Title: Holographic correlators from bootstrap and supersymmetric localization

Speakers: Silviu Pufu

Series: Quantum Fields and Strings

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Abstract: In this talk, I will describe some of the recent progress on computing holographic correlators using analytic bootstrap techniques combined with supersymmetric localization. From taking a certain flat space limit of the holographic correlators, one can obtain scattering amplitudes of gravitons in string theory, and one can then reproduce some of the known results for these scattering amplitudes. I will focus mostly on the case of the 4d {\cal N} = 4 super-Yang-Mills theory, but I will also mention related work in the 3d ABJM theory.

Holographic correlators from bootstrap and supersymmetric localization

Silviu S. Pufu, Princeton University

Based on:

- arXiv:1804.00949 with S. Chester and X. Yin
- arXiv:1808.10554 with D. Binder and S. Chester
- arXiv:1902.06263 **with D. Binder, S. Chester, and Y. Wang**
- arXiv:1906.07195 with D. Binder and S. Chester

Perimeter Institute, October 1, 2019

Motivation

- The AdS/CFT correspondence can be used in two ways:
  - learn about quantum gravity / string theory / M-theory from CFT
  - learn about strongly-interacting QFTs from the bulk
- Challenges:
  - The CFTs are strongly coupled, so calculations are almost impossible.
  - Not much is known about string theory / M-theory in AdS beyond the supergravity approximation, and not much has been done beyond tree level.
- **This talk:** Using analytic bootstrap + supersymmetric localization
  $\Longrightarrow$ explore AdS/CFT beyond SUGRA
  $\Longrightarrow$ New results about holographic correlators.

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AdS/CFT

- Most well-established examples of AdS/CFT:
  - 4d $SU(N)$ $\mathcal{N} = 4$ SYM at large N and large 't Hooft coupling $\lambda = g^2_{\text{YM}} N$ / type IIB strings on $AdS_5 \times S^5$
  - 3d $U(N)_k \times U(N)_{-k}$ ABJM theory at large N / M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$.
- Focus on $\mathcal{N} = 4$ SYM / type IIB string for most of the talk.

Analytic bootstrap for holographic correlators

- **Main idea:** Holographic correlators are
  - simple in Mellin space (“simple” = can be bootstrapped)
  - “flat space limit” $\Longrightarrow$ scattering amplitudes in flat space

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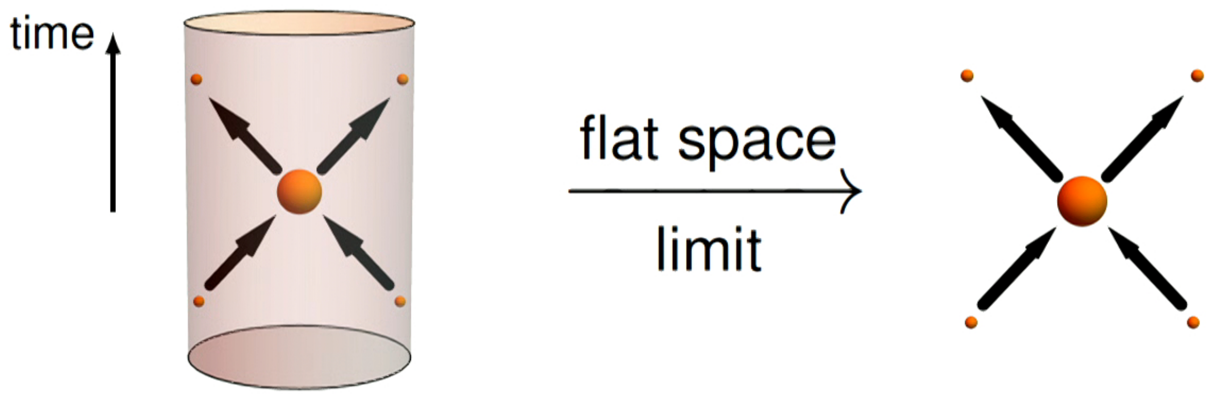

Mellin amplitude

- **Example:** Ops $\mathcal{O}_i(\vec{x})$ w/ $\Delta_i = \Delta$ have 4-pt function

$$\langle \mathcal{O}_1(\vec{x}_1)\cdots\mathcal{O}_4(\vec{x}_4)\rangle = \frac{1}{|\vec{x}_{12}|^{2\Delta}|\vec{x}_{34}|^{2\Delta}} G(U,V)\,,$$

where $\vec{x}_{ij} \equiv \vec{x}_i - \vec{x}_j$ and

$$U \equiv \frac{\vec{x}_{12}^2\vec{x}_{34}^2}{\vec{x}_{13}^2\vec{x}_{24}^2}\,, \qquad V \equiv \frac{\vec{x}_{14}^2\vec{x}_{23}^2}{\vec{x}_{13}^2\vec{x}_{24}^2}$$

