Title: Strings on the Keldysh Contour

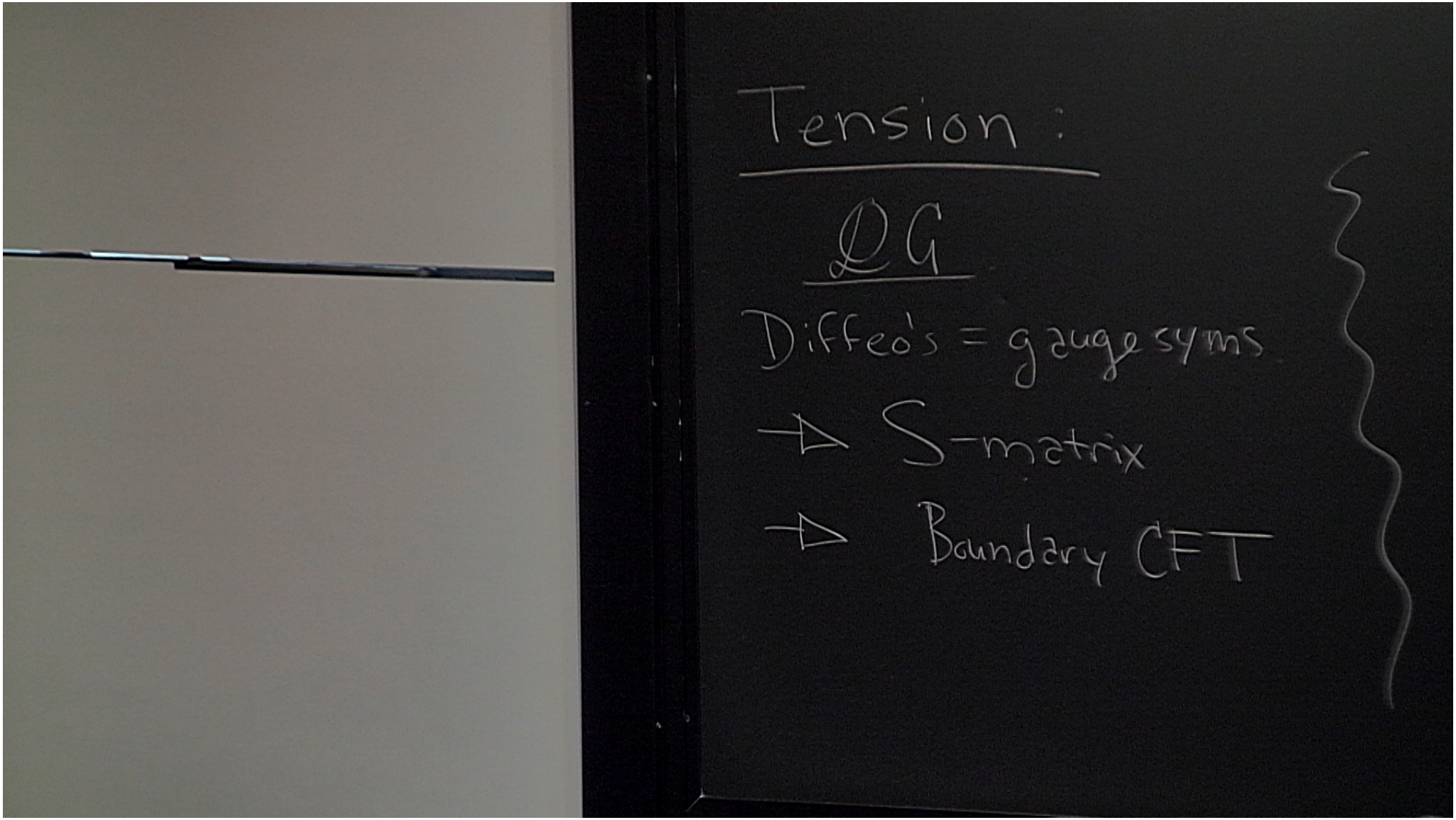
Speakers: Christopher Mogni

Series: Quantum Fields and Strings

Date: October 15, 2019 - 2:30 PM

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Abstract: There is a fundamental tension between what string theory computes (S-matrix elements) and what you can compute in QFT on a time-dependent backgrounds (real time operator expectation values). The prescription for computing time dependent expectation values in QFT involves a path integral defined on a closed time path, known as the Keldysh contour. In this talk, we'll discuss how a worldsheet formulation of string theory must perceive the Keldysh contour and how this modifies critical string theory at tree level in the string coupling.


Tension :
QG
Diffeo's = gauge syms
⇒ S-matrix
⇒ Boundary CFT

Non-eq. / Time-dependent

adiabatic thm breaks down

1) $H = H_0 + V(t)$

2) $\hat{\rho}(-\infty) = \sum_{\Phi} p_\Phi |\Phi\rangle\langle\Phi|$

↑ not nec. eigenstates of H_{em}

⇒ finite, real-time op expect. vals.

→ Boundary CFT

2) $\hat{\rho}(-\infty) = \sum_{\Phi} p_\Phi |\Phi\rangle\langle\Phi|$

↑ not nec. eigenstates of H_{sm}

⇒ finite, real-time qp expect. vals.

QFT Keldysh contour

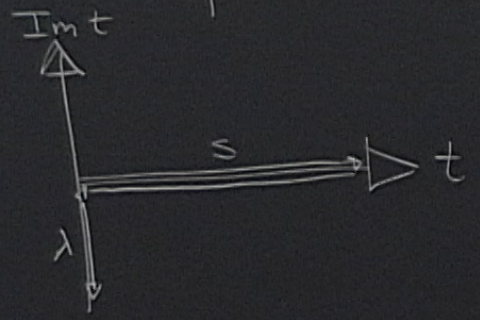
$$\langle \hat{O}(t) \rangle_{\hat{\rho}} = \frac{1}{\mathrm{Tr}(\rho(-\infty))} \mathrm{Tr}\left(\hat{O}\hat{\rho}(t)\right)$$

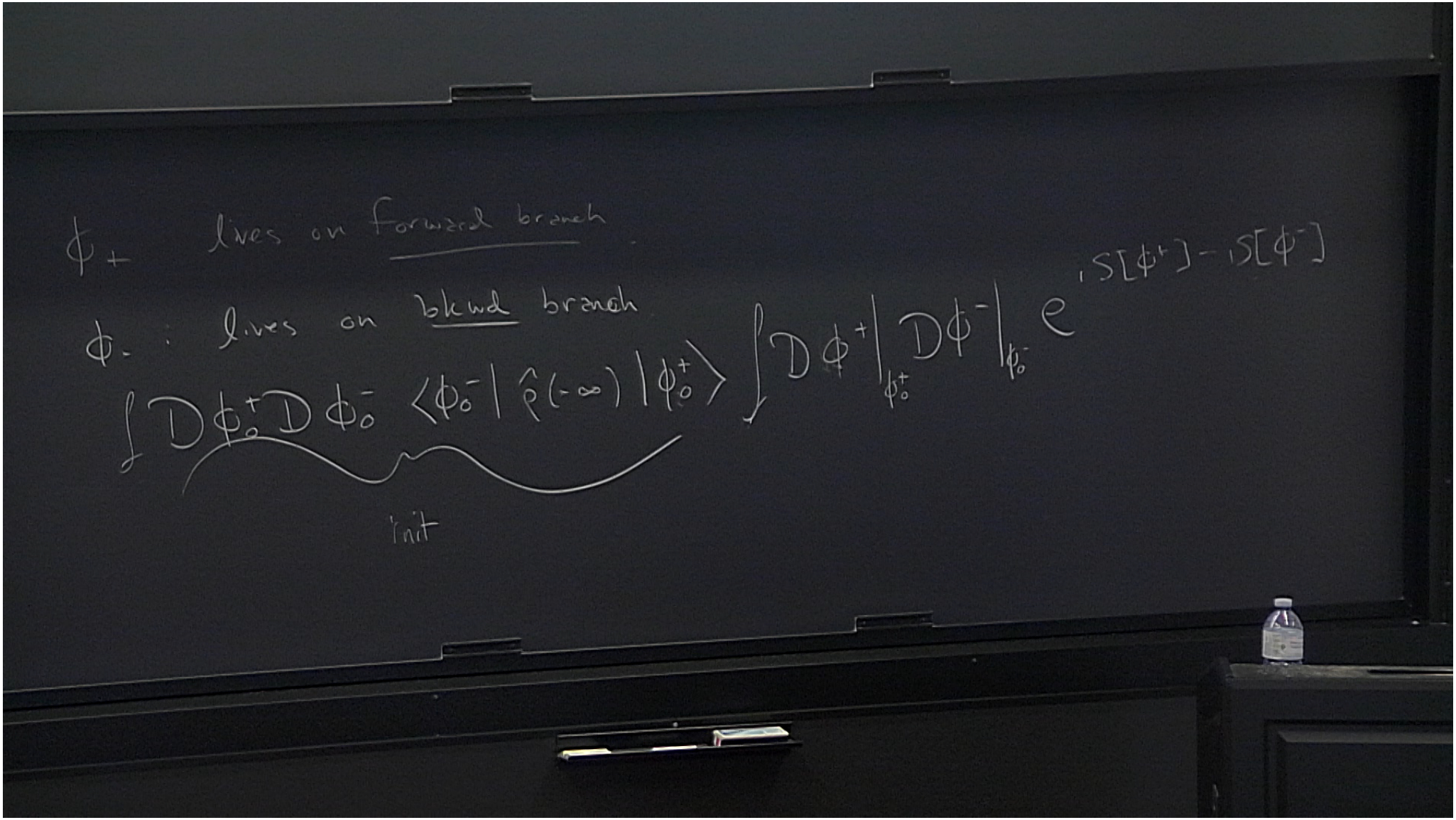
$$= \frac{1}{\quad} \sum_{m,n} \rho_{mn} \langle n | \left(\overline{T} e^{i \int_{-\infty}^{t} dt' H_I(t') dt'} \right) O_I(t) \left(T e^{-i \int_{-\infty}^{t} H_I(t') dt'} \right) | m \rangle$$

