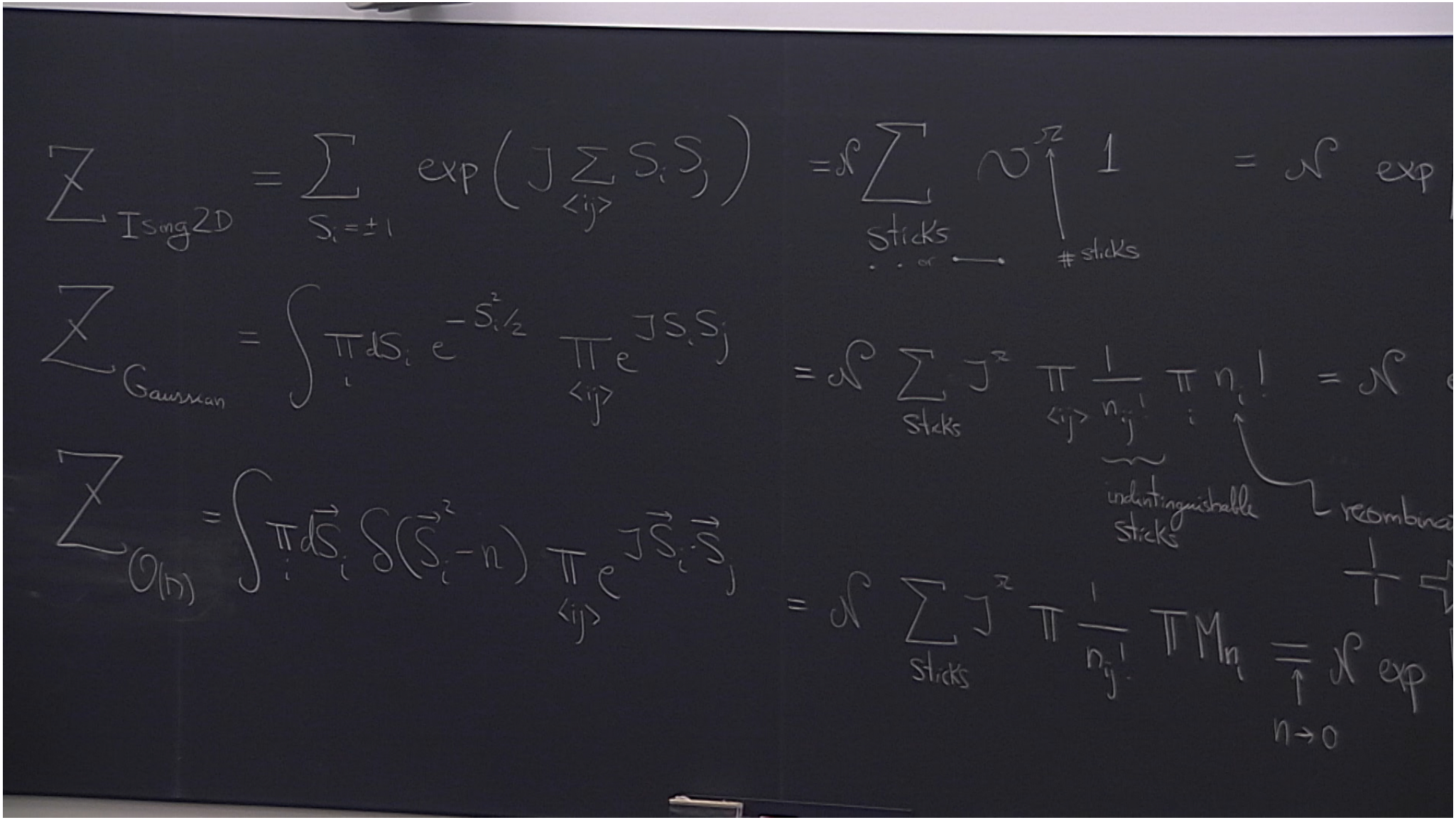
Title: PSI 2019/2020 - Statistical Mechanics (Vieira) - Lecture 14

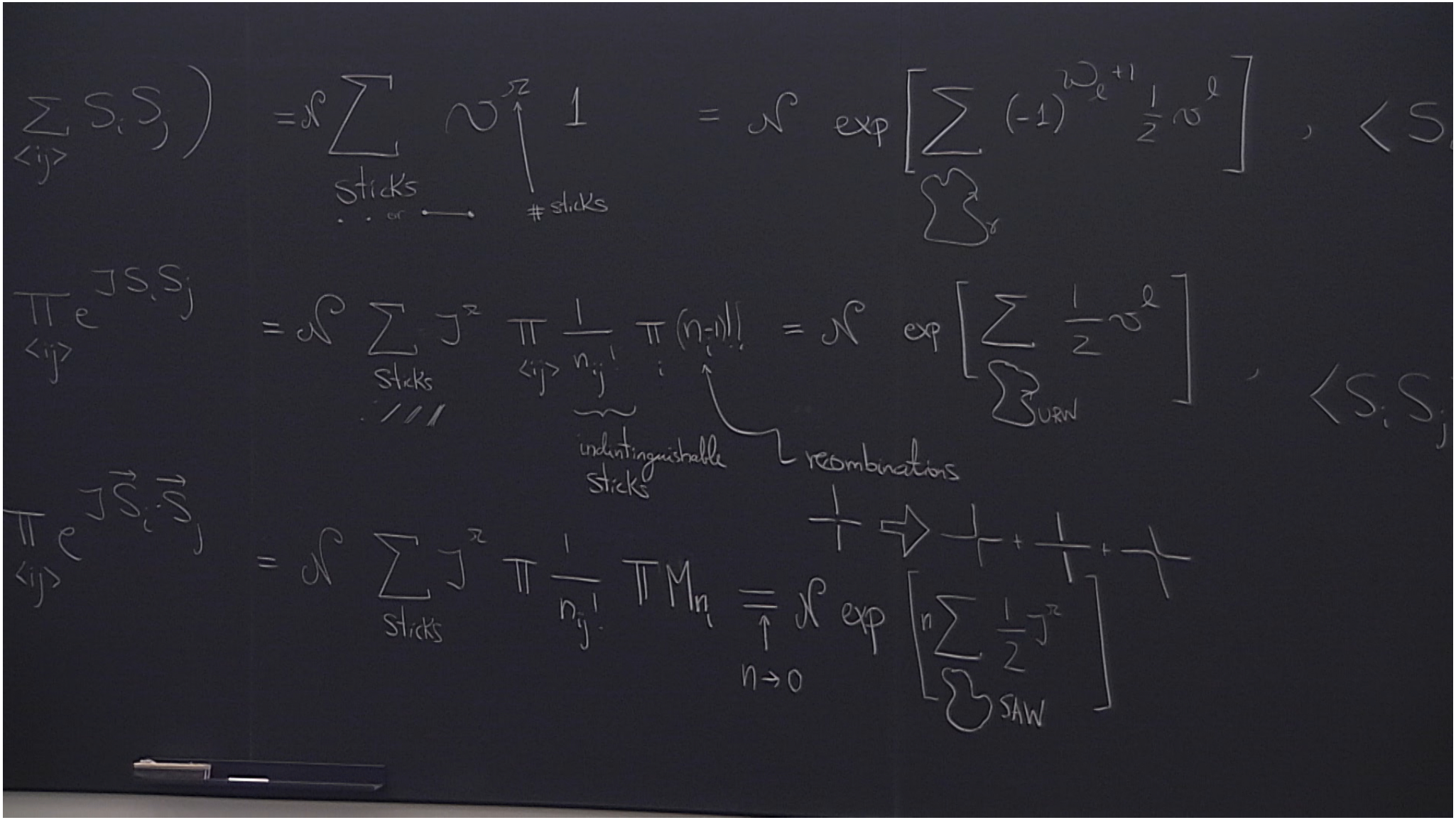
Speakers: Pedro Vieira

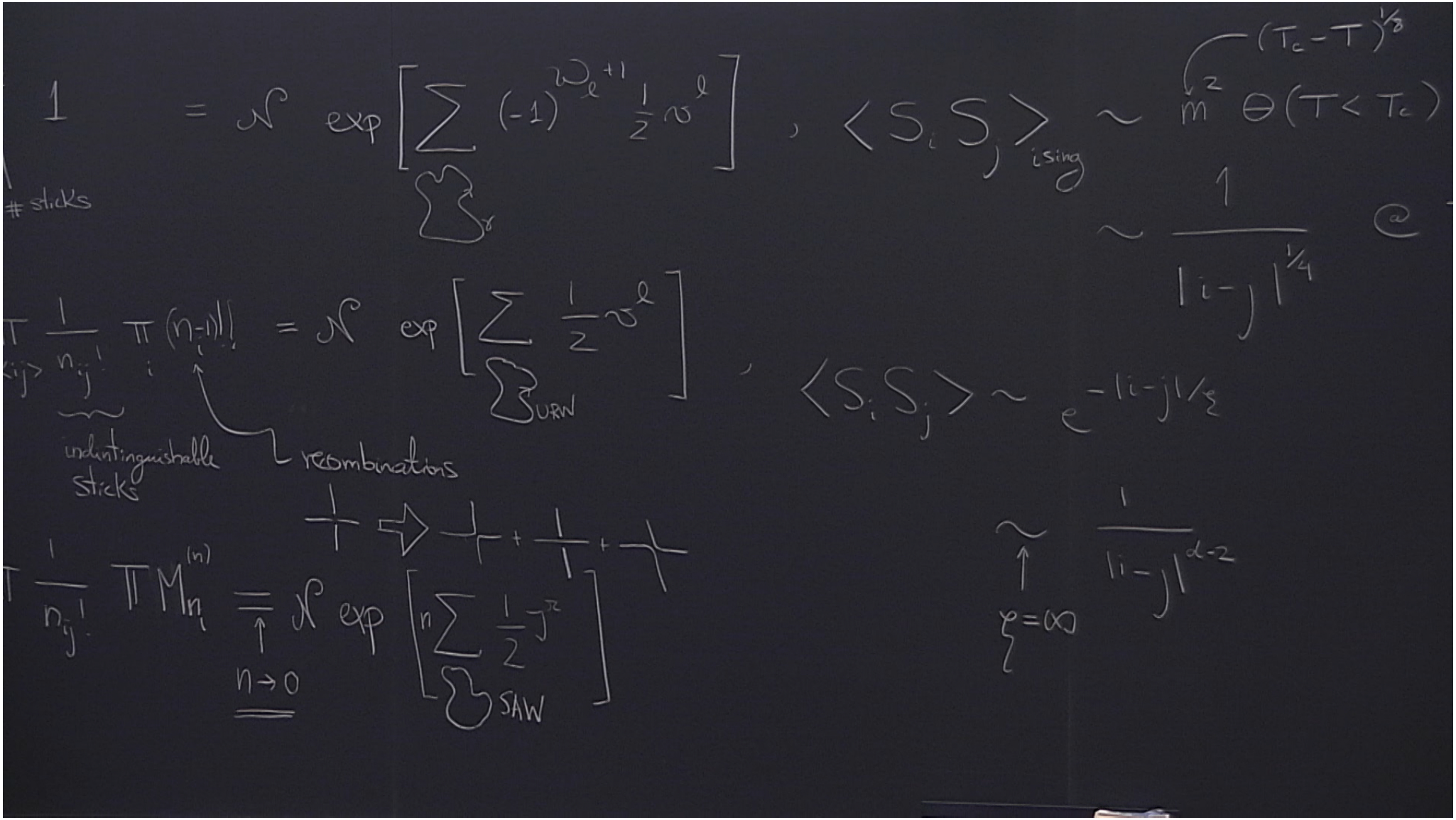
Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