- Mellin amplitude:

$$G_{\text{conn}}(U,V) = \frac{\Gamma\left(\frac{4\Delta-d}{2}\right)\pi^{d/2}}{\Gamma(\Delta)^4}\int \frac{ds\,dt}{(4\pi i)^2} U^{s/2} V^{u/2-\Delta}$$
$$\times\,\Gamma^2\left(\Delta-\frac{s}{2}\right)\Gamma^2\left(\Delta-\frac{t}{2}\right)\Gamma^2\left(\Delta-\frac{u}{2}\right) M(s,t)$$

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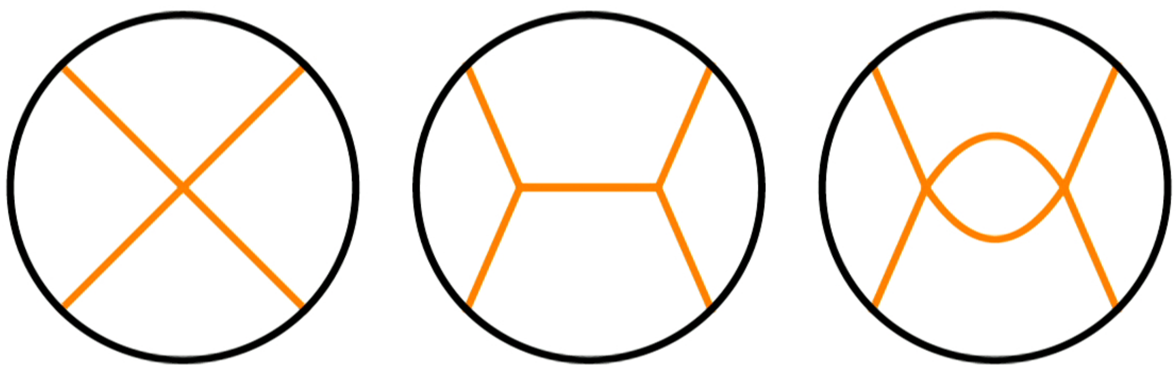
Properties of Mellin amplitudes

Properties of Mellin amplitudes:

1) Analytic structure:

- Contact Witten diagrams $\Longrightarrow$ polynomial Mellin amplitude in s, t of degree $= (\#\text{ of ders})/2$.
- Exchange Witten diagrams $\Longrightarrow$ Mellin amplitudes with poles.

  (E.g. s-channel scalar exchange gives poles at $s = \Delta_{\text{exch}} + 2n$ where Δ_{exch} is the dimension of the op dual to the exchanged field).



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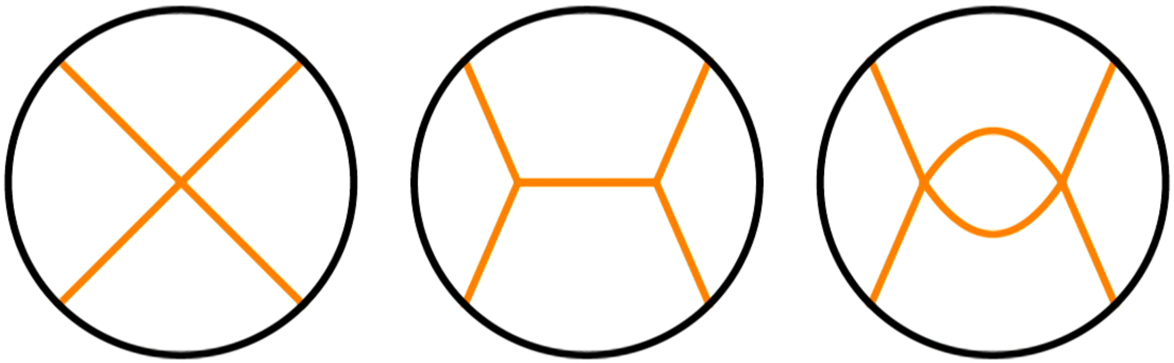

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$$A(S,T) = \lim_{L\to\infty} \frac{\Gamma(2\Delta - \frac{d}{2})}{L^{3-d}} \int_{c-i\infty}^{c+i\infty} \frac{d\alpha}{2\pi i} \alpha^{\frac{d}{2}-2\Delta} e^{\alpha} M\left(\frac{L^2 S}{2\alpha}, \frac{L^2 T}{2\alpha}\right) .$$

Here, $A(S,T)$ is the scattering amplitude of massless particles in flat space:

$$m^2 = \frac{\Delta(\Delta - d)}{L^2} \to 0 \quad \text{as } L \to \infty$$

and S, T, U with $S + T + U = 0$ are the Mandelstam invariants.

