Title: Ignorance is Cheap: From Black Hole Entropy To Energy-Minimizing States In QFT

Speakers: Arvin Shahbazi-Moghaddam

Series: Quantum Fields and Strings

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Abstract: Behind certain marginally trapped surfaces one can construct a geometry containing an extremal surface of equal, but not larger area. This construction underlies the Engelhardt-Wall proposal for explaining the Bekenstein-Hawking entropy as a coarse-grained entropy. The construction can be proven to exist classically but fails if the Null Energy Condition is violated. Here we extend the coarse-graining construction to semiclassical gravity. Its validity is conjectural, but we are able to extract an interesting nongravitational limit. Our proposal implies Wallâ€™s ant conjecture on the minimum energy of a completion of a quantum field theory state on a halfspace. It further constrains the properties of the minimum energy state; for example, the minimum completion energy must be localized as a shock at the cut. We verify that the predicted properties hold in a recent explicit construction of Ceyhan and Faulkner, which proves our conjecture in the nongravitational limit.

Quantum Coarse Graining of Black Holes

Arvin Shahbazi-Moghaddam

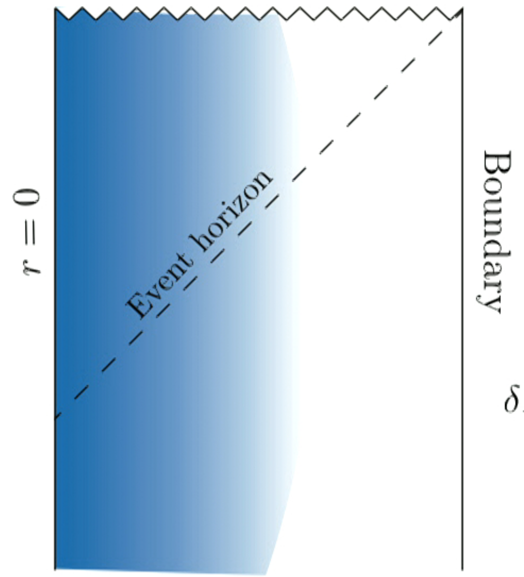
with Raphael Bousso and Ven Chandrasekaran

Berkeley

October 2019

Black hole second law

Black holes are thermodynamic objects with $S_{BH} = \frac{A}{4G\hbar}$ [Hawking, Bekenstein]



$$\delta A_{EH} \geq 0$$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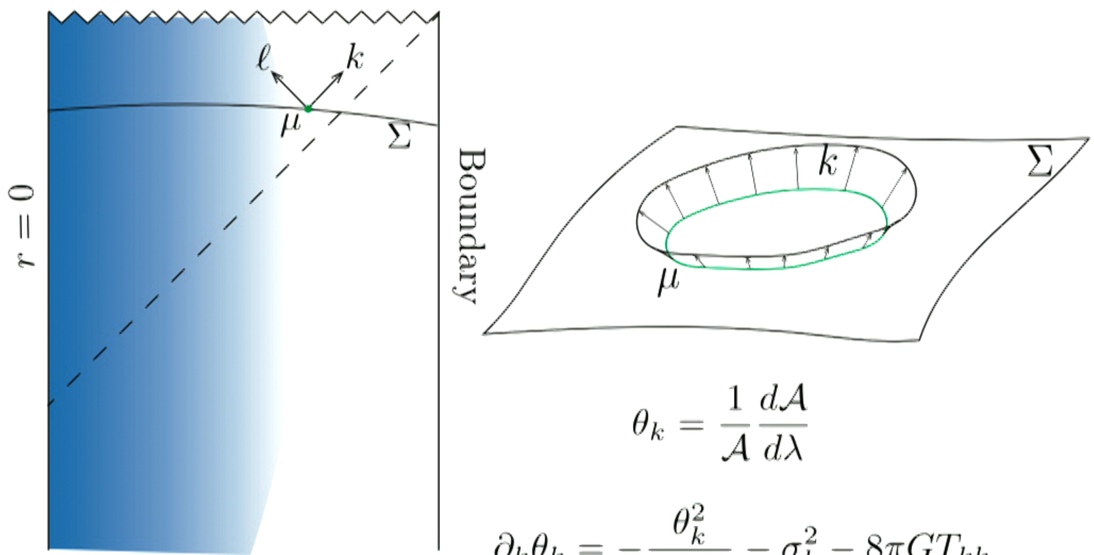
Black hole second law

More local definition of a black hole



$$\theta_k = \frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{d\lambda}$$

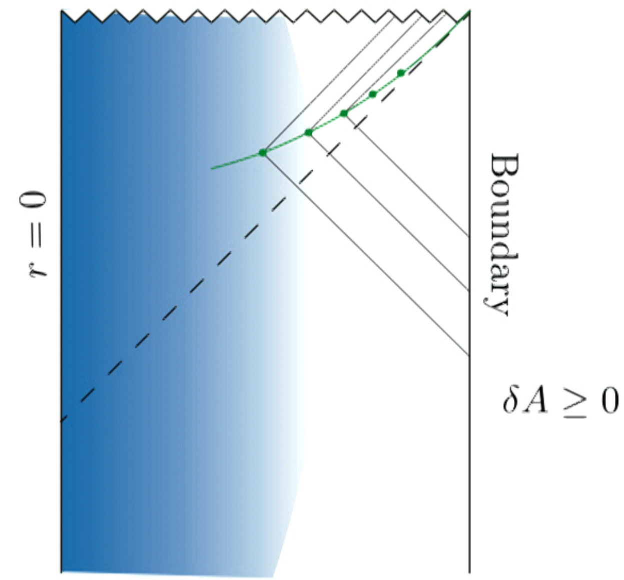
$$\partial_k \theta_k = -\frac{\theta_k^2}{d-2} - \sigma_k^2 - 8\pi G T_{kk}$$

$$T_{kk} \geq 0 \text{ (NEC)}$$

μ : Marginally trapped surface $(\theta_k = 0, \theta_\ell < 0)$

Black hole second law

Dynamical horizons have an area law



Black hole second law

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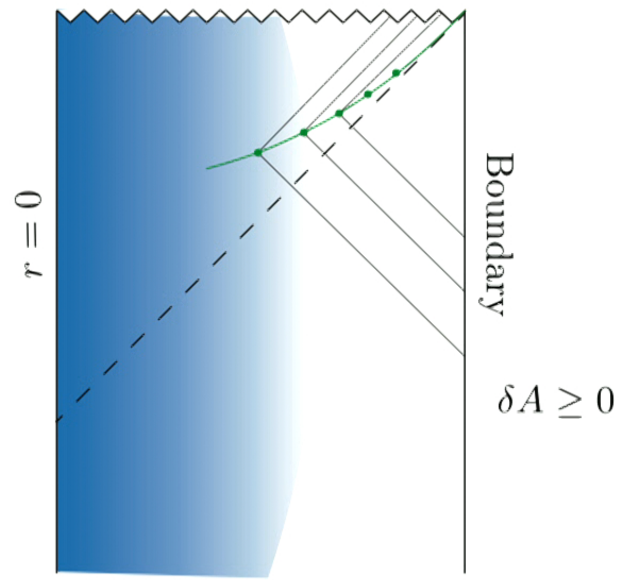

Is $A[\mu]/4G\hbar$ a coarse-grained entropy?

