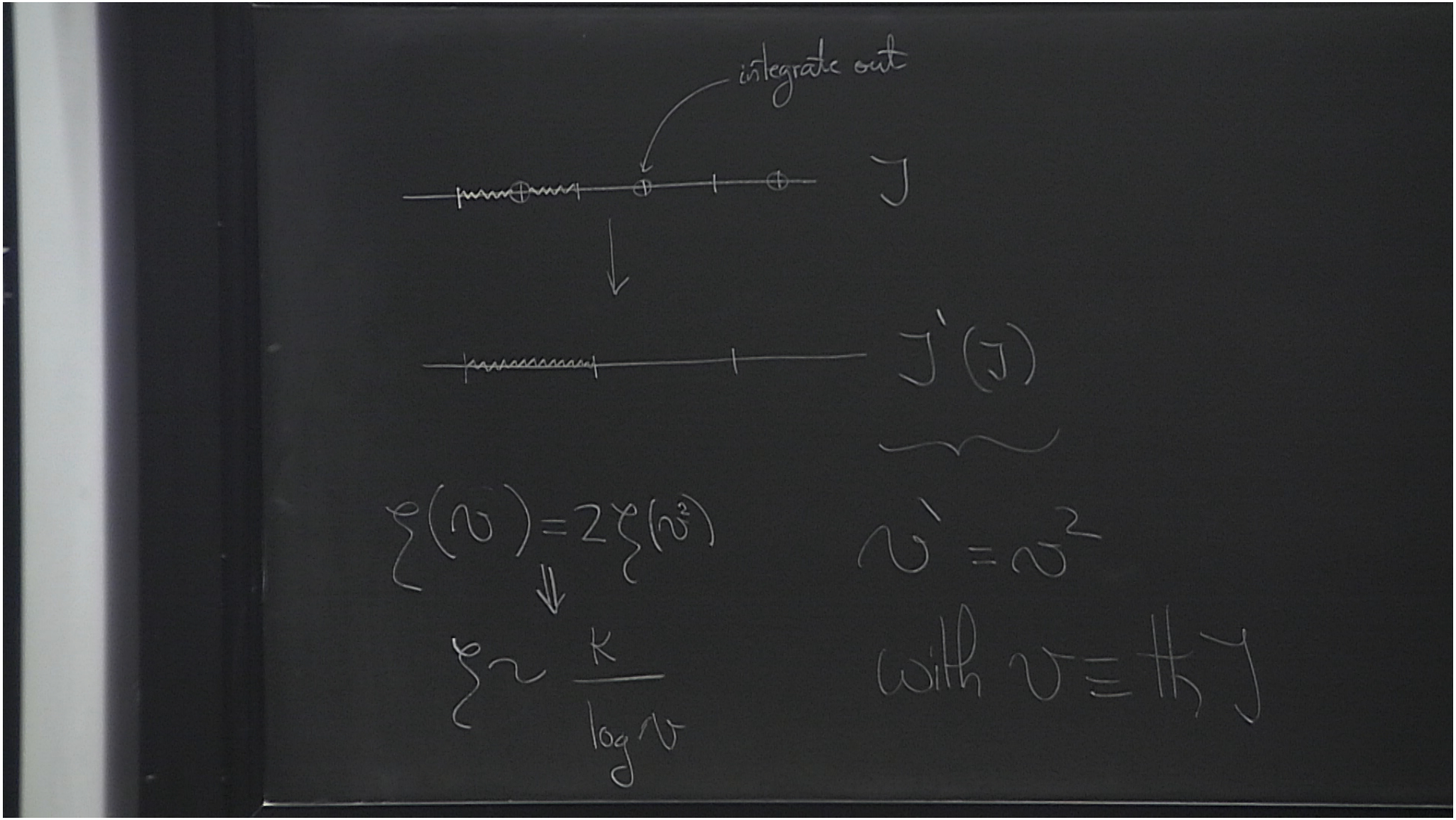
Title: PSI 2019/2020 - Statistical Mechanics (Vieira) - Lecture 12