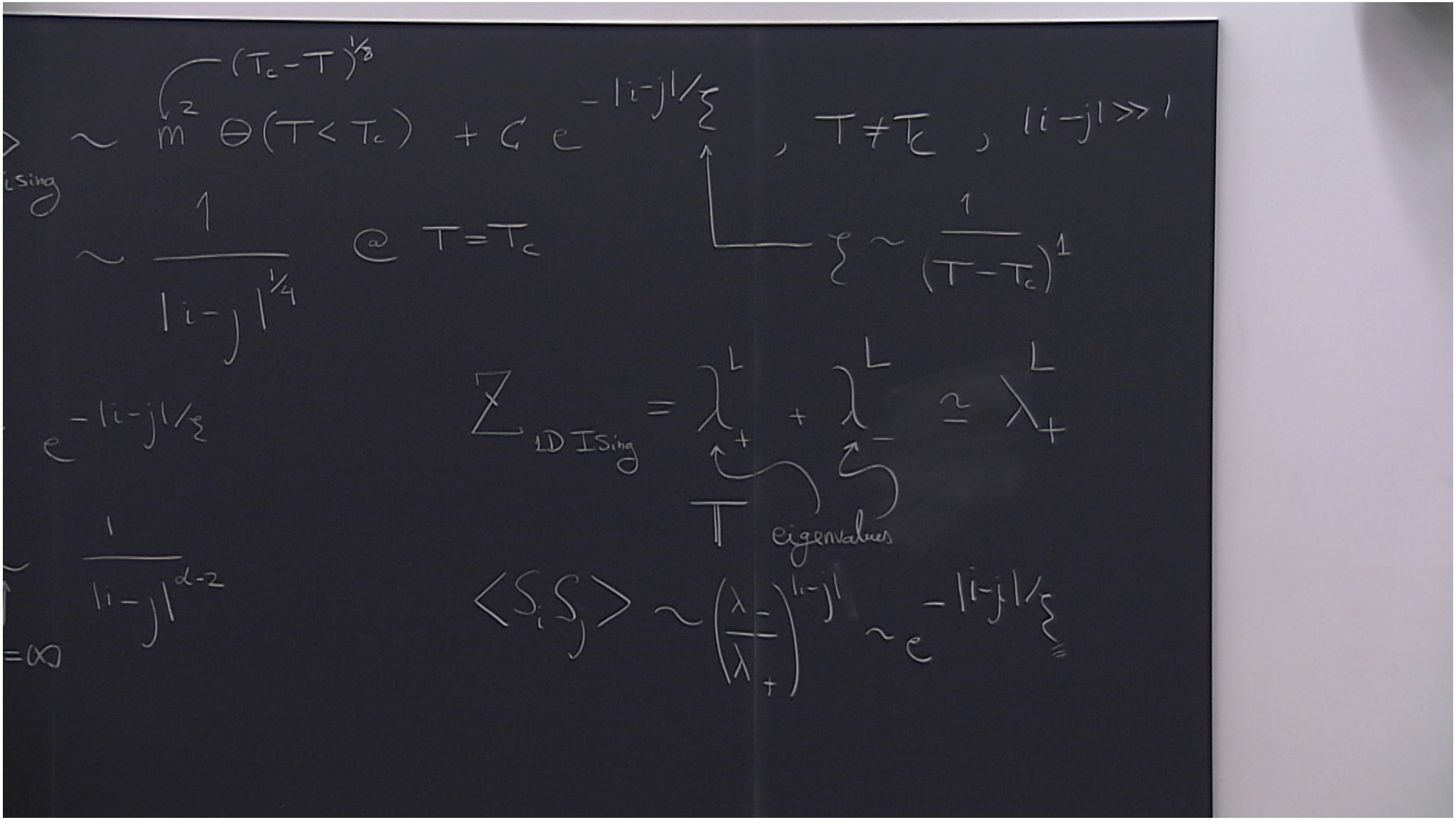
Date: October 25, 2019 - 10:45 AM

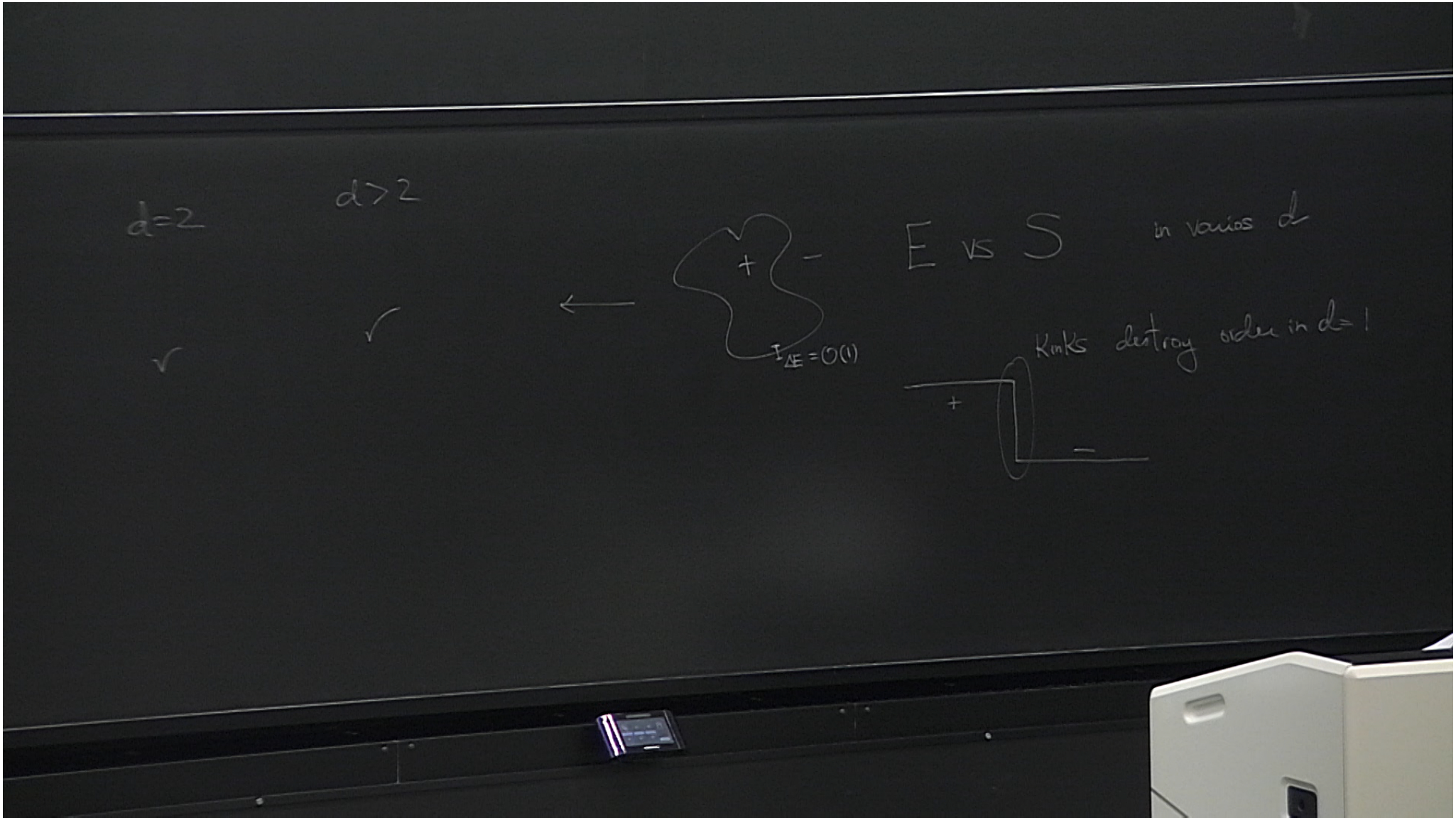
URL: http://pirsa.org/19100038

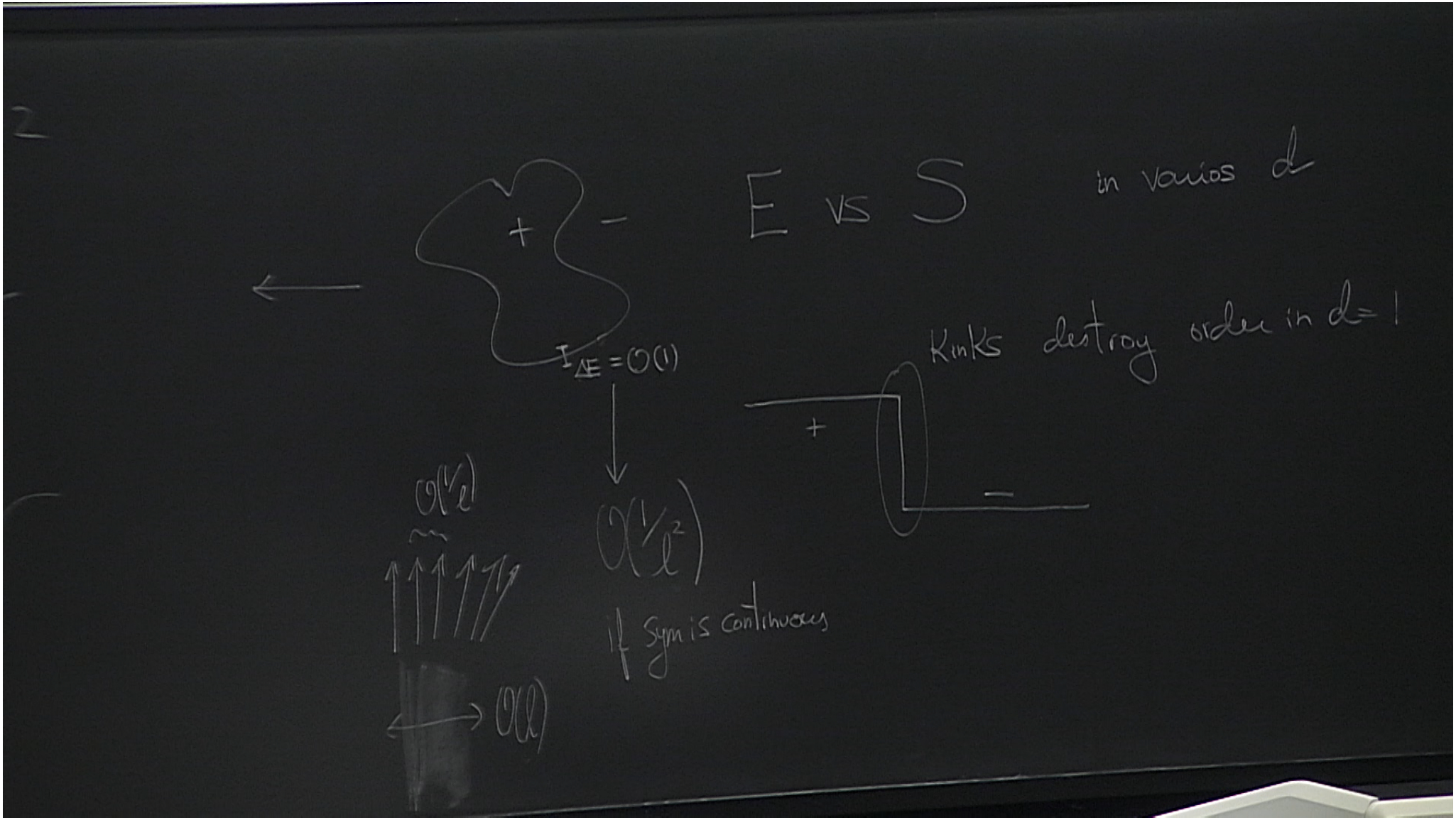

$Z_{Ising2D} = \sum_{S_i=\pm 1} \exp\left(J \sum_{\langle ij \rangle} S_i S_j\right) = \sum_{sticks} \cdots$
sticks
or
# sticks
1
$Z_{Gaussian} = \int \prod_i dS_i \, e^{-S_i^2/2} \prod_{\langle ij \rangle} e^{J S_i S_j}$
Sticks
indistinguishable sticks
recombina
$Z_{O(n)} = \int \prod_i d\vec{S}_i \, \delta(\vec{S}_i^2 - n) \prod_{\langle ij \rangle} e^{J \vec{S}_i \cdot \vec{S}_j}$
$n \to 0$

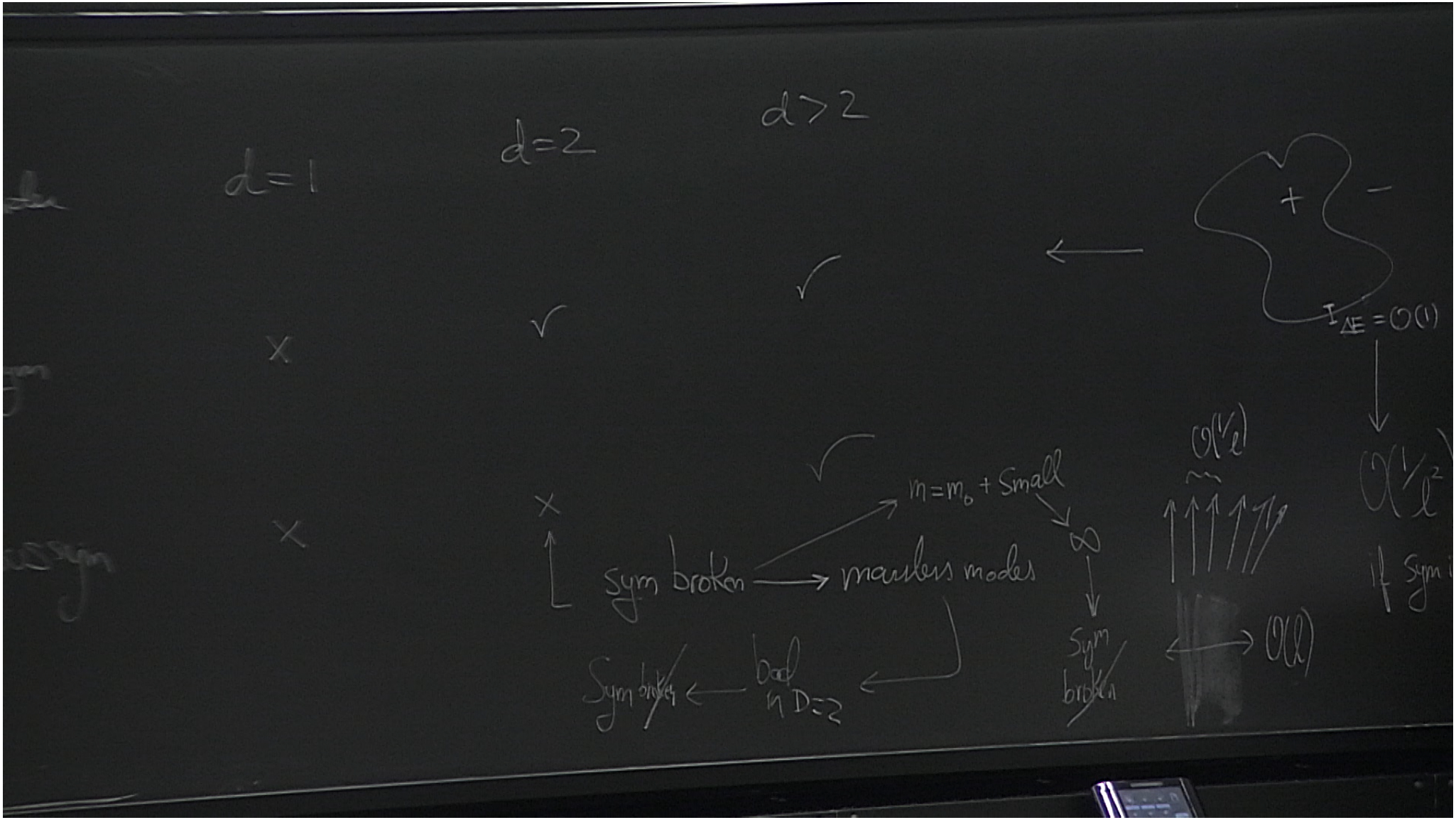

sticks
or
# sticks
indistinguishable sticks
recombinations
URW
SAW
n→0

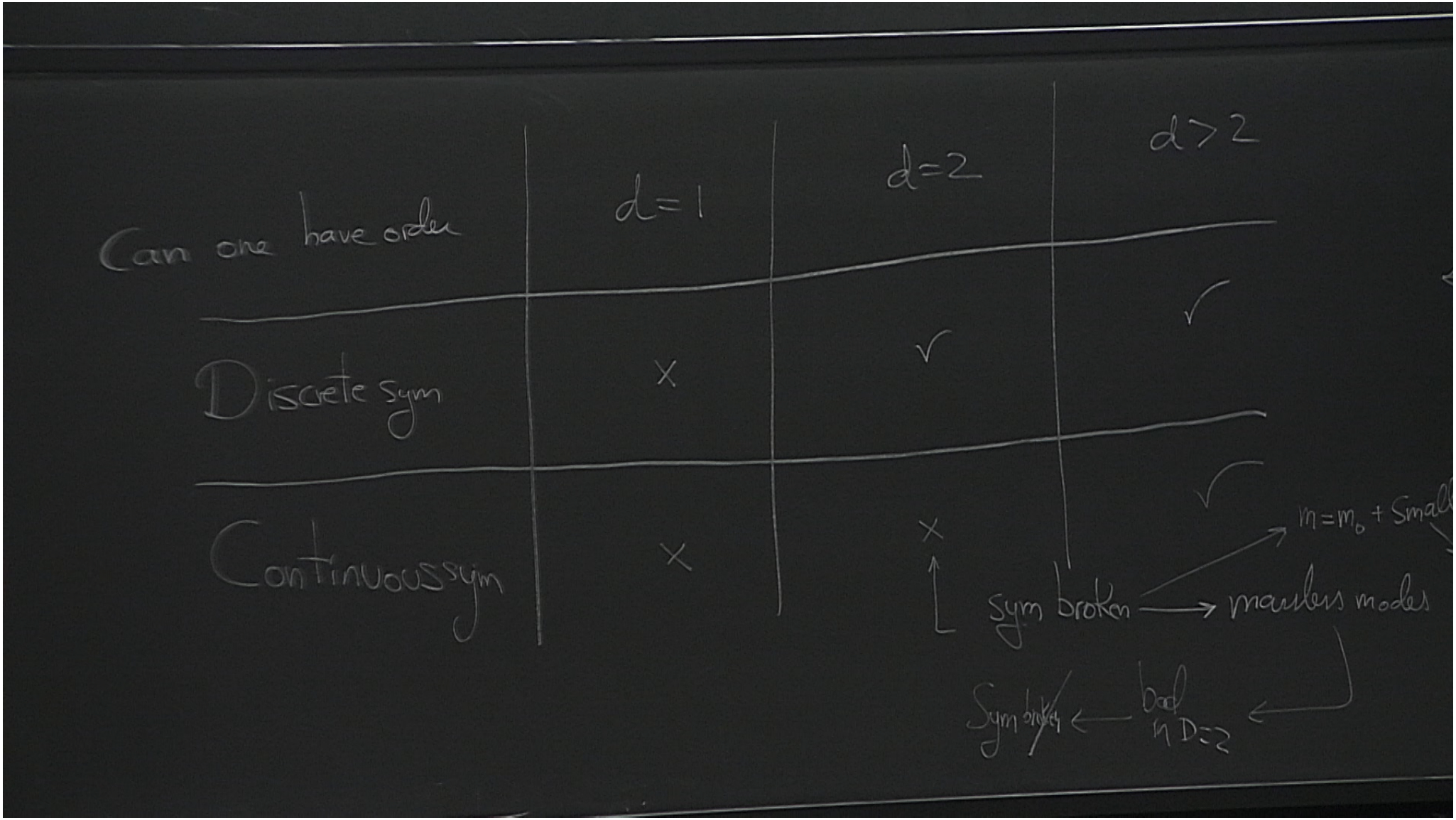

1 = N exp [ Σ_γ (-1)^{ω_ℓ+1} 1/2 v^ℓ ]
# sticks
< S_i S_j >_ising ~ m^2 Θ(T < T_c)
(T_c - T)^{1/8}
~ 1 / |i-j|^{1/4}
@
Π 1/n_ij! Π (n_i-1)!
= N exp [ Σ_URW 1/2 v^ℓ ]
< S_i S_j > ~ e^{-|i-j|/ξ}
indistinguishable sticks
recombinations
~ 1 / |i-j|^{d-2}
ξ = ∞
Π M_{n_i}^{(n)}
= N exp [ n Σ_SAW 1/2 J^2 ]
n → 0

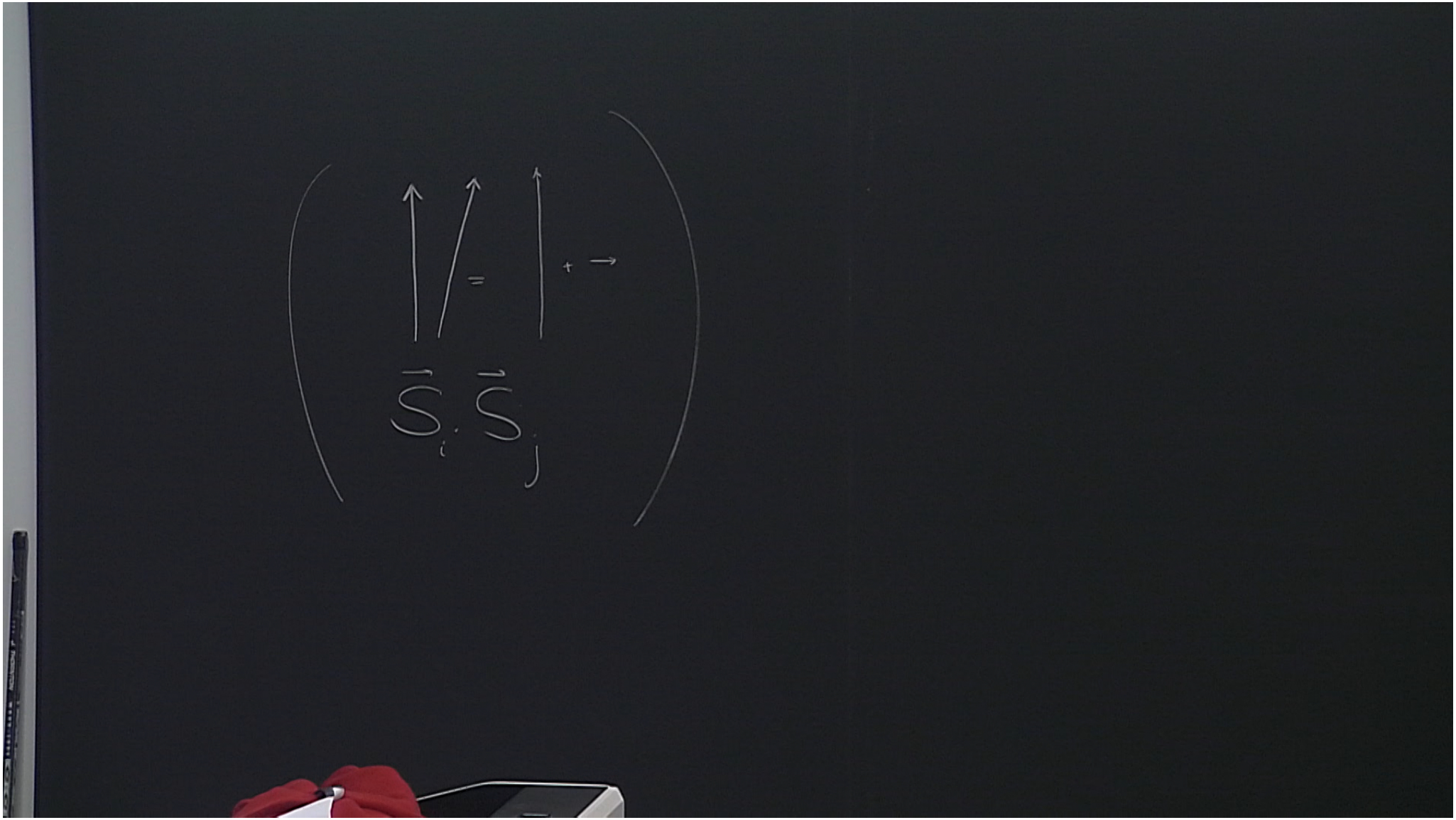

$\sim m^2 \sim (T_c - T)^{1/8}$
$\Theta(T < T_c)$
$+ C e^{-|i-j|/\xi}$, $T \neq T_c$, $|i-j| \gg 1$
$\xi \sim \frac{1}{(T-T_c)^1}$
$\sim \frac{1}{|i-j|^{1/4}}$ @ $T = T_c$
$e^{-|i-j|/\xi}$
$\frac{1}{|i-j|^{d-2}}$
$Z_{1D\ Ising} = \lambda_+^L + \lambda_-^L \simeq \lambda_+^L$
eigenvalues
$\langle S_i S_j \rangle \sim \left(\frac{\lambda_-}{\lambda_+}\right)^{|i-j|} \sim e^{-|i-j|/\xi}$