Black hole second law

Engelhardt-Wall answered this question for classical black holes

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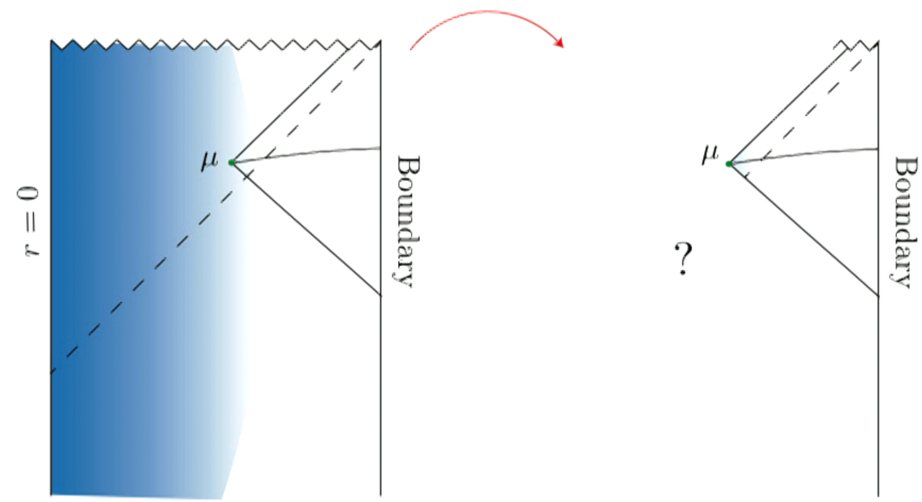
We need a microscopic theory+prescription for coarse-graining

Black hole second law

Engelhardt-Wall answered this question for classical black holes
We need a microscopic theory+prescription for coarse-graining
AdS/CFT!

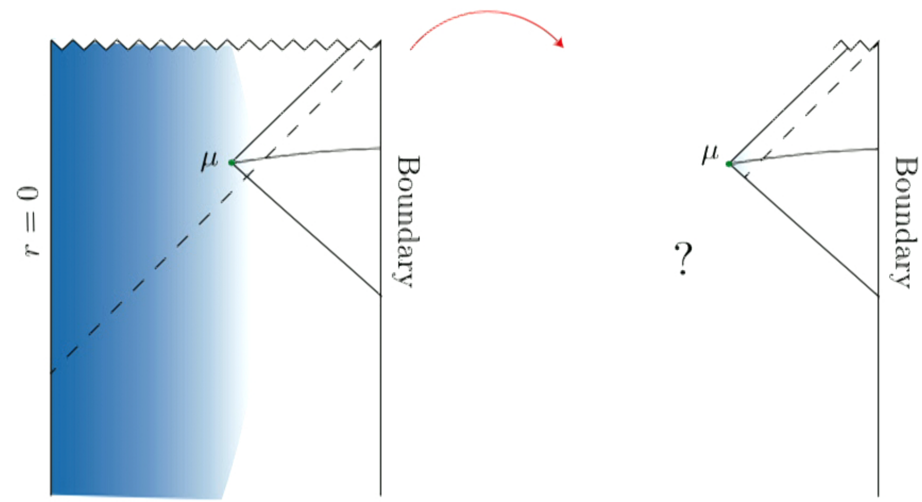
Review of Engelhardt-Wall construction

Coarse-graining prescription in AdS/CFT



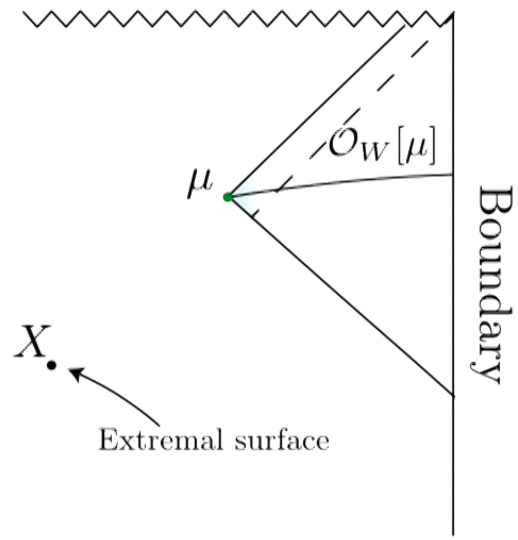
Review of Engelhardt-Wall construction

Coarse-graining prescription in AdS/CFT



Ryu-Takayanagi prescription $S_{\mathrm{CFT}} = \frac{A[X]}{4G\hbar}$
where X is an extremal surface ($\theta_k = \theta_l = 0$)

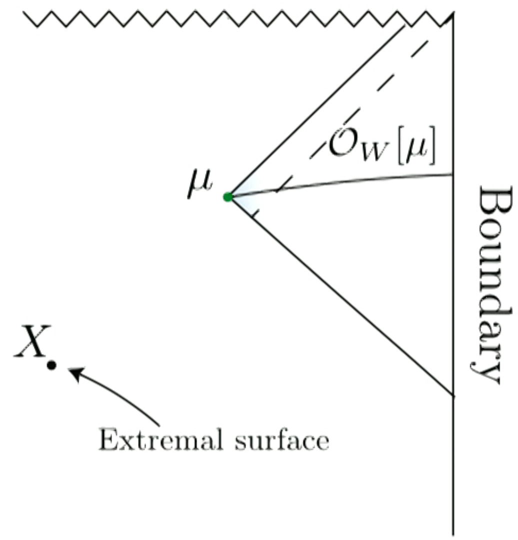
Review of Engelhardt-Wall construction



$$\max A[X] : \text{holding } \mathcal{O}_W[\mu] \text{ fixed}$$

$$\implies S_{\text{coarse}}[\mu] = \frac{A[X]}{4G\hbar}$$

Review of Engelhardt-Wall construction

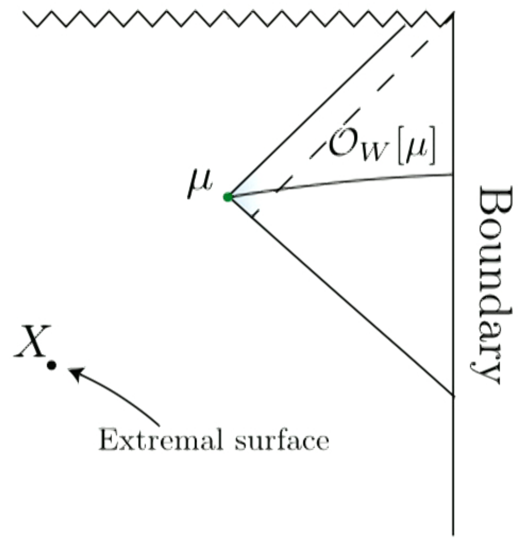


$$\max A[X] : \text{holding } \mathcal{O}_W[\mu] \text{ fixed}$$

$$\Longrightarrow S_{\text{coarse}}[\mu] = \frac{A[X]}{4G\hbar}$$

Can show $A[X] \leq A[\mu]$ (classical focusing $\Leftarrow$ null energy condition)