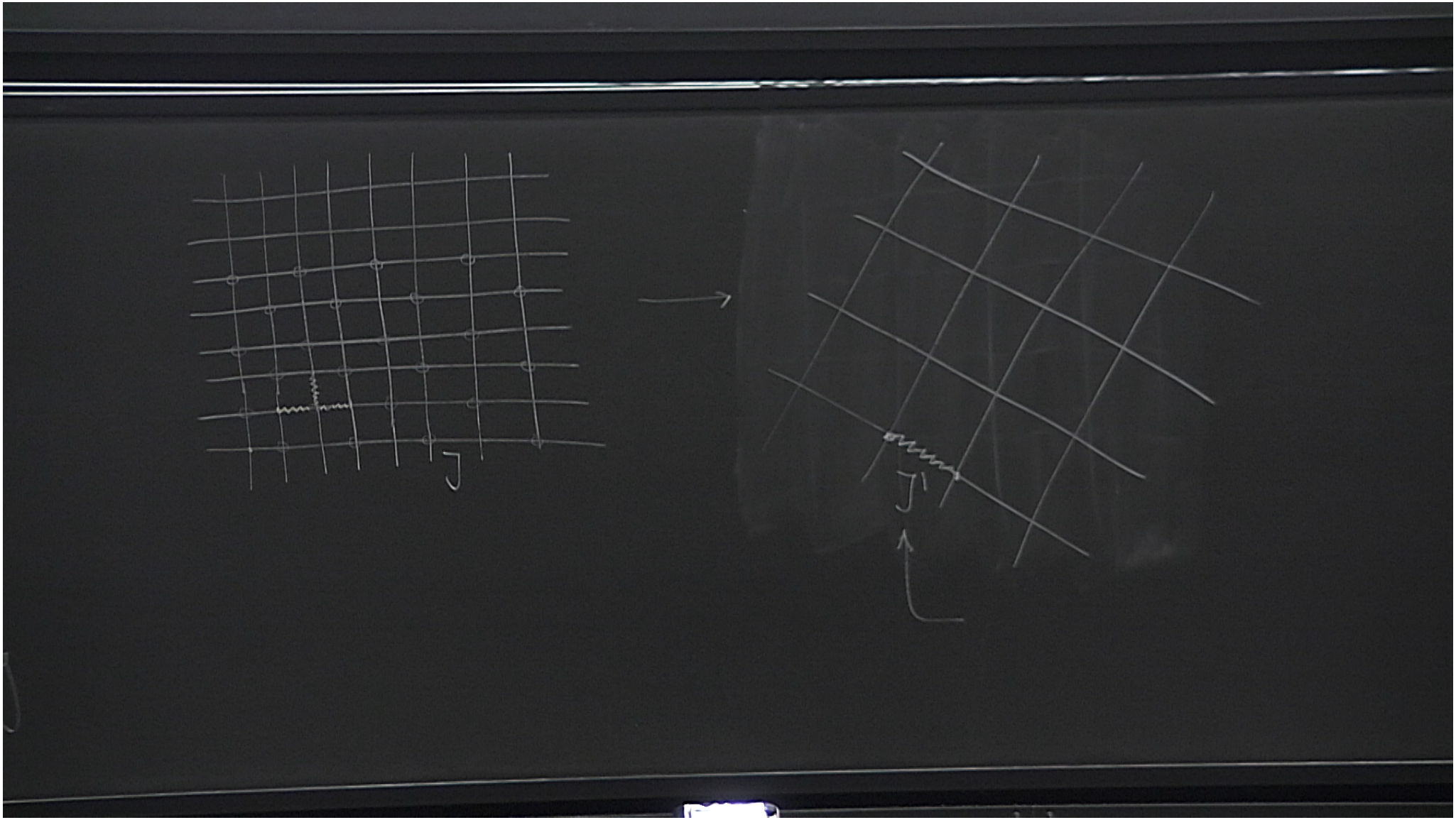
Speakers: Pedro Vieira

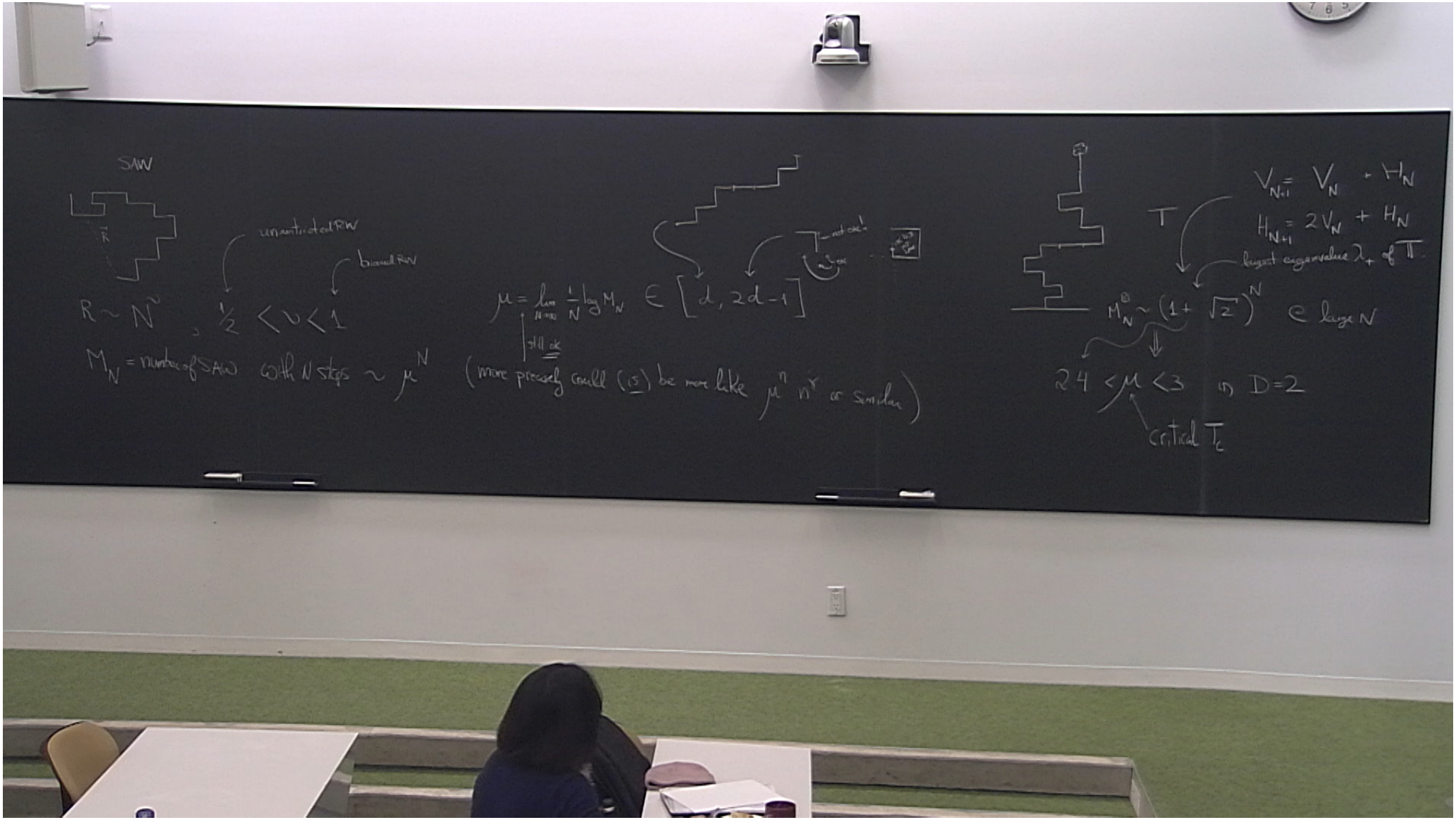
Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