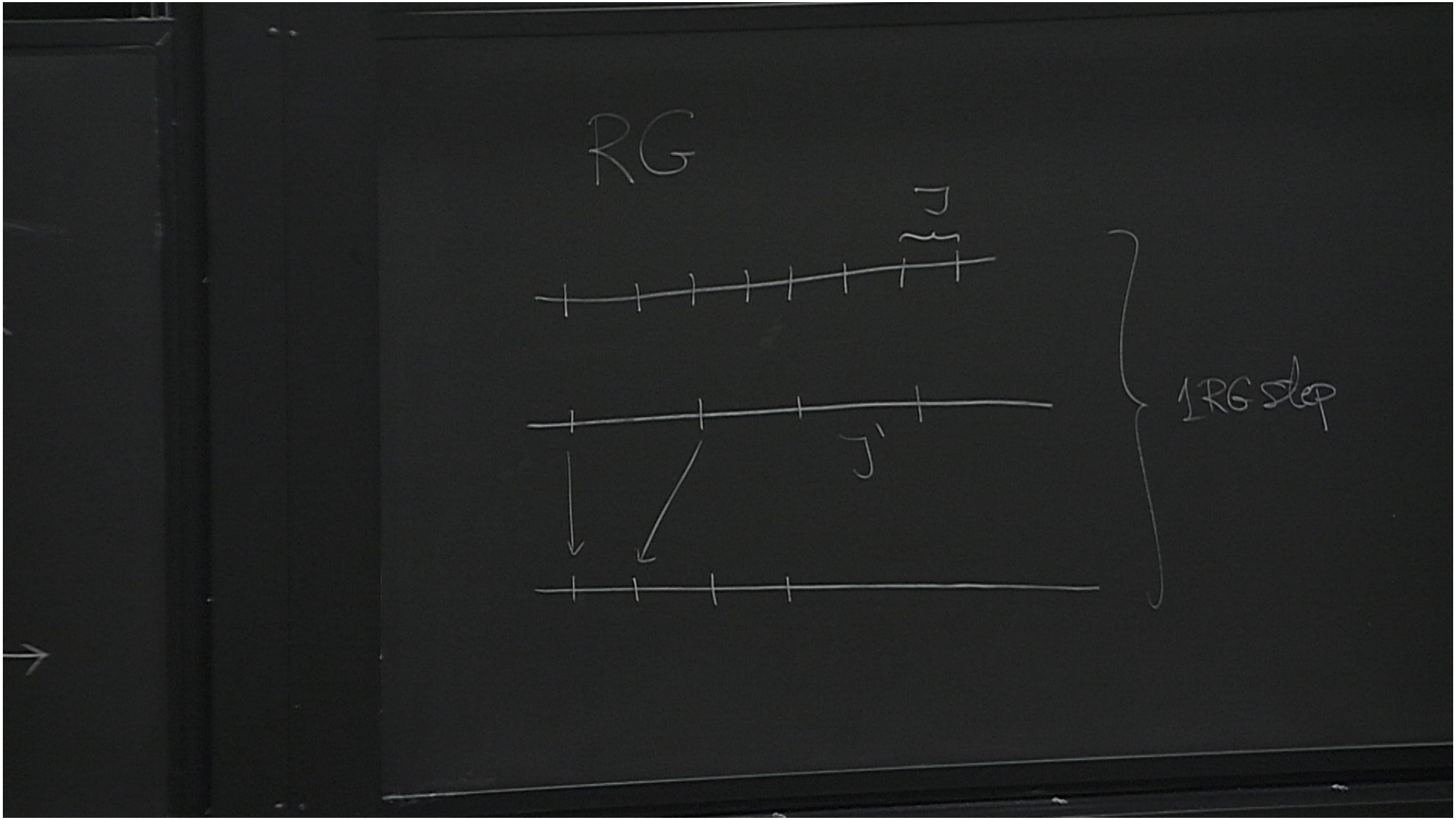

d=2
d>2
E vs S
in various d
Kinks destroy order in d=1

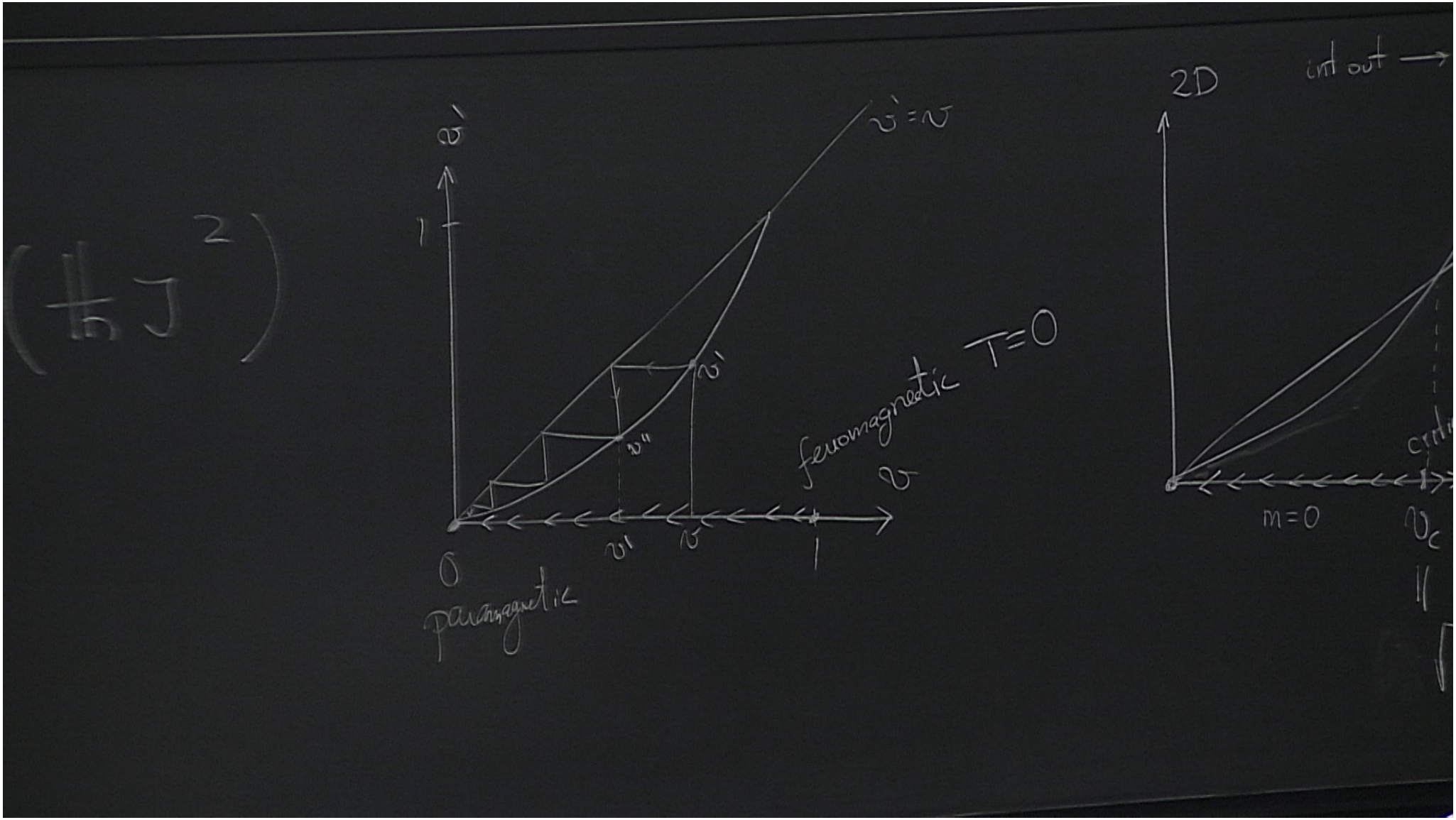

E vs S
in various d
+
−
$I_{\Delta E} = O(1)$
Kinks destroy order in d = 1
+
−
$O(1/\ell)$
$O(1/\ell^2)$
if sym is continuous
$O(\ell)$

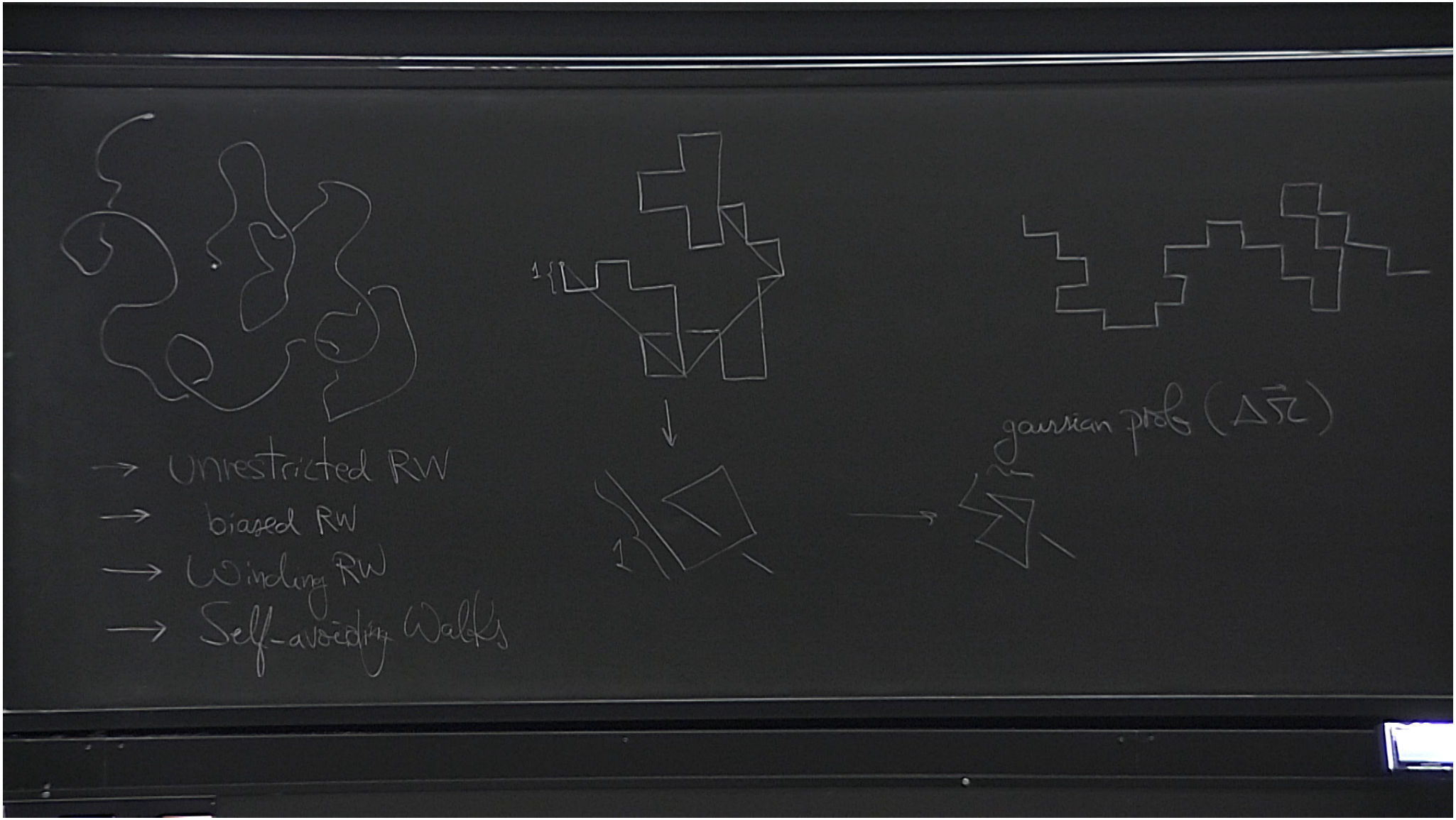

d=1
d=2
d>2
+
−
$I_{\Delta E} = O(1)$
$m = m_0 + \text{small}$
sym broken
massless modes
$\infty$
$O(1/\ell)$
$O(1/\ell^2)$
Sym broken
bad in D=2
Sym broken
O(2)

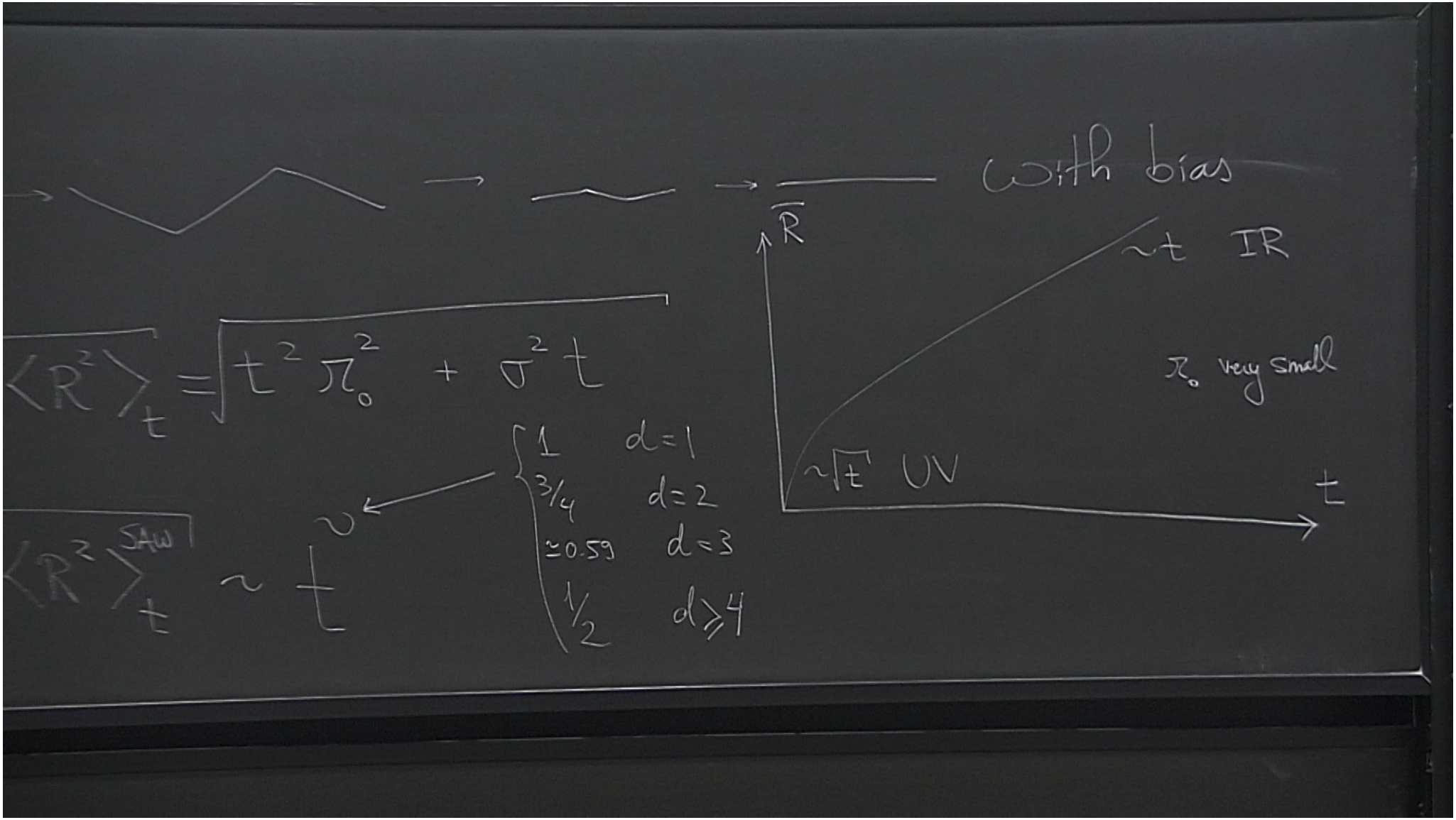

Can one have order
d=1
d=2
d>2
Discrete sym
×
✓
✓
Continuous sym
×
×
✓
sym broken
massless modes
bad in D=2