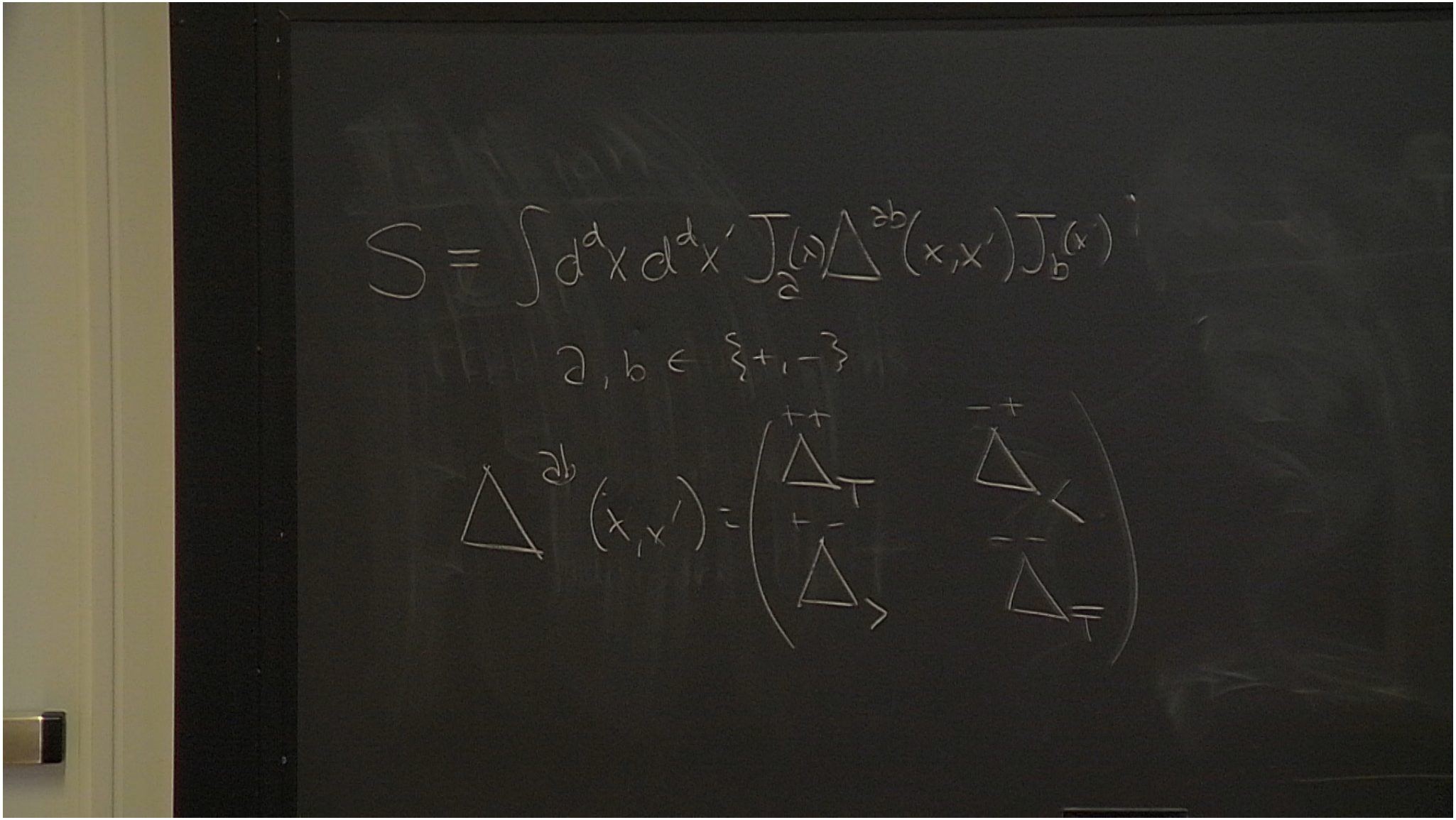
$$\int \mathcal{D}\phi\, e^{i S_c[\phi]}\, \phi(t_1)\cdots\phi(t_n) = \langle \phi(t_1)\cdots\phi(t_n)\rangle$$

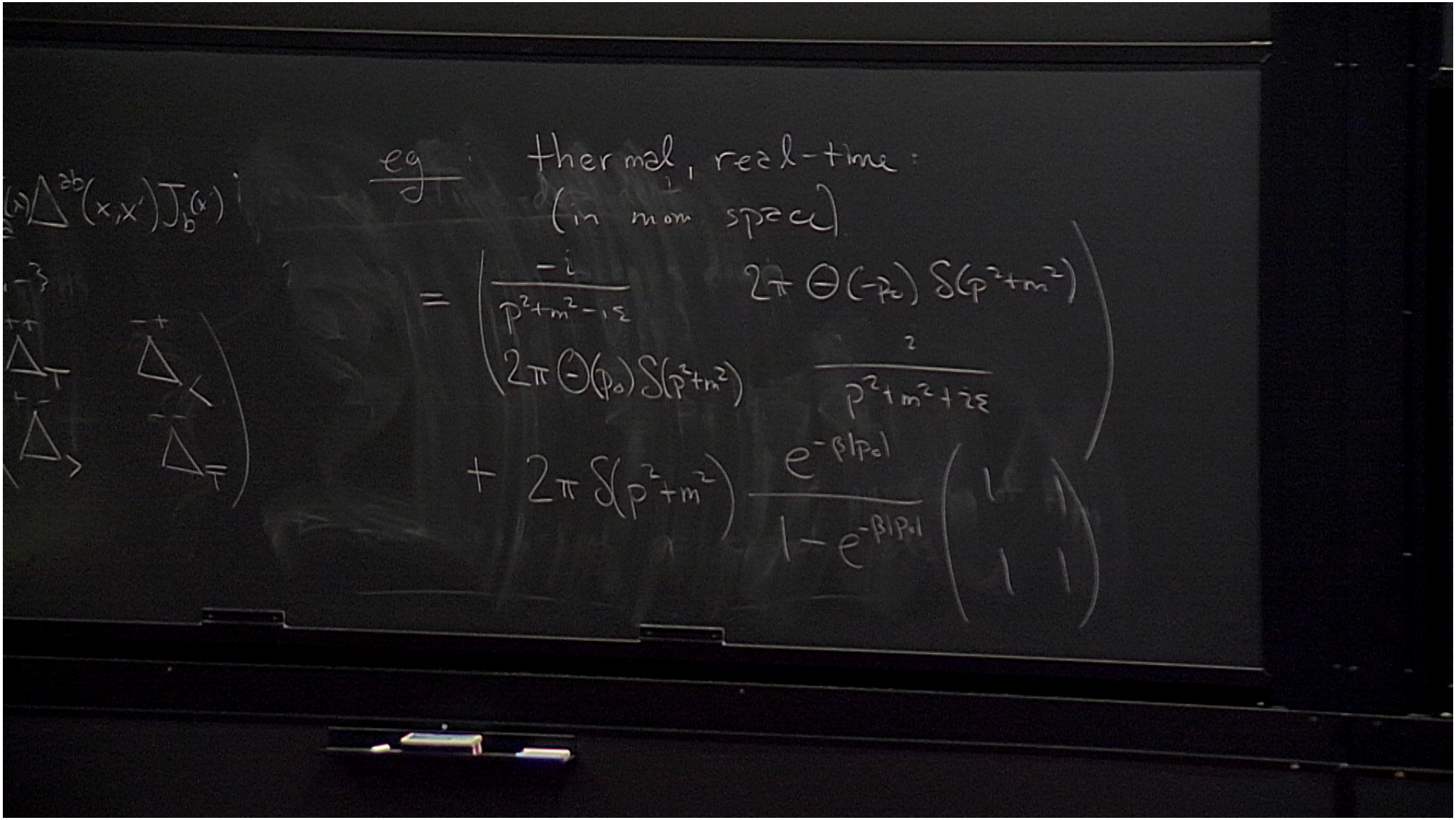
$$\hat{\rho}(-\infty) = e^{-\lambda K[\phi]}$$

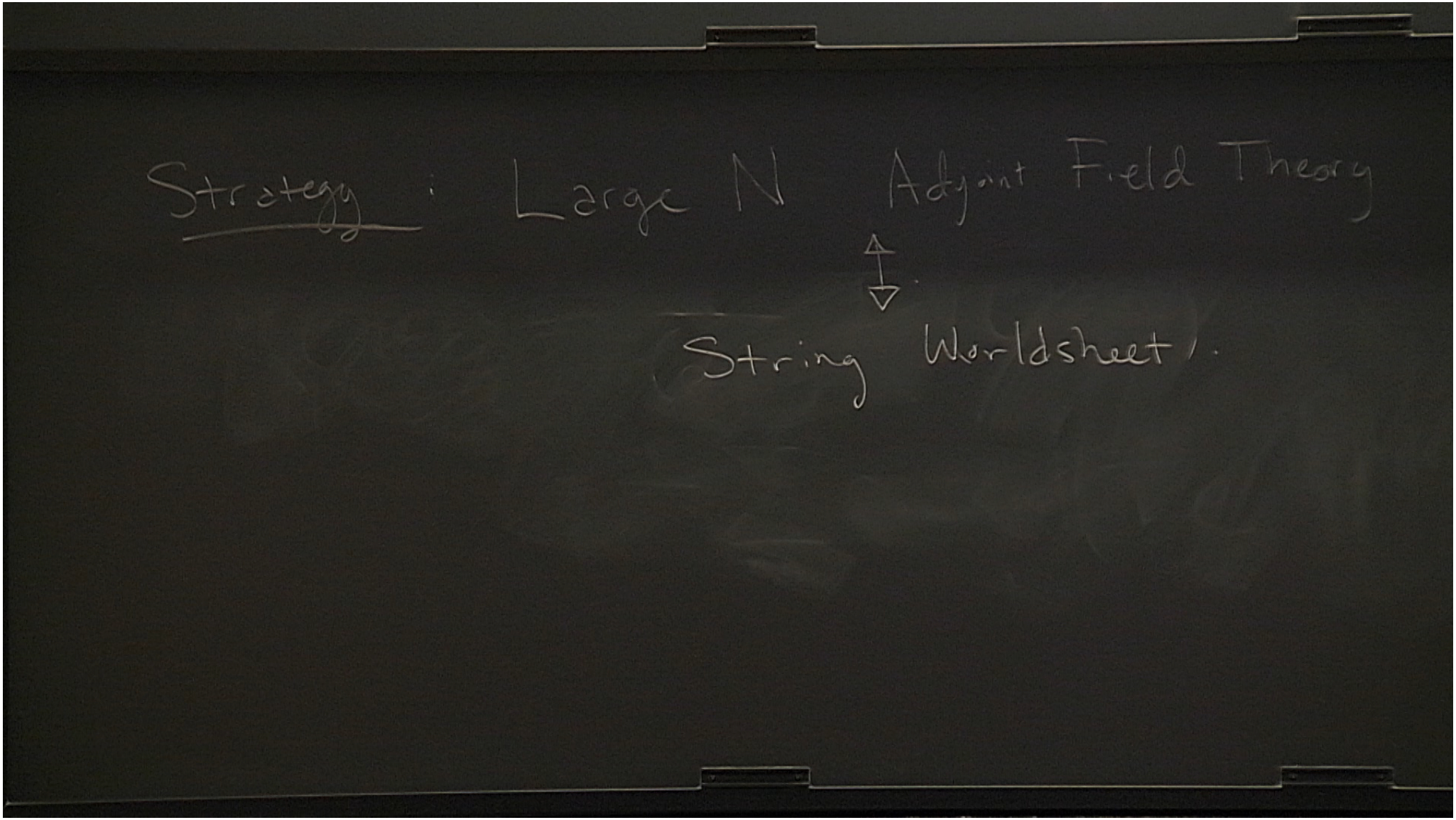
in eq. $\lambda = \beta$, $K = H$




ϕ_+ lives on forward branch
ϕ_- : lives on bkwd branch
$\int D\phi_0^+ D\phi_0^- \langle \phi_0^- | \hat{\rho}(-\infty) | \phi_0^+ \rangle \int D\phi^+ \Big|_{\phi_0^+} D\phi^- \Big|_{\phi_0^-} e^{iS[\phi^+] - iS[\phi^-]}$
init