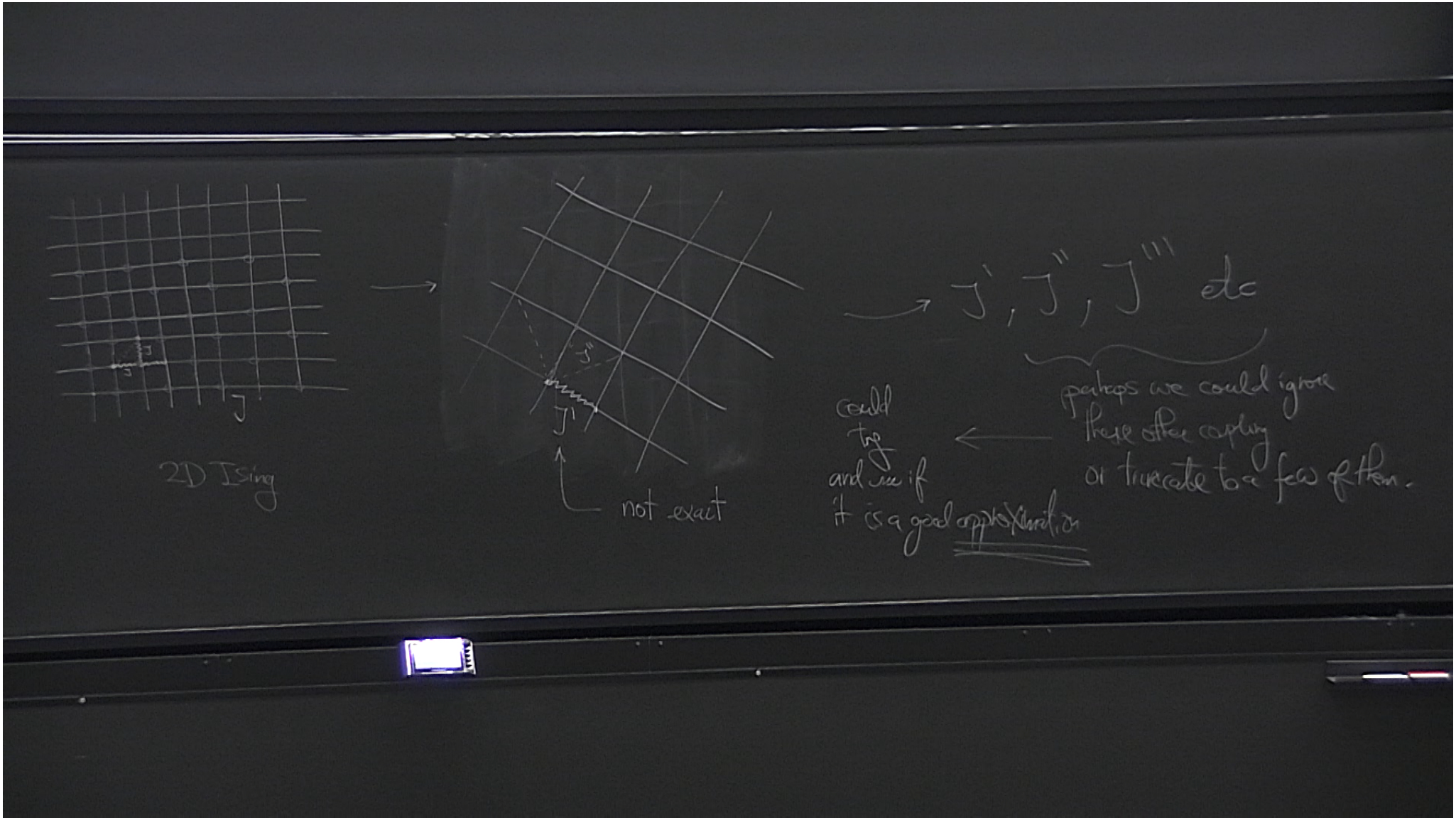
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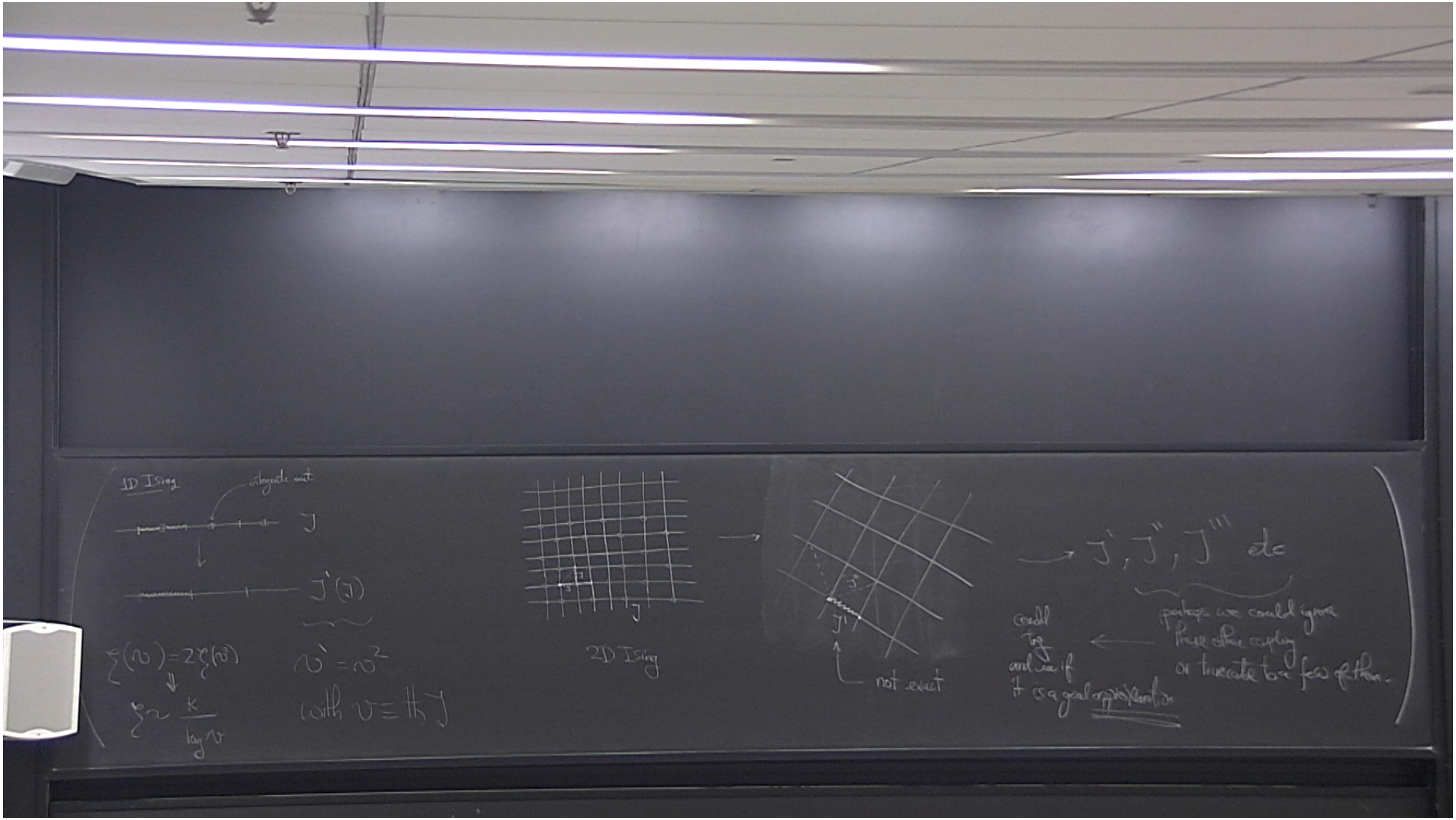
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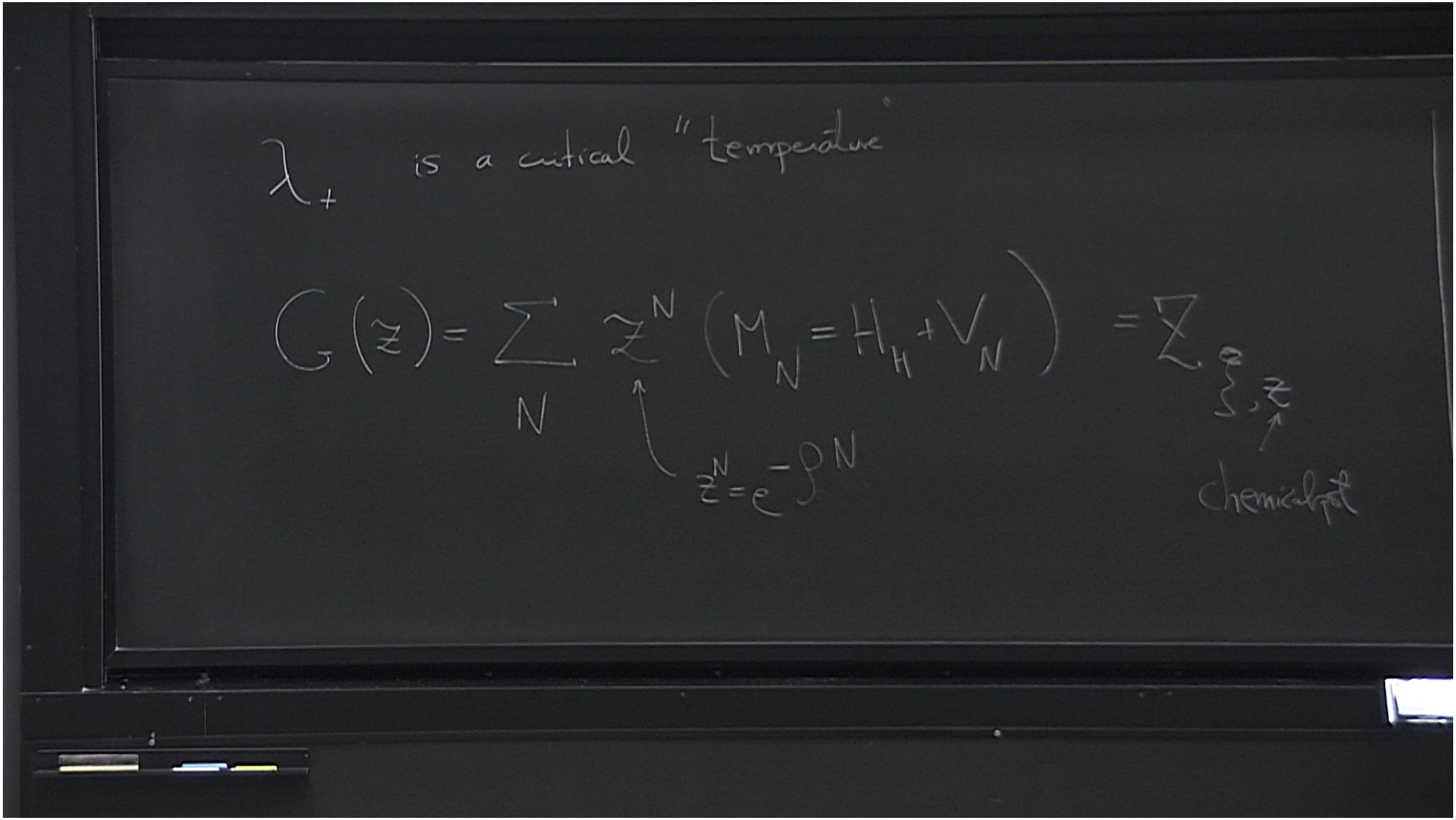