Review of Engelhardt-Wall construction



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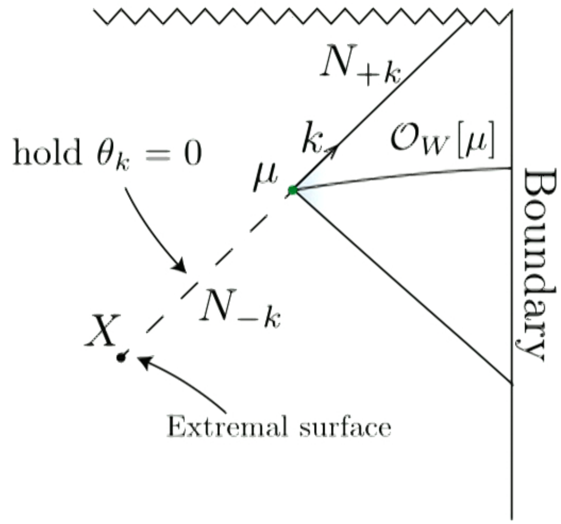
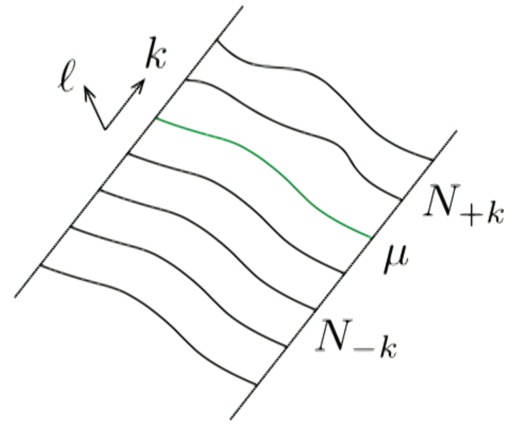
Review of Engelhardt-Wall construction

Engelhardt-Wall's explicit construction





$$N_{-k} \text{ stationary} : \theta_k = \sigma_k = 0$$

$$\partial_k \theta_\ell = \text{constant} < 0$$

$$\partial_k \theta_k = -\frac{\theta_k^2}{d-2} - \sigma_k^2 - 8\pi G T_{kk}$$

X was found such that $A[X] = A[\mu] \implies S_{coarse} = \frac{A[\mu]}{4G\hbar}$!

Generalized entropy

Area law can be violated quantum-mechanically! e.g. Hawking evaporation

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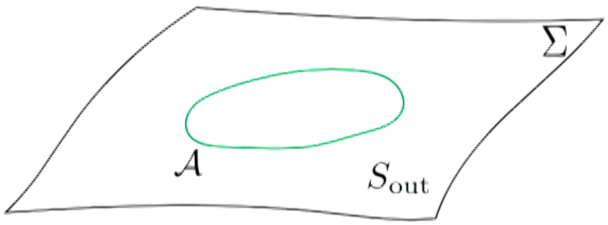
Add to the area the von Neumann entropy of matter fields outside:
Generalized entropy! [Bekenstein]



$$\frac{\mathcal{A}}{4G\hbar} \to \frac{\mathcal{A}}{4G\hbar} + S_{out} = S_{\text{gen}}$$

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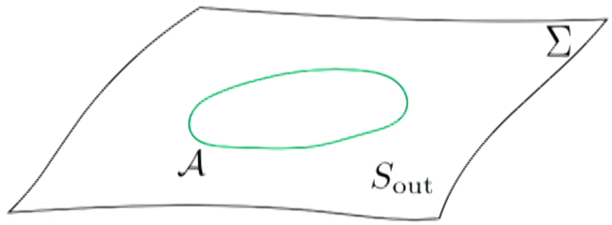
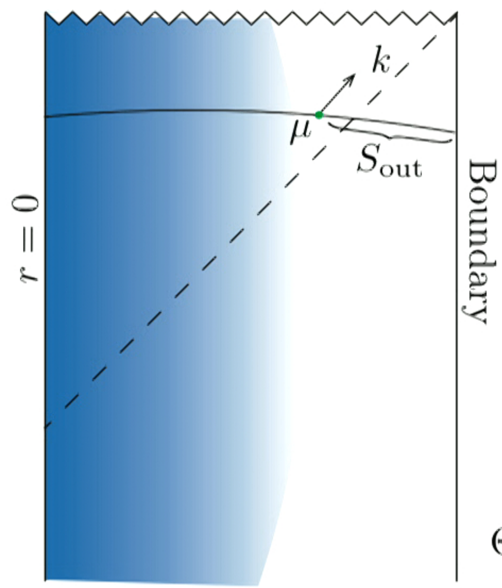
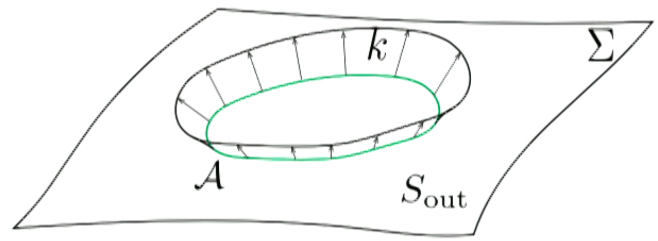


$$\frac{\mathcal{A}}{4G\hbar} \to \frac{\mathcal{A}}{4G\hbar} + S_{out} = S_{\rm gen}$$

Area law of causal horizons → Generalized second law
Classical focusing → Quantum focusing conjecture
Null energy condition → Quantum null energy condition
Classical RT prescription → Quantum extremal surfaces
Penrose's classical singularity theorem → Quantum singularity theorem
[Balakrishnan, Bousso, Casini, Ceyhan, Engelhardt, Faulkner, Fisher, Koeller, Leichenauer, Lewkowycz, Maldacena, Susskind, Uglum, Wall]