- This formula implies that leading order in large s, t:

$$M(s,t) \propto \text{flat space scattering amplitude}$$

and, in particular, $M(s,t)$ has the same growth at large s, t as the flat space scattering amplitude.

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Procedure

Procedure for computing holographic correlators: Instead of computing Witten diagrams, use

- analytic structure
- growth at large s and t
- crossing symmetry
- SUSY

to determine the Mellin amplitudes up to a few numerical constants.

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Cases studied

Applied this procedure to:

- 4d $\mathcal{N} = 4$ SYM (1/2-BPS ops) $\longrightarrow$ **(super)graviton** scattering in type IIB string theory [Binder, Chester, SSP, Wang '19; Chester '19]
- 6d A_{N-1} (2, 0) theory (1/2-BPS ops) $\longrightarrow$ **(super)graviton** scattering in M-theory [Chester, Perlmutter '18]
- 3d $\mathcal{N} = 8$ ABJM $U(N)_k \times U(N)_{-k}$ with $k = 1, 2$ (1/2-BPS ops) $\longrightarrow$ **(super)graviton** scattering in M-theory [Chester, SSP, Yin '18]
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- More SUSic localization constraints in 3d.

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Scattering amplitudes in 10d and 11d

- Effective action in 10d:

$$S_{10\text{d}} = \int d^{10}x\,\sqrt{g}\left[R + \ell_s^6 R^4 + \ell_s^{10} D^4 R^4 + \cdots\right] + \text{(SUSic completion)}$$

where ℓ_s is the string length.

- The terms w/ > 2 derivatives are not fully known!
- This gives the amplitude

$$\mathcal{A}(S,T) = \delta^{16}(Q)\left[\frac{1}{STU} + \frac{\zeta(3)}{32}\ell_s^6 + \frac{\zeta(5)}{2^{10}}(S^2+T^2+U^2)\ell_s^{10} + \cdots + O(g_s^2)\right]$$

$$= \mathcal{A}_{\text{SG, tree}}(S,T) f(S,T)\,, \qquad \mathcal{A}_{\text{SG, tree}} = \frac{\delta^{16}(Q)}{STU}\,.$$

- The full leading term in g_s is a ratio of Gamma functions.
- In 11d, the effective action is similar, with $\ell_s \to \ell_p$ (and no g_s), but terms only up to $D^6 R^4$ are known.

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Scattering amplitudes in type IIB string theory

- One can also write the scattering amplitude w/o expanding in g_s:

$$\mathcal{A}(S,T) = \delta^{16}(Q)\left[\frac{1}{STU} + \frac{g_s^{3/2}}{64}\mathcal{E}_{3/2}(\tau,\bar{\tau}) + \cdots\right]$$

where $\tau = \chi + i/g_s$ and $\mathcal{E}_r$ is a non-holomorphic Eisenstein series

$$\mathcal{E}_r = \sum_{(m,n)\neq(0,0)} \frac{g_s^{-r}}{|m+n\tau|^{2r}}\,.$$

- For $\mathcal{E}_{3/2}$ when $\chi = 0$, one has a finite number of perturbative contributions + non-perturbative contributions:

$$\mathcal{E}_{3/2} = \frac{2\zeta(3)}{g_s^{3/2}} + \frac{2\pi^2}{3}g_s^{1/2} + e^{-\frac{2\pi}{g_s}}\left[4\pi + \frac{3g_s}{4} + \cdots\right] + e^{-\frac{4\pi}{g_s}}\left[\cdots\right] + \cdots\,.$$

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Holographic correlators in $\mathcal{N} = 4$ SYM

- In $\mathcal{N} = 4$ SYM, focus on the $\mathbf{20}'$ ops $S_{IJ} = \text{tr}\left(\phi_I\phi_J - \frac{\delta_{IJ}}{6}\phi_K\phi^K\right)$.
- At large N, $\lambda = g_{\text{YM}}^2 N$, it is dual to string theory on $AdS_5 \times S^5$ of curvature radius L.

$$\lambda = \frac{L^4}{\ell_s^4}\,, \qquad \frac{g_{\text{YM}}^2}{4\pi} = g_s\,.$$