$S = \int d^d x\, d^d x'\, J_a(x) \Delta^{ab}(x,x') J_b(x')$
$a, b \in \{+,-\}$
$\Delta^{ab}(x,x') = \begin{pmatrix} \Delta_T & \Delta_< \\ \Delta_> & \Delta_{\bar{T}} \end{pmatrix}$


eg: thermal, real-time:
(in mom space)
$\Delta^{ab}(x,x')J_b(x')$
$\Delta_T \quad \Delta_<$
$\Delta_> \quad \Delta_{\bar{T}}$
$= \begin{pmatrix} \frac{-i}{p^2+m^2-i\varepsilon} & 2\pi\,\theta(-p_0)\,\delta(p^2+m^2) \\ 2\pi\,\theta(p_0)\,\delta(p^2+m^2) & \frac{i}{p^2+m^2+i\varepsilon} \end{pmatrix}$
$+ 2\pi\,\delta(p^2+m^2)\,\frac{e^{-\beta|p_0|}}{1-e^{-\beta|p_0|}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$


Strategy : Large N Adjoint Field Theory
String Worldsheet

$S_{YM}[A; g] \to \frac{1}{g^2} S_{YM}[A; 1]$

$\lambda \equiv g^2 N$ fixed

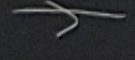
$g \to 0$ $\quad g \to \infty$

Vert (V)



$\sim \frac{N}{\lambda}$

Prop (E)