integrate out
J
J'(J)
$\xi(v) = 2\xi(v^2)$
$v' = v^2$
$\xi \sim \frac{K}{\log v}$
with $v \equiv \tanh y$

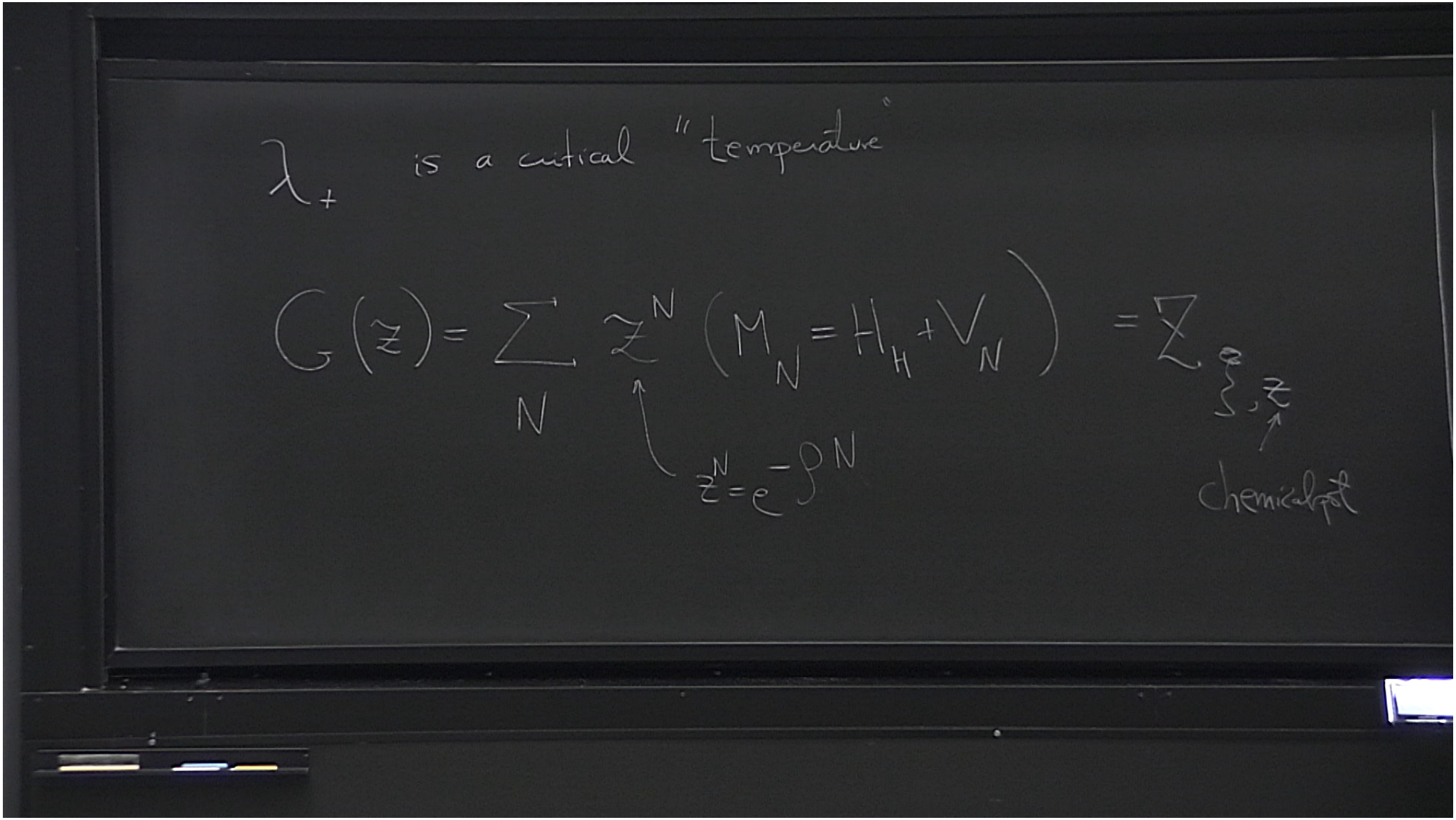

J
J'


