \sim \psi_1$	$J_1 \sim f_0$	J_2	J_3	J_4

affine Yangian generators

$$e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad \psi(z) = 1 + \sigma_3 \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

Affine Yangian of $\mathfrak{gl}_1$

Def: Associative algebra with generators e_j, f_j and ψ_j, $j = 0, 1, \ldots$

- Generators

$$\psi(z) = 1 + (h_1 h_2 h_3) \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

- Parameters (h_1, h_2, h_3) with $h_1 + h_2 + h_3 = 0$
- One S_3 invariant function $\varphi_3(z) = \frac{(z+h_1)(z+h_2)(z+h_3)}{(z-h_1)(z-h_2)(z-h_3)}$
- Defining relations

$$[e(z)\,, f(w)] = -\frac{1}{h_1 h_2 h_3} \frac{\psi(z) - \psi(w)}{z - w}$$

$$\psi(z)\, e(w) \sim \varphi_3(z-w)\, e(w)\, \psi(z) \qquad \psi(z)\, f(w) \sim \varphi_3(w-z)\, f(w)\, \psi(z)$$

$$e(z)\, e(w) \sim \varphi_3(z-w)\, e(w)\, e(z) \qquad f(z)\, f(w) \sim \varphi_3(w-z)\, f(w)\, f(z)$$

- Initial conditions

$$[\psi_{0,1}, e_j] = 0 \quad [\psi_2, e_j] = 2e_j \quad [\psi_{0,1}, f_j] = 0 \quad [\psi_2, f_j] = -2f_j$$

- Serre relation

$$\mathrm{Sym}_{(j_1,j_2,j_3)}\, [e_{j_1}, [e_{j_2}, e_{j_3+1}]] = 0 \qquad \mathrm{Sym}_{(j_1,j_2,j_3)}\, [f_{j_1}, [f_{j_2}, f_{j_3+1}]] = 0$$

Schiffmann Vasserot '12 Maulik Okounkov '12 Tsymbaliuk '14

Advantages of affine Yangian over $\mathcal{W}_\infty$

1. number of generators
   - $\mathcal{W}_\infty$: ∞

$$J(z)\,,T(z)\,,W^{(3)}(z)\,,W^{(4)}(z)\,\ldots$$

   - affine Yangian of $\mathfrak{gl}_1$: only 3

$$\psi(z)\,,e(z)\,,f(z)$$

2. Defining relations
   - $\mathcal{W}_\infty$:

non-linear, fixed order by order by Jacobi-identities

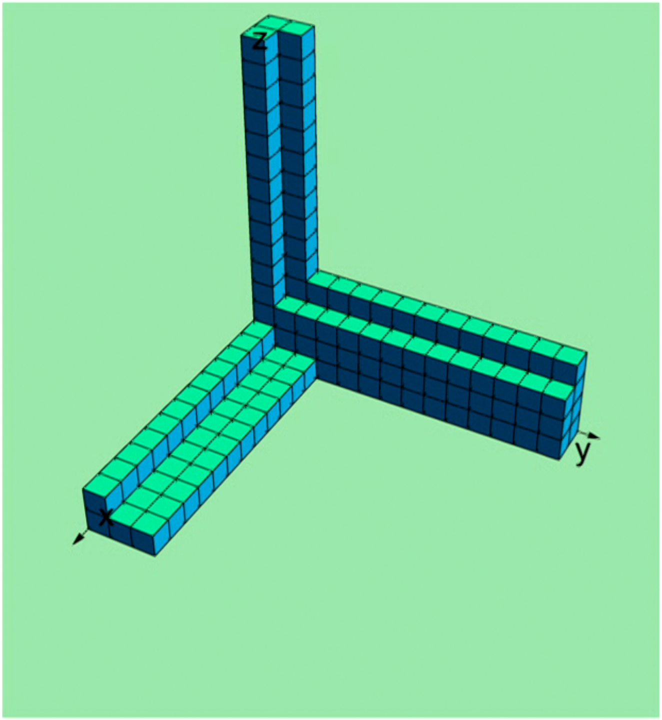
   - affine Yangian of $\mathfrak{gl}_1$:

linear, given explicitly

3. $\mathcal{S}_3$ invariance
   - $\mathcal{W}_\infty$: Hidden
   - affine Yangian of $\mathfrak{gl}_1$: manifest
4. Plane partition representation