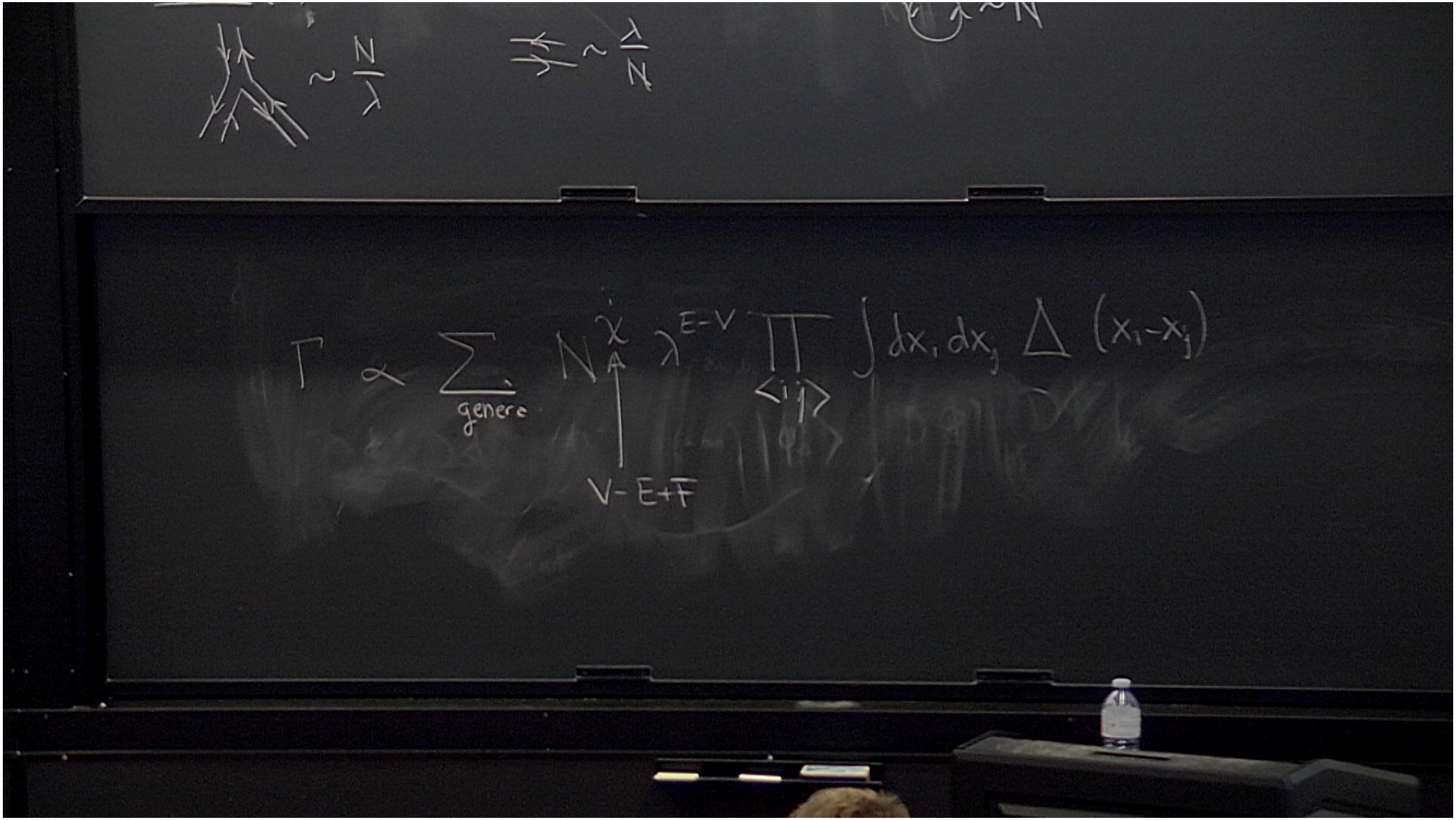


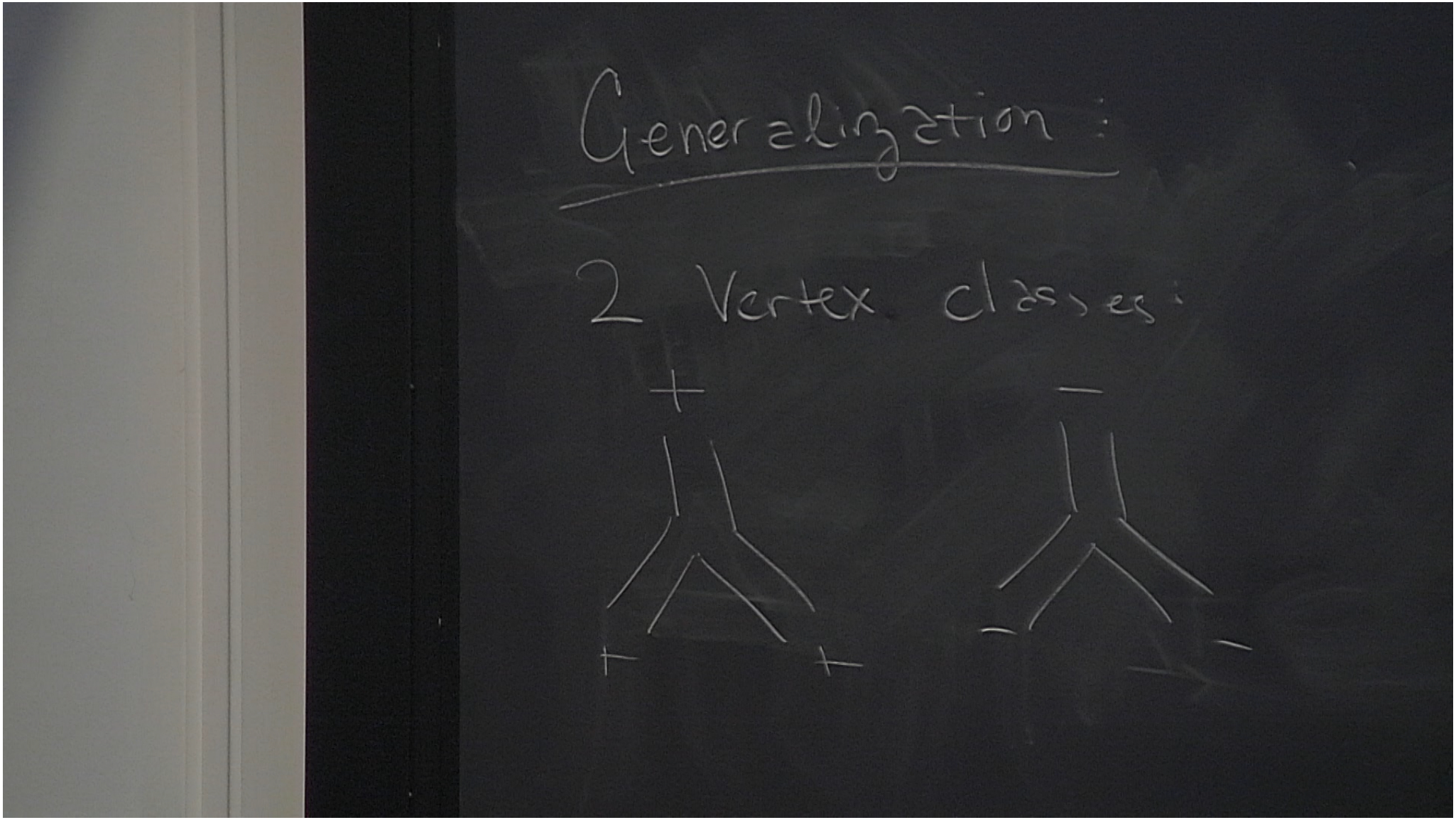
$\sim \frac{\lambda}{N}$

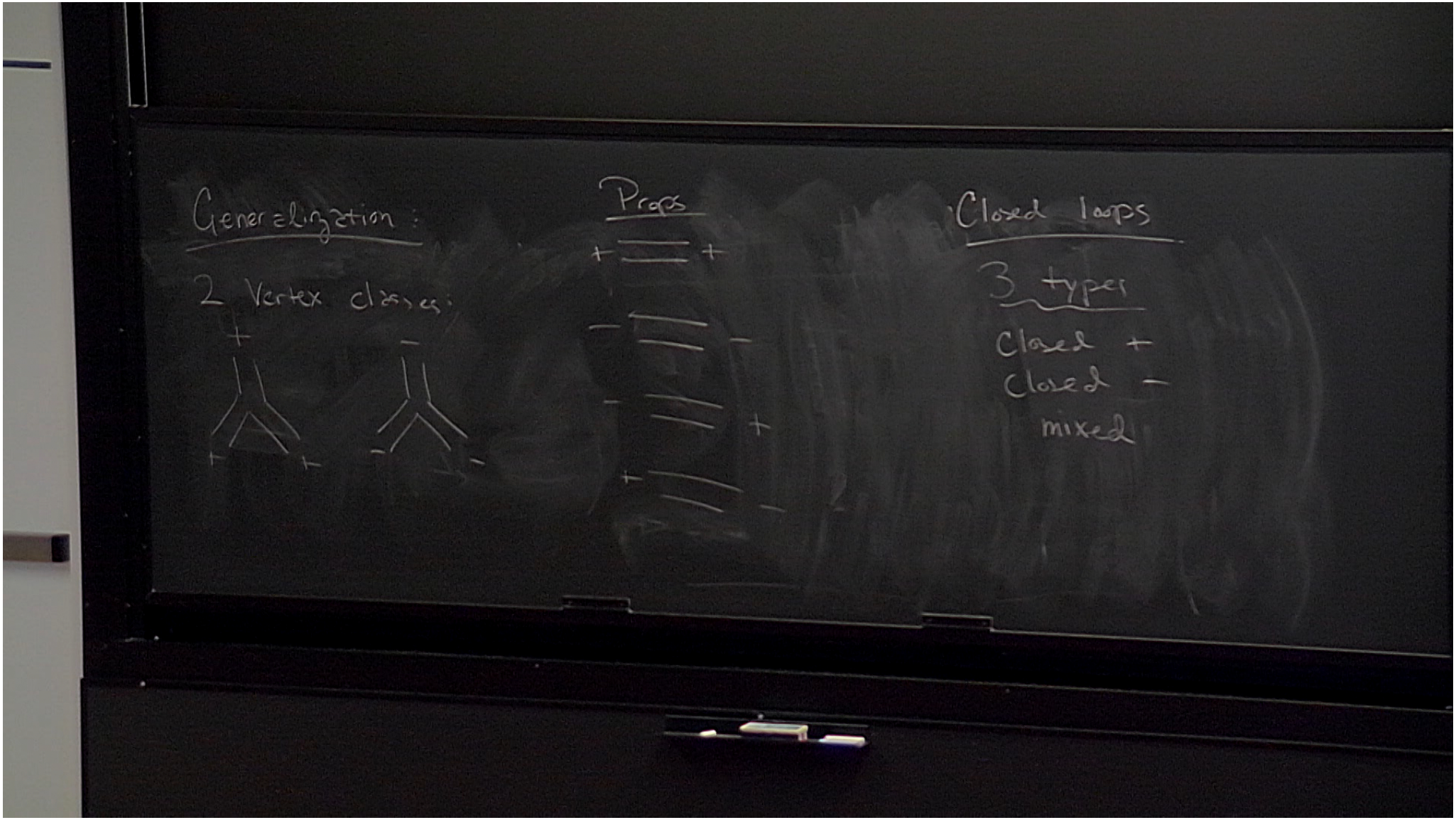
Closed color (F)