Generalized entropy





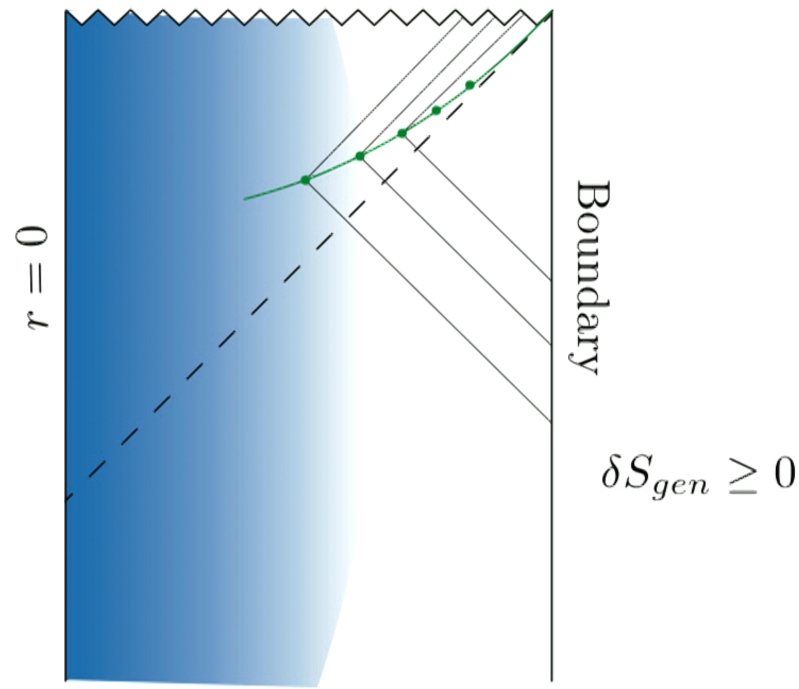
$$\frac{\mathcal{A}}{4G\hbar} \to \frac{\mathcal{A}}{4G\hbar} + S_{out} = S_{\text{gen}}$$

$$\Theta_k = 4G\hbar\, k^\alpha \frac{\delta S_{\text{gen}}}{\delta X^\alpha} = \theta_k + 4G\hbar \partial_k S$$

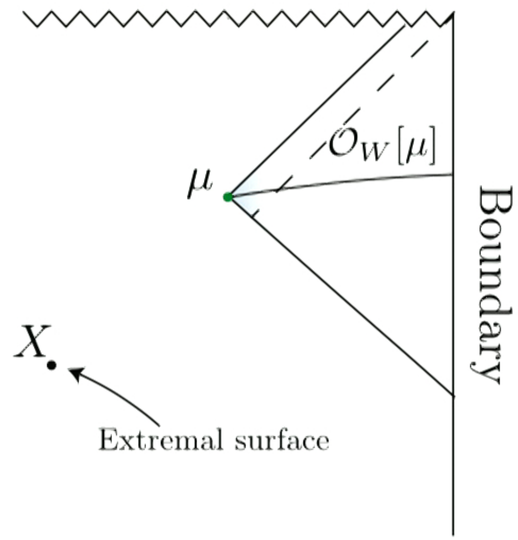
μ : Quantum marginally trapped surface $\Theta_k = 0$

Generalized entropy

Quantum dynamical horizons ($\Theta_k = 0$) satisfy a generalized second law
[Bousso, Engelhardt]



Review of Engelhardt-Wall construction



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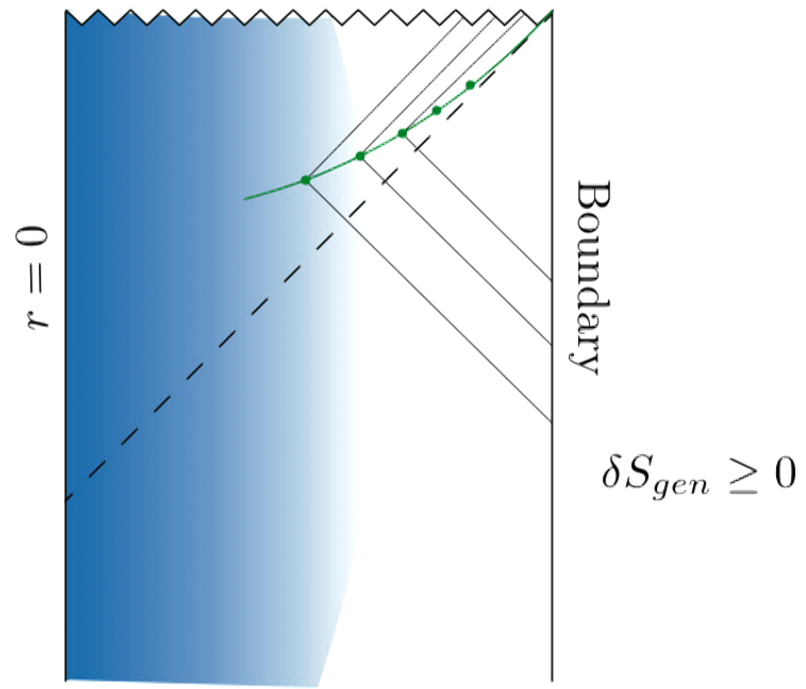
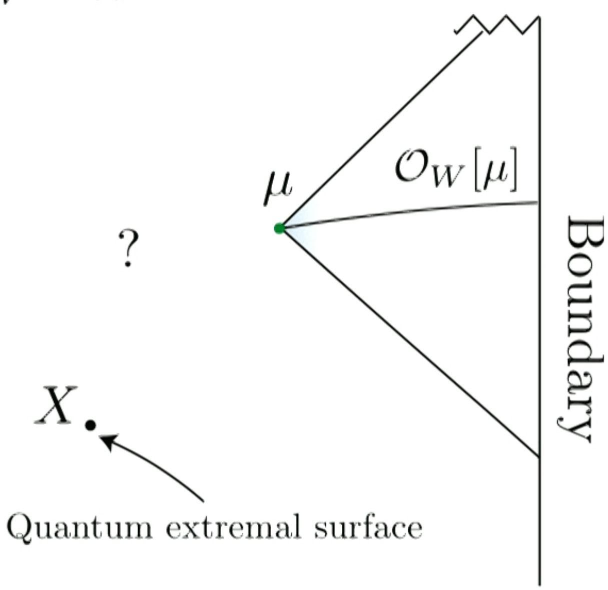


Quantum coarse-graining?