Plane partition with non-trivial asymptotics

Ground state of $(\Lambda_x, \Lambda_y, \Lambda_z)$



Action of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$ on a plane partition

- $\psi(z)$ acts diagonally *Tsymbaliuk '14, Prochazka '15*

$$\psi(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$$

$$\psi_\Lambda(z) \equiv \left(1 + \frac{\psi_0 \sigma_3}{z}\right) \prod_{\square \in (\Lambda)} \varphi_3(z - h(\square))$$

$$h(\square) = h_1 x(\square) + h_2 y(\square) + h_3 z(\square)$$

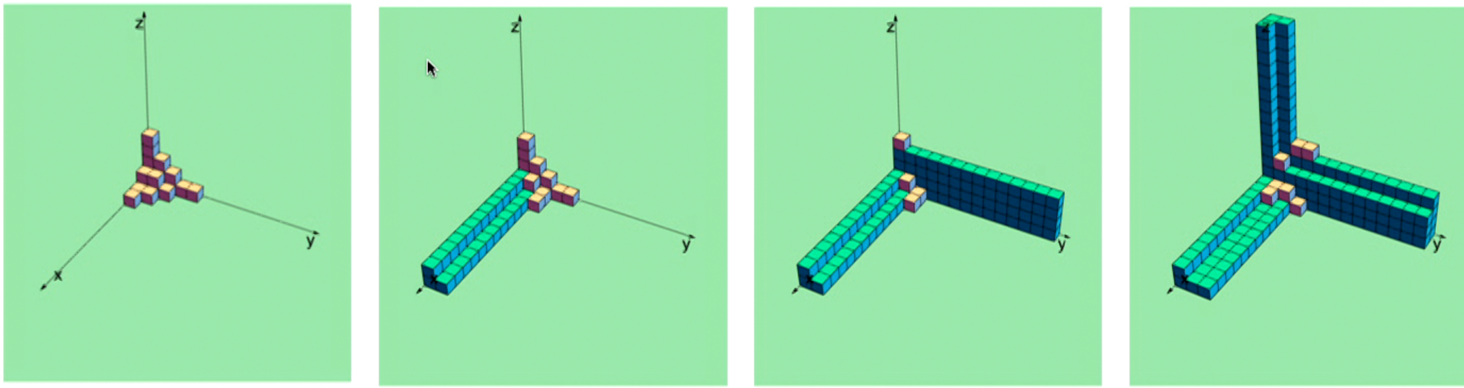
- $e(z)$ adds one box

$$e(z)|\Lambda\rangle = \sum_{\square \in \mathrm{Add}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)} \psi_\Lambda(w)\right]^{\frac{1}{2}}}{z - h(\square)} |\Lambda + \square\rangle$$

- $f(z)$ removes one box

$$f(z)|\Lambda\rangle = \sum_{\square \in \mathrm{Rem}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)} \psi_\Lambda(w)\right]^{\frac{1}{2}}}{z - h(\square)} |\Lambda - \square\rangle$$

Plane partition as representations of W



Trivial b.c.	$(\Lambda_x;0)=(\Lambda;0)$	$(\Lambda_x;\Lambda_y)=(\Lambda_+;\Lambda_-)$	$(\Lambda_x;\Lambda_y;\Lambda_z)$
vacuum	perturbative in Vasiliev	non-perturbative in Vasiliev	new representation