- The 4-pt function $\langle SSSS\rangle$ has an expansion in $1/N$ (or $1/c$ with $c = (N^2-1)/4$) and $1/\lambda$:

$$\langle SSSS\rangle = \text{disconnected} + \frac{1}{c}\left[1 + \frac{1}{\lambda^{\frac{3}{2}}} + \frac{1}{\lambda^{\frac{5}{2}}} + \cdots\right] + \frac{1}{c^2}\left[\lambda^{\frac{1}{2}} + 1 + \frac{1}{\lambda^{\frac{3}{2}}} + \cdots\right] + \frac{1}{c^3}\left[\lambda^{\frac{3}{2}} + \lambda^{\frac{1}{2}} + 1 + + \cdots\right] + \cdots$$

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$$\langle SSSS \rangle = \text{disconnected} + \frac{1}{c}\left[1 + \frac{1}{\lambda^{\frac{3}{2}}} + \frac{1}{\lambda^{\frac{5}{2}}} + \cdots\right]$$
$$+ \frac{1}{c^2}\left[\lambda^{\frac{1}{2}} + 1 + \frac{1}{\lambda^{\frac{3}{2}}} + \cdots\right] + \frac{1}{c^3}\left[\lambda^{\frac{3}{2}} + \lambda^{\frac{1}{2}} + 1 + + \cdots\right] + \cdots$$

corresp to tree-level: SG, R^4, $D^4 R^4$; one-loop ; etc.

- **All of the highlighted terms are known completely!**

Holographic correlators in $\mathcal{N} = 4$ SYM

Two comments:

- One can compute $\langle SSSS \rangle$ to the orders mentioned even though the higher derivative corrections to SUGRA are not completely known!
- We can also consider a large N expansion at fixed g_{YM} ("very strong coupling" expansion):

$$\langle SSSS \rangle = \text{disc} + \frac{1}{c} + \frac{(\text{fn of } g_{\text{YM}})}{c^{\frac{7}{4}}} + \frac{1}{c^2} + \frac{(\text{fn of } g_{\text{YM}})}{c^{\frac{9}{4}}} + \cdots ,$$

where the fns of g_{YM} should involve Eisenstein series.

1/2-BPS single trace primaries

- $\mathcal{N} = 4$ SYM field content: gauge field A_μ, fermions λ_a in **4** of $SU(4)_R$, scalars ϕ_I in **6** of $SU(4)_R$.
- 1/2-BPS single trace primaries:

$$S_p \propto \operatorname{tr} \phi^p \,, \qquad \Delta = p \,, \qquad [0p0] \text{ of } SU(4)_R \,.$$

- 4-pt functions $\langle S_p S_q S_r S_s \rangle$ have been extensively studied. (In SUGRA limit, full answer is known [Rastelli, Zhou] .)
- **For now:** $\langle S_2 S_2 S_2 S_2 \rangle$. Conformal sym and $SU(4)_R \Longrightarrow$

$$\langle S_2 S_2 S_2 S_2 \rangle = \frac{1}{x_{12}^4 x_{34}^4} \times (6 \text{ functions of } U, V)$$

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Ward identity

- SUSY Ward identities
  $\Longrightarrow$ *algebraic* relations between the 6 functions of U, V
  $\Longrightarrow$ all 6 fns can be written in terms of a single function $T(U, V)$!
- Roughly,

$$\langle S_2 S_2 S_2 S_2 \rangle = (\text{free part}) + \frac{1}{\vec{x}_{12}^4 \vec{x}_{34}^4} \begin{pmatrix} V \\ UV \\ U \\ \vdots \end{pmatrix} \times T(U, V)\,.$$

- This is special to 4d $\mathcal{N} = 4$ SUSY.

Mellin space

Can relate $T(U,V)$ to the Mellin transform $\mathcal{M}(s,t)$:

$$T(U,V) = \int \frac{ds\,dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{u}{2}-3} \Gamma^2\left(2-\frac{s}{2}\right) \times \Gamma^2\left(2-\frac{t}{2}\right)\Gamma^2\left(2-\frac{u}{2}\right)\mathcal{M}(s,t)\,,$$

where $u \equiv 4 - s - t$.

$\mathcal{M}(s,t)$ can be determined up to a few constants using:

- crossing symmetry

$$\mathcal{M}(s,t) = \mathcal{M}(s,u)\,, \qquad \mathcal{M}(s,t) = \mathcal{M}(u,t)\,.$$

- pole structure
- asymptotic growth at large s, t

Analytic bootstrap for Mellin amplitude

The consistency conditions give:

$$\mathcal{M}(s,t) = \frac{8}{c}\left[\frac{1}{(s-2)(t-2)(u-2)} + \frac{c_1}{\lambda^{\frac{3}{2}}} + \frac{c_2(s^2+t^2+u^2)+c_3}{\lambda^{\frac{5}{2}}} + \cdots\right] + O(1/c^2)\,.$$

where $c_1, c_2, c_3, \ldots$ are arbitrary constants.