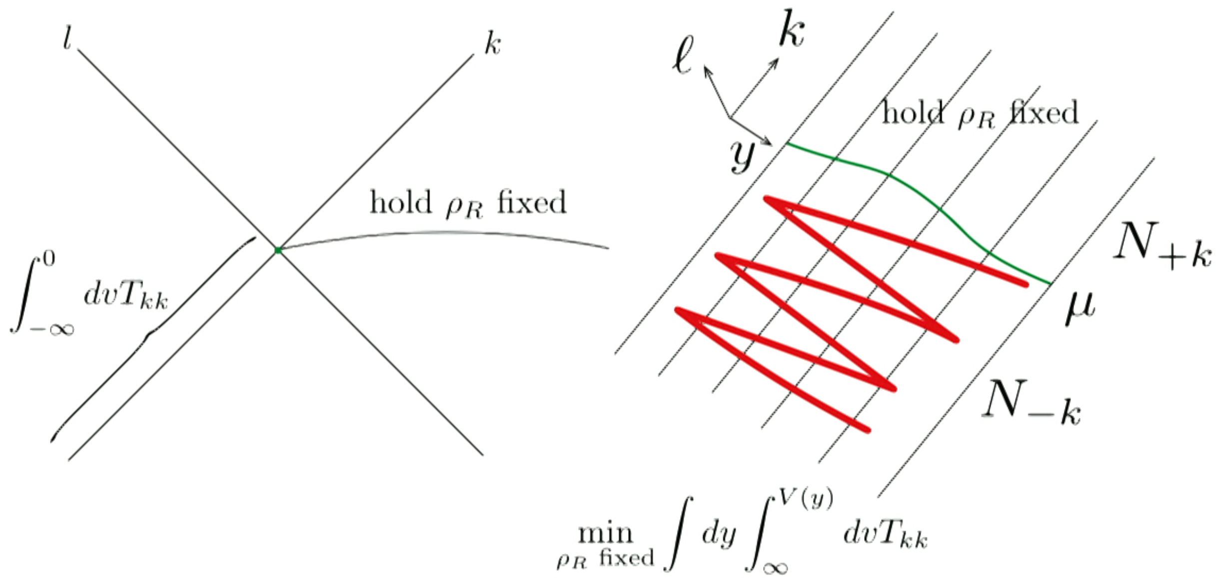
Quantum-corrected RT formula[FLM, Engelhardt-Wall]:
$S_{\mathrm{CFT}} = S_{\mathrm{gen}}[X]$ for X quantum extremal ($\Theta_k = \Theta_\ell = 0$)

$$\max_{\mathcal{O}_W \text{ fixed}} S_{\mathrm{gen}}[X] \stackrel{?}{=} S_{\mathrm{gen}}[\mu]$$



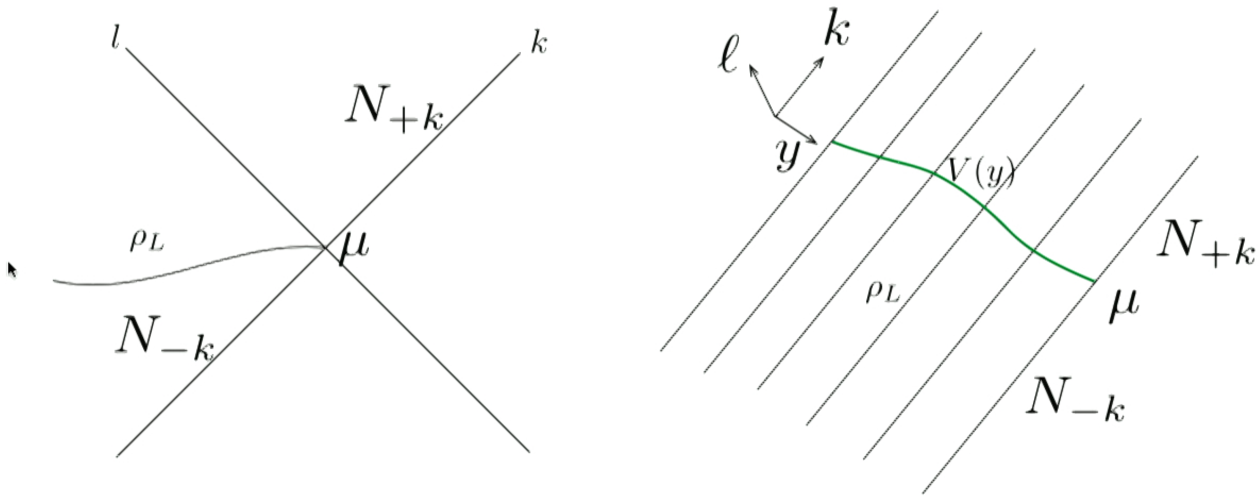
Energy-minimizing states

Aron Wall's thought experiment [1701.03196]



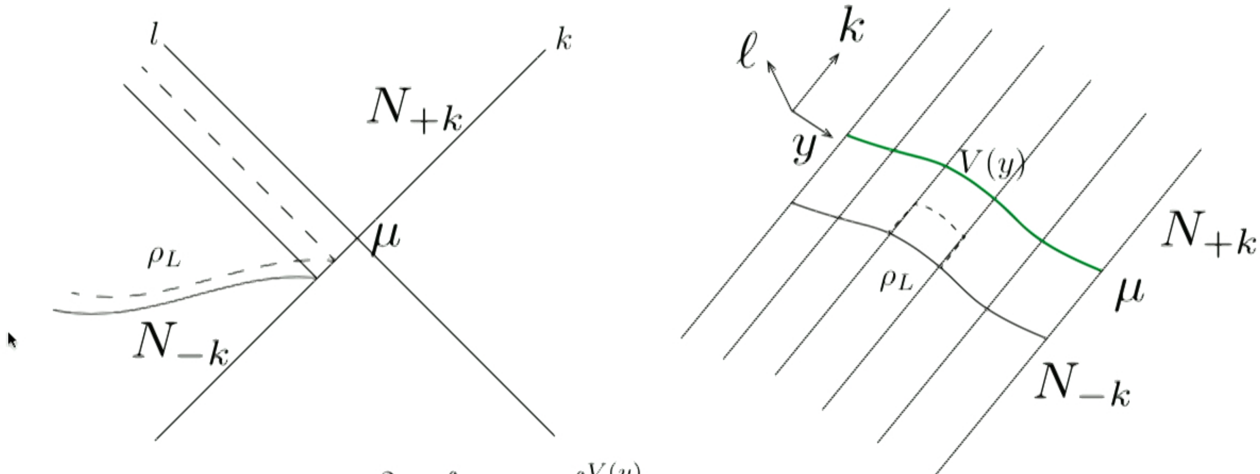
Relative entropy in QFT

$S_{\mathrm{rel}}(\rho_L|\sigma_L) = tr_L(\rho_L \log \rho_L - \rho_L \log \sigma_L)$ [Casini]



$$S_{\mathrm{rel}}(\rho_L|\sigma_L) = \underbrace{\frac{2\pi}{\hbar}\int dy \int_{-\infty}^{V(y)} dv (v - V(y)) T_{kk}(v,y)}_{\text{Modular Hamiltonian: Boost energy}} - \Delta S_L \geq 0$$