character of $\mathcal{W}_{1+\infty}$ = generating function of plane partition

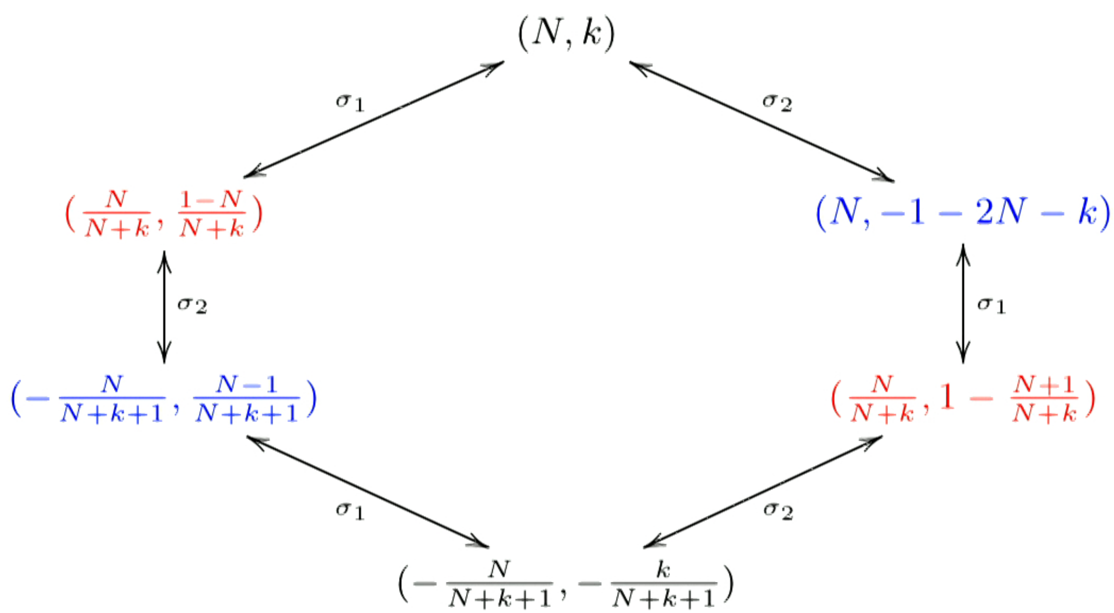
Feigin Jimbo Miwa Mukhin '10-11

$\mathcal{S}_3$ action on $\mathcal{W}_{N,k}$ coset

$\mathcal{W}_{N,k}$ coset

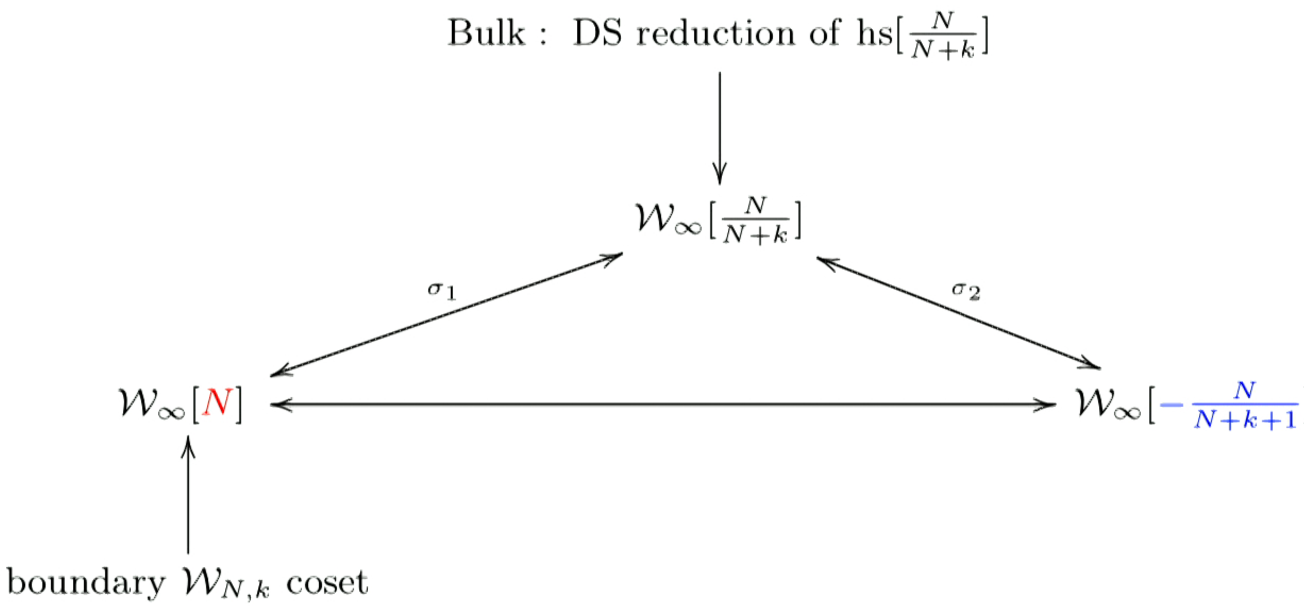
$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

had hidden $\mathcal{S}_3$



Triality symmetry for higher spin holography

For fixed c, three $\mathcal{W}_\infty[\lambda]$ are isomorphic *Gaberdiel Gopakumar '12*