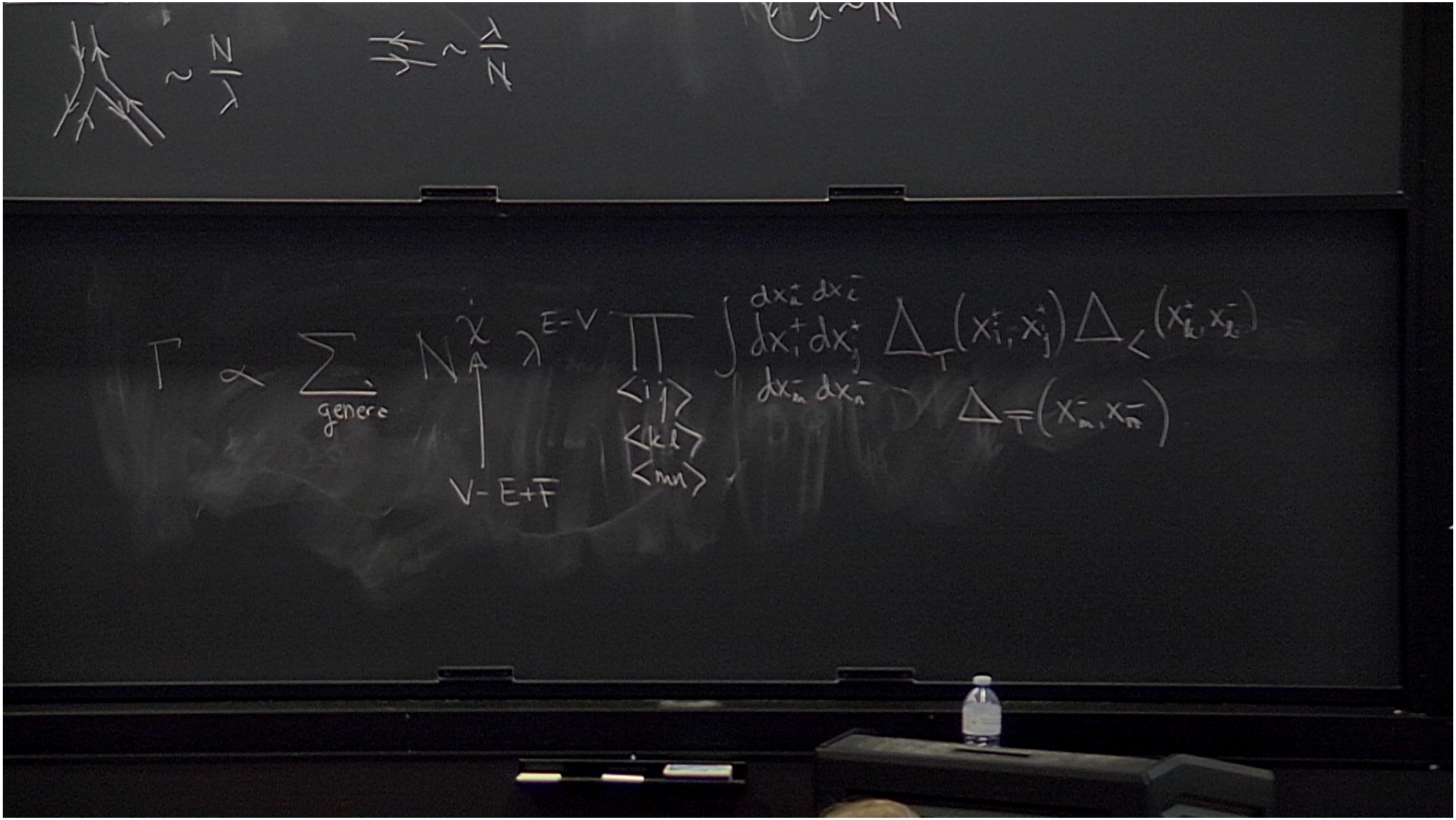


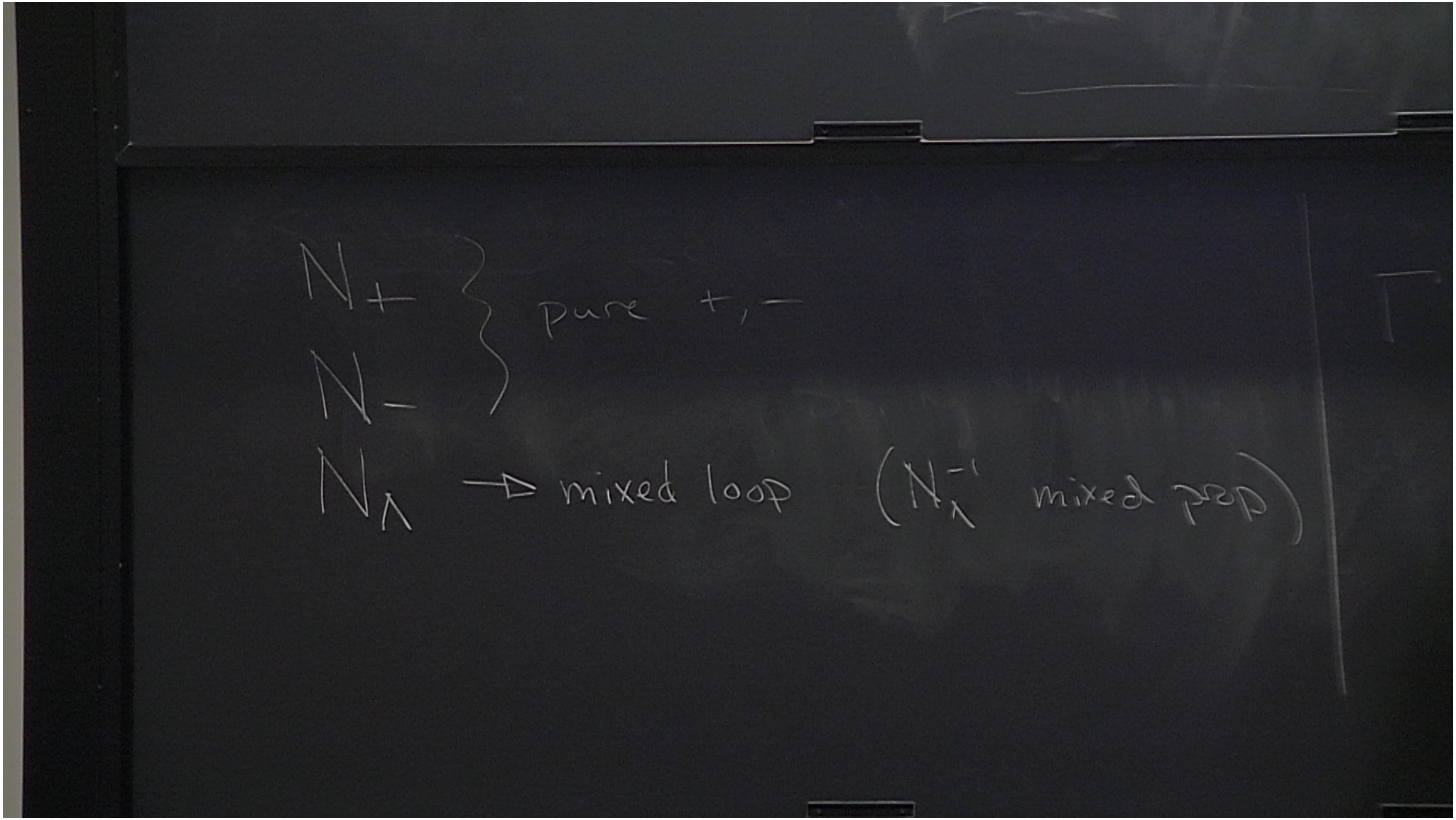
$\sim N$

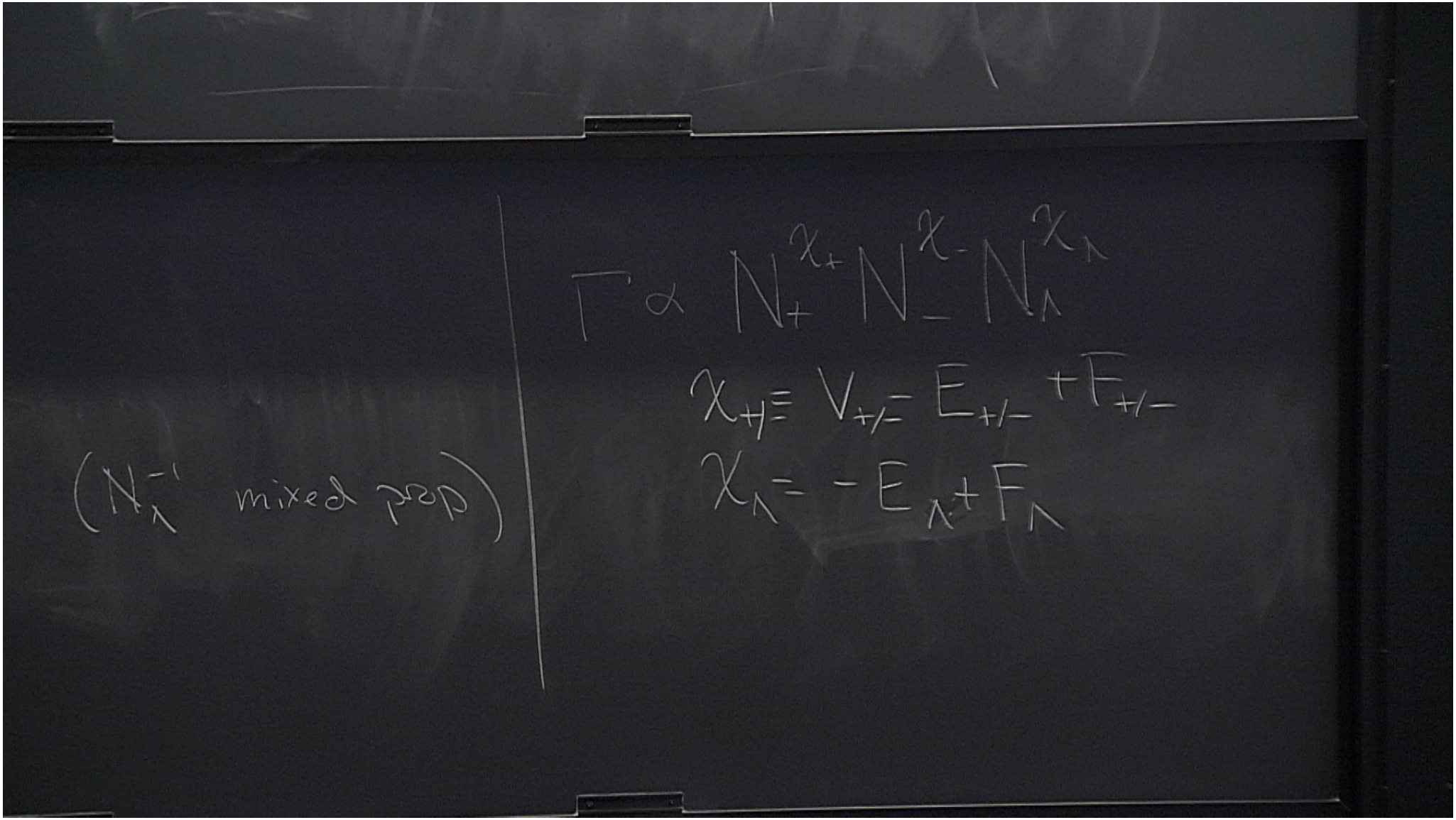

$\sim \frac{N}{\lambda}$
$\sim \frac{\lambda}{N}$
$\sim N$
$\Gamma \propto \sum_{\text{genere}} N^{\chi_A} \lambda^{E-V} \prod_{\langle ij \rangle} \int dx_i dx_j \, \Delta(x_i - x_j)$
$V - E + F$


Generalization :
2 Vertex classes :
+
−
+
+
−
−


Generalization:
2 Vertex classes:
Props
Closed loops
3 types
Closed +
Closed −
mixed

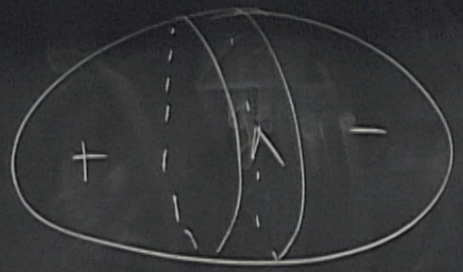

$\sim \frac{N}{\lambda}$
$\sim \frac{\lambda}{N}$
$\Gamma \propto \sum_{\text{genere}} N_A^{\chi} \lambda^{E-V} \prod_{\langle ij \rangle \langle kl \rangle \langle mn \rangle} \int dx_i^+ dx_i^- dx_j^+ dx_j^- dx_m^+ dx_m^- \Delta_+(x_i^+, x_j^+) \Delta_-(x_k^-, x_l^-) \Delta_+(x_m^-, x_n^-)$
$V - E + F$


N_+
N_-
pure +, −
N_Λ → mixed loop
(N_Λ^{-1} mixed pop)

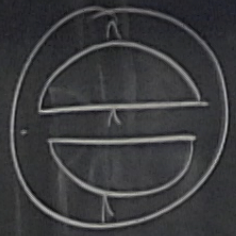

$\Gamma \propto N_+^{\chi_+} N_-^{\chi_-} N_\Lambda^{\chi_\Lambda}$
$\chi_{+/-} = V_{+/-} - E_{+/-} + F_{+/-}$
$\chi_\Lambda = -E_\Lambda + F_\Lambda$
$(N_\Lambda^{-1}$ mixed prop$)$

Under the rug : e^{-N}

ex:

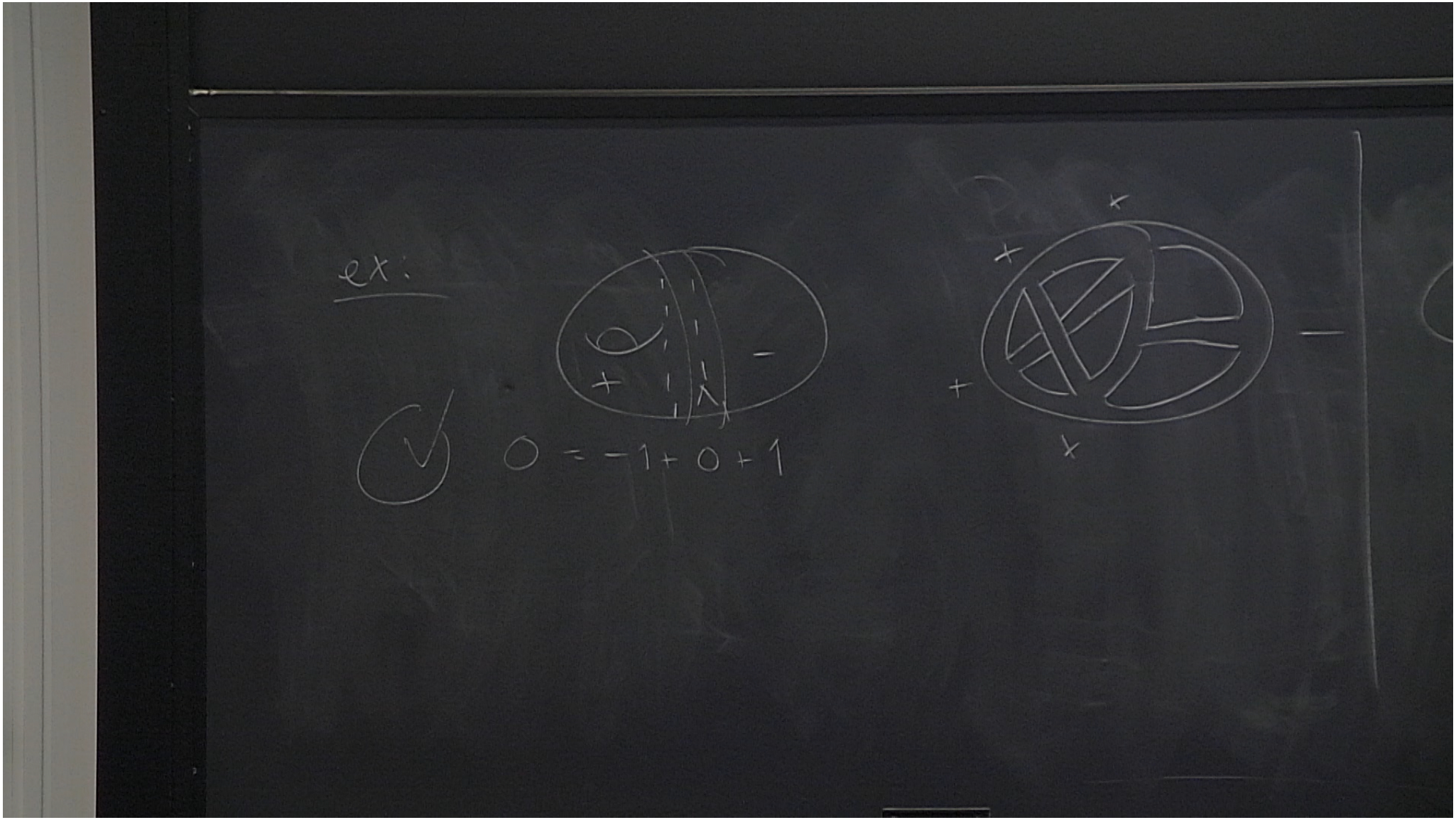


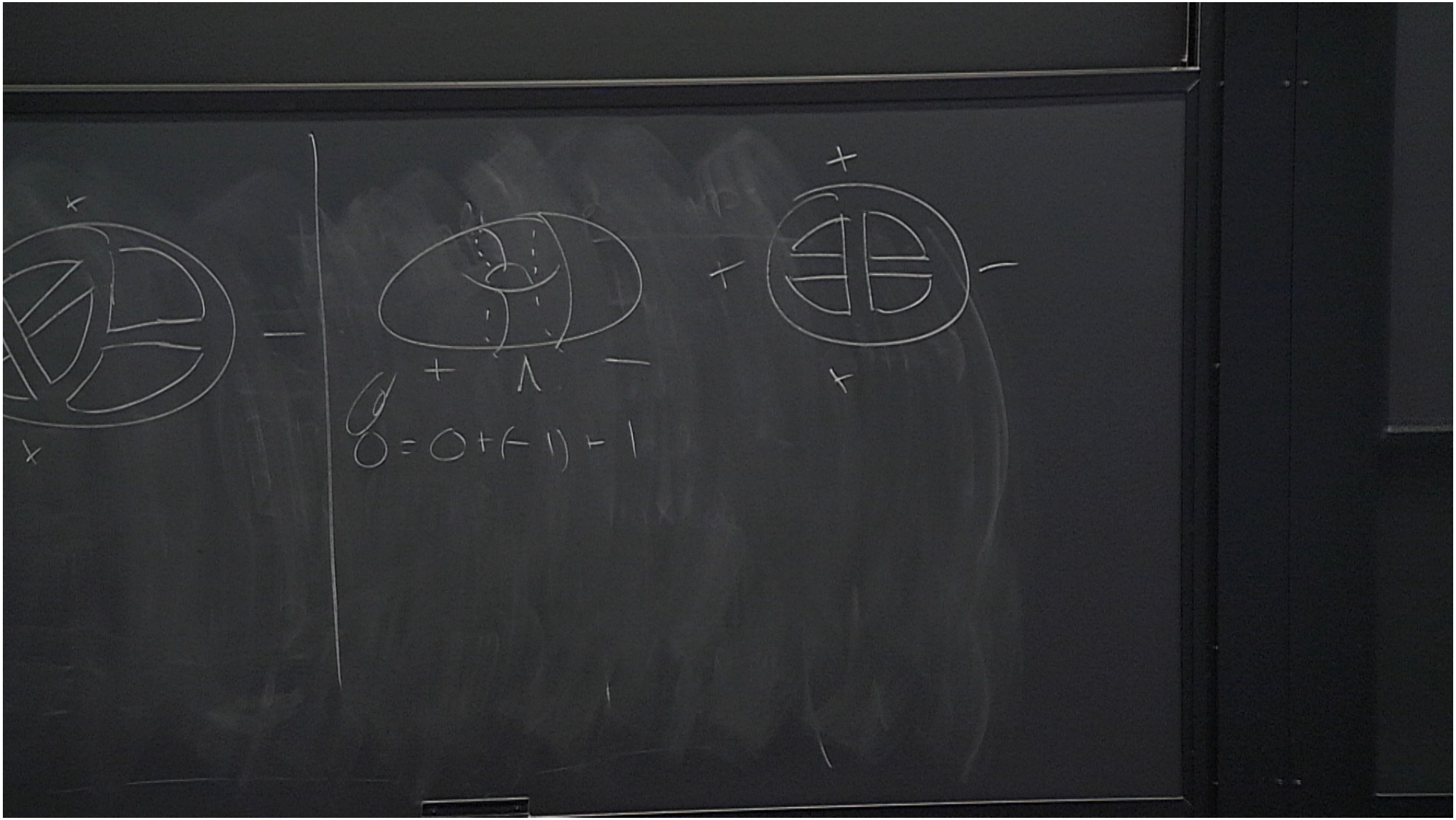
$\sim$ +

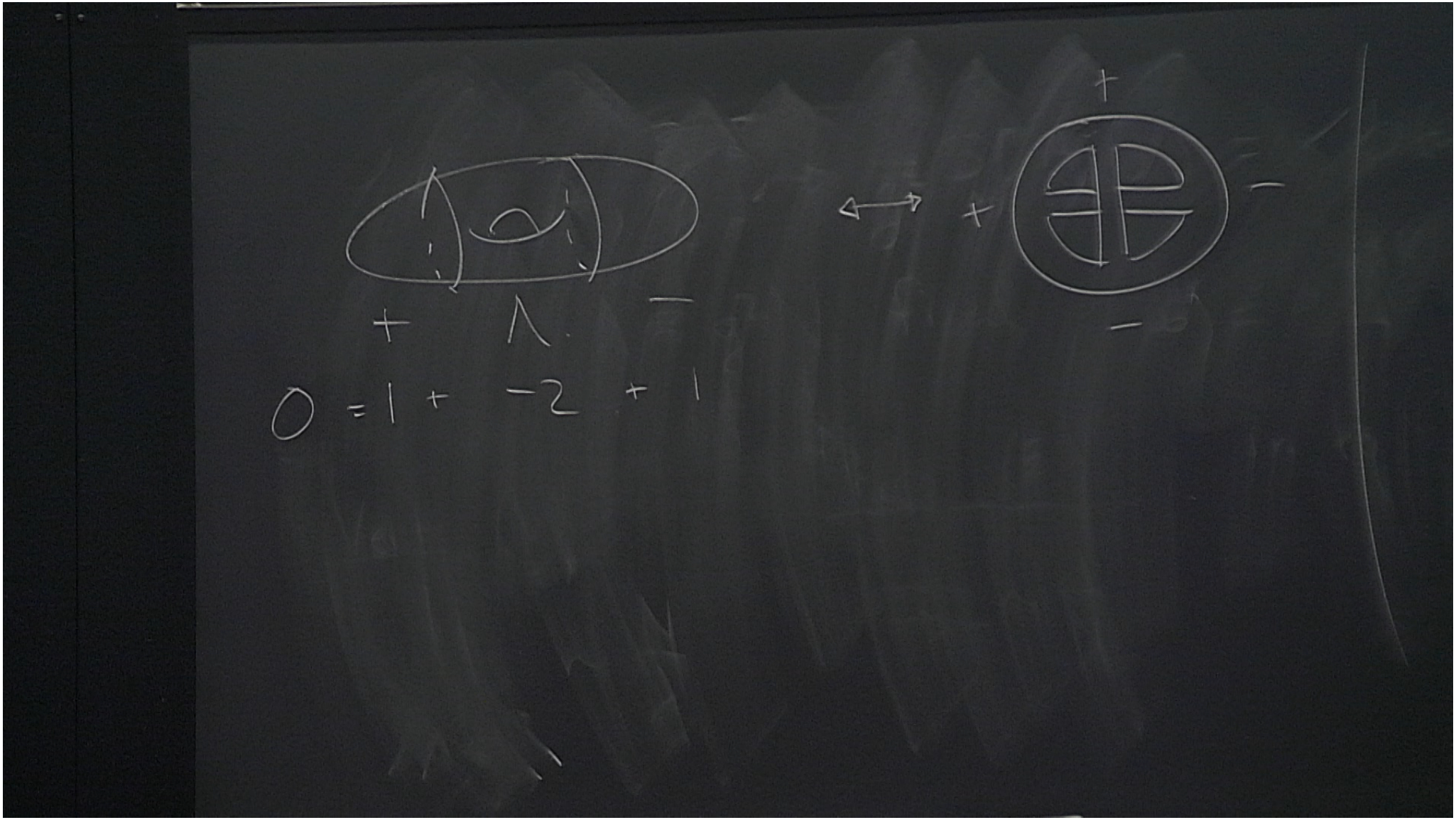


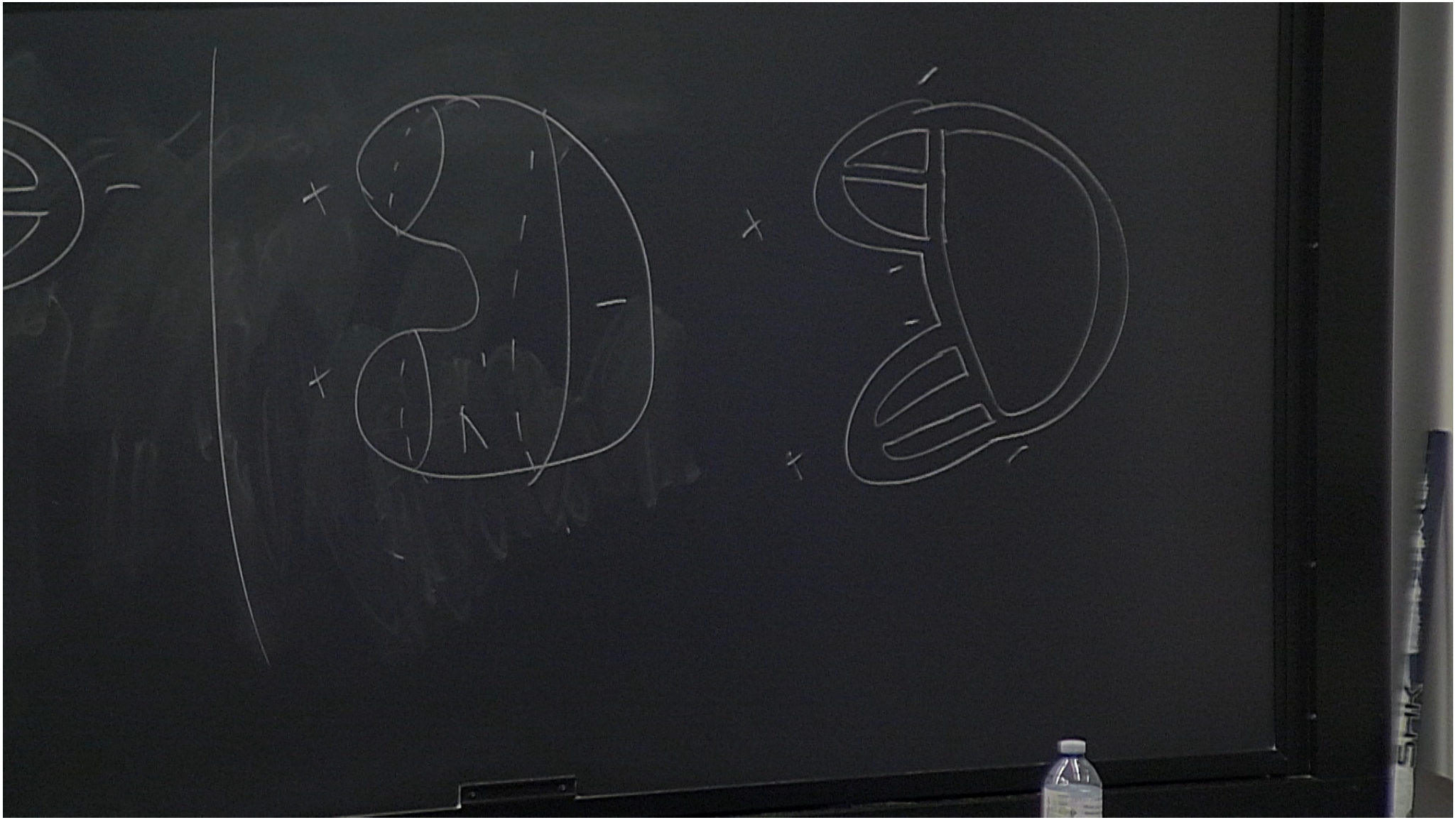
−

$2 = 1 + 0 + 1$


ex:
+
−
0 = −1 + 0 + 1
+
+
+
+
−


+
−
+
+
−
+
Λ
−
+
+
−
+
g
0 = 0 + (−1) + 1


+
+
−
+
−
+
−
0 = 1 + −2 + 1



$$\int D\phi^E D\phi^+ D\phi^- \; e^{iS[\phi^+] - iS[\phi^-] - \lambda K[\phi^E]}$$