Relative entropy in QFT

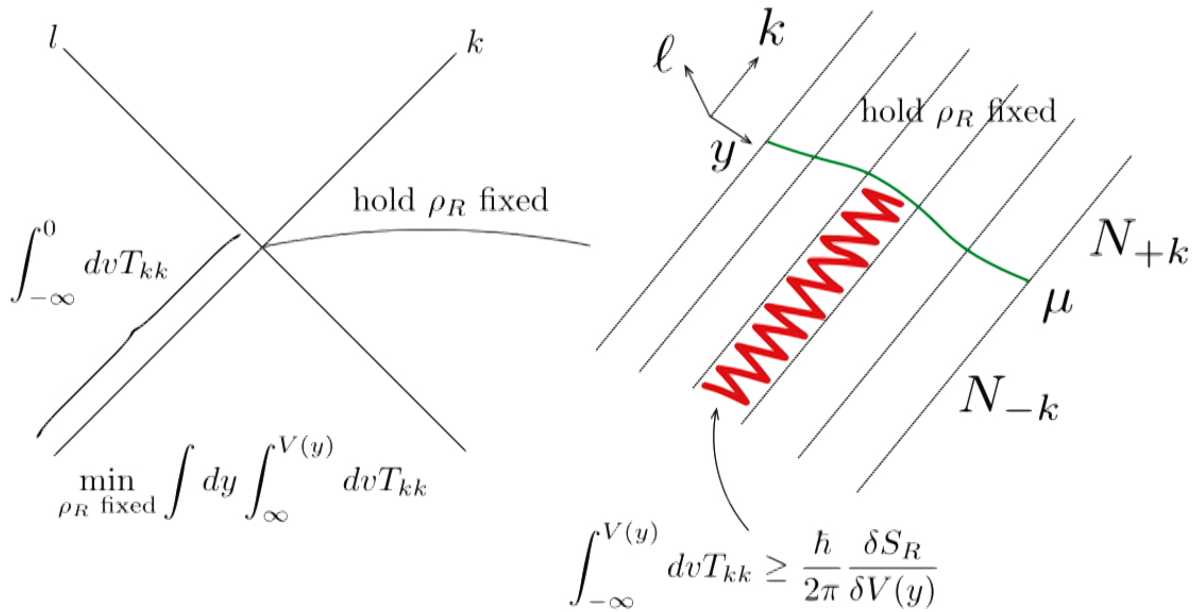


$$S_{\rm rel}(\rho_L|\sigma_L) = \frac{2\pi}{\hbar}\int d^{d-2}y \int_{-\infty}^{V(y)} dv(v - V(y))T_{kk}(v,y) - \Delta S(\rho_L) \geq 0$$

$$\frac{\delta}{\delta V(y)} S_{\rm rel}(\rho_L|\sigma_L) = \int_{\infty}^{V(y)} T_{kk}(v,y) - \frac{\delta S_L}{\delta V(y)} \geq 0$$

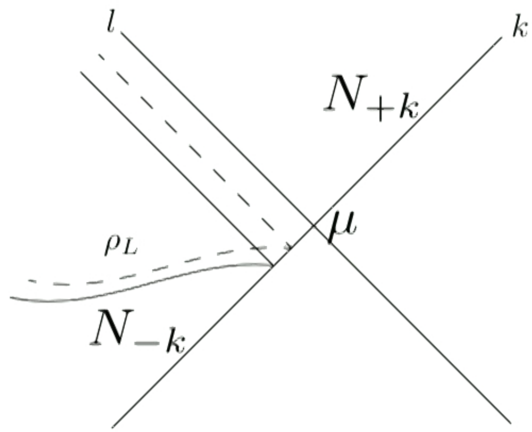
$$\frac{\delta^2}{\delta V(y)\delta V(y')} S_{\rm rel}(\rho_L|\sigma_L) \geq 0 \text{ (QNEC)}$$

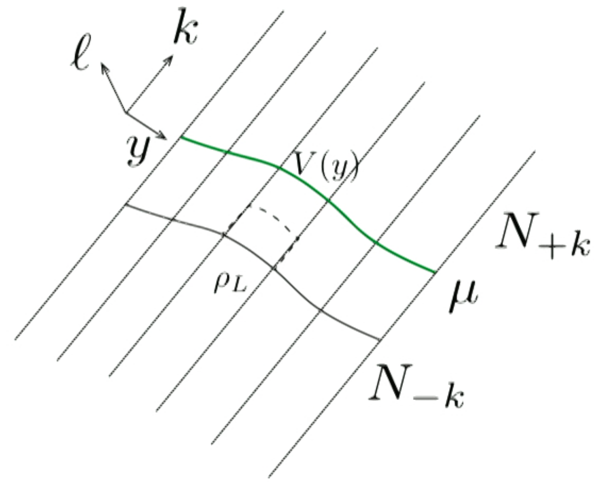
Energy-minimizing states



$$\int_{-\infty}^{V(y)} T_{kk}(v,y) = \frac{\hbar}{2\pi} \frac{\delta S_R}{\delta V(y)} \quad \to \quad \frac{\delta}{\delta V(y)} S_{\text{rel}}(\rho_L | \sigma_L) \Big|_{\mu} = 0$$

Relative entropy in QFT





$$S_{\rm rel}(\rho_L|\sigma_L) = \frac{2\pi}{\hbar}\int d^{d-2}y \int_{-\infty}^{V(y)} dv(v - V(y))T_{kk}(v,y) - \Delta S(\rho_L) \geq 0$$

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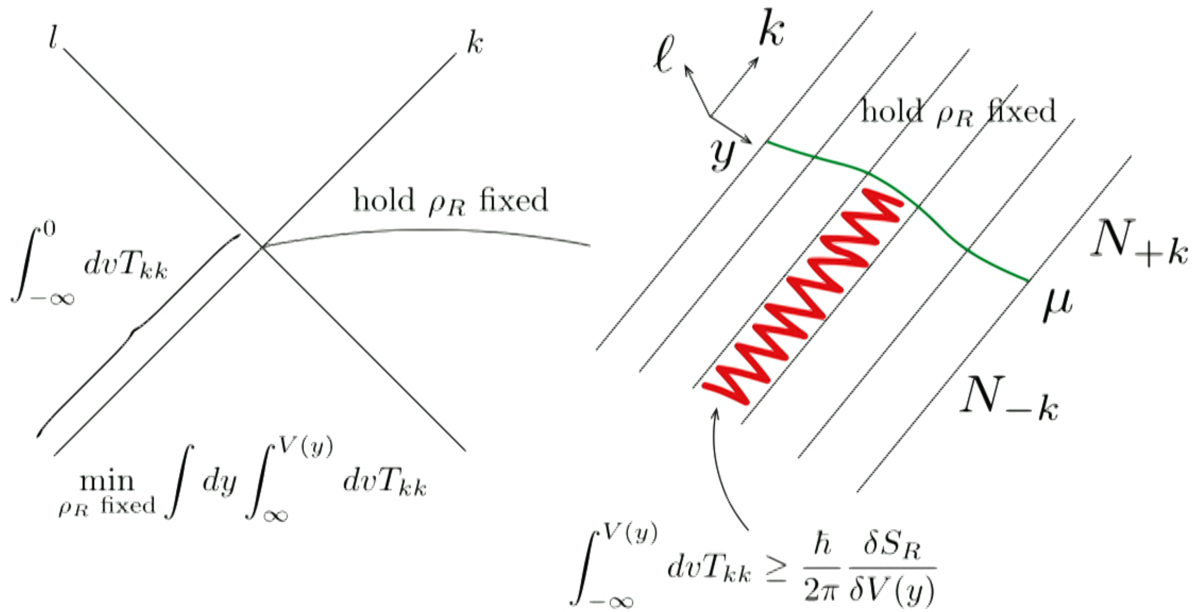
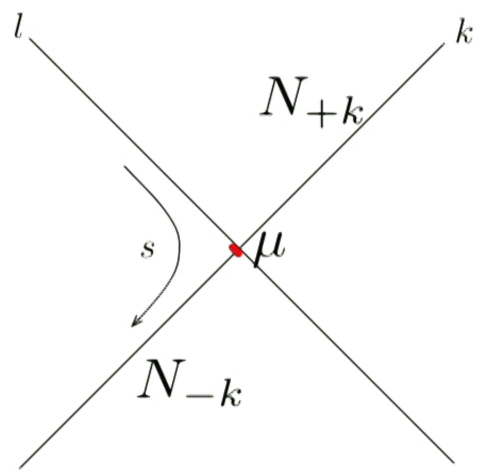
Energy-minimizing states