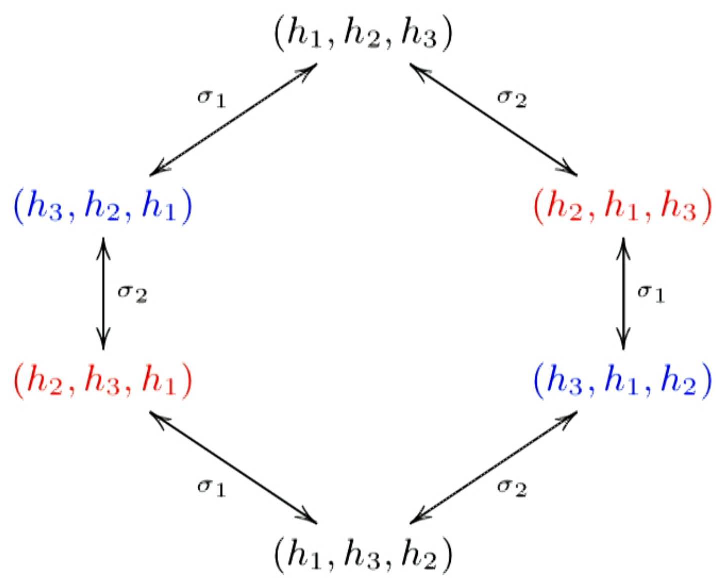


Crucial for Vasiliev theory in $\text{AdS}_3 = \mathcal{W}_{N,k}$ coset

$\mathcal{S}_3$ symmetry in $\mathcal{Y}[\mathfrak{gl}_1]$

- $\mathcal{S}_3$ symmetry in $\mathcal{W}_\infty$CFT is highly non-trivial (UV-IR)
  - hard to check/prove *Gaberdiel Gopakumar '12, Linshaw '17*
- Manifest in $\mathcal{Y}[\widehat{\mathfrak{gl}_1}]$