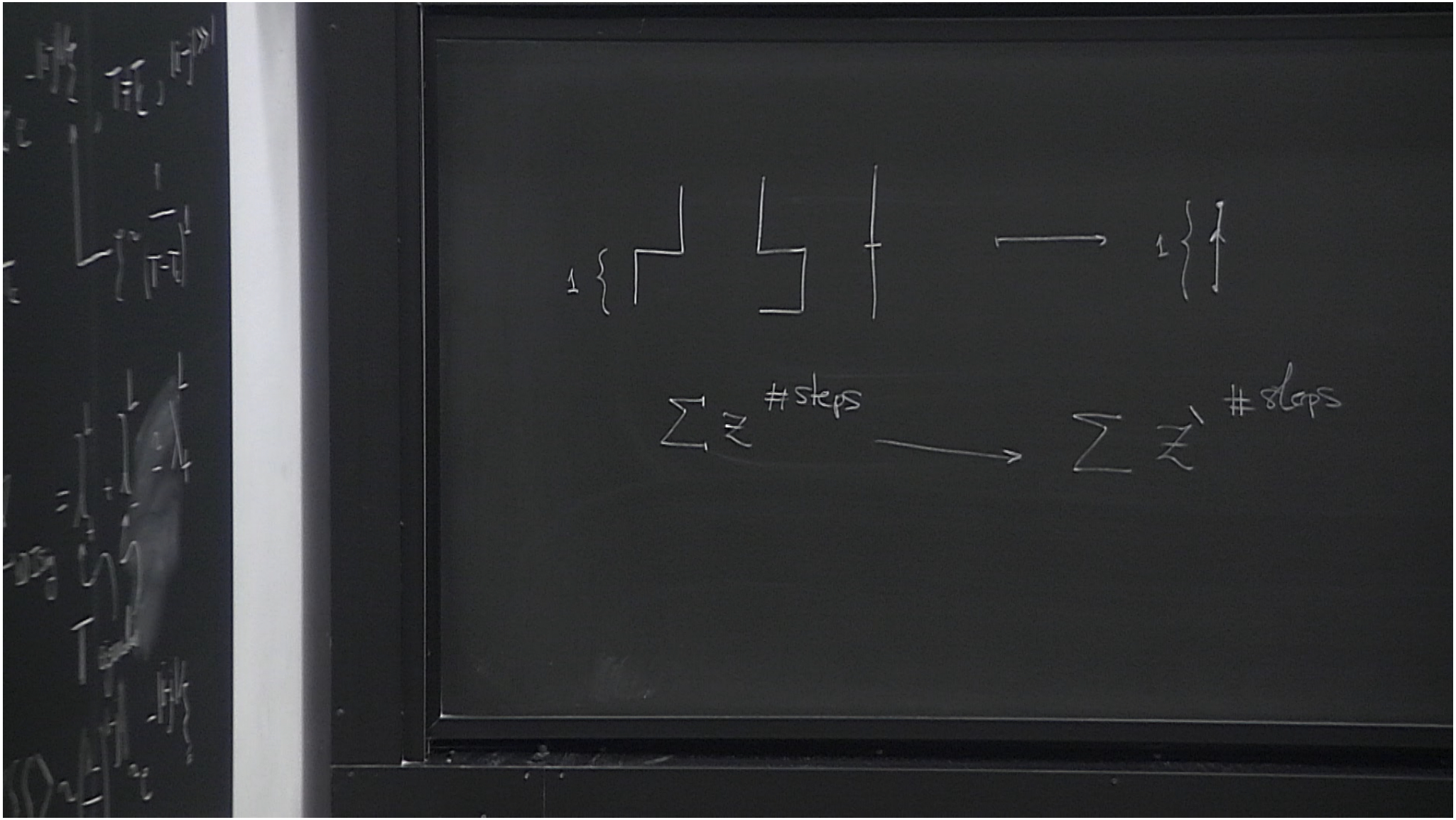

$\vec{S}_i \cdot \vec{S}_j$

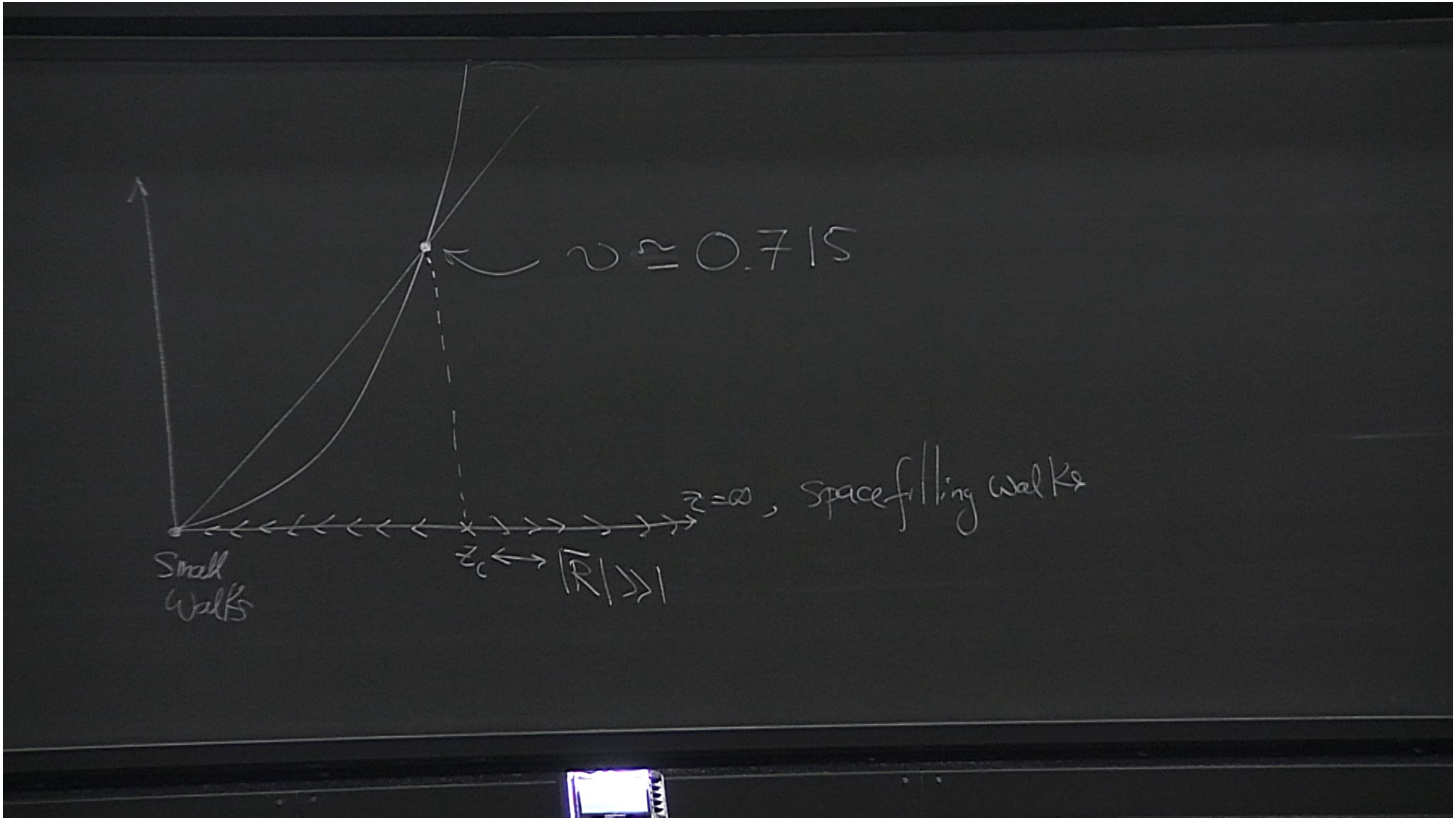

RG
J
J'
1 RG step

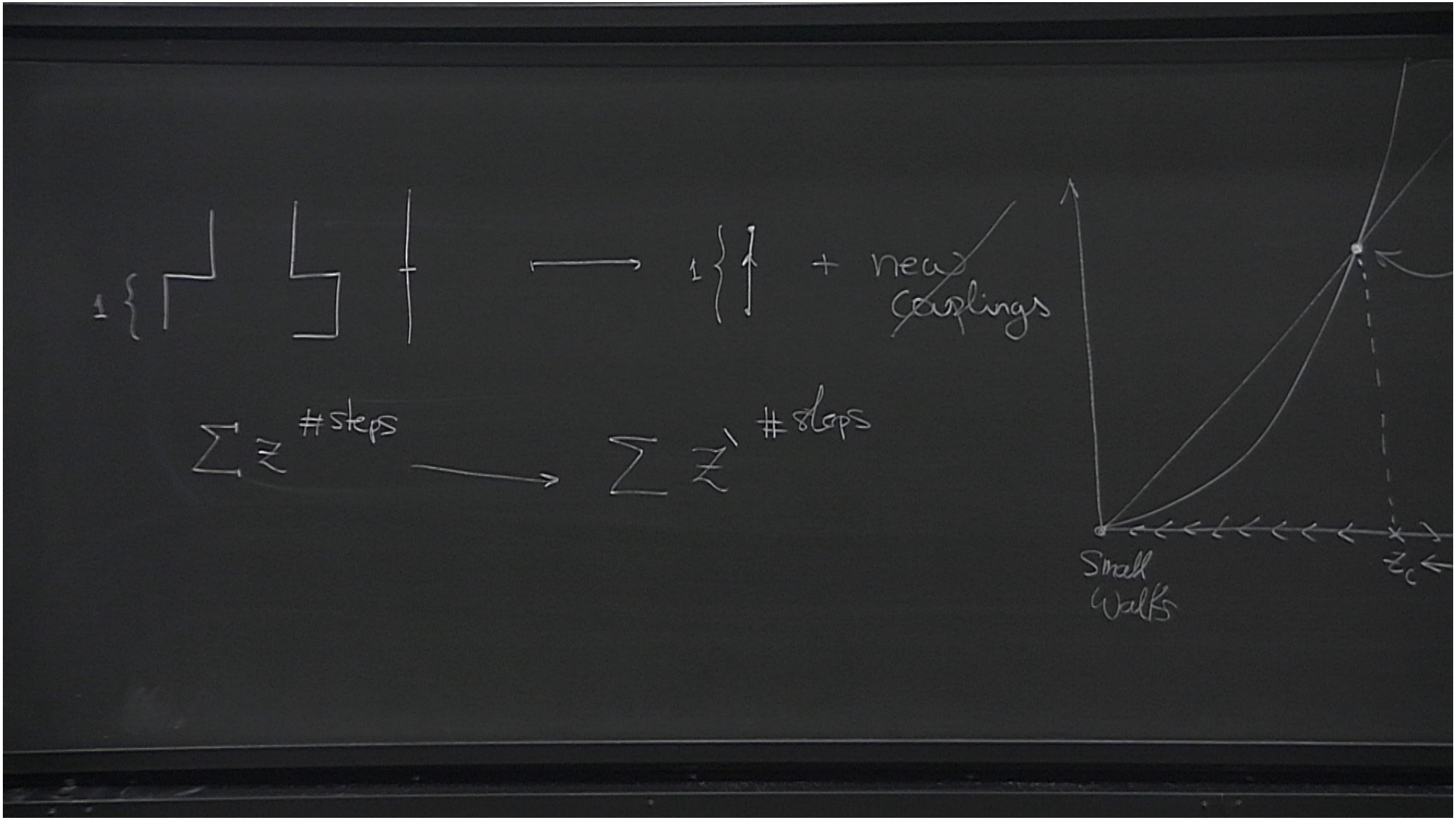

$(\frac{1}{h}J^2)$
$v'$
$v'=v$
1
$v'$
$v''$
ferromagnetic T=0
$v$
0
$v_1$
$v^*$
1
paramagnetic
2D
int out
m=0
$v_c$


→ Unrestricted RW
→ biased RW
→ Winding RW
→ Self-avoiding Walks
gaussian prob (Δr⃗)

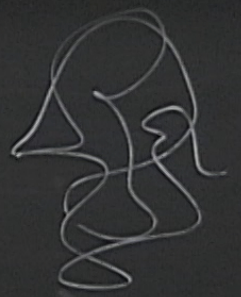

with bias
$\overline{R}$
$\sim t$ IR
$r_0$ very small
$\sim\sqrt{t}$ UV
$t$
$\overline{R^2}_t = \sqrt{t^2 r_0^2 + \sigma^2 t}$
$\overline{R^2}^{SAW}_t \sim t^{\nu}$
1 d=1
3/4 d=2
≃0.59 d=3
1/2 d≥4


1
1
$\sum z^{\#\text{steps}}$
$\sum z'^{\#\text{steps}}$


$\nu \simeq 0.715$
$z = \infty$, space filling walks
Small walks
$z_c \longleftrightarrow |R| \gg 1$

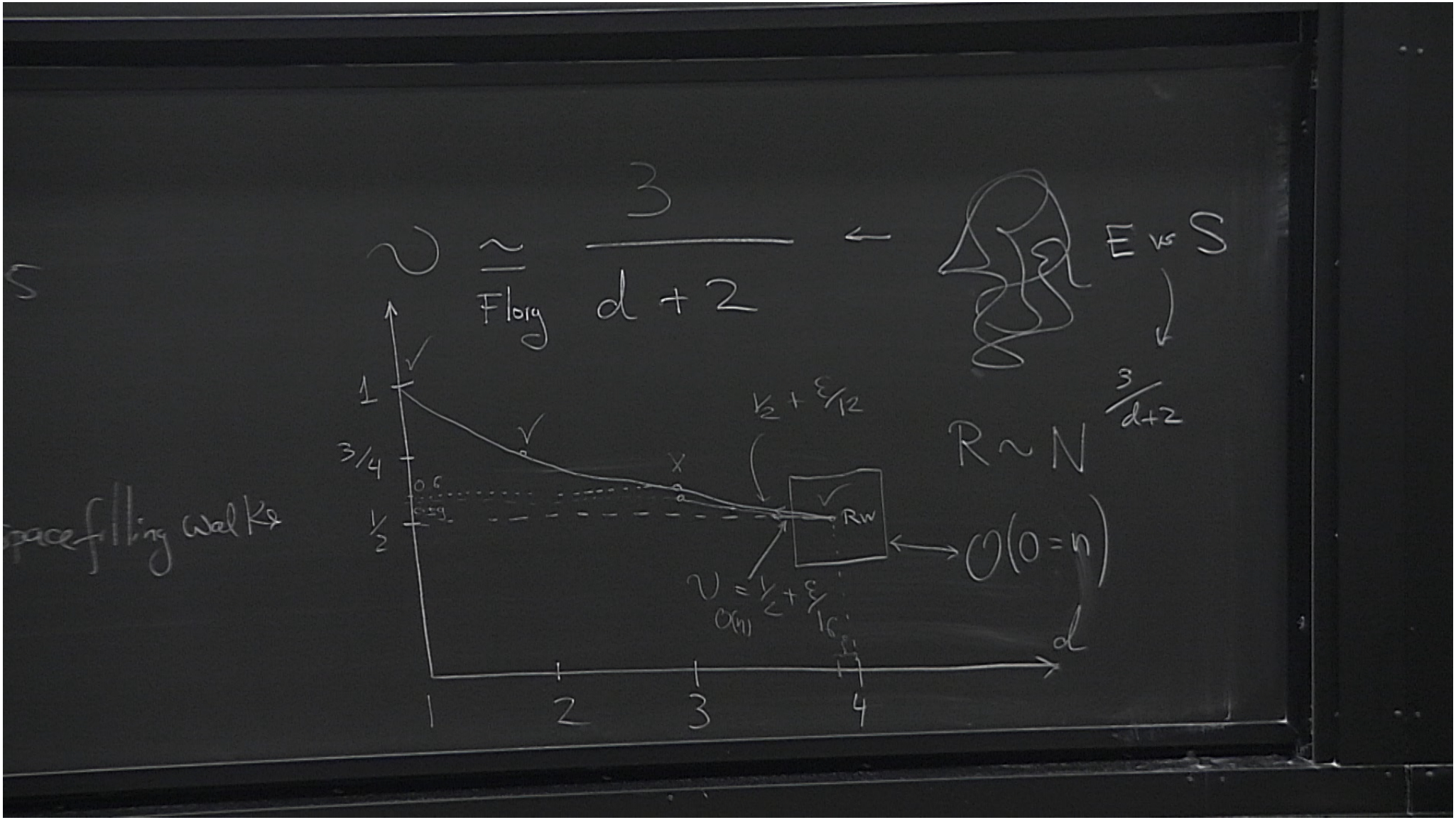

+ new couplings
$\sum z^{\#\text{steps}} \longrightarrow \sum z'^{\#\text{steps}}$
Small Walks
$z_c$