7 region types

E
+ } pure
−

N_E
N_+
N_-

(V, E, F)

+, −
+, E
−, E

(E, F)

+, −, E

(F)

Critical, Bosonic S.T. M^4 $T=0$

1) We know the V.O.'s
2) We know how to do this in operator formalism

Critical, Bosonic S.T. Mink T=0

1) We know the V.O's
2) We know how to do this in operator formalism

$$(L_0 - 1)|\phi\rangle = 0$$

$$\Delta_E = (L_0 - 1)^{-1} = \int_0^\infty dt\, e^{-t(L_0-1)} = \int_0^1 dz\, z^{L_0-2}$$

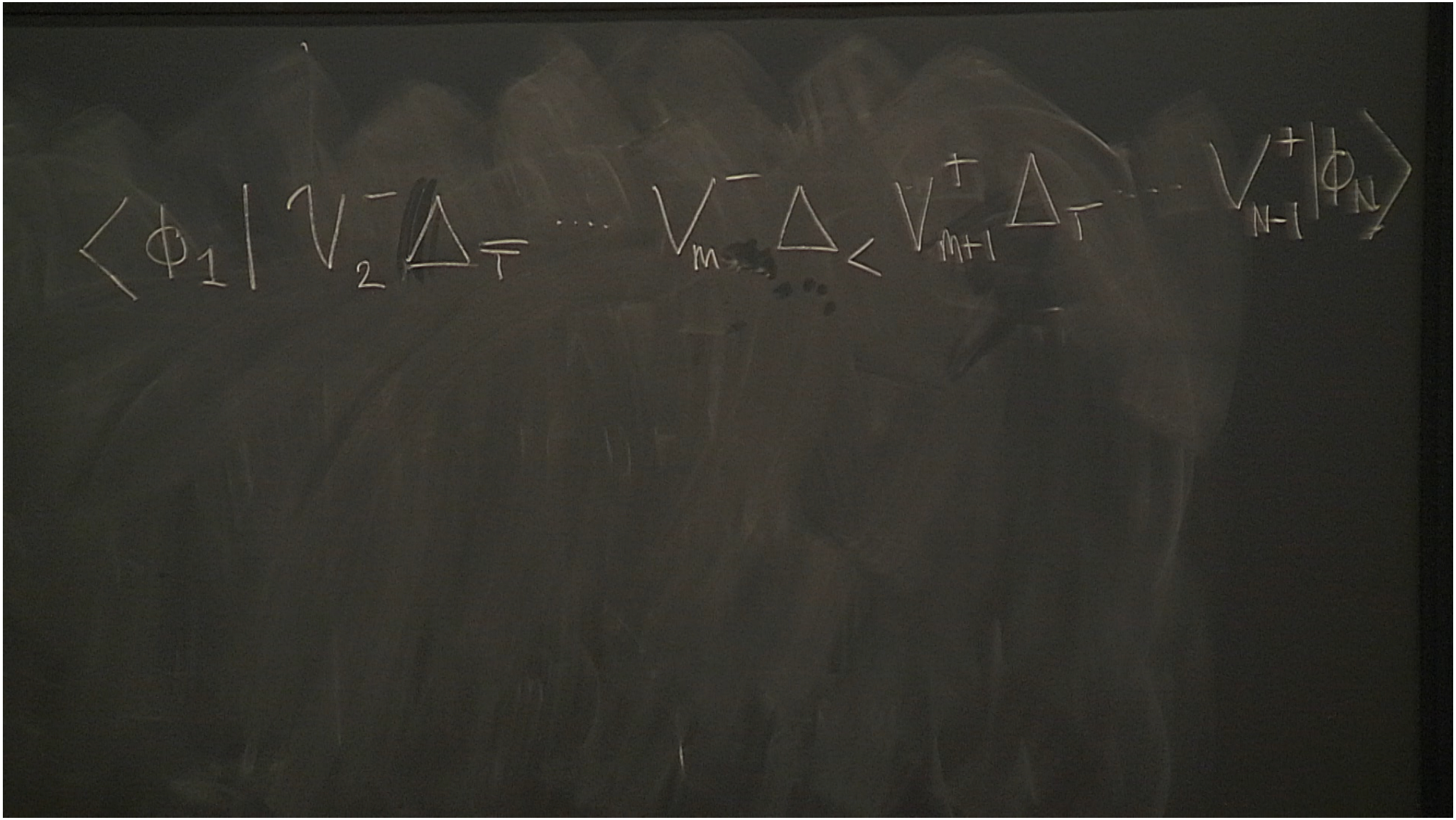
$$A_M \sim g^{M-2} \langle \phi_1 | V_2(k_2) \Delta_E V(k_3) \cdots V_{M-1}(k_{M-1}) | \phi_M \rangle$$

$2 = 1 + 0 + 1$

$$\Delta_T = \frac{-i}{L_0 - 1 - i\varepsilon}$$

$$\Delta_{\bar{T}} = \frac{i}{L_0 - 1 + i\varepsilon}$$

$$\Delta_<$$


$\langle \phi_1 | V_2^- \Delta_T \cdots V_m^- \Delta_< V_{m+1}^+ \Delta_T \cdots V_{N-1}^+ | \phi_N \rangle$

$$\Delta_T = \frac{-i}{L_0 - 1 - i\varepsilon}$$

$$\Delta_{\bar{T}} = \frac{i}{L_0 - 1 + i\varepsilon}$$

$$\Delta_< = 2\pi\,\Theta(-P_0)\,\delta(L_0 - 1)$$

$$\Delta_< + \Delta_> = \Delta_T + \Delta_{\bar{T}}$$

$\langle \phi_1$

Critical, Bosonic S.T. Mink T=0

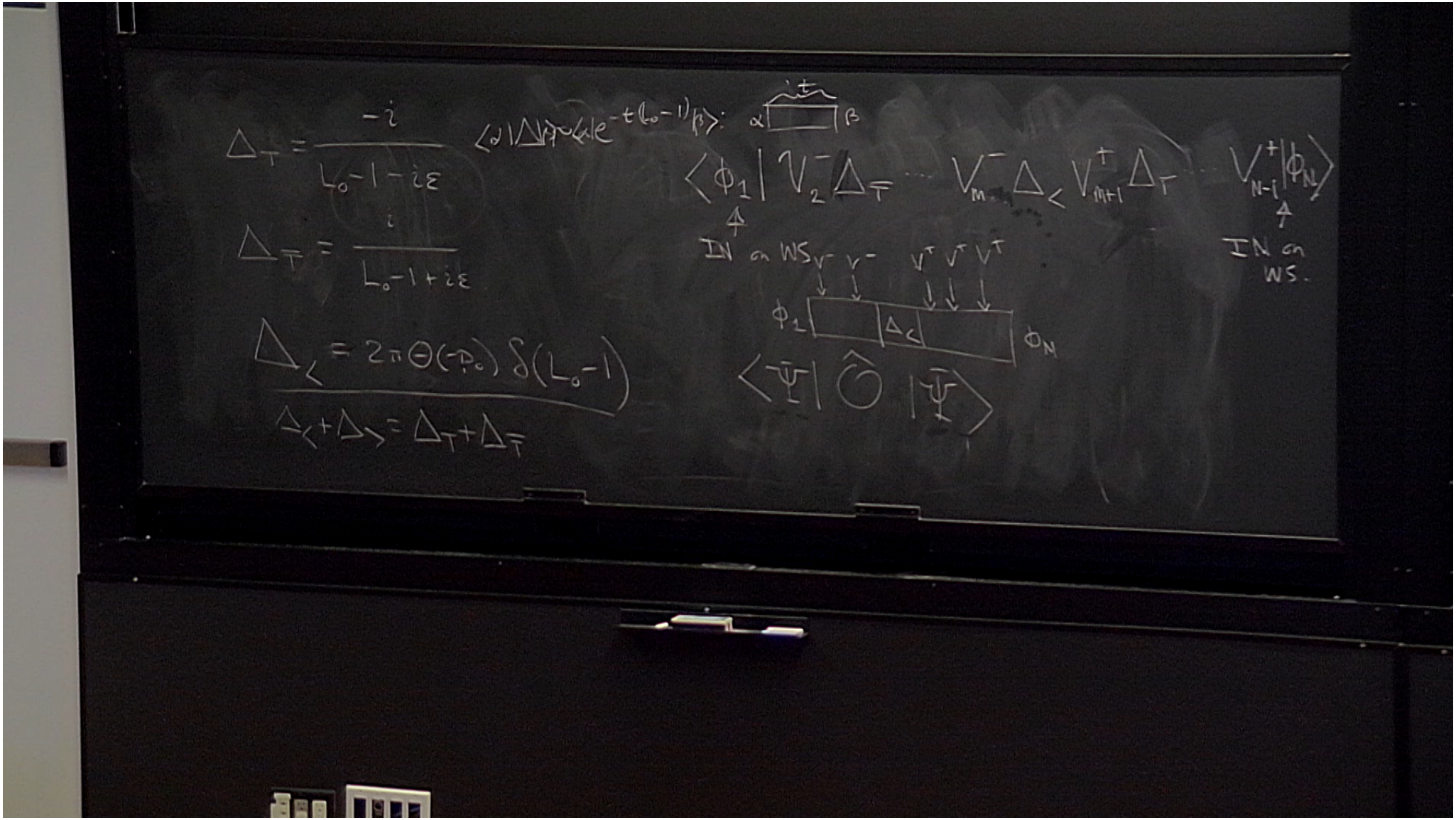
1) We know the V.O's
2) We know how to do this in operator formalism