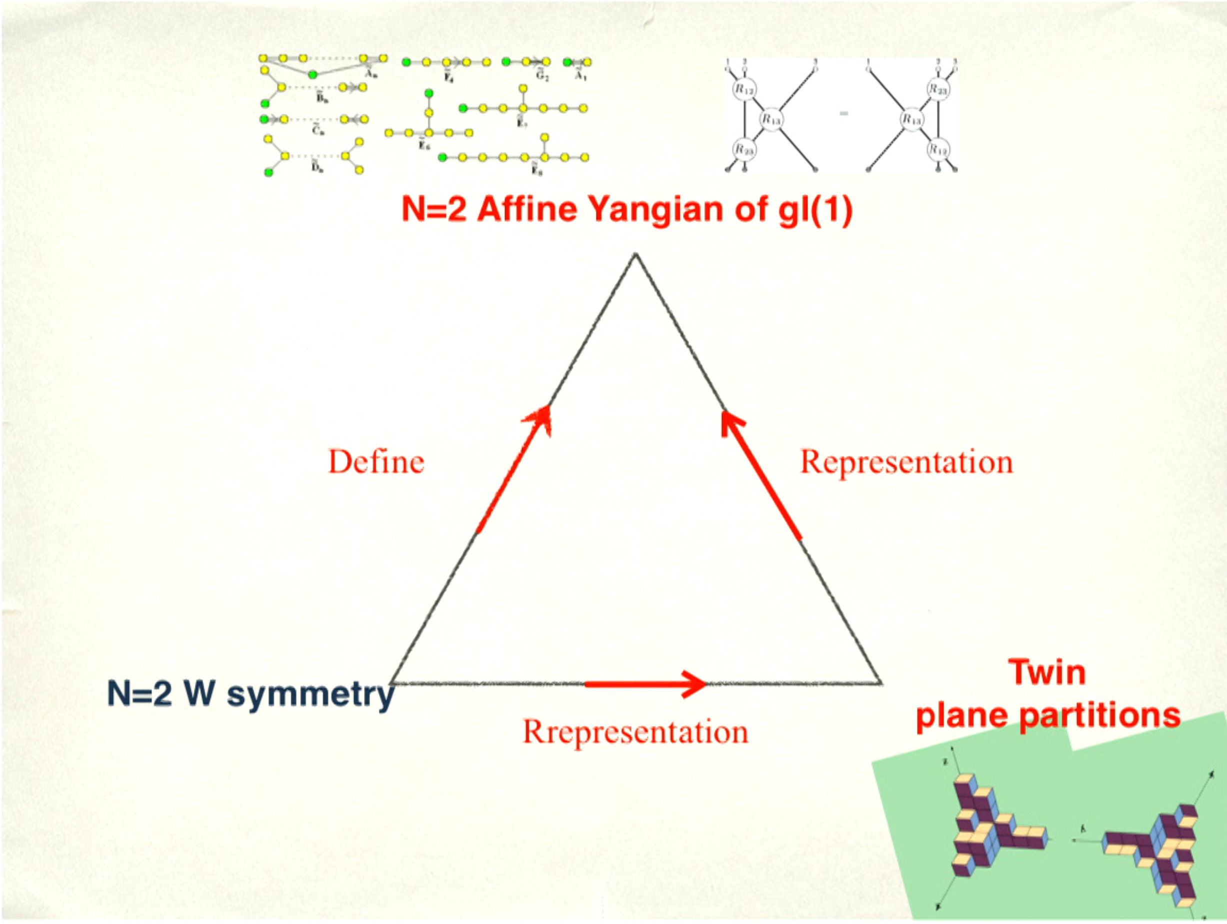
Under $\mathcal{S}_3$ transformation on (N, k)



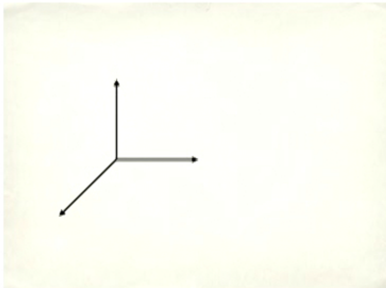
Procházka '15, Gaberdiel Gopakumar Li Peng '17

$\mathcal{N} = 2$ triangle

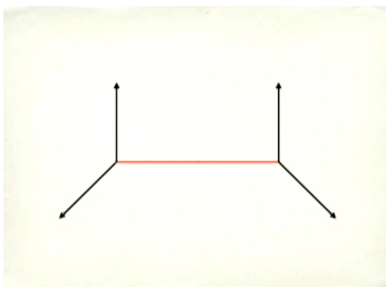
Gaberdiel WL Peng Zhang '17, Gaberdiel WL Peng '18



Building blocks and gluing



1. Algebra: $\mathcal{W}_{1+\infty}$ $\Rightarrow$ affine Yangian of $\mathfrak{gl}_1$
2. Representation: plane partitions