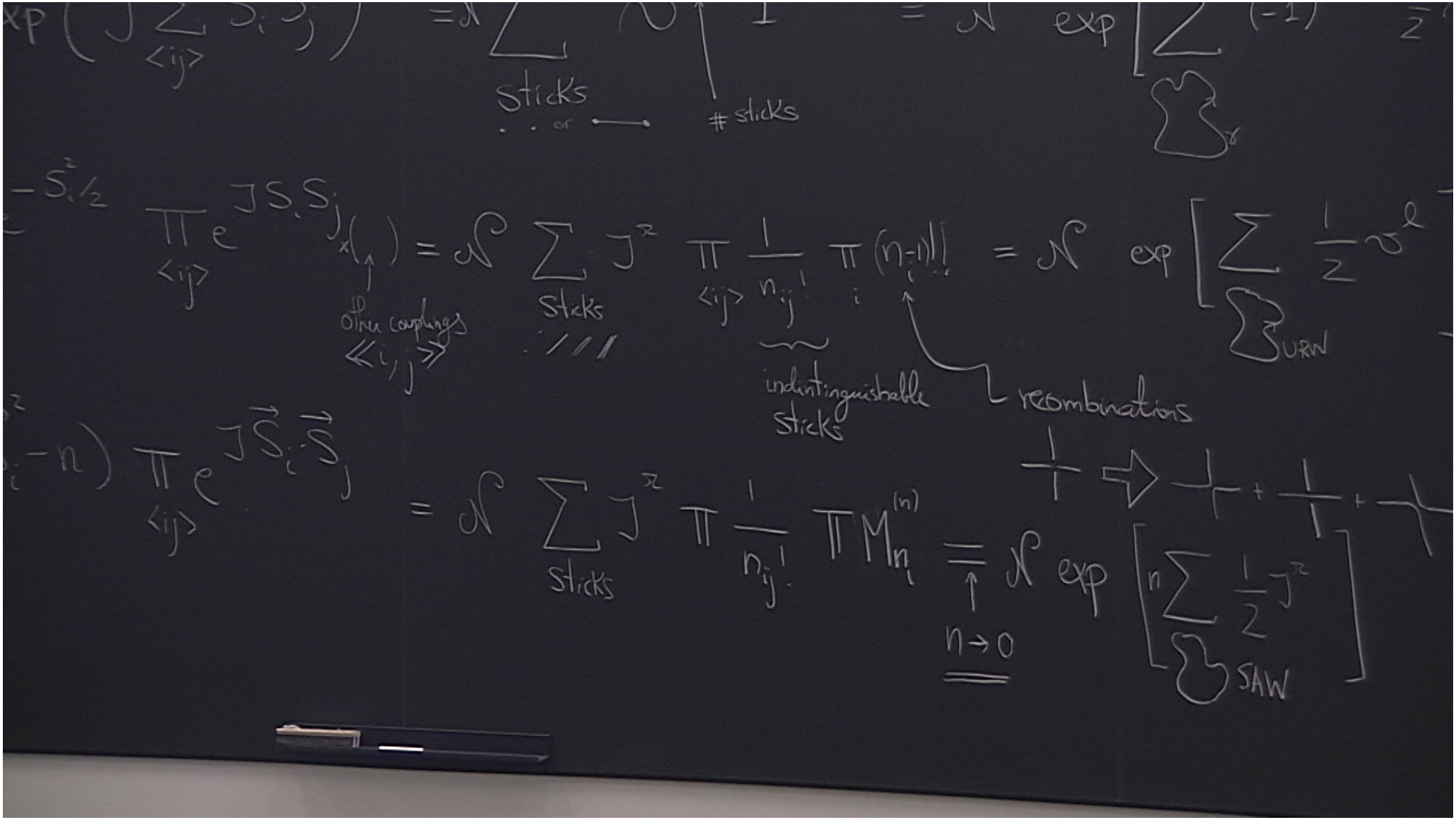
$\nu \simeq \frac{3}{d+2}$ Flory

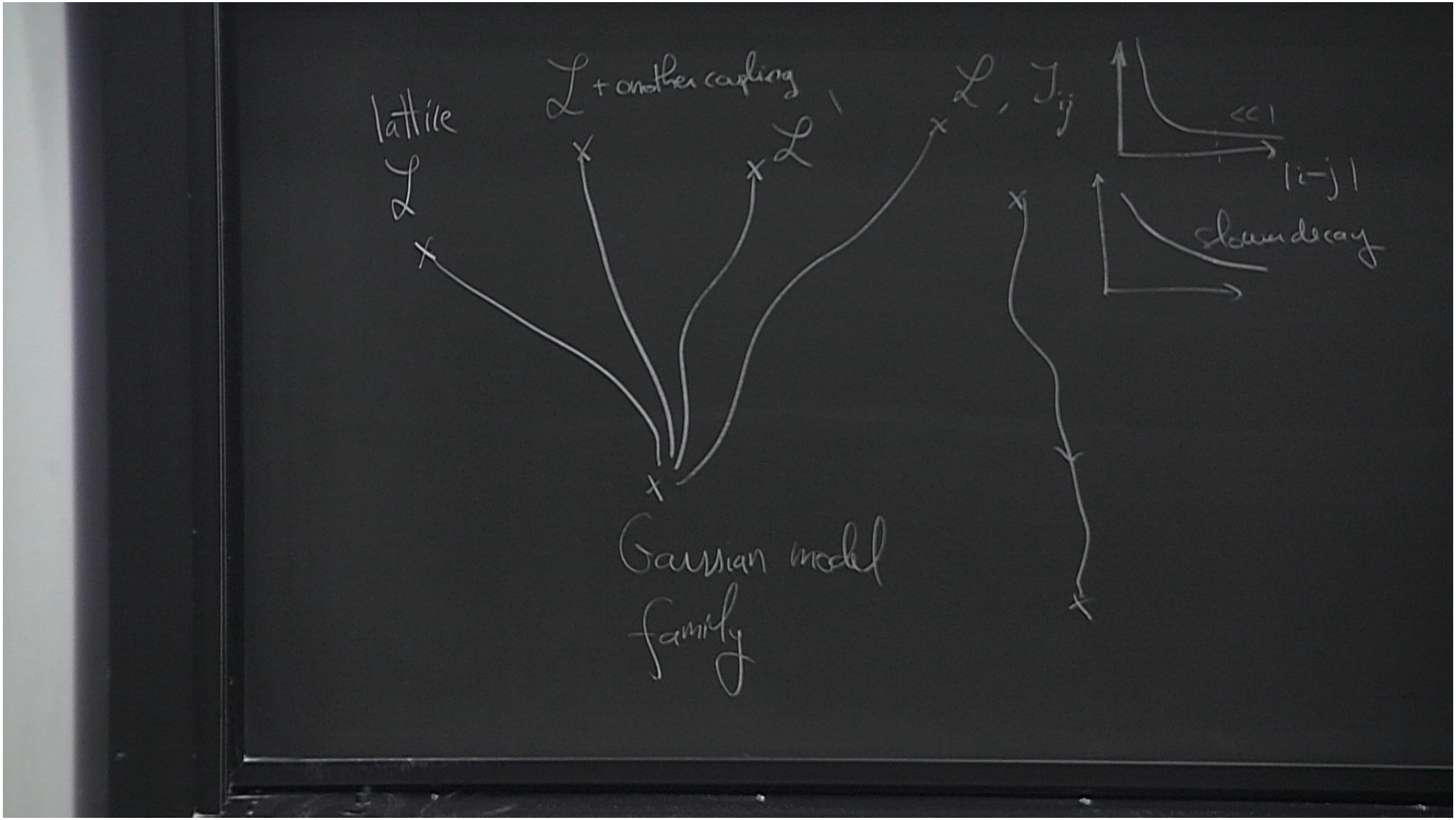


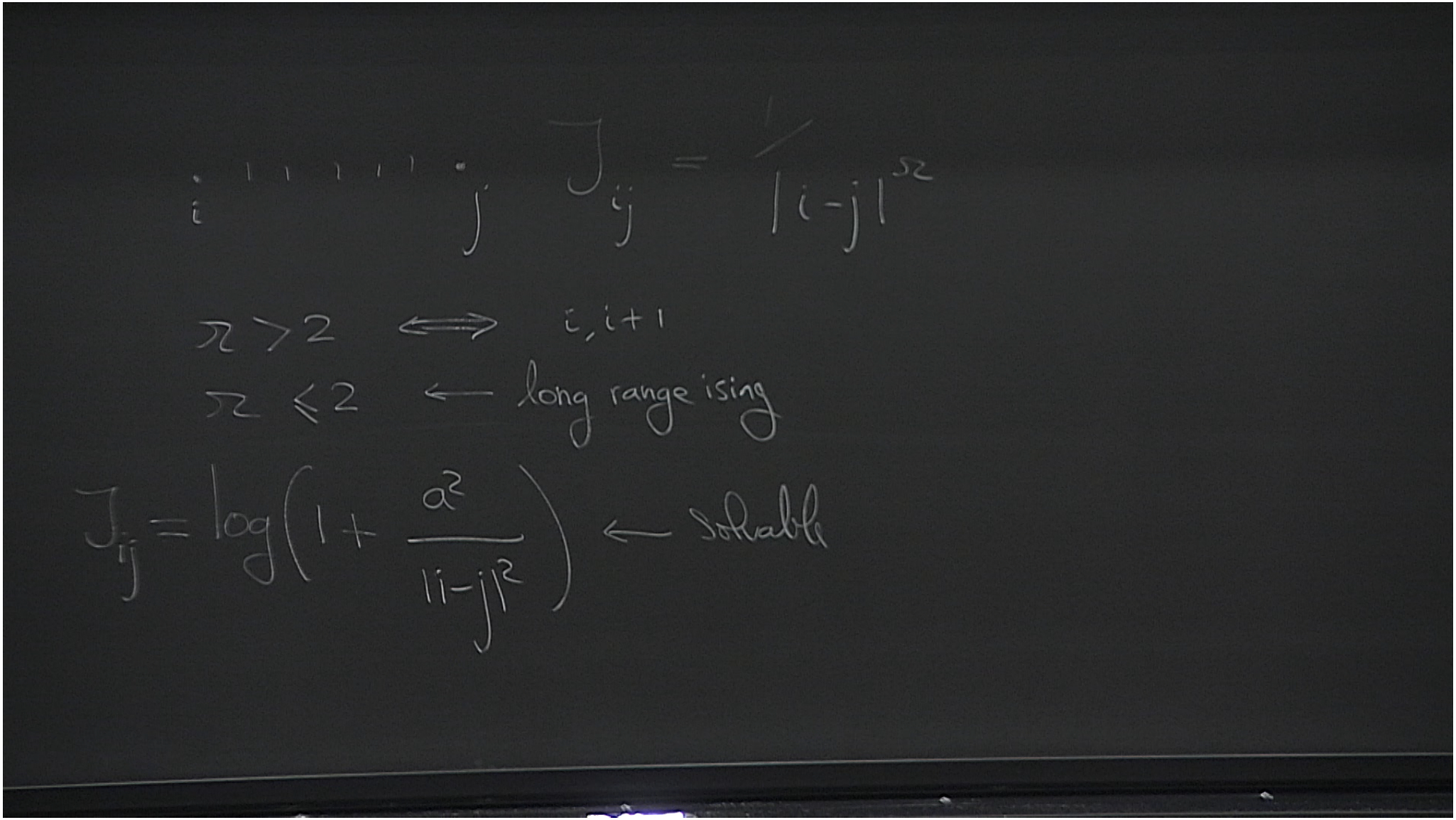

$R \sim N^{\frac{3}{d+2}}$

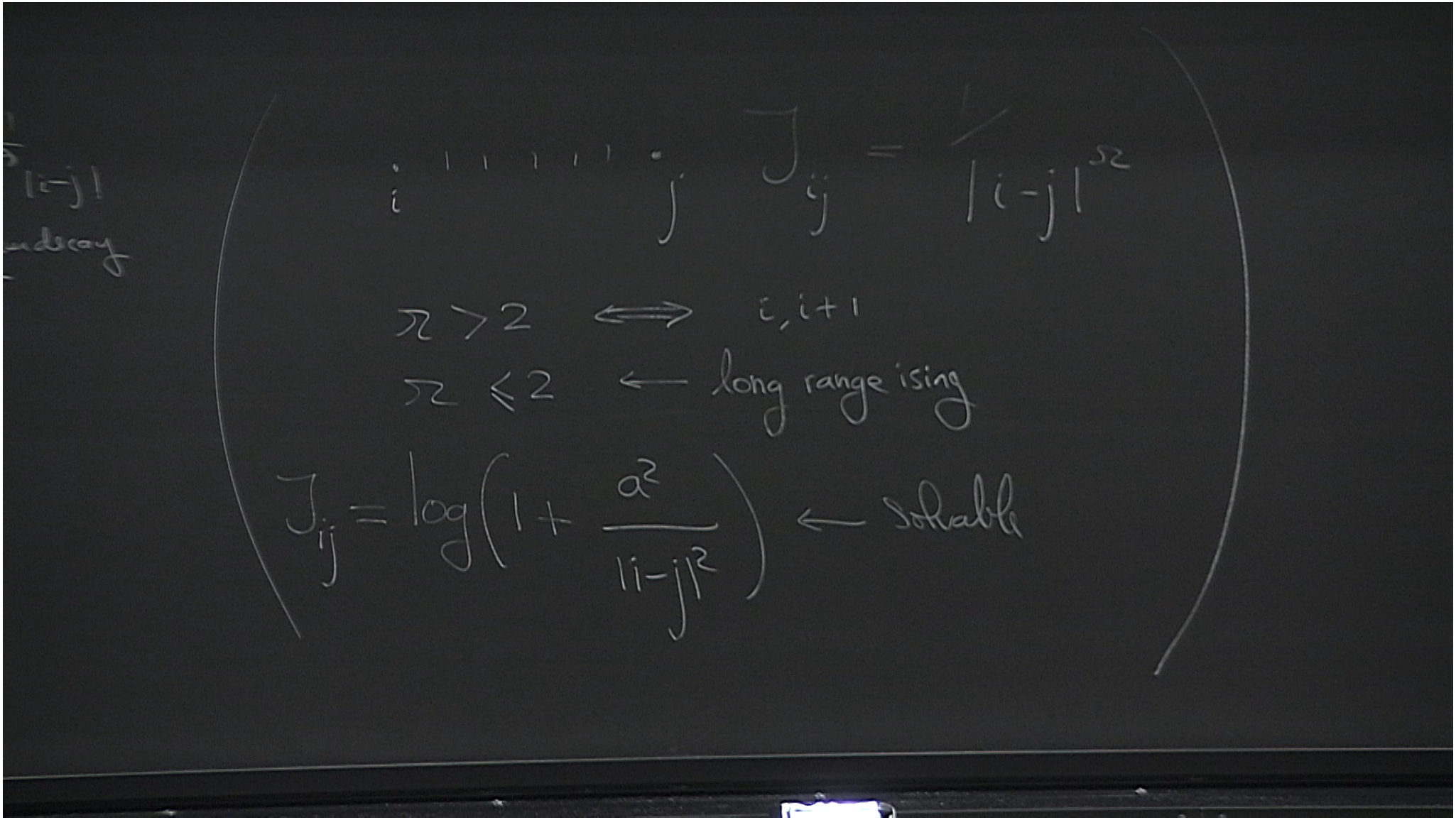
space filling walks

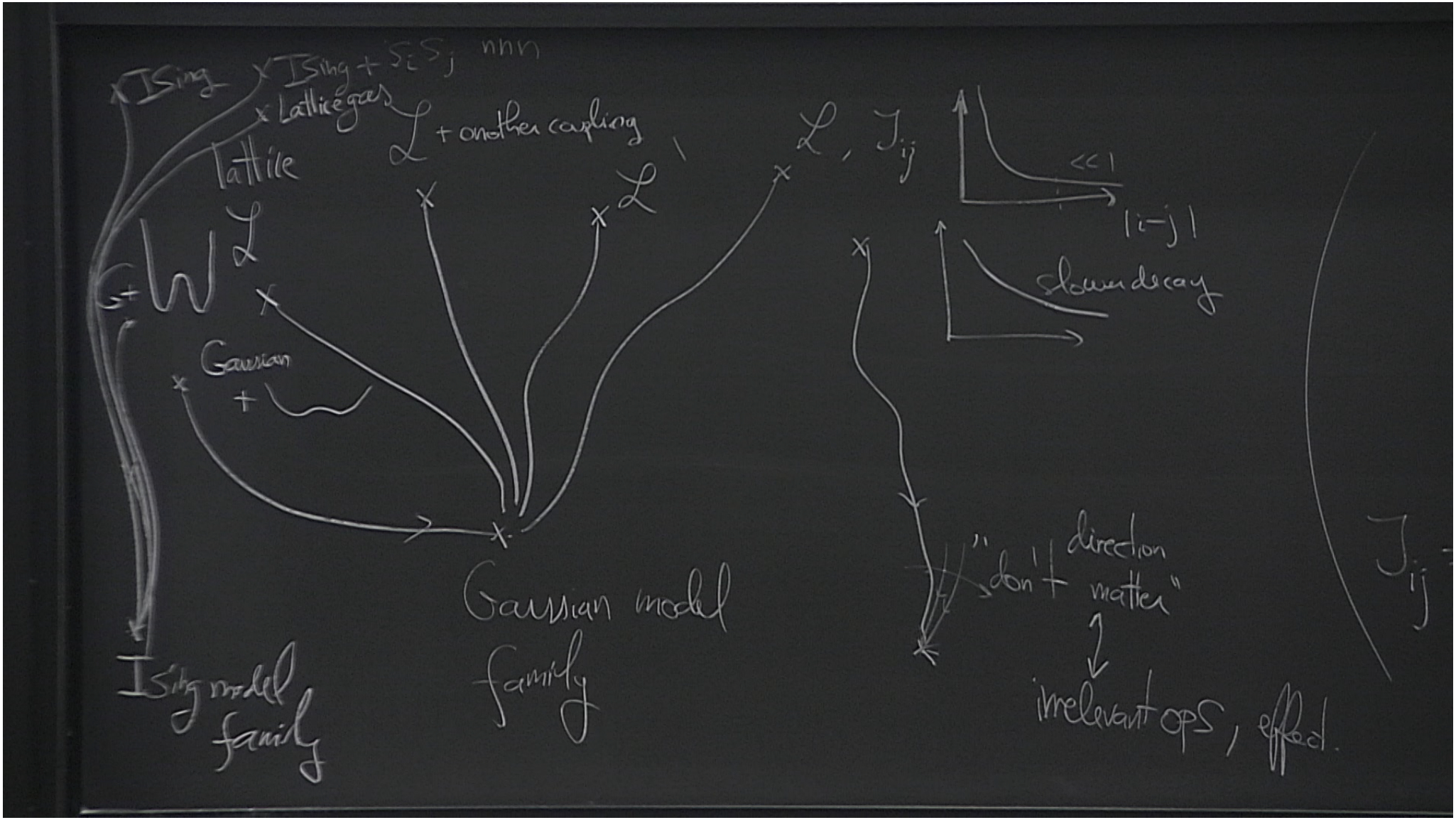

$\nu \simeq \frac{3}{d+2}$
Flory
E vs S
$\frac{3}{d+2}$
$R \sim N$
1
3/4
1/2
0.6
0.59
$\nu$
X
$\frac{1}{2} + \varepsilon/12$
RW
$\nu = \frac{1}{2} + \frac{\varepsilon}{16}$
O(n)
O(0=n)
d
1
2
3
4
space filling walks

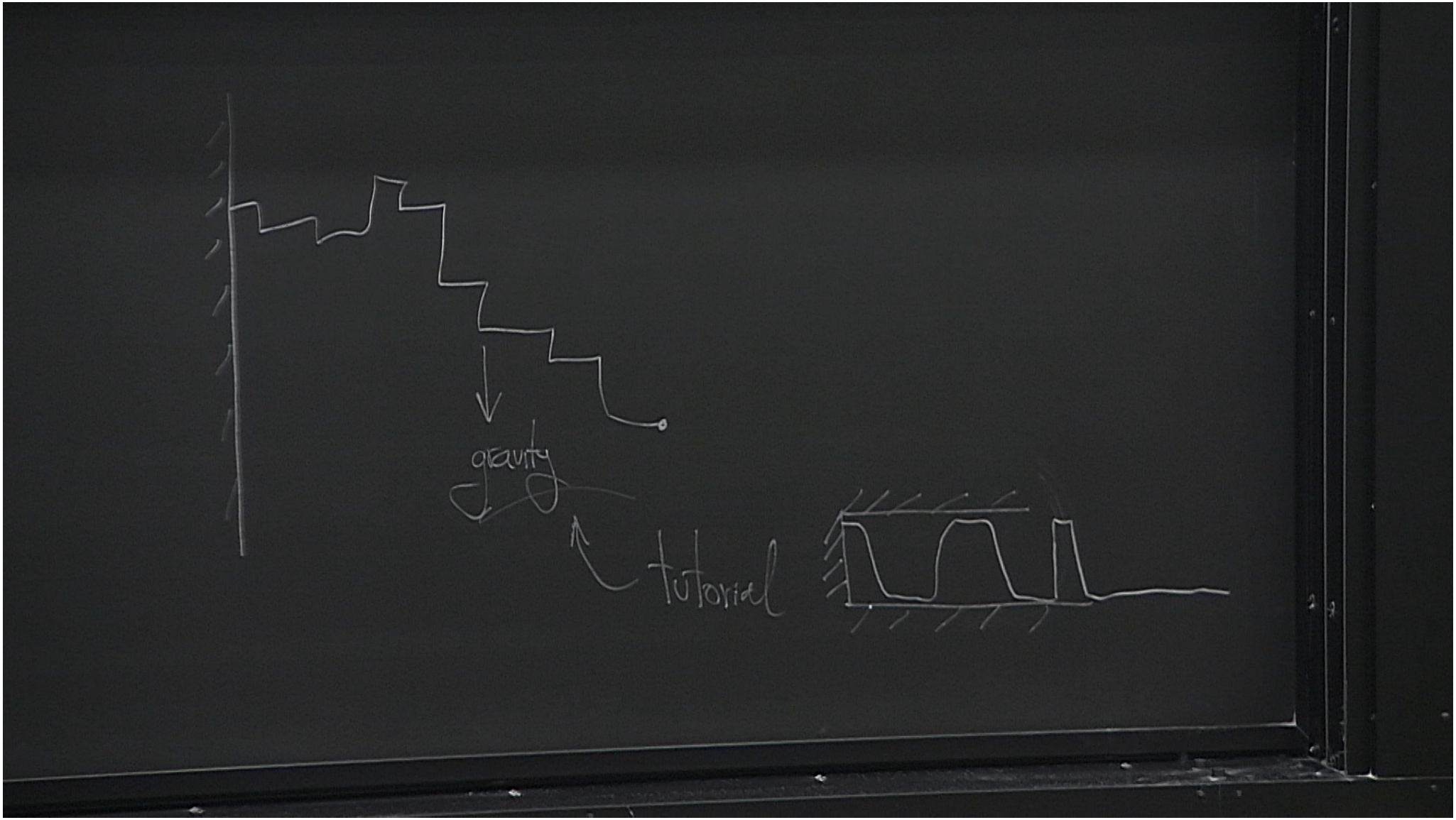

sticks
or
# sticks
other couplings
sticks
indistinguishable sticks
recombinations
loops
SAW
n → 0


lattice
$\mathcal{L}$
$\mathcal{L}$ + another coupling
$\mathcal{L}'$
$\mathcal{L}$, $J_{ij}$
$\ll 1$
$|i-j|$
slow decay
Gaussian model
family


i
j
$J_{ij} = \frac{1}{|i-j|^{sz}}$
$sz > 2 \iff i, i+1$
$sz \leq 2 \leftarrow$ long range ising
$J_{ij} = \log\left(1 + \frac{a^2}{|i-j|^2}\right) \leftarrow$ solvable