- Constraints on $c_1, c_2, c_3, \ldots$ come from
  - the flat space limit
  - SUSic localization
- The flat space limit

$$f(S,T) = \frac{STU}{2^{11}\pi^2 g_s^2 \ell_s^8} \lim_{\frac{L}{\ell_s}\to\infty} \int \frac{d\alpha}{2\pi i} e^{\alpha} \alpha^{-6} \mathcal{M}\left(\frac{L^2 S}{2\alpha}, \frac{L^2 T}{2\alpha}\right)$$

gives $\boxed{c_1 = 15\zeta(3)}$; $\boxed{c_2 = \frac{315}{4}\zeta(5)}$; ...

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- This is special to 4d $\mathcal{N} = 4$ SUSY.

Scattering amplitudes in 10d and 11d

- Effective action in 10d:

$$S_{10\text{d}} = \int d^{10}x\,\sqrt{g}\left[R + \ell_s^6 R^4 + \ell_s^{10} D^4 R^4 + \cdots\right] + (\text{SUSic completion})$$

  where ℓ_s is the string length.
- The terms w/ > 2 derivatives are not fully known!
- This gives the amplitude

$$\mathcal{A}(S,T) = \delta^{16}(Q)\left[\frac{1}{STU} + \frac{\zeta(3)}{32}\ell_s^6 + \frac{\zeta(5)}{2^{10}}(S^2+T^2+U^2)\ell_s^{10} + \cdots + O(g_s^2)\right]$$

$$= \mathcal{A}_{\text{SG, tree}}(S,T) f(S,T)\,, \qquad \mathcal{A}_{\text{SG, tree}} \equiv \frac{\delta^{16}(Q)}{STU}\,.$$

- The full leading term in g_s is a ratio of Gamma functions.
- In 11d, the effective action is similar, with $\ell_s \to \ell_p$ (and no g_s), but terms only up to $D^6 R^4$ are known.

Analytic bootstrap for Mellin amplitude

The consistency conditions give:

$$\mathcal{M}(s,t) = \frac{8}{c}\left[\frac{1}{(s-2)(t-2)(u-2)} + \frac{c_1}{\lambda^{\frac{3}{2}}} + \frac{c_2(s^2+t^2+u^2)+c_3}{\lambda^{\frac{5}{2}}} + \cdots\right] + O(1/c^2)\,.$$

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Supersymmetric localization

- SUSic localization gives exact results in SUSic theories, but generally *not* for correlation functions of local ops.
- Idea:
$$Z = \int DX\, e^{-S} = \int DX\, e^{-S - t\{Q,\Psi\}}$$
is independent of t. If $\{Q, \Psi\}_{\text{bos}} \geq 0$, then take t large and
$$Z = \int_{\substack{X \text{ such that} \\ \{Q,\Psi\}=0}} e^{-S} \times (\text{one-loop determinant})$$
(Ideally, this is a finite-dimensional integral.)
- Can insert Q-invariant observables.
- Generically, $Q\mathcal{O}(\vec{x}) \neq 0$ b/c $Q^2 =$ translation $+$ R-symm rotation and $Q^2\mathcal{O}(\vec{x}) \neq 0$ for a local operator $\mathcal{O}(\vec{x})$.

3. Supersymmetric localization: observables

So, SUSic observables can be:

- integrated operators
- local operators inserted at points where $Q^2 = 0$.

In this talk: we'll relate two of the S_2's to integrated SUSic operators, and we'll place the other two S_2's at points where $Q^2 = 0$.

Specific setup:

$\mathcal{N} = 4$ SYM on $\mathbb{R}^4$ $\longrightarrow$ $\mathcal{N} = 4$ SYM on S^4 $\longrightarrow$ deform while preserving $Q \in \mathcal{N} = 2$ superalgebra

$Q^2 = 0$ only at N and S poles of S^4

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Localization setup in 4d $\mathcal{N} = 2$ SCFTs

$\mathcal{N} = 2$ SCFT: R-symmetry is $SU(2)_R \times U(1)_R$.