SAW
unrestricted RW
biased RW
$R \sim N^{\tilde{\nu}}$, $\frac{1}{2} < \nu < 1$
M_N = number of SAW with N steps $\sim \mu^N$
$\mu = \lim_{N\to\infty} \frac{1}{N} \log M_N \in [d, 2d-1]$
still ok
(more precisely could (is) be more like $\mu^n n^r$ or similar)
not ok!
$V_{N+1} = V_N + H_N$
$H_{N+1} = 2V_N + H_N$
T
largest eigenvalue λ_+ of T
$M_N \sim (1+\sqrt{2})^N$ e large N
$2.4 < \mu < 3$ in $D=2$
critical T_c


J
J
J
2D Ising
J'
J''
not exact
J', J'', J''' etc
perhaps we could ignore
these other coupling
or truncate to a few of them.
could
try
and see if
it is a good approximation


1D Ising
integrate out
J
J'(J)
ξ(v)=2ξ(v²)
ξ ~ K / log v
v' = v²
with v ≡ tanh J
2D Ising
J
J'
not exact
J', J'', J''' etc
perhaps we could ignore these other couplings or truncate to a few of them.
could try and see if it is a good approximation


λ_+ is a critical "temperature"
$G(z) = \sum_N z^N \left(M_N = H_H + V_N \right) = \sum_{\xi, z}$
$z^N = e^{-\rho N}$
chemical pot


λ_+ is a critical "temperature"
$G(z) = \sum_N z^N \left(M_N = H_H + V_N \right) = \Xi_{\xi, z}$
$z^N = e^{-\beta N}$
chemical pot

not exact # is a good approximation

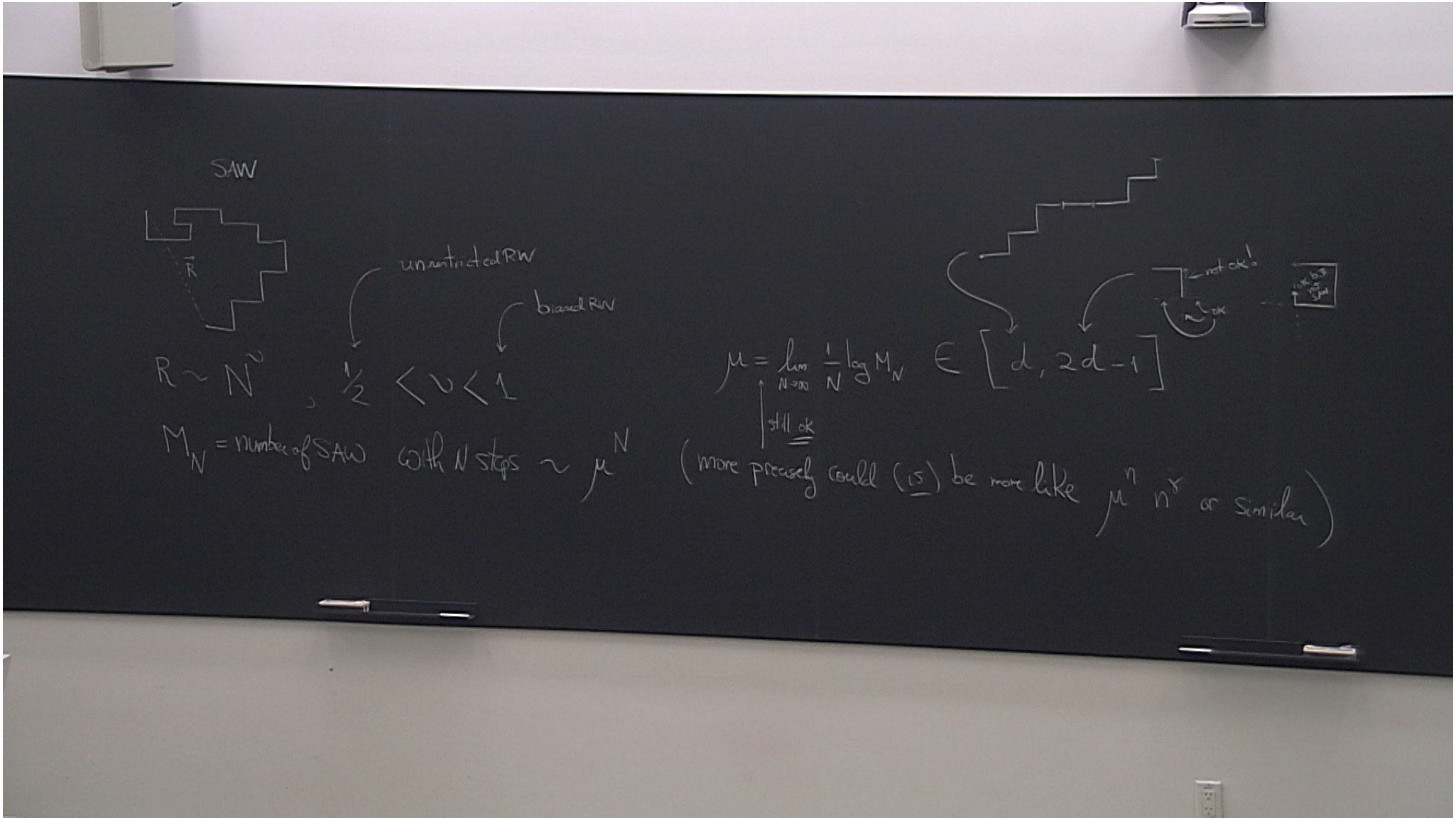
$$\frac{1}{z} Z^{N+1} V_{N+1} = Z^N V_N + Z^N H_N$$

$$\begin{cases} \frac{1}{z}\left(V(z) - z\right) = V(z) + H(z) \\ \frac{1}{z} H(z) = 2V(z) + H(z) \end{cases}$$

$\rightarrow$ $V = \dots$

$H = \dots$

$G = V + H$


SAW
$\vec{R}$
unrestricted RW
biased RW
$R \sim N^{\nu}$, $\frac{1}{2} < \nu < 1$
M_N = number of SAW with N steps $\sim \mu^N$
$\mu = \lim_{N\to\infty} \frac{1}{N} \log M_N \in [d, 2d-1]$
still ok
(more precisely could (is) be more like $\mu^n n^{\gamma}$ or similar)
not ok!
ok
not ok!