$|i-j|$
$J_{ij} = \frac{1}{|i-j|^{\alpha}}$
$\alpha > 2 \iff i, i+1$
$\alpha \leq 2 \leftarrow$ long range ising
$J_{ij} = \log\left(1 + \frac{a^2}{|i-j|^2}\right) \leftarrow$ solvable


Ising
Ising + $S_i S_j$ nnn
+ Lattice gas
Lattice
+ another coupling
$\mathcal{L}$
$\mathcal{L}'$
$\mathcal{L}, J_{ij}$
$\ll 1$
$|i-j|$
slow decay
Gaussian
Gaussian model family
Ising model family
"don't direction matter"
irrelevant ops, effect.
$J_{ij}$


gravity
tutorial

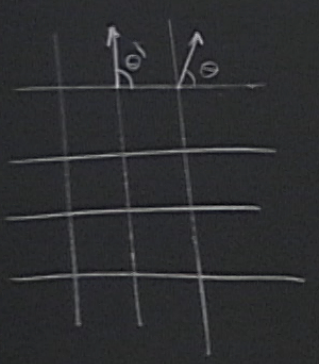
O(2) model

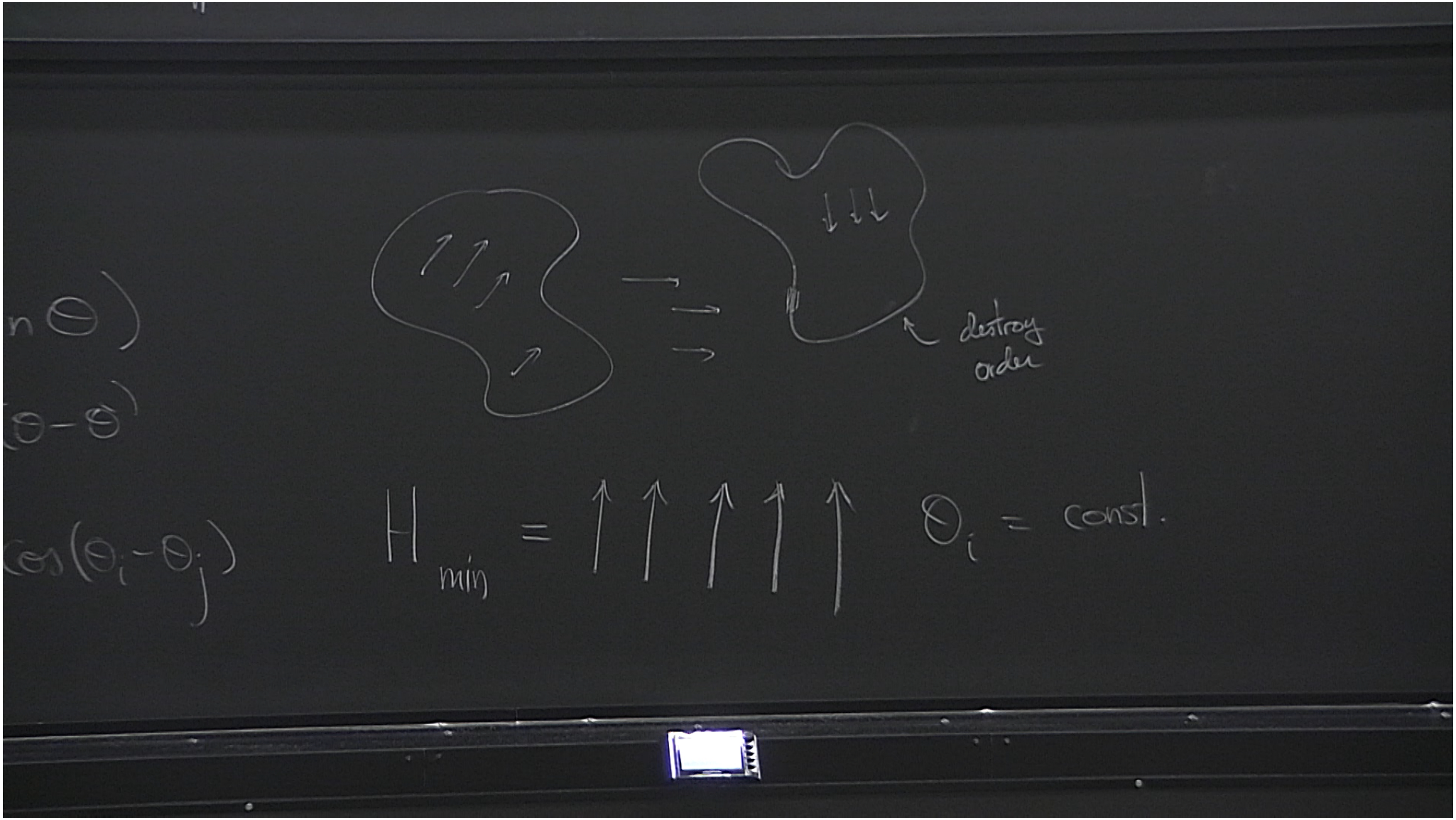
$\vec{S}(\theta) = (\cos\theta, \sin\theta)$

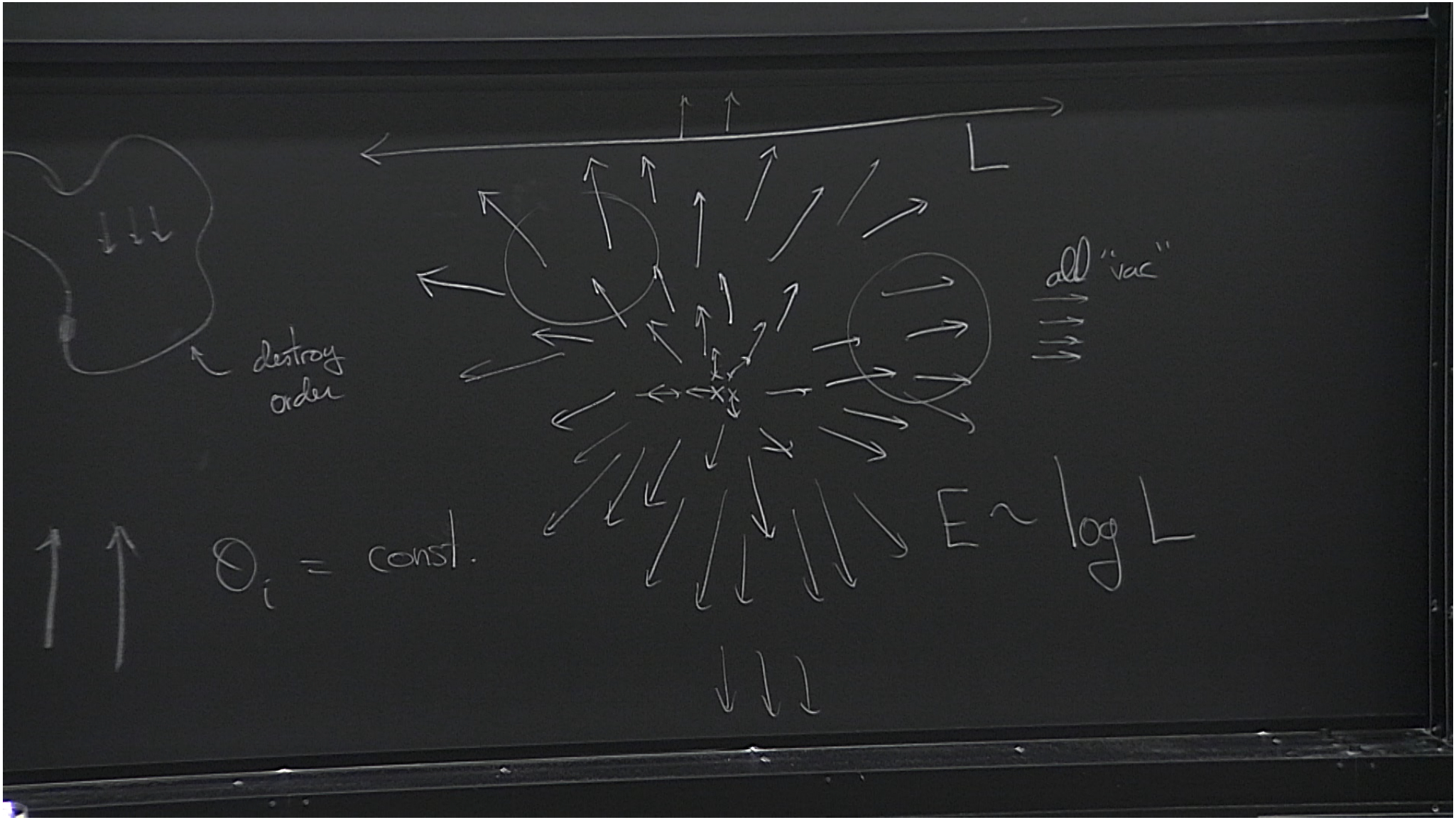
$\vec{S}\cdot\vec{S} = \cos(\theta - \theta')$

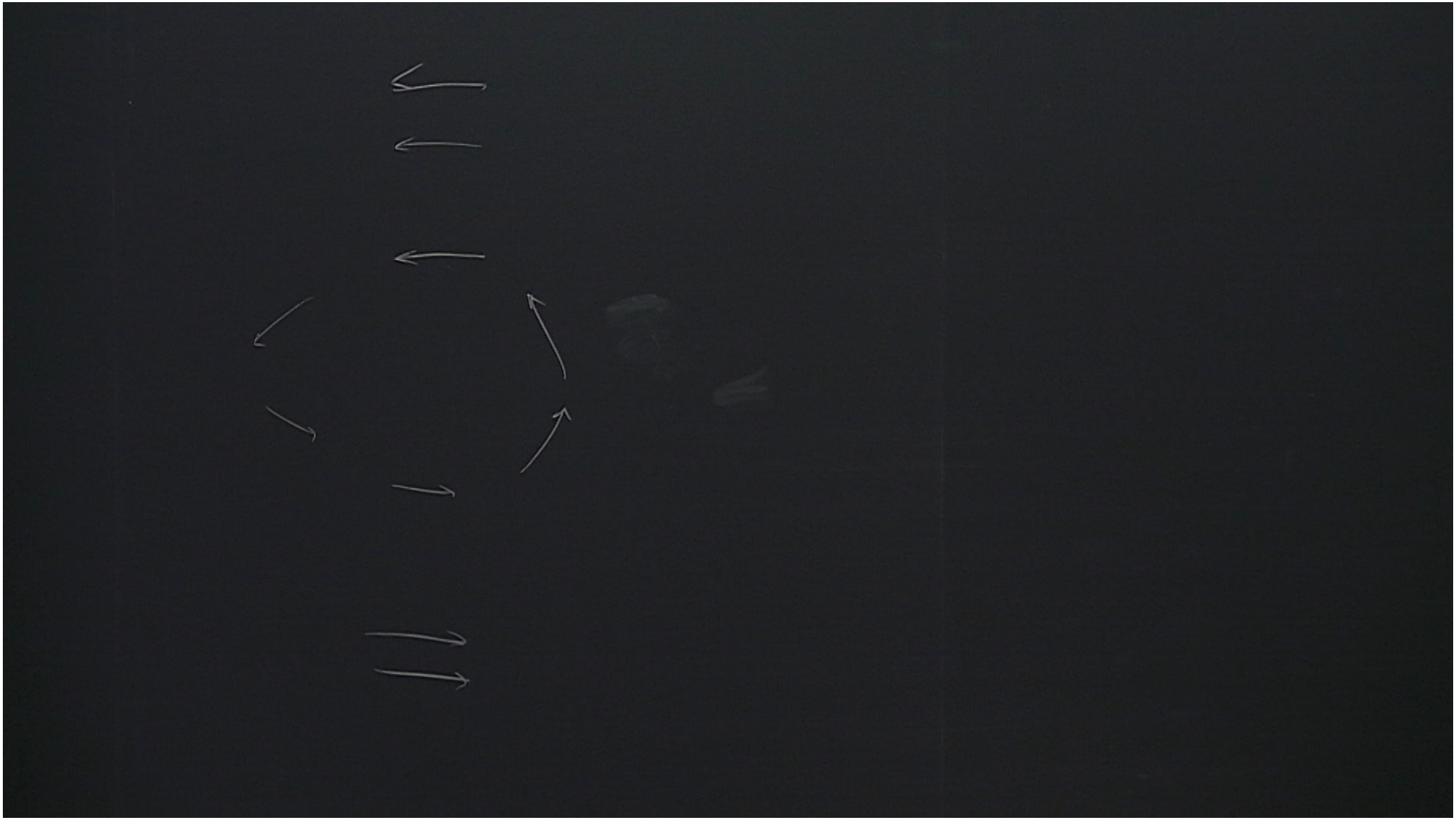
$-H = \sum_{\langle ij \rangle} J \cos(\theta_i - \theta_j)$