We will consider two multiplets:

- conserved current: $(J, K, \ldots, j_\mu)$

$$J : \mathbf{3}_0\,, \quad \Delta = 2 \quad \text{(scalar bilinear)}$$
$$K : \mathbf{1}_{\pm 2}\,, \quad \Delta = 3 \quad \text{(fermion bilinear)}$$

- chiral / anti-chiral multiplet: $(A_p, \ldots) \leftarrow$ interested in $p = 2$

$$A_p, \bar{A}_p : \mathbf{1}_{\pm p}\,, \quad \Delta = p$$

On S^4, deformations preserving Q:

- Real mass (using certain components of J and K):

$$m \int d^4x \sqrt{g}\, [iJ + K]$$

- $A_p(N)$; $\bar{A}_p(S)$.

$\mathcal{N} = 4$ SYM as an $\mathcal{N} = 2$ SCFT

$$SU(4)_R \qquad\qquad SU(2)_R \times U(1)_R \times SU(2)_F$$

$$S_2: \quad \mathbf{20}' \qquad \underbrace{(\mathbf{1},\mathbf{1})_{\pm 2}}_{A_2,\,\bar{A}_2} + \underbrace{(\mathbf{3},\mathbf{3})_0}_{J} + (\mathbf{1},\mathbf{1})_0 + (\mathbf{2},\mathbf{2})_{\pm 1}$$

So consider

$$S_{\substack{\mathcal{N}=4 \\ \text{SYM} \\ \text{on } S^4}}(\tau,\bar{\tau}) + \delta\tau\, A_2(N) + \delta\bar{\tau}\, \bar{A}_2(S) + m \int (iJ + K) \ ,$$

where $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\text{YM}}^2}$.

- Actually, up to Q-exact terms, $\delta\tau\, A_2(N) = \int d^4x\, \sqrt{g}\delta\tau\, \mathcal{O}_\tau$ and $\delta\bar{\tau}\, A_2(S) = \int d^4x\, \sqrt{g}\delta\tau\, \bar{\mathcal{O}}_{\bar{\tau}}$, where $\mathcal{O}_\tau$ and $\bar{\mathcal{O}}_{\bar{\tau}}$ are marginal ops [Gerchkovitz, Gomis, Komargodski '14; Gomis, Ishtiaque '16]:

$$Z(\tau,\bar{\tau},\delta\tau,\delta\bar{\tau},m) = Z(\tau+\delta\tau,\bar{\tau}+\delta\bar{\tau},m) \ .$$

- It is enough to compute $Z(\tau,\bar{\tau},m)$ (part fn of $\mathcal{N} = 2^*$ on S^4).

$\mathcal{N} = 4$ SYM as an $\mathcal{N} = 2$ SCFT

$$SU(4)_R \qquad\qquad SU(2)_R \times U(1)_R \times SU(2)_F$$

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Matrix model for $\mathcal{N} = 2^*$ partition function

Pestun computed the S^4 partition function of the $\mathcal{N} = 2^*$ theory [Pestun '07] :

$$Z = \int da \left[\prod_{i<j} \frac{(a_i - a_j)^2 H^2(a_i - a_j)}{H(a_i - a_j + m) H(a_i - a_j - m)} \right] e^{-\frac{N}{\lambda} \sum_i a_i^2} |Z_{\text{inst}}|^2$$

where H is the product of two Barnes G-functions, and Z_{inst} represents the contribution of instantons localized at the N and S poles of S^4.

- Perturbatively at large N (and fixed λ) one can ignore the instanton contributions.

$\mathcal{N} = 4$ SYM as an $\mathcal{N} = 2$ SCFT

- Then

$$\left.\frac{\partial^4 \log Z}{\partial m^2 \partial\tau \partial\bar\tau}\right|_{m=0} = \left\langle A_2(N)\bar A_2(S) \int_{S^4} (iJ+K) \int_{S^4} (iJ+K) \right\rangle \qquad (*)$$

- Using Ward identities + integration by parts + group theory to relate A, $\bar A$, J, K to operators in $\mathcal{N} = 4$ SYM, we have

$$\left.\frac{\frac{\partial^4 \log Z}{\partial m^2 \partial\tau\partial\bar\tau}}{\frac{\partial^2 Z}{\partial\tau\partial\bar\tau}}\right|_{m=0} = -\frac{16c}{\pi}\int dr\, d\theta\, r^3 \sin^2\theta \left.\frac{T(U,V)}{U^2}\right|_{\substack{U=1+r^2-2r\cos\theta \\ V=r^2}}$$

$$\approx 8\int_0^\infty d\omega \frac{\omega}{\sinh^2\omega}\left[J_1\left(\frac{\sqrt{\lambda}\omega}{\pi}\right)^2 - J_2\left(\frac{\sqrt{\lambda}\omega}{\pi}\right)^2\right]$$

where the last equality is at leading order in $1/c$.