$$G(z) = \frac{z(1+z)}{1-2z-z^2}$$

starts small at $z=0$

∞ if $\rho \gg 1$

What is μ? $(M_N \sim \mu^N)$

λ_+ is a critical "temperature"

$$G(z) = \sum_N z^N \left(M_N = H_N + V_N\right) = \sum_{\rho, z}$$

$z^N = e^{-\beta N}$

Chemical pt

$$\frac{1}{z} Z^{N+1} V_{N+1} = z^N V_N + z^N H_N$$

$$\begin{cases} \frac{1}{z}\left(V(z) - z\right) = V(z) + H(z) \\ \frac{1}{z} H(z) = 2V(z) + H(z) \end{cases} \longrightarrow \begin{array}{l} V = \dots \\ H = \dots \\ G = V + H \end{array}$$

$$\boxed{G(z) = \frac{z(1+z)}{1-2z-z^2}}$$

starts small at $z=0$

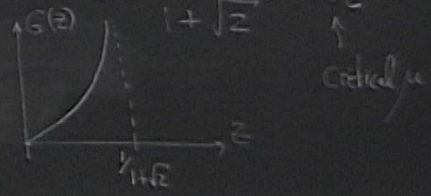
$\neq$ for $\rho \gg 1$

What is μ? $\left(M_N \sim \mu^N\right)$

$1 - 2z - z^2 = 0 \Longrightarrow \frac{1}{z} = 1 \pm \sqrt{2}$

$$G = G' + \frac{C'}{z - z_1} + \frac{C''}{z - z_2}$$

smallest $z = \frac{1}{1+\sqrt{2}} \equiv z_c$

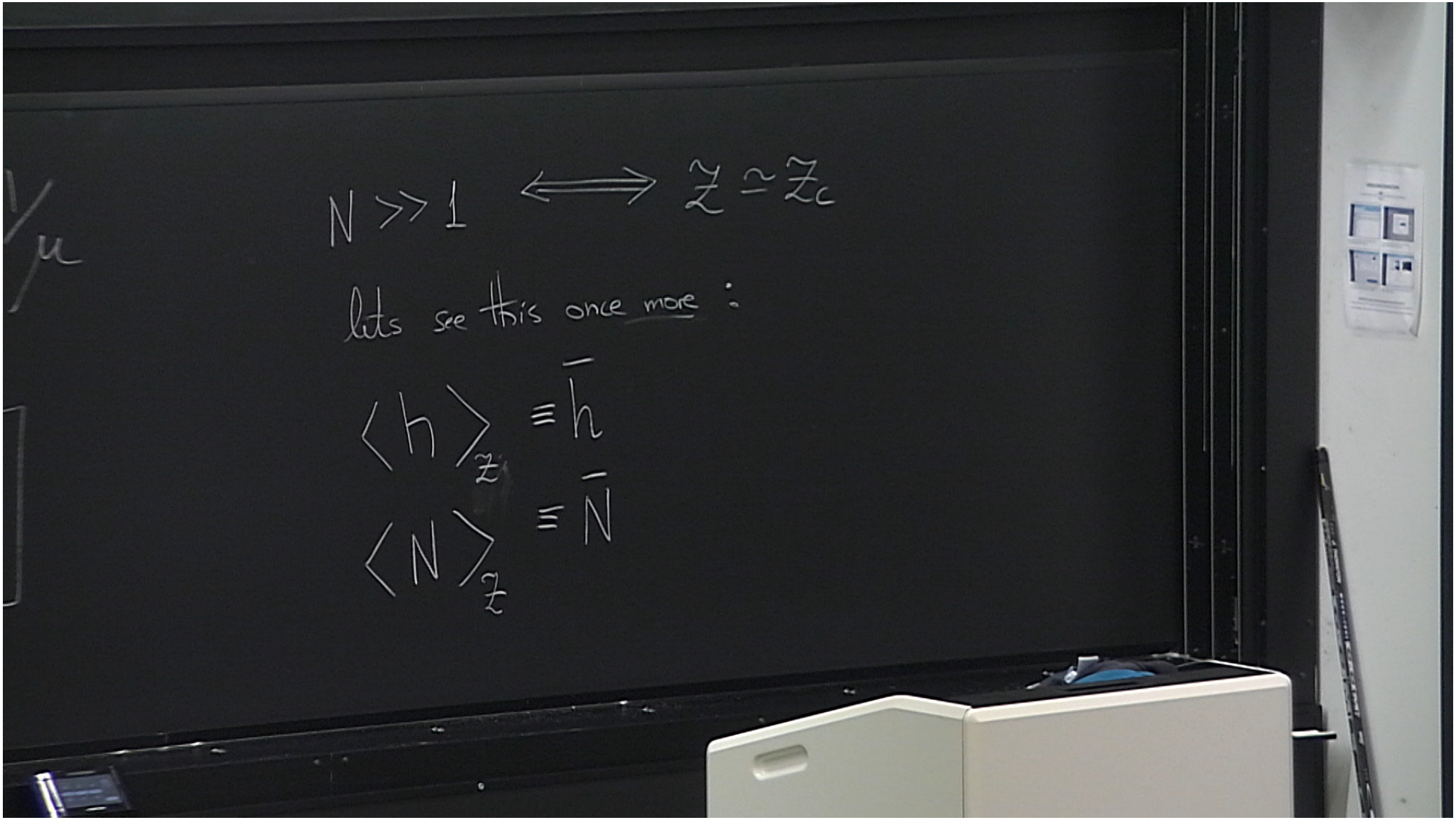


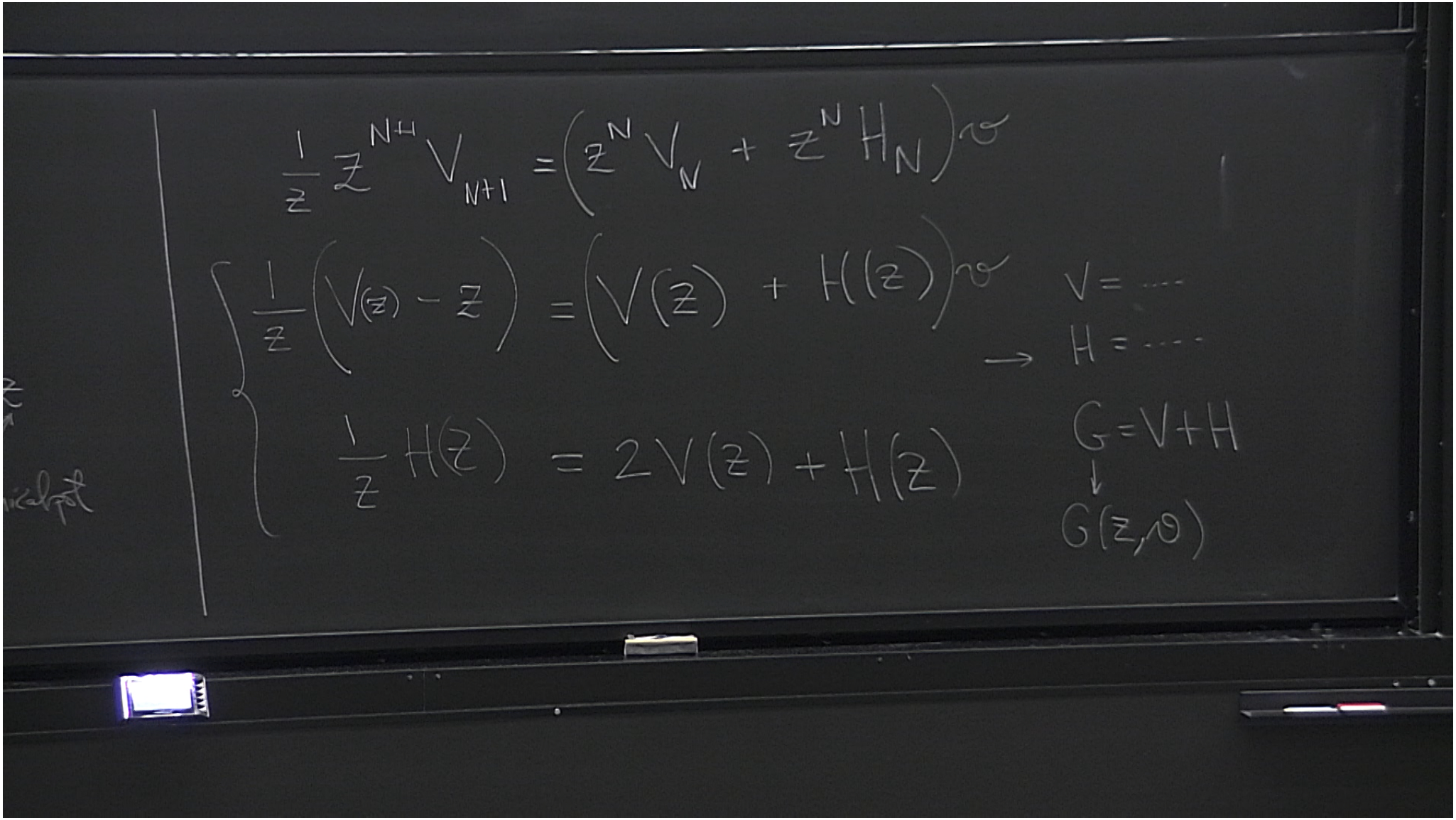
$$G(z) = \sum z^N M_N \quad \leftarrow \text{radius of convergence} \equiv 1/\mu$$