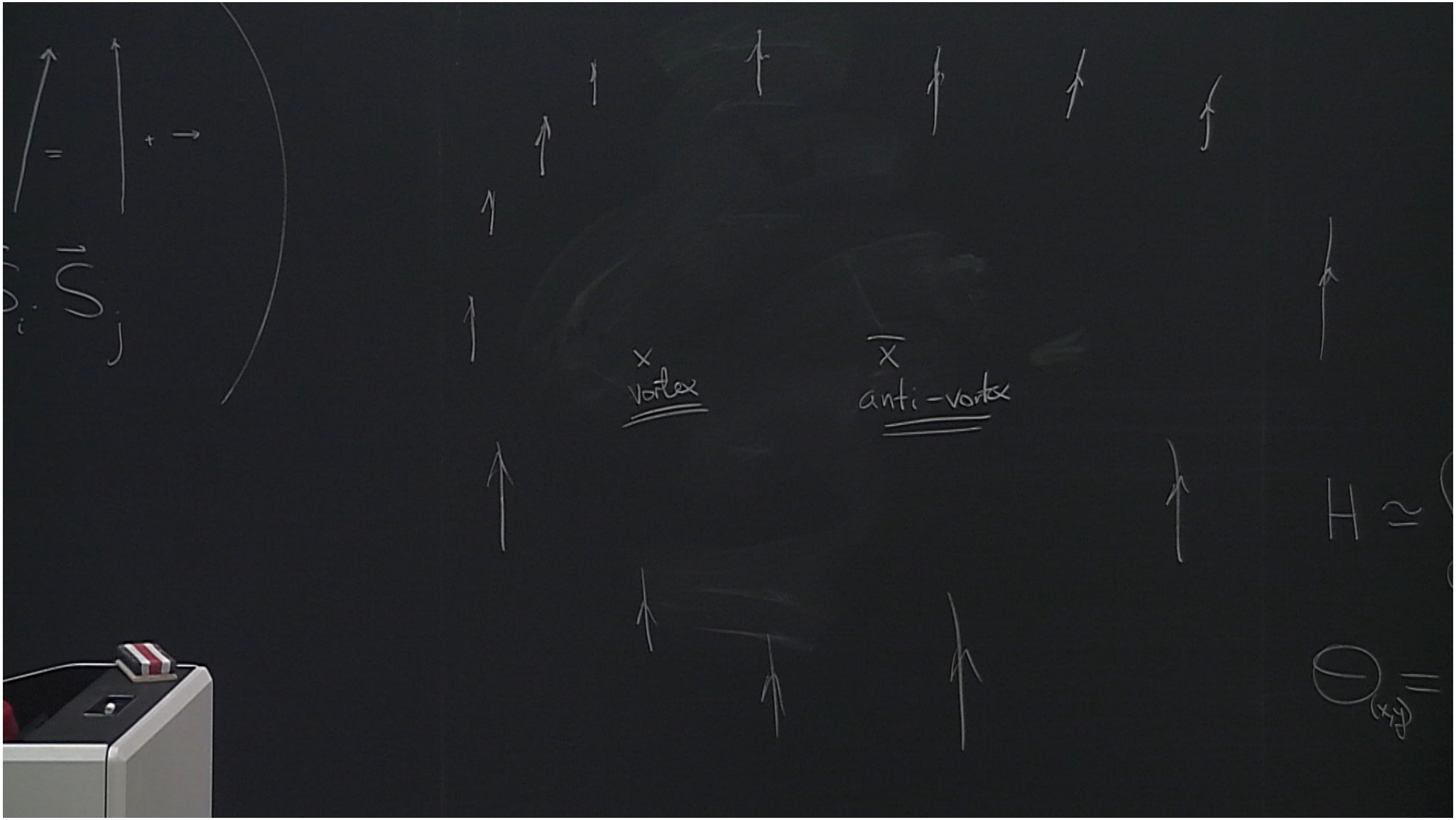
$H_{min} =$ ↑ ↑ ↑ ↑ ↑

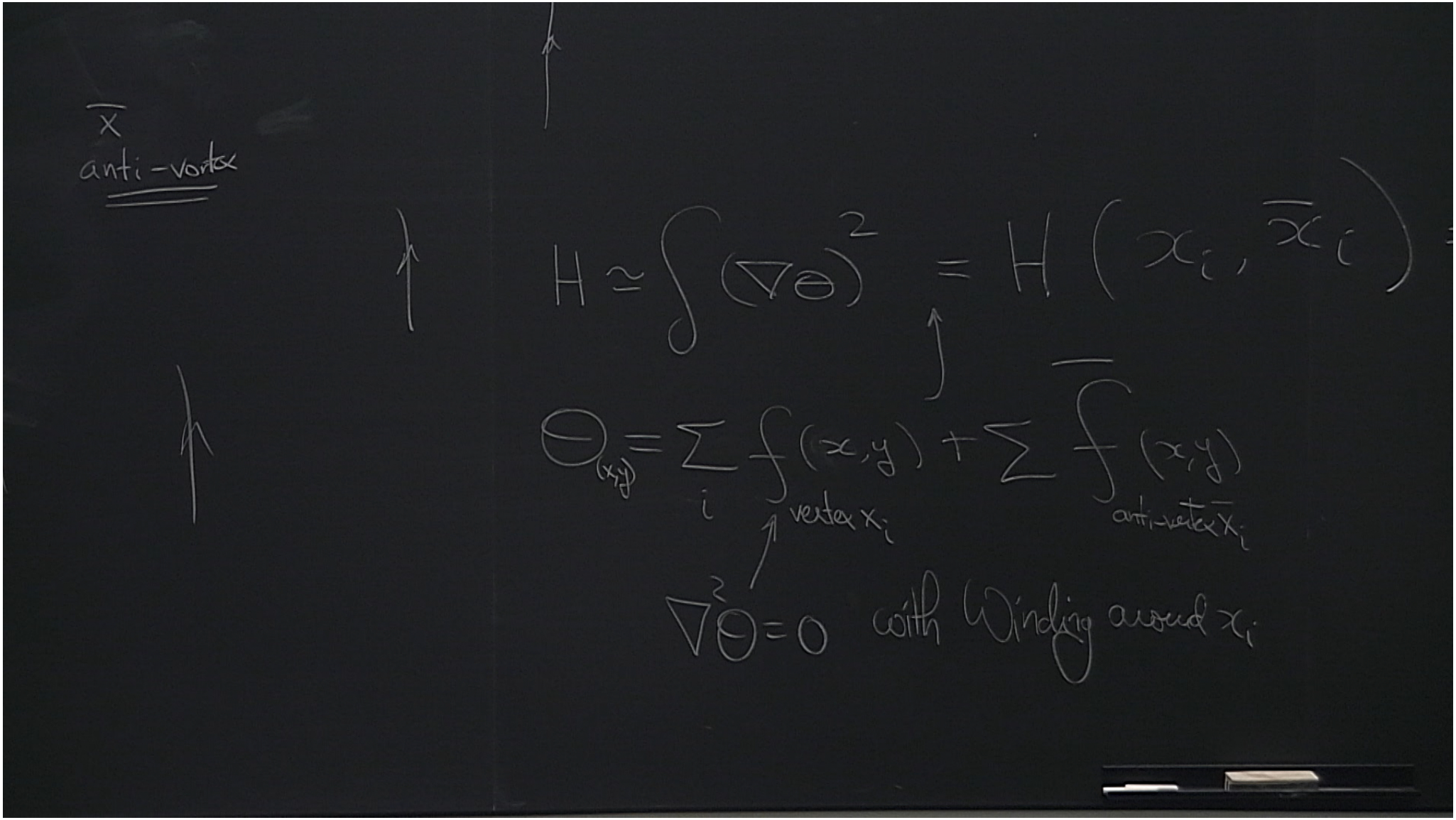


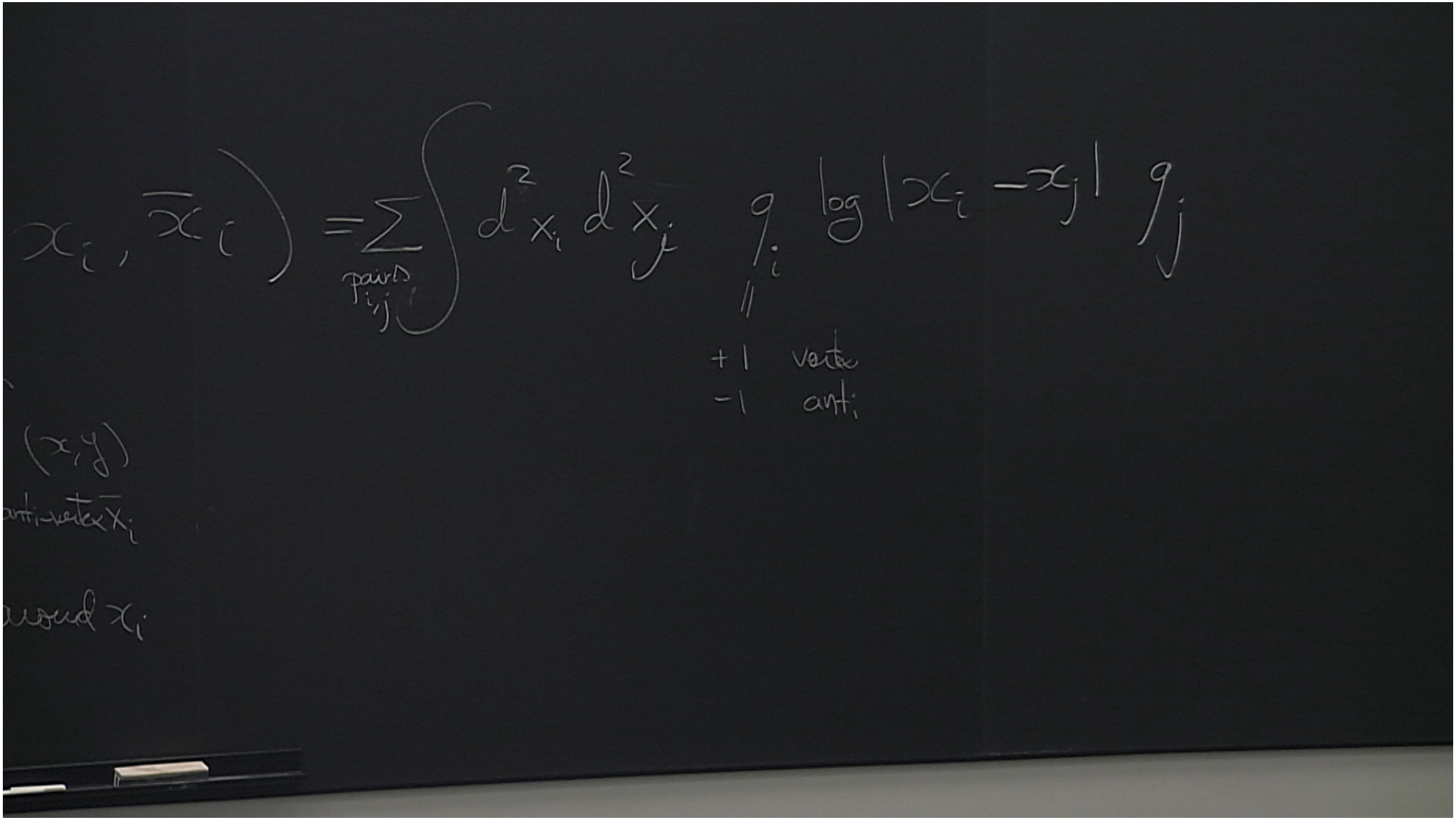

destroy order
$H_{min}$ = $\theta_i$ = const.