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Localization constraints

- Expanding in $1/\lambda$, we obtain

$$c_1 = 15\zeta(3) \longleftarrow \textbf{same as from flat space limit!}$$
$$64c_2 + 28c_3 = -1575\zeta(5)\,, \quad \text{etc.}$$

Comments:

- c_1 derived in two ways $\longrightarrow$ **check of AdS/CFT beyond SUGRA**.
- Combining flat space limit + localization, we obtain

$$c_2 = -\frac{1}{3}c_3 - \frac{315}{4}\zeta(5)\,.$$

- To fully fix all the constants at higher orders in the tree-level $1/\lambda$ expansion, one would need more constraints.
  - There are potentially more quantities that can be computed using SUSic loc: $\partial^4 \log Z/\partial m^4\big|_{m=0}$, squashed sphere partition function.

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Generalization to $\langle S_2 S_2 S_p S_p \rangle$

- One can generalize the localization constraints to $\langle S_2 S_2 S_p S_p \rangle$ (where $S_p = \operatorname{tr} \phi^p$) because:
  - S_p contains an $\mathcal{N} = 2$ chiral / anti-chiral multiplet w/ scalar ops A_p and $\bar{A}_p$ that can be inserted at N and S poles of S^4.
  - The $\mathcal{N} = 2^*$ partition function further deformed by these insertions can be computed by a modification of Pestun's matrix model [Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, SSP '16] :

$$Z = \int da \left[\prod_{i<j} \frac{(a_i - a_j)^2 H^2(a_i - a_j)}{H(a_i - a_j + m) H(a_i - a_j - m)} \right] e^{-\frac{N}{\lambda} \sum_i a_i^2 + i(\tau_p' - \bar{\tau}_p') \sum_i a_i^p} |Z_{\text{inst}}|^2$$

which gives at leading order in $1/N$:

$$\frac{\partial_m^2 \partial_{\tau_p} \partial_{\bar{\tau}_p} \log Z}{\partial_{\tau_p} \partial_{\bar{\tau}_p} \log Z} \Bigg|_{\substack{m=0 \\ \tau_p = \bar{\tau}_p = 0}} \approx 4p \int_0^\infty d\omega\, \omega \frac{J_1(\sqrt{\lambda}\omega/\pi)^2 - J_p(\sqrt{\lambda}\omega/\pi)^2}{\sinh^2 \omega}$$

Very strong coupling limit

- So far we worked in the 't Hooft strong coupling limit.
- "Very strong" coupling limit: $N \to \infty$, g_{YM} fixed. Use $c = (N^2 - 1)/4$ instead of N as before.
- For $\langle S_2 S_2 S_2 S_2 \rangle$, expect:

$$\mathcal{M}(s,t) = \frac{8}{c}\frac{1}{(s-2)(t-2)(u-2)} + b(\tau)\frac{1}{c^{7/4}} + \frac{1}{c^2}\mathcal{M}_{\text{1-loop}}(s,t) + \frac{1}{c^{9/4}}\left[d_1(\tau)(s^2+t^2+u^2) + d_2(\tau)\right] + \cdots ,$$

  where $\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\text{YM}}^2}$ as before.
- These correspond to R, $f(\tau)R^4$, $g(\tau)D^4R^4$ in the flat space limit.

Prediction for Z_{S^4} in the very strong coupling limit

- Flat space limit requires

$$b(\tau) = \frac{60}{(\mathrm{Im}\,\tau)^{3/2}} \mathcal{E}_{3/2}(\tau, \bar{\tau}) \,,$$

where $\mathcal{E}_{3/2}(\tau, \bar{\tau}) \equiv \sum_{(m,n)\neq(0,0)} \frac{g_s^{-3/2}}{|m+n\tau|^{3/2}}$ is an Eisenstein series.

- Relation between $\langle S_2 S_2 S_2 S_2 \rangle$ and S^4 partition function implies

$$\left. \frac{\partial^2 \log Z}{\partial m^2} \right|_{m=0} = 4c \log g_{\text{YM}} - \frac{\sqrt{2} c^{1/4}}{\pi^{3/2}} \mathcal{E}_{3/2}(\tau, \bar{\tau}) + \cdots$$
$$+ \,(\text{hol} + \text{anti-hol ambiguity}) \,.$$

- To check the $c^{1/4}$ term we would need to sum the instantons in the S^4 partition function of the mass-deformed $\mathcal{N} = 4$ SYM theory [Chester, Green, SSP, Wang, Wen (work in progress)].