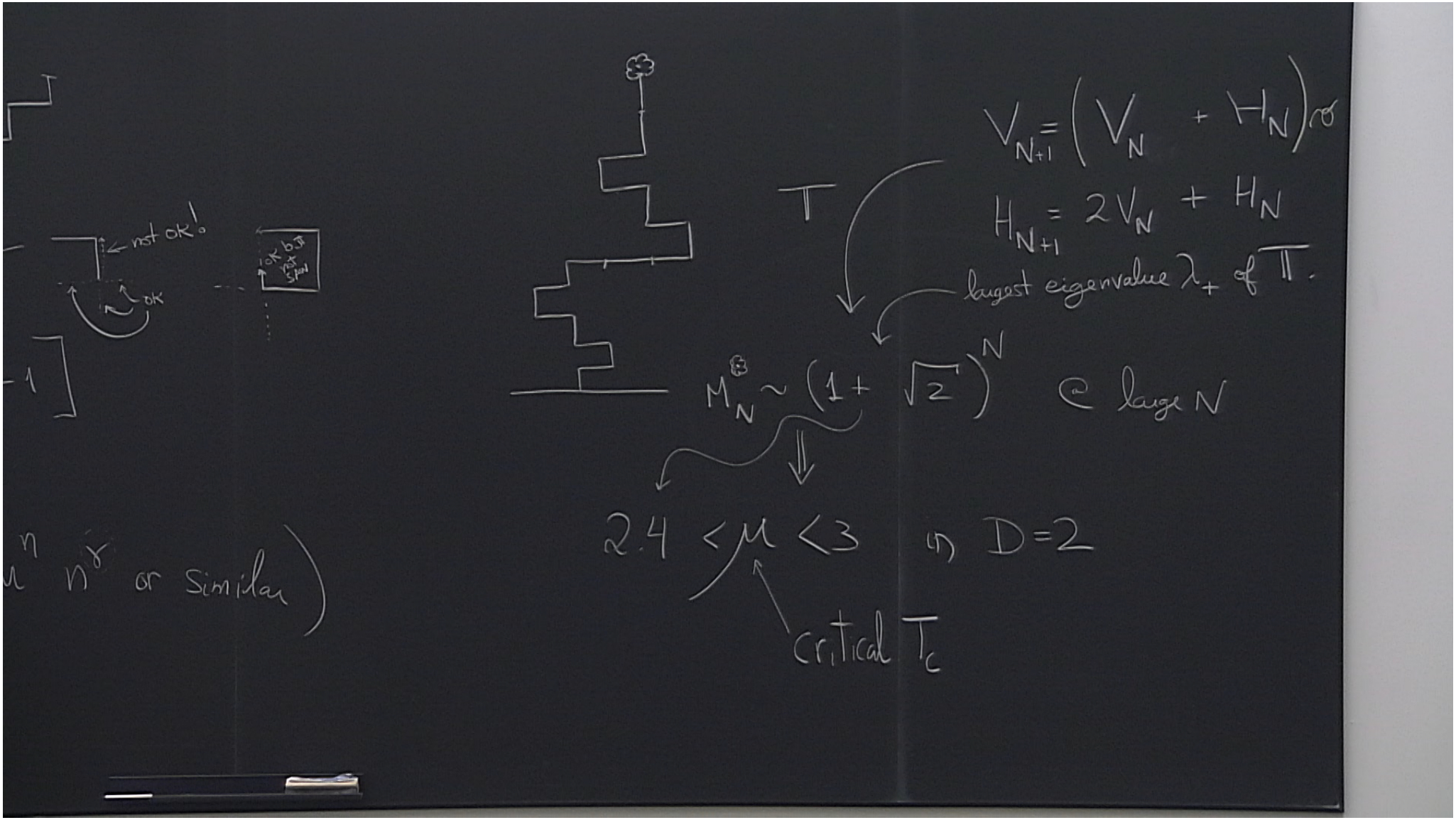
$$M_N \sim \mu^N$$

$$\sim \frac{1}{1 - z\mu} + \text{reg} \quad \text{as } z \simeq 1/\mu$$

$$\boxed{\mu = 1/z_c}$$


$N \gg 1 \iff z \simeq z_c$
lets see this once more :
$\langle h \rangle_z \equiv \bar{h}$
$\langle N \rangle_z \equiv \bar{N}$


$\frac{1}{z} z^{N+1} V_{N+1} = \left(z^N V_N + z^N H_N \right) \sigma$
$\frac{1}{z}\left(V(z) - z \right) = \left(V(z) + H(z) \right) \sigma$
$V = \cdots$
$\rightarrow H = \cdots$
$\frac{1}{z} H(z) = 2V(z) + H(z)$
$G = V + H$
$G(z, 0)$


not ok!
ok
ok but not stay
T
$V_{N+1} = (V_N + H_N)$ ⊗
$H_{N+1} = 2V_N + H_N$
largest eigenvalue λ_+ of $\mathbb{T}$.
$M_N \sim (1+\sqrt{2})^N$ @ large N
$2.4 < \mu < 3$ ⇒ D=2
critical T_c
or Similar)

$$G = \frac{z\nu\left(z z + \nu(1-z)\right)}{1 - z(\nu+1) - \nu z^2} \longrightarrow \text{previous.}$$