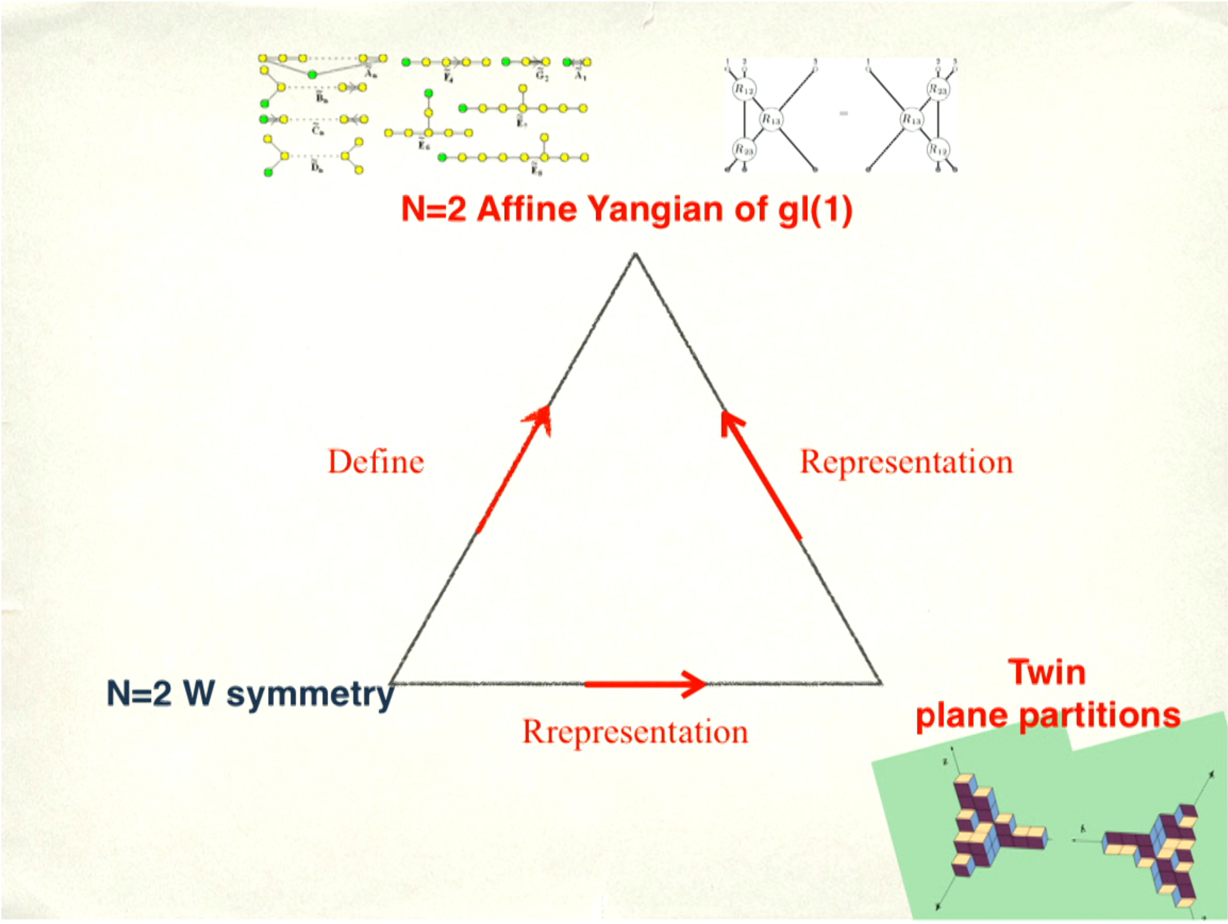
1. Algebra: internal leg $\Rightarrow$ additional operators
   conformal dimension
2. Irrep: bi-module $\Rightarrow$ non-trivial b.c. for vertices
   relative representation

Two parameter family of glued Yangian!

(Algebraic relations are fixed by demanding that the algebra acts on twin-plane-partition faithfully.)