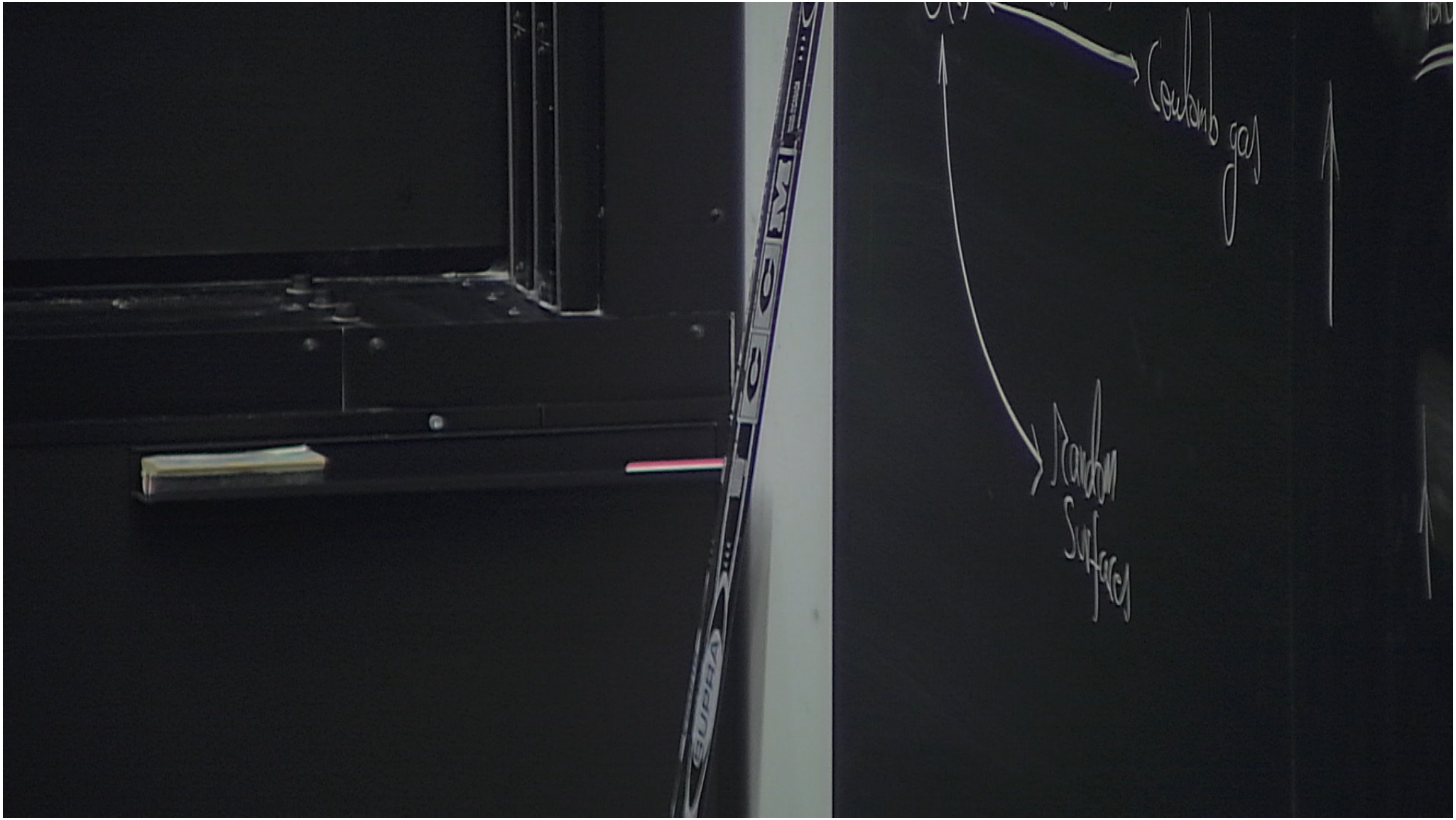

L
all "vac"
destroy order
$\theta_i$ = const.
$E \sim \log L$

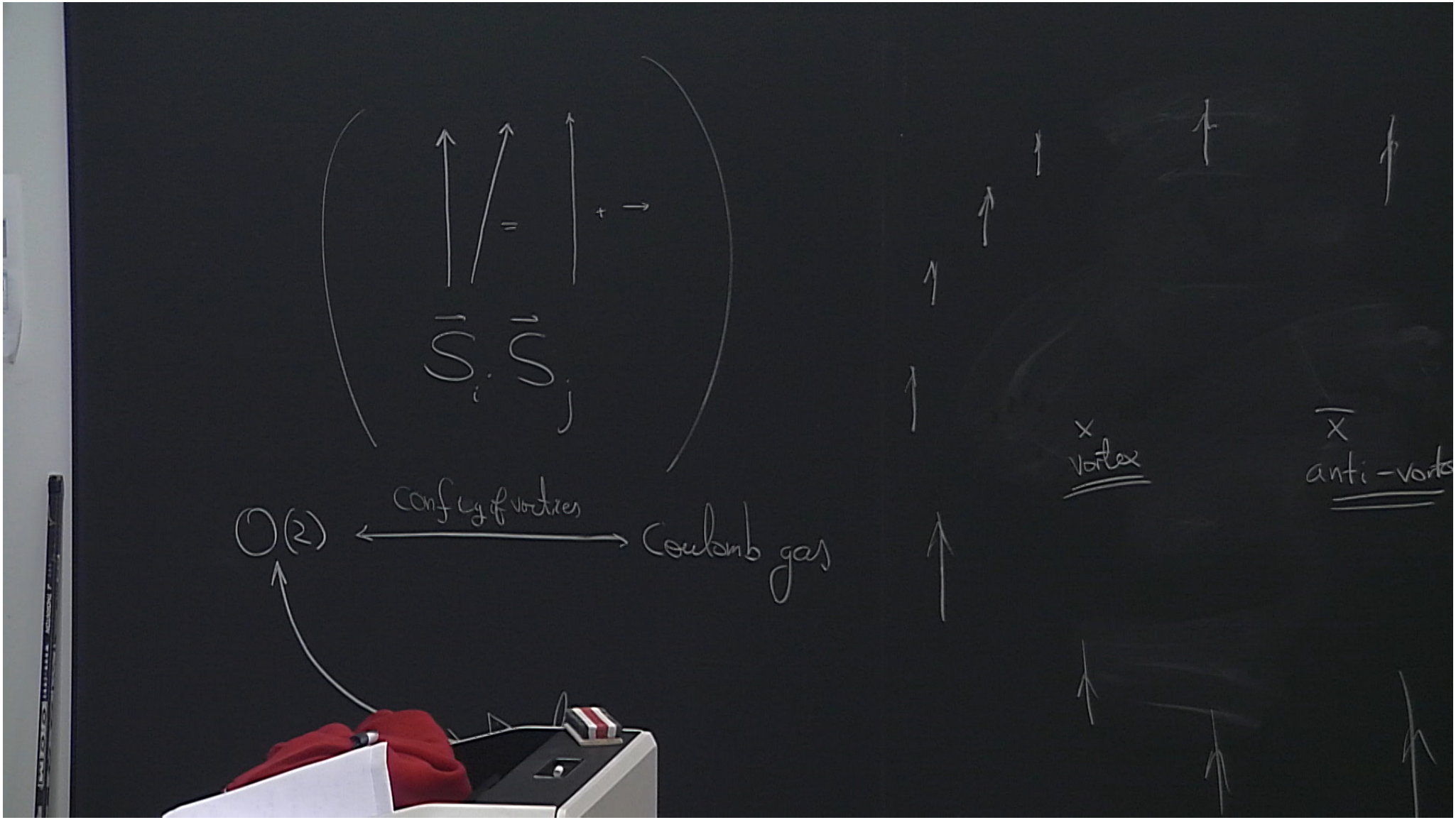


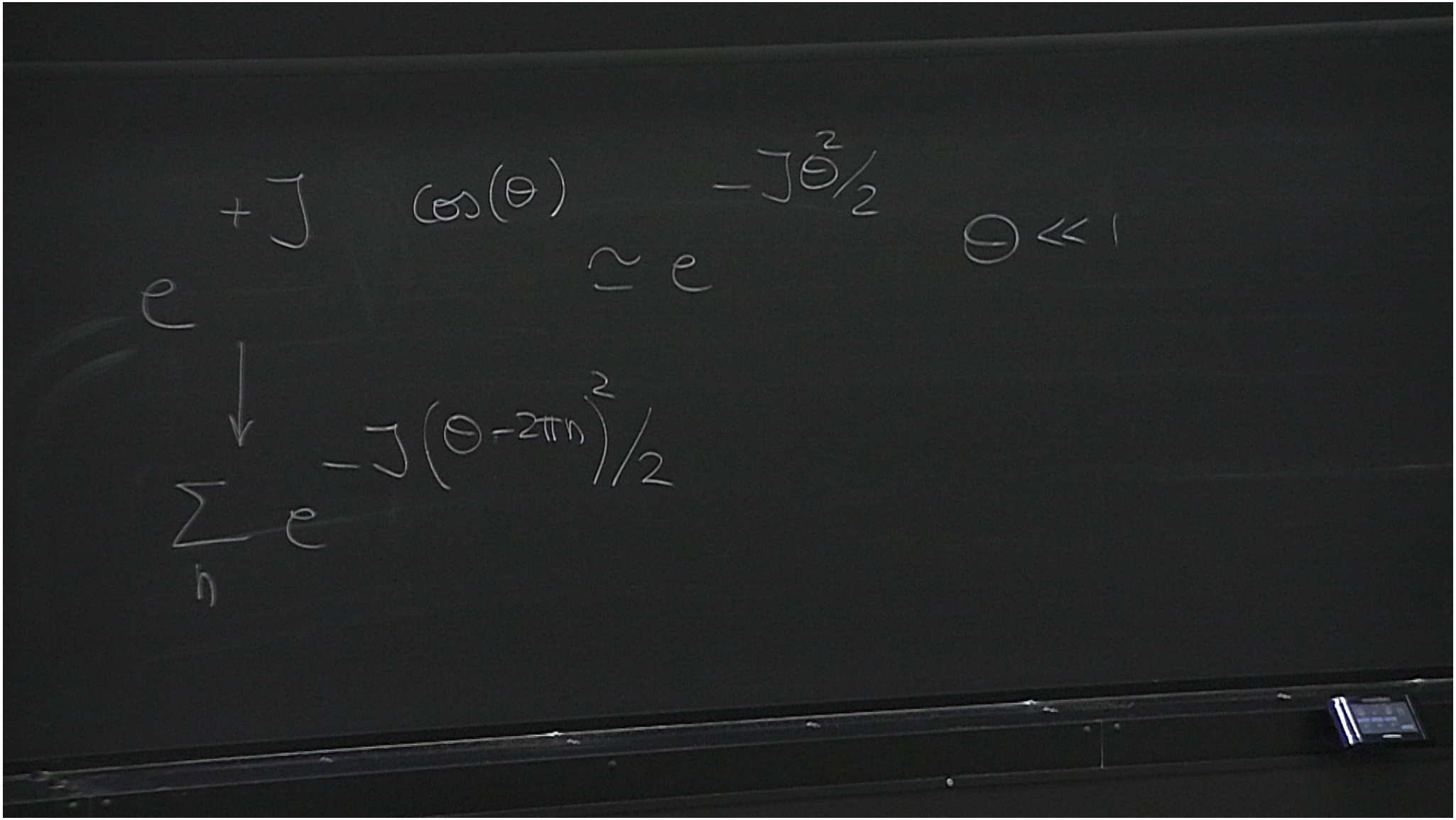

vortex
anti-vortex

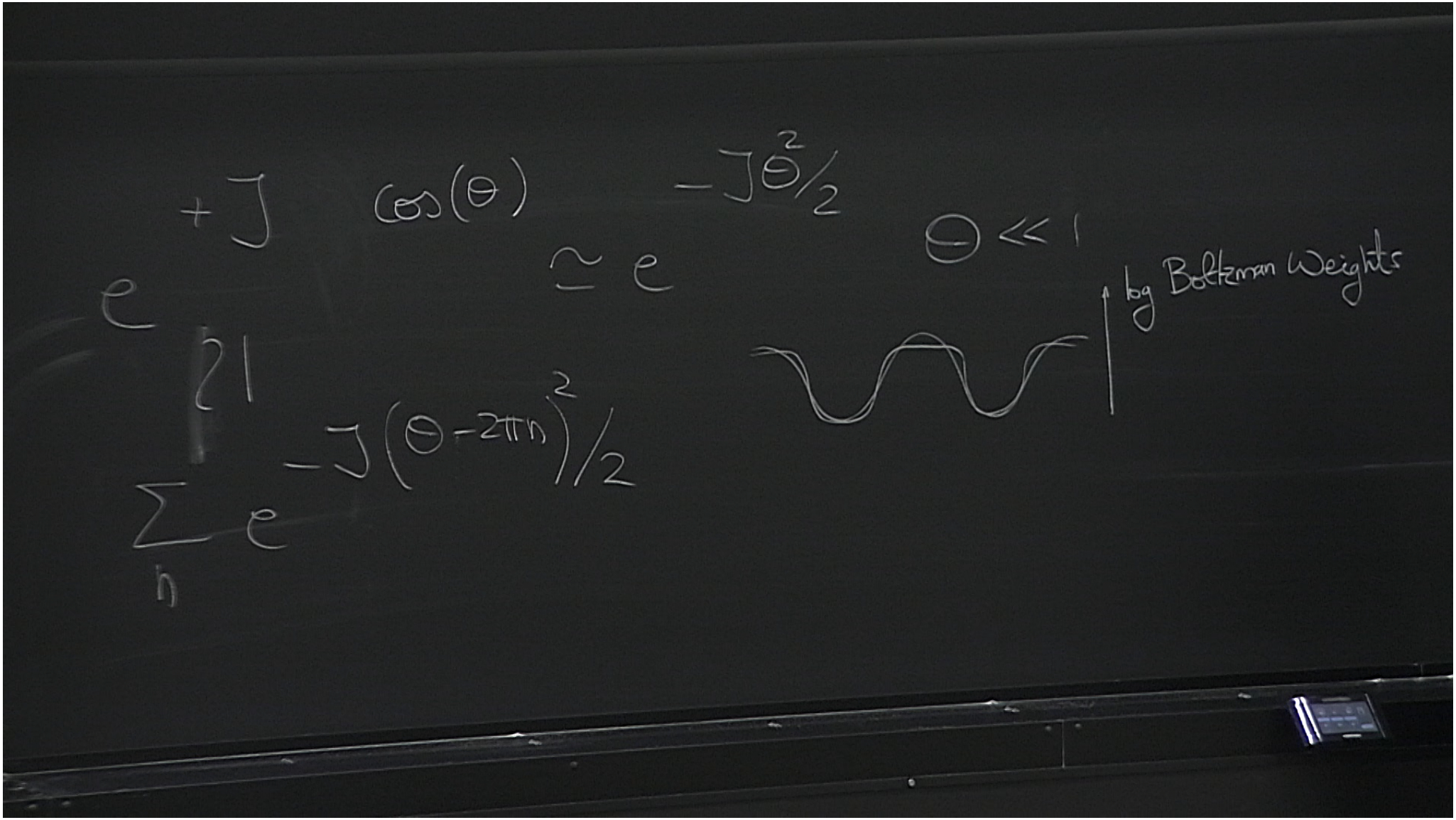

$\bar{x}$
anti-vortex
$H \simeq \int (\nabla\theta)^2 = H(x_i, \bar{x}_i)$
$\theta_{(x,y)} = \sum_i \int_{\text{vortex } x_i} (x,y) + \sum \int_{\text{anti-vortex } \bar{x}_i} (x,y)$
$\nabla^2\theta = 0$ with Winding around $x_i$


$x_i, \bar{x}_i) = \sum_{\text{pairs } i,j} \int d^2x_i\, d^2x_j\; q_i \log|x_i - x_j|\, q_j$
$q_i$
$+1$ vertex
$-1$ anti
$(x, y)$
anti-vertex $\bar{x}_i$
around $x_i$


Coulomb gas
Random
Surfaces


$\vec{S}_i \cdot \vec{S}_j$
O(2)
Config of vortices
Coulomb gas
vortex
anti-vorte


$+J\cos(\theta)$
$-J\theta^2/2$
$\theta \ll 1$
$\simeq e$
$e$
$\sum_n e^{-J(\theta-2\pi n)^2/2}$


$+J\cos(\theta)$
$-J\theta^2/2$
$\theta \ll 1$
$\simeq e$
$e$
$\sum_n e^{-J(\theta-2\pi n)^2/2}$
log Boltzmann Weights

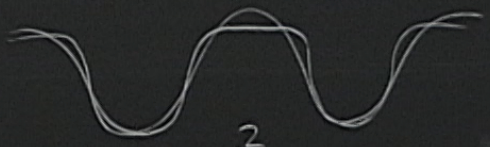
$$Z = \int \prod_i d\theta_i \sum_{\{n_{ij} \in \mathbb{Z}\}} \prod_{\langle ij \rangle} e^{-J(\theta_i - \theta_j - 2\pi n_{ij})^2/2}$$

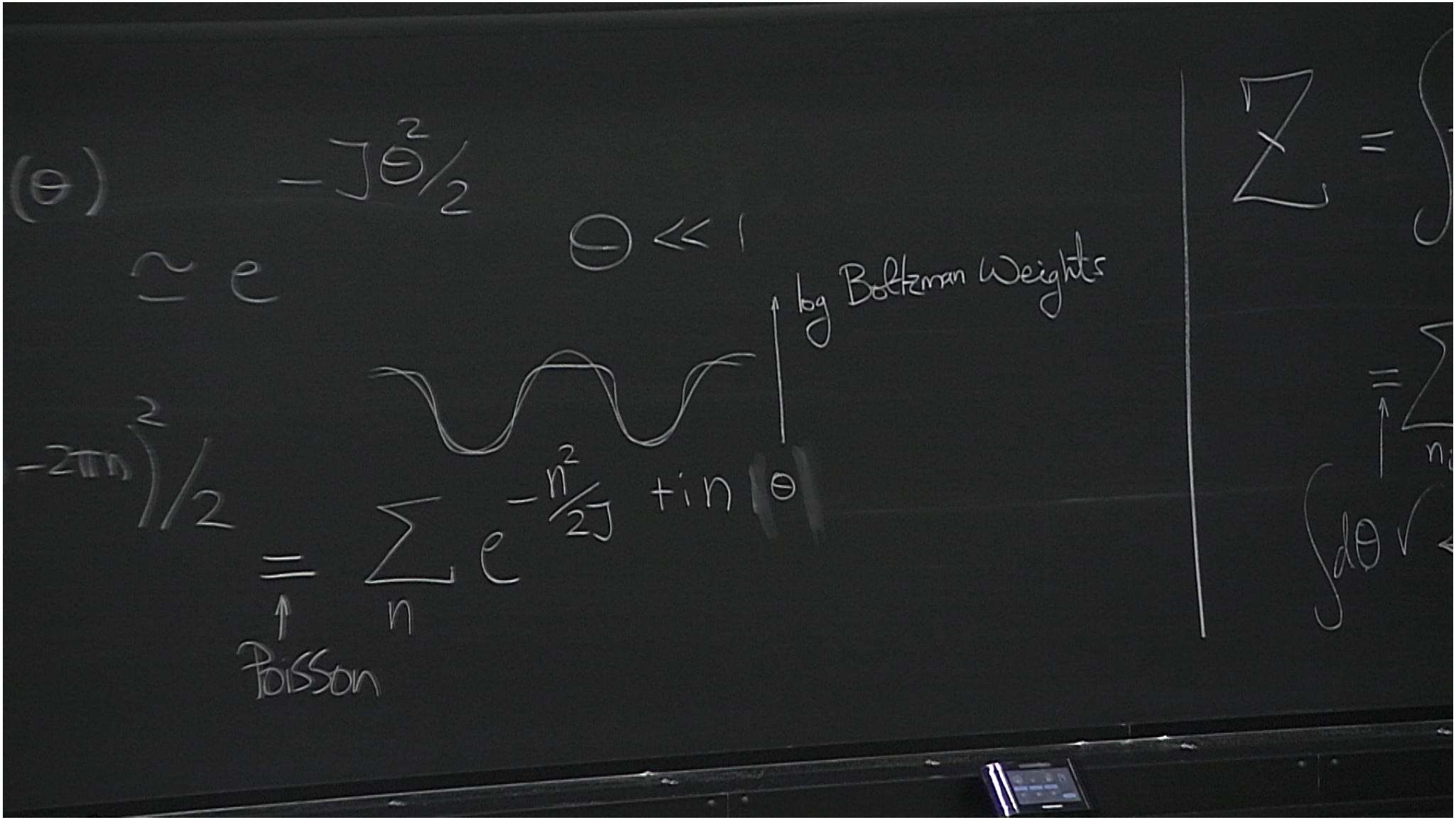
$$e^{+J\cos(\theta)} \simeq e^{-J\theta^2/2} \qquad \theta \ll 1$$

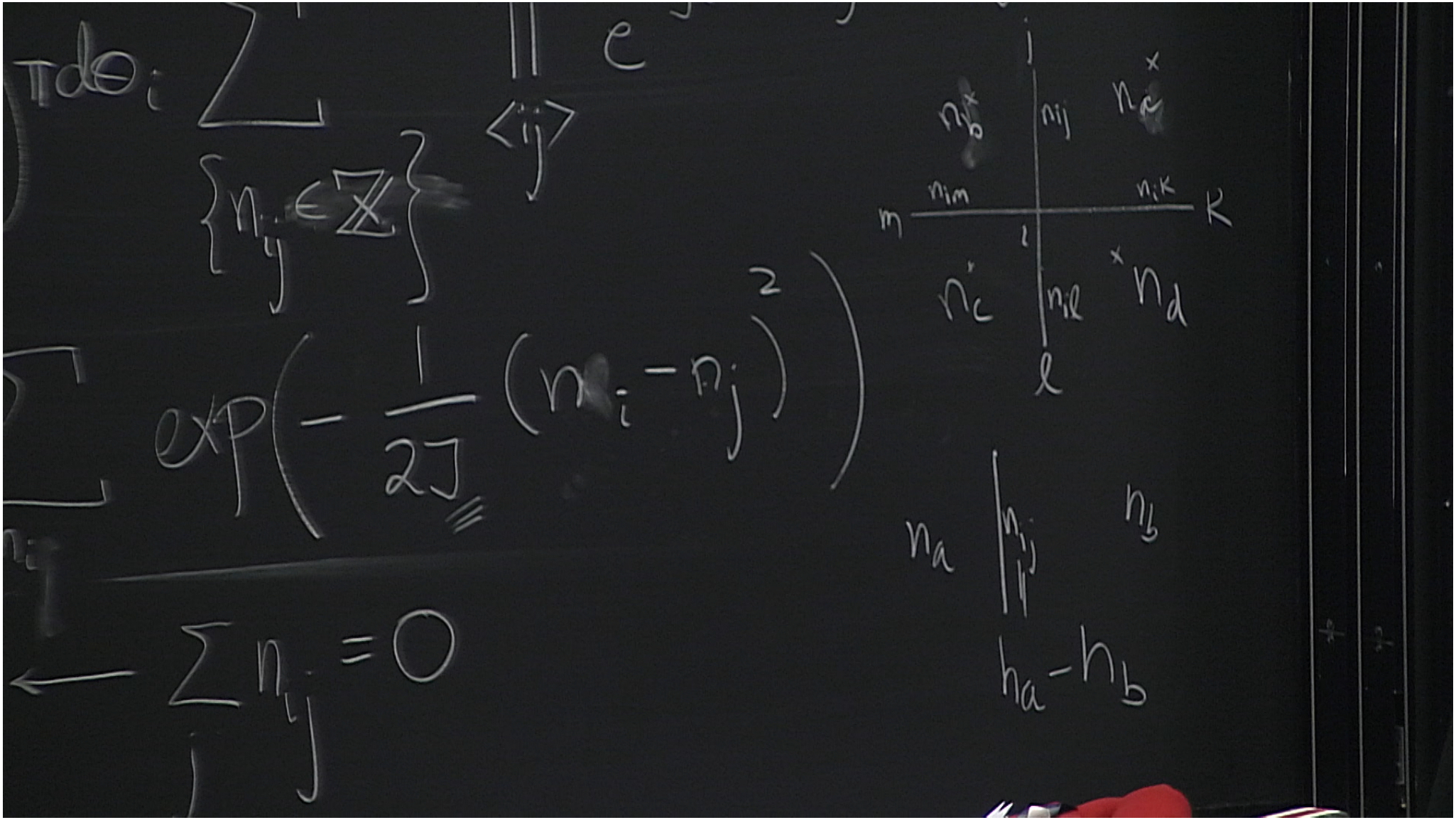
$$\sum_n e^{-J(\theta-2\pi n)^2/2} = \sum_n e^{-\frac{n^2}{2J}+in\theta}$$

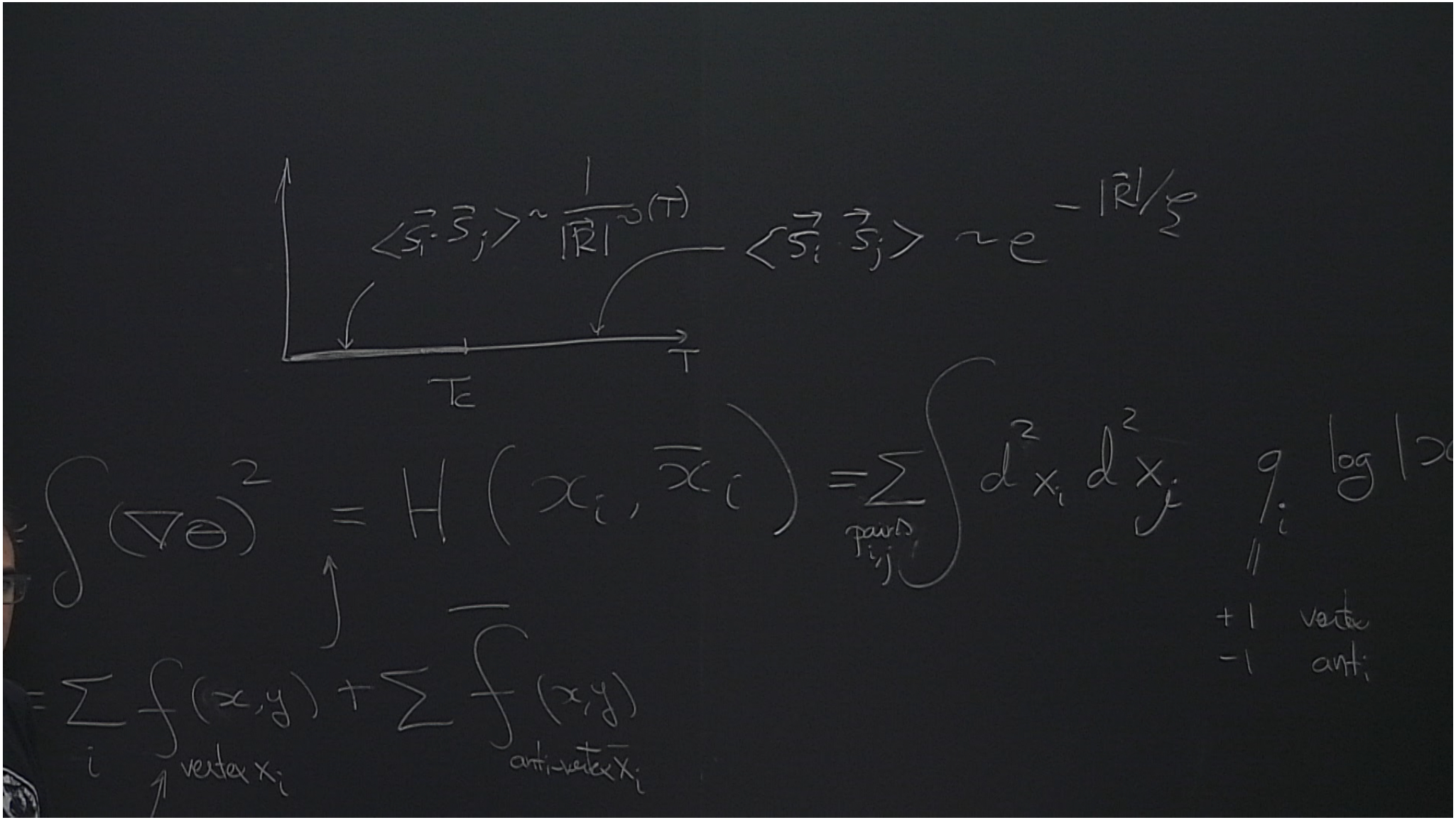
↑ Poisson

↑ log Boltzman Weights




$\theta \ll 1$
by Boltzman Weights
Poisson
$\sum_n e^{-\frac{n^2}{2J} + in\theta}$
$Z = \int d\theta$


$\pi d\theta_i$
$\sum$
$\langle ij \rangle$
$e^{-}$
$\{n_{ij} \in \mathbb{Z}\}$
$\exp\left(-\frac{1}{2J}(n_i - n_j)^2\right)$
$\sum n_{ij} = 0$
$n_{ij}$
$n_{ik}$
$n_{im}$
$n_{il}$
$n_a$
$n_b$
$h_a - h_b$


$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim \frac{1}{|\vec{R}|^{\eta(T)}}$
$\langle \vec{S}_i \vec{S}_j \rangle \sim e^{-|\vec{R}|/\xi}$
$T_c$
$T$
$\int (\nabla\theta)^2 = H(x_i, \bar{x}_i) = \sum_{\text{pairs } i,j} \int d^2x_i \, d^2x_j \; q_i \log |x$
$q_i$
$+1$ vortex
$-1$ anti
$= \sum_i \int_{\text{vortex } x_i} (x,y) + \sum \int_{\text{anti-vortex } \bar{x}_i} (x,y)$