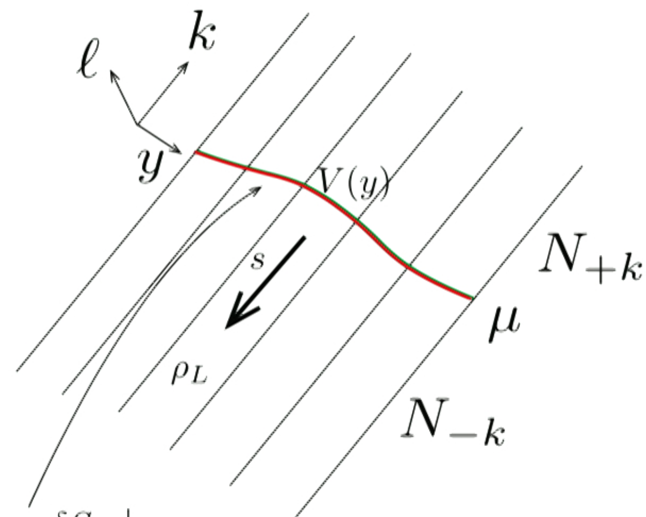


$$\int_{-\infty}^{V(y)} T_{kk}(v,y) = \frac{\hbar}{2\pi} \frac{\delta S_R}{\delta V(y)} \quad \rightarrow \quad \frac{\delta}{\delta V(y)} S_{\rm rel}(\rho_L|\sigma_L)\Big|_{\mu} = 0$$

Energy-minimizing states: Connes co-cycle flow

Faulkner-Ceyhan [1812.04683]



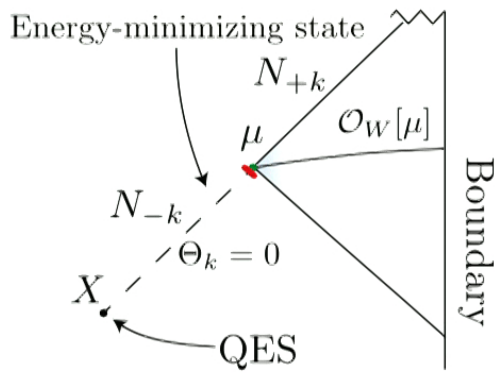


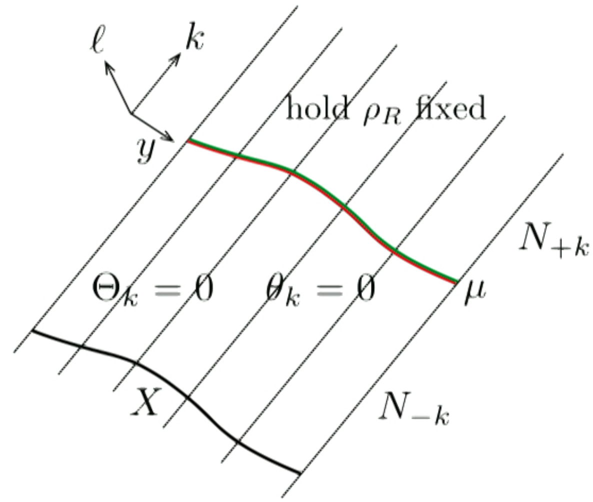
$$\langle T_{kk}\rangle_{\psi_s} = (1-e^{-s})\frac{\hbar}{2\pi}\,\frac{\delta S}{\delta V(y)}\bigg|_{\mu}\,\delta(v - V(y))$$

$$|\psi_s\rangle = (\rho_L^{\sigma})^{-is}(\rho_L^{\psi})^{is}\,|\psi\rangle \quad \longrightarrow \quad N_{-k}: \begin{cases} \langle T_{kk}(v,y)\rangle_{\psi_s} = e^{-2s}\langle T_{kk}(ve^{-s},y)\rangle_{\psi} \\ \partial_k S_R(\psi_s) = e^{-s}\partial_k S_R(\psi) \end{cases}$$

Put them together: quantum coarse-graining

$$\max_{\mathcal{O}_W \text{ fixed}} S_{\text{gen}}[X] \stackrel{?}{=} S_{\text{gen}}[\mu]$$





$$\Theta_k[\mu] = 0$$

$$\Downarrow$$

$$\theta_k[\mu] = -4G\hbar\partial_k S[\mu] \xrightarrow[\partial_k\theta_k \supset 8\pi G T_{kk}]{T_{kk} = \left(\frac{\hbar}{2\pi}\partial_k S[\mu]\right)\delta(v)} N_{-k} : \begin{cases} \theta_k = 0 \\ \partial_k S = 0 \end{cases}$$

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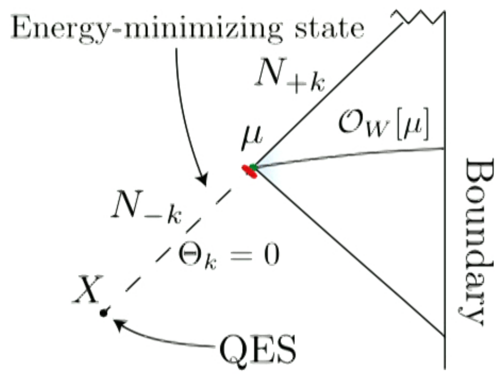