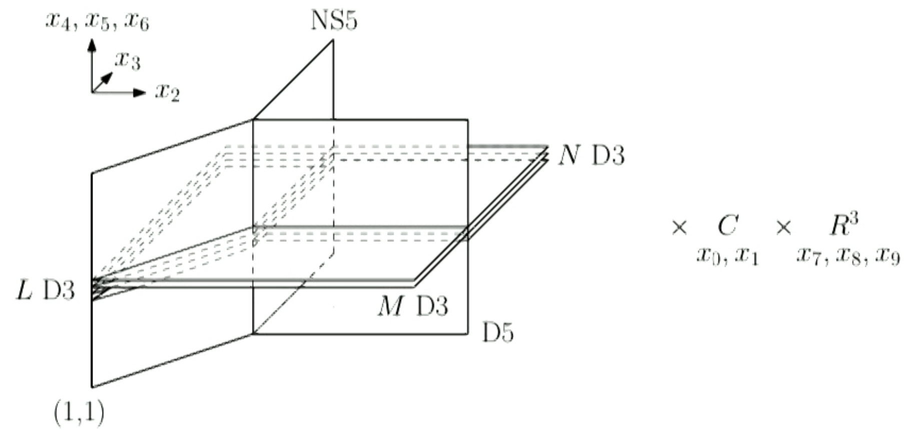
$\mathcal{N} = 2$ triangle

Gaberdiel WL Peng Zhang '17, Gaberdiel WL Peng '18



Y-junction of 4D SYM

picture: Gaiotto Rapcak '17



1. a Y-junction between $U(L)$, $U(M)$, $U(N)$ SYM.
2. Kapustin-Witten twist $\Longrightarrow$ localize to Junction — *Kapustin Witten '06*
3. Along the edges (2+1 dim): Chern Simons with supergroup $U(N_i|N_j)$ — *Mikhaylov Witten '14*
4. At the junction (1+1 dim): BPS operators form a vertex operator algebra

VOA on the 2D junction is a truncation of $\mathcal{W}_{1+\infty}$

Gaiotto Rapcak '17

Truncation and gluing

Gluing of these finite truncations of $\mathcal{W}_{1+\infty}$ should give chiral algebra of Y-junction webs *Rapcak Prochazka'17*

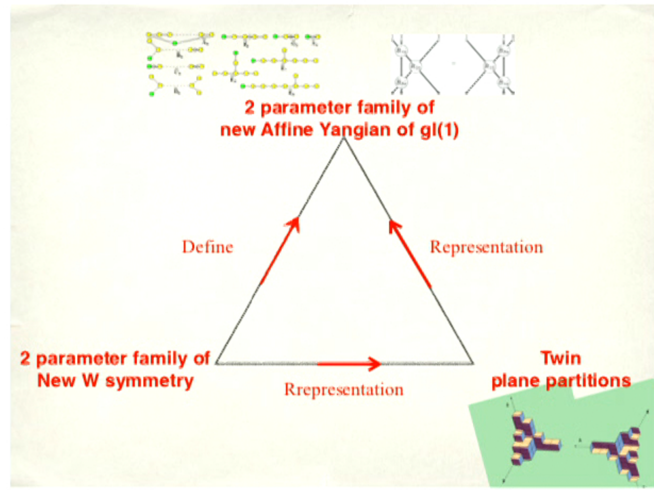
Finite truncation of $\mathcal{W}_{1+\infty}$ is easier to study as truncation of affine Yangian of $\mathfrak{gl}_1$ *Fukuda Matsuo Nakamura Zhu '15, Prochazka '15*

WL Longhi'19

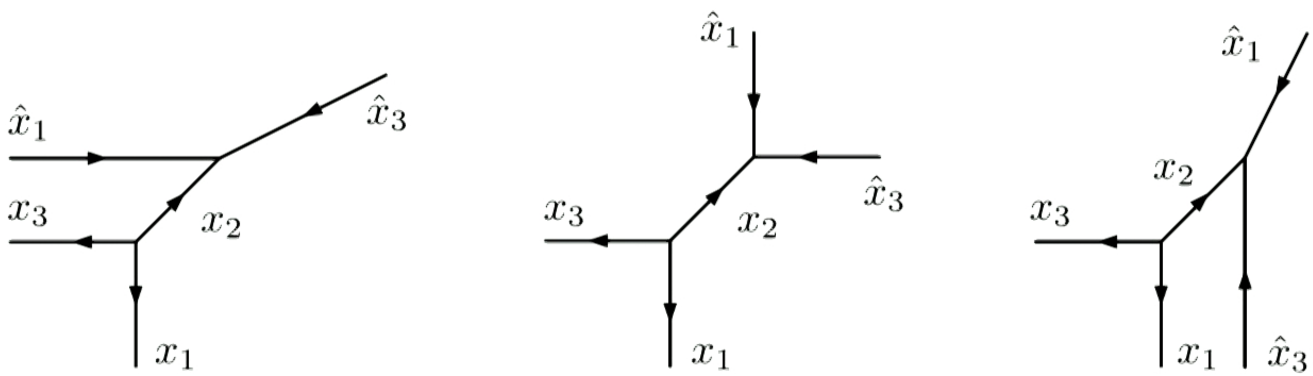
1. Can use affine Yangian of $\mathfrak{gl}_1$ basis instead of $\mathcal{W}_{1+\infty}$ basis
2. Can first glue and then truncate

New W and Yangian algebras via gluing

WL Longhi '19



The 4 constraints from twin plane partition
match one-to-one
to the 4 constraints from (p, q) web.