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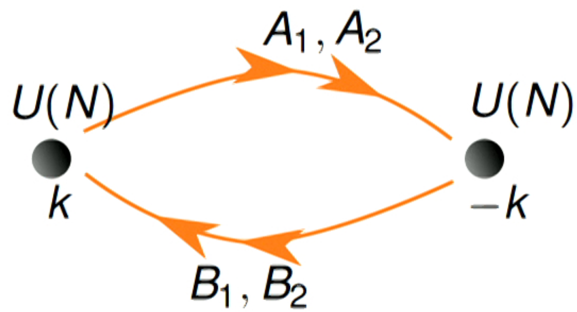
ABJM theory at $k = 1, 2$



- The 3d $U(N)_k \times U(N)_{-k}$ ABJM theory is dual to **M-theory** on $AdS_4 \times S^7/\mathbb{Z}_k$ and has $\mathcal{N} = 8$ SUSY for $k = 1, 2$ and $\mathcal{N} = 6$ SUSY for $k > 2$.
- Two params: (N, k). Instead of N use

$$c_T \propto k^{1/2} N^{3/2} \propto \frac{1}{k} \left(\frac{L}{\ell_p} \right)^9$$

Here, $\langle T_{\mu\nu} T_{\rho\sigma} \rangle \propto c_T$.

ABJM theory at $k = 1, 2$

- $\mathcal{N} = 8$ case ($k = 1, 2$): the analog of the $\mathbf{20}'$ op from $\mathcal{N} = 4$ SYM is the superconf. primary of the stress tensor multiplet:

$$S_2: \qquad \Delta = 1\,, \qquad j = 0\,, \qquad \mathbf{35}_c \text{ of } SO(8)_R\,.$$

- 4-pt function:

$$\langle S_2 S_2 S_2 S_2 \rangle = (\text{disc.}) + \frac{1}{c_T} + \frac{1}{c_T^{\frac{5}{3}}} + \frac{1}{c_T^2} + \frac{1}{c_T^{\frac{19}{9}}} + \frac{1}{c_T^{\frac{7}{3}}} + \frac{1}{c_T^{\frac{23}{9}}} + \cdots\,.$$

corresp to: SG tree , R^4 (1 polyn soln), $D^4 R^4$ (2 polyn solns), $D^6 R^4$ (3 polyn solns), etc.

- To fix constants, use
  - the flat space limit
  - SUSic localization: $\boxed{Z_{S^3}(m_1, m_2)}$ known to all orders in $1/N$!!
- Use both $\partial^4 Z/\partial m_1^4$ and $\partial^4 Z/\partial m_1^2 \partial m_2^2$.

ABJM theory at $k > 2$ and fixed $\lambda = N/k$

- In the large N fixed $\lambda = N/k$ limit $\to$ type IIA on $AdS_4 \times \mathbb{CP}^3$:

$$\frac{L^8}{\ell_s^8} = 4\pi^2\lambda^2 \,, \qquad g_s^2 \approx \frac{512\lambda^2}{3c_T} \,.$$

- S_2 is a $\Delta = 1$ op. in **15** of $SO(6)_R$. Stress tensor multiplet is only 1/3-BPS.
- 4-pt function

$$\langle S_2 S_2 S_2 S_2 \rangle = \text{disc.} + \frac{1}{c_T}\left[1 + \frac{1}{\lambda^{\frac{3}{2}}} + \frac{1}{\lambda^{\frac{5}{2}}} + \cdots \right] + \frac{1}{c_T^2}\left[\sqrt{\lambda} + 1 + \cdots\right] + \cdots \,.$$

corresp to: SG tree, R^4 (2 polyn solns), etc.

- $\frac{1}{c_T \lambda^{3/2}}$ and $\frac{\sqrt{\lambda}}{c_T^2}$ multiply different functions of s, t.

Conclusion

- A combination of techniques (supersymmetric localization, SUSY Ward identities, analytic bootstrap in Mellin space) can be used to study holographic correlators beyond the SUGRA approximation in both 4d $\mathcal{N} = 4$ SYM and 3d ABJM theory.

For the future:

- Explore other constraints from supersymmetric localization, e.g. squashed sphere partition function (ongoing work).
- Loops in AdS and loop scattering amplitudes (1-loop in $\mathcal{N} = 4$ SYM done [Alday, Bissi '17; Aprile, Drummond, Heslop, Paul '17, '18; Chester '19]).
- Combine integrated constraints with numerical bootstrap (ongoing work).