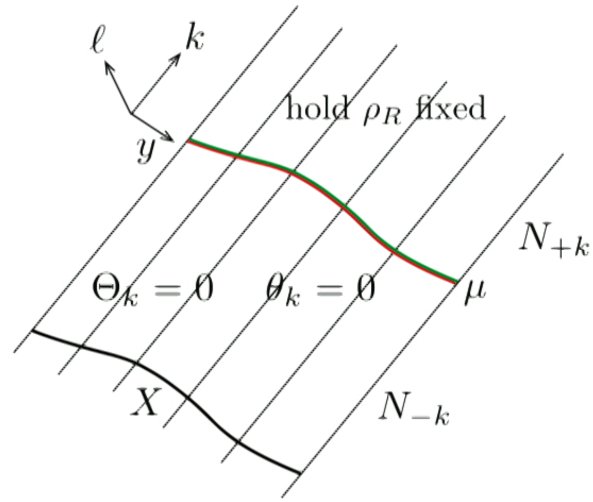

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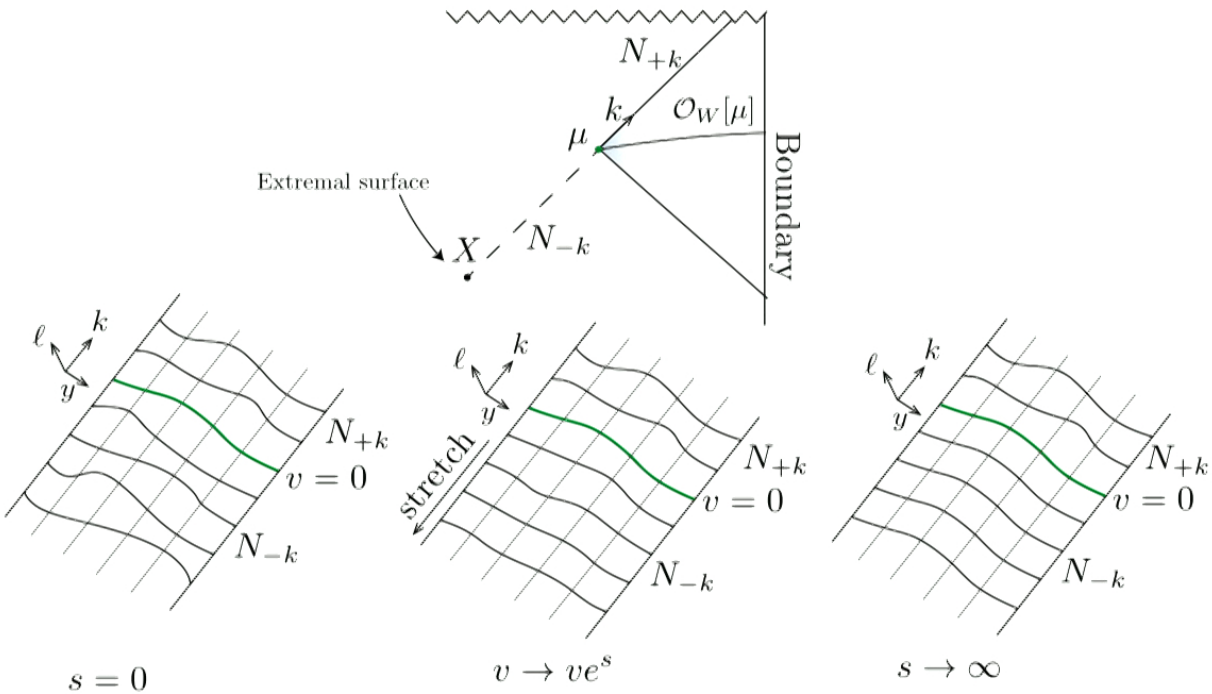
$$S_{gen}[X] = S_{gen}[\mu] \implies S_{coarse} = S_{gen}[\mu]$$

Conjecture for the non-perturbative case



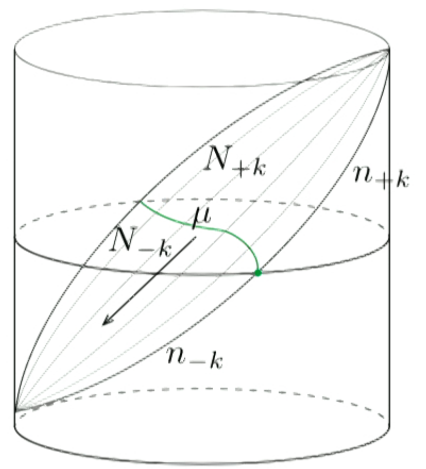
$$\theta_k'(v, y) = e^{-s}\theta_k(ve^{-s}, y) \qquad \theta_k'(v, y) = 0$$

$$\sigma_k'(v, y) = e^{-s}\sigma_k(ve^{-s}, y) \qquad \sigma_k'(v, y) = 0$$

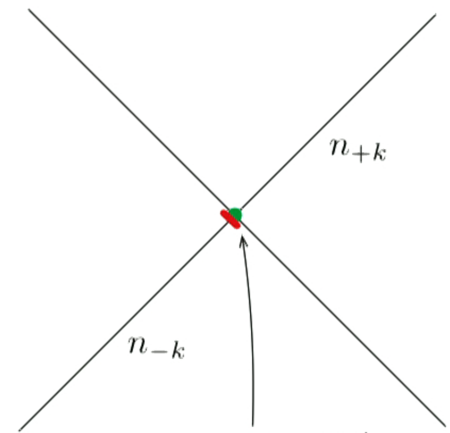
Evidence: Boundary co-cycle flow

(w.i.p. with Raphael, Ven, and Pratik Rath)

Bulk (z emergent direction)



Boundary ($z = 0$)



$$v \to ve^{s}$$

$$N_{-k}\begin{cases} \theta'_k(v,y) = e^{-s}\theta_k(ve^{-s},y) \\ \sigma'_k(v,y) = e^{-s}\sigma_k(ve^{-s},y) \end{cases}$$

$$T_{kk}|_{\text{boundary}} = (1-e^{-s})\frac{\hbar}{2\pi}\left.\frac{\delta S}{\delta V}\right|_{\text{boundary}}\delta(v)$$

$$\uparrow$$

$$C_{kzkz}|_{\text{bulk}} \sim T_{kk}|_{\text{boundary}}$$

$$\Delta\theta_k|_{\mu} = 0$$

$$\Delta\sigma_k|_{\mu} = (1-s^{-s})\sigma_k|_{\mu}$$

$$\xrightarrow{\partial_k(\sigma_k)_{zz} \supset C_{kzkz}}$$

$$(\sigma_k)_{zz}|_{\text{bulk}} \sim \left.\frac{\delta S}{\delta V}\right|_{\text{boundary}}$$

Summary and future

Summary:

- Derived the generalized entropy of black holes as a coarse-grained entropy
- Explained the generalized second law of black holes from a microscopic viewpoint
- proposed a non-perturbative construction of semiclassical states (possibly) dual to boundary Connes co-cycle flow

Future:

- Understand microscopically the origin of the causal horizon generalized second law
- Non-perturbative JLMS