Higher spin gravity and holography

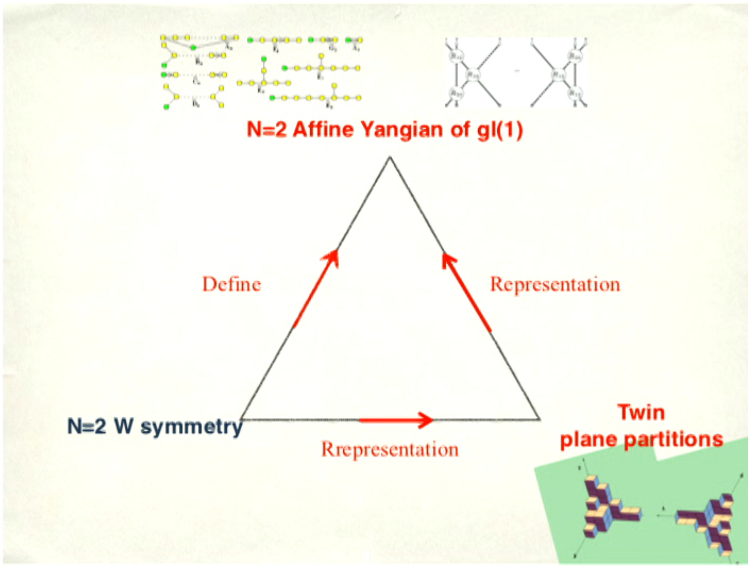
1. It is possible to define gravity coupled to higher spin gauge symmetry in AdS_d or dS_d
2. Simplest example: $\mathfrak{sl}(N) \oplus \mathfrak{sl}(N)$ Chern-Simons
3. higher spin holography is special

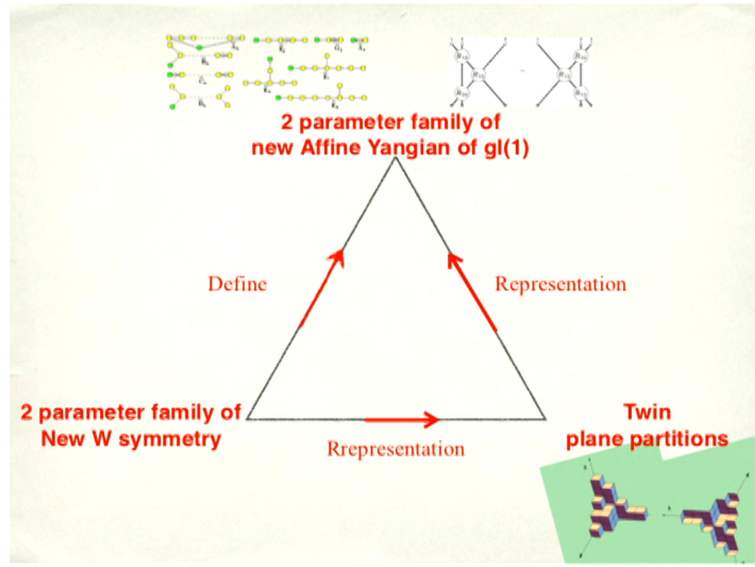
   simple, weak/weak, no need for supersymmetry

Higher spin symmetry v.s. stringy symmetry

1. Vasiliev's higher spin gravity can be embedded into string theory
2. Tensionless limit $\longrightarrow$ stringy symmetry
3. Vasiliev higher spin symmetry is subalgebra of stringy symmetry (from leading Regge trajectory)
4. In AdS_3: additional symmetry enhancement
5. Tensionless + AdS_3: strongly constrain the theory

New W and Yangian algebras via gluing plane partitions





Plane partition is also very useful in the gluing process

- visualize Fock space
- Define algebra by faithful representation