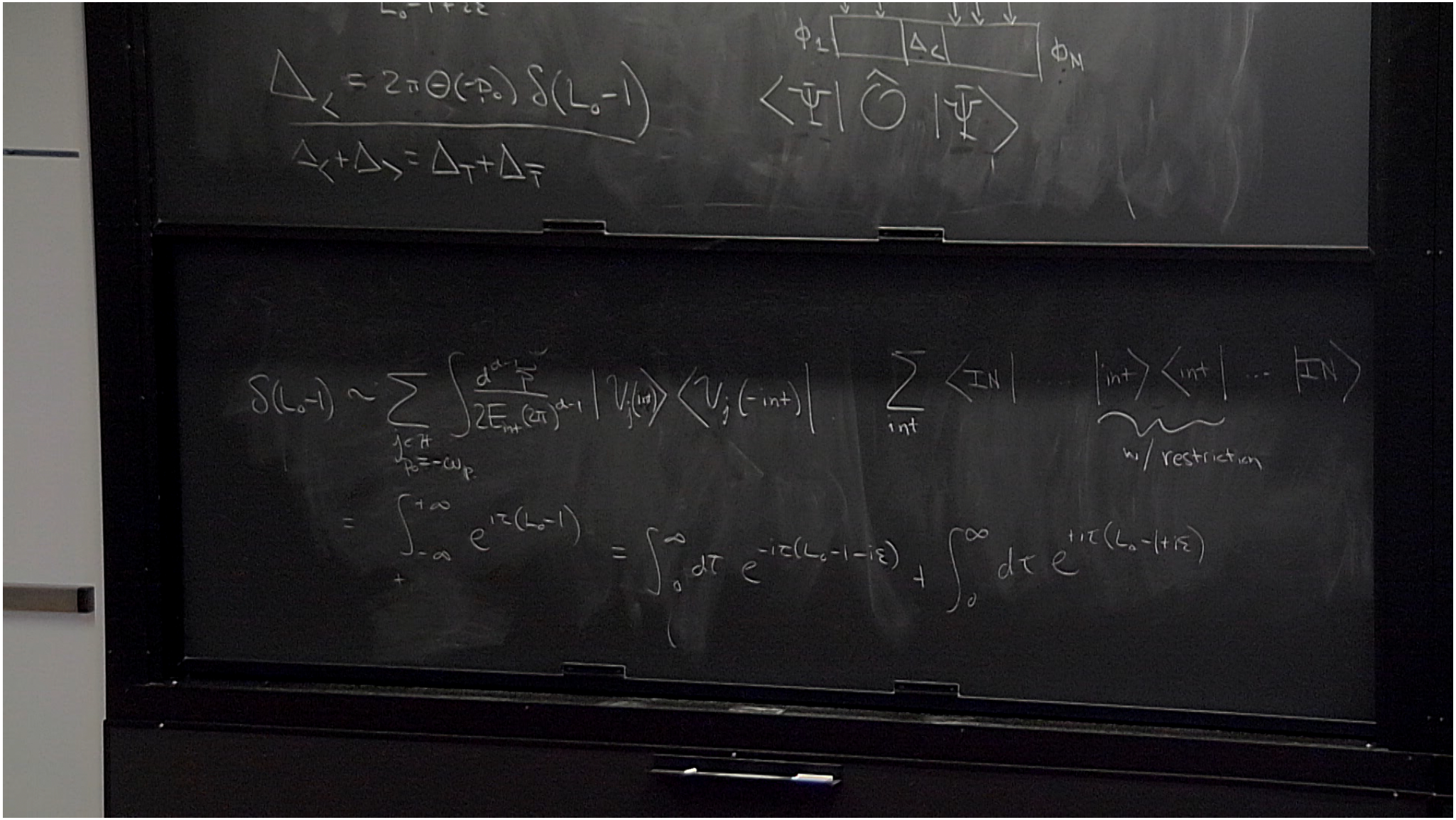
$$(L_0 - 1)|\phi\rangle = 0$$

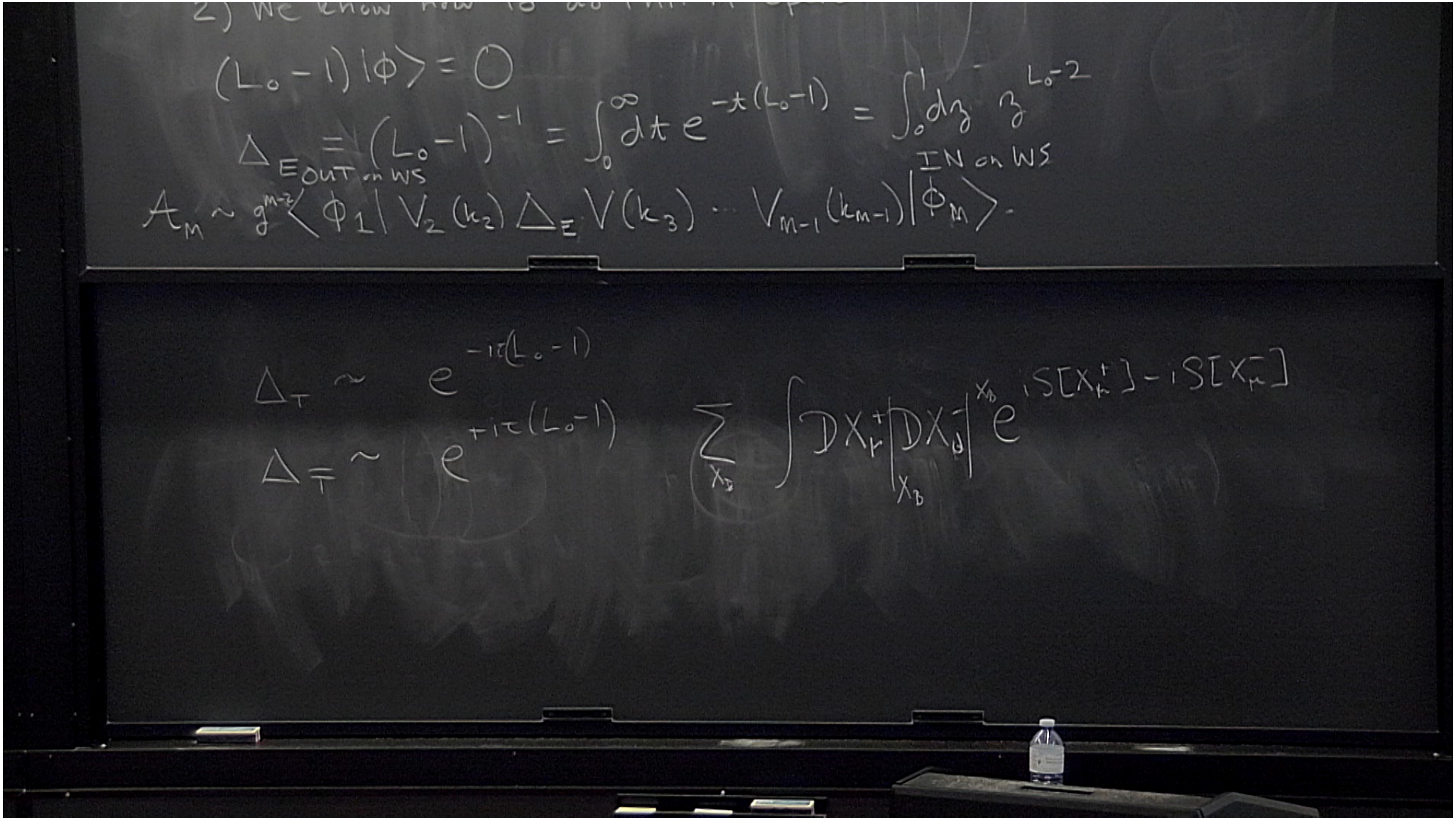
$$\Delta_{E_{OUT\ on\ WS}} = (L_0 - 1)^{-1} = \int_0^\infty dt\, e^{-t(L_0-1)} = \int_0 dz\, z^{L_0 - 2}$$

$$A_M \sim g^{M-2} \langle \phi_1 | V_2(k_2) \Delta_E V(k_3) \cdots V_{M-1}(k_{M-1}) | \phi_M \rangle_{IN\ on\ WS}$$

$2 = 1 + 0 + 1$


$\Delta_T = \frac{-i}{L_0 - 1 - i\varepsilon}$
$\Delta_{\bar{T}} = \frac{i}{L_0 - 1 + i\varepsilon}$
$\Delta_< = 2\pi\Theta(-P_0)\,\delta(L_0 - 1)$
$\Delta_< + \Delta_> = \Delta_T + \Delta_{\bar{T}}$
$\langle \alpha | \Delta^{n-1} c\, e^{-t(L_0-1)} | \beta \rangle$:
$\langle \phi_1 | V_2^- \Delta_{\bar{T}} \cdots V_m^- \Delta_< V_{m+1}^+ \Delta_T \cdots V_{N-1}^+ | \phi_N \rangle$
IN on WS
IN on WS.
ϕ_1
ϕ_N
$\langle \bar{\Psi} | \hat{O} | \Psi \rangle$


$\Delta_< = 2\pi\Theta(-p_0)\,\delta(L_0-1)$
$\Delta_< + \Delta_> = \Delta_T + \Delta_{\bar{T}}$
ϕ_1
$\Delta_<$
ϕ_N
$\langle\bar{\Psi}|\,\hat{\mathcal{O}}\,|\bar{\Psi}\rangle$
$\delta(L_0-1) \sim \sum_{\substack{j<N \\ p_0=-\omega_p}} \int \frac{d^{d-1}\vec{p}}{2E_{int}(2\pi)^{d-1}} |V_j(int)\rangle\langle V_j(-int)| \sum_{int} \langle IN|\ \ldots\ |int\rangle\langle int|\ \ldots\ |IN\rangle$
w/ restriction
$= \int_{-\infty}^{+\infty} e^{i\tau(L_0-1)} = \int_0^\infty d\tau\, e^{-i\tau(L_0-1-i\varepsilon)} + \int_0^\infty d\tau\, e^{+i\tau(L_0-(1+i\varepsilon)}$


2) We know
$(L_0 - 1)|\phi\rangle = 0$
$\Delta_E = (L_0 - 1)^{-1} = \int_0^\infty dt\, e^{-t(L_0-1)} = \int_0^1 dz\, z^{L_0-2}$
OUT on WS
IN on WS
$A_M \sim g^{M-2} \langle \phi_1 | V_2(k_2) \Delta_E V(k_3) \cdots V_{M-1}(k_{M-1}) | \phi_M \rangle$
$\Delta_T \sim e^{-i\tau(L_0-1)}$
$\Delta_{\bar{T}} \sim e^{+i\tau(L_0-1)}$
$\sum_{X_0} \int \mathcal{D}X_\mu^+ \mathcal{D}X_\mu^- \Big|_{X_0}^{X_0} e^{iS[X_\mu^+] - iS[X_\mu^-]}$