$\nu = 1$

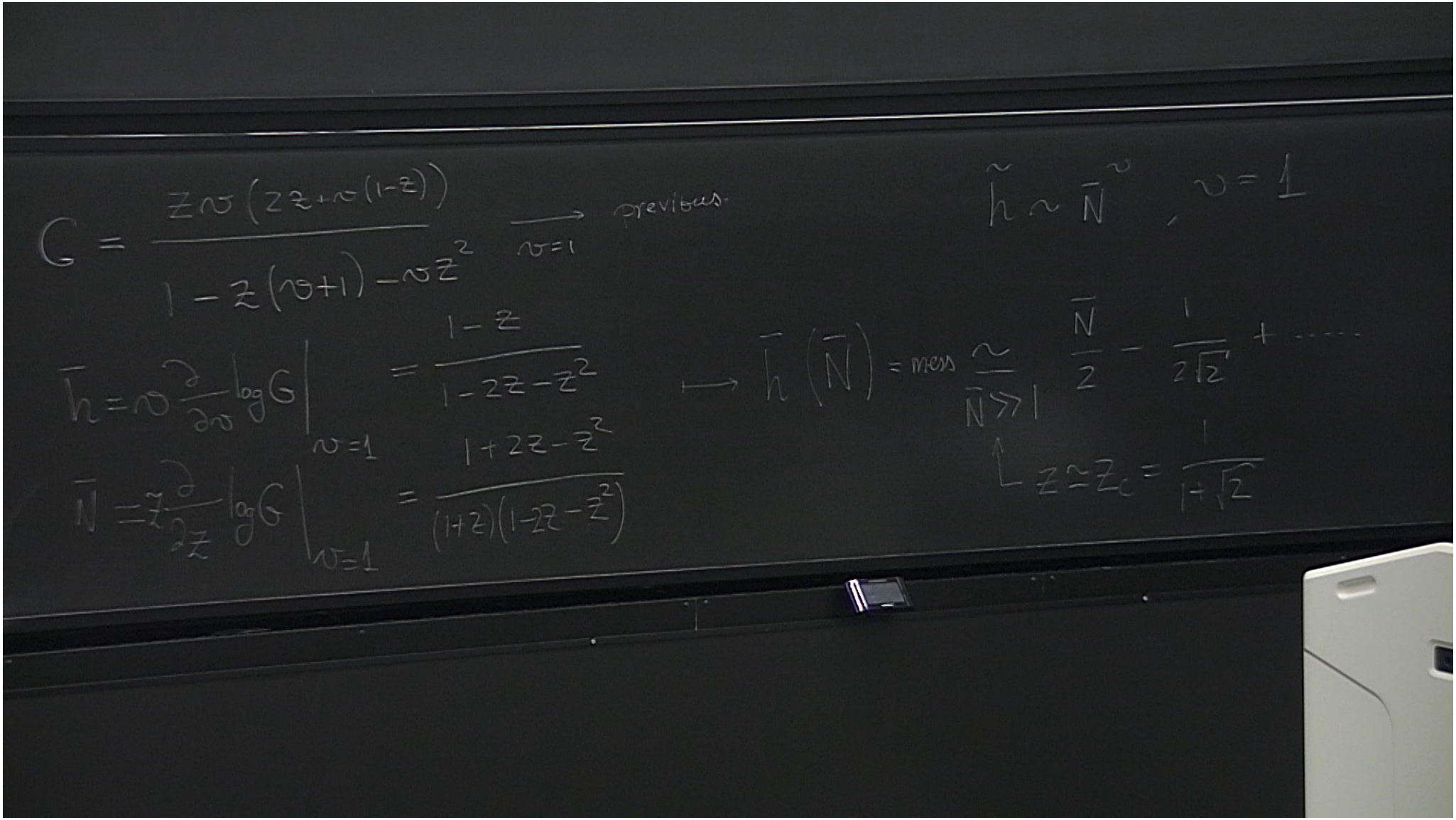
$$\bar{h} = \nu \frac{\partial}{\partial \nu} \log G \Big|_{\nu=1} = \frac{1-z}{1-2z-z^2}$$

$$\bar{N} = z \frac{\partial}{\partial z} \log G \Big|_{\nu=1} = \frac{1+2z-z^2}{(1+z)(1-2z-z^2)}$$

$$G = \frac{z\nu\left(z z + \nu(1-z)\right)}{1 - z(\nu+1) - \nu z^2} \xrightarrow[\nu=1]{} \text{previous.}$$

$$\bar{h} = \nu \frac{\partial}{\partial \nu} \log G \Big|_{\nu=1} = \frac{1-z}{1-2z-z^2} \longrightarrow \bar{h}(\bar{N})$$

$$\bar{N} = z \frac{\partial}{\partial z} \log G \Big|_{\nu=1} = \frac{1+2z-z^2}{(1+z)(1-2z-z^2)}$$


$G = \frac{z\nu(z+\nu(1-z))}{1-z(\nu+1)-\nu z^2} \xrightarrow[\nu=1]{} \frac{1-z}{1-2z-z^2}$
previous
$\tilde{h} \sim \bar{N}^{\nu}, \quad \nu = 1$
$\bar{h} = \nu \frac{\partial}{\partial \nu} \log G \Big|_{\nu=1}$
$\bar{N} = z \frac{\partial}{\partial z} \log G \Big|_{\nu=1} = \frac{1+2z-z^2}{(1+z)(1-2z-z^2)}$
$\bar{h}(\bar{N}) = \text{meas} \underset{\bar{N} \gg 1}{\simeq} \frac{\bar{N}}{2} - \frac{1}{2\sqrt{2}} + \dots$
$z \simeq z_c = \frac{1}{1+\sqrt{2}}$

previous:

$$\bar{h} \sim \bar{N}^{\nu}, \quad \nu = 1$$

expected!

bias

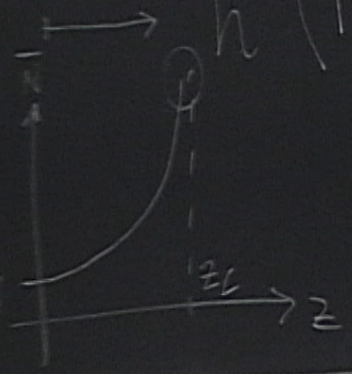
$$\bar{h}(\bar{N}) = \text{mess} \underset{\bar{N} \gg 1}{\simeq} \frac{\bar{N}}{2} - \frac{1}{2\sqrt{2}} + \cdots$$

$$z \simeq z_c = \frac{1}{1+\sqrt{2}}$$

$$\bar{h} \simeq \frac{\#}{(z - z_c)^{\nu}} \quad \nu \neq 1$$

$$\bar{h} \sim \bar{N}^{\nu}$$

$$\bar{N} \simeq \frac{\#}{z - z_c}$$



$$\langle h \rangle_z \equiv \bar{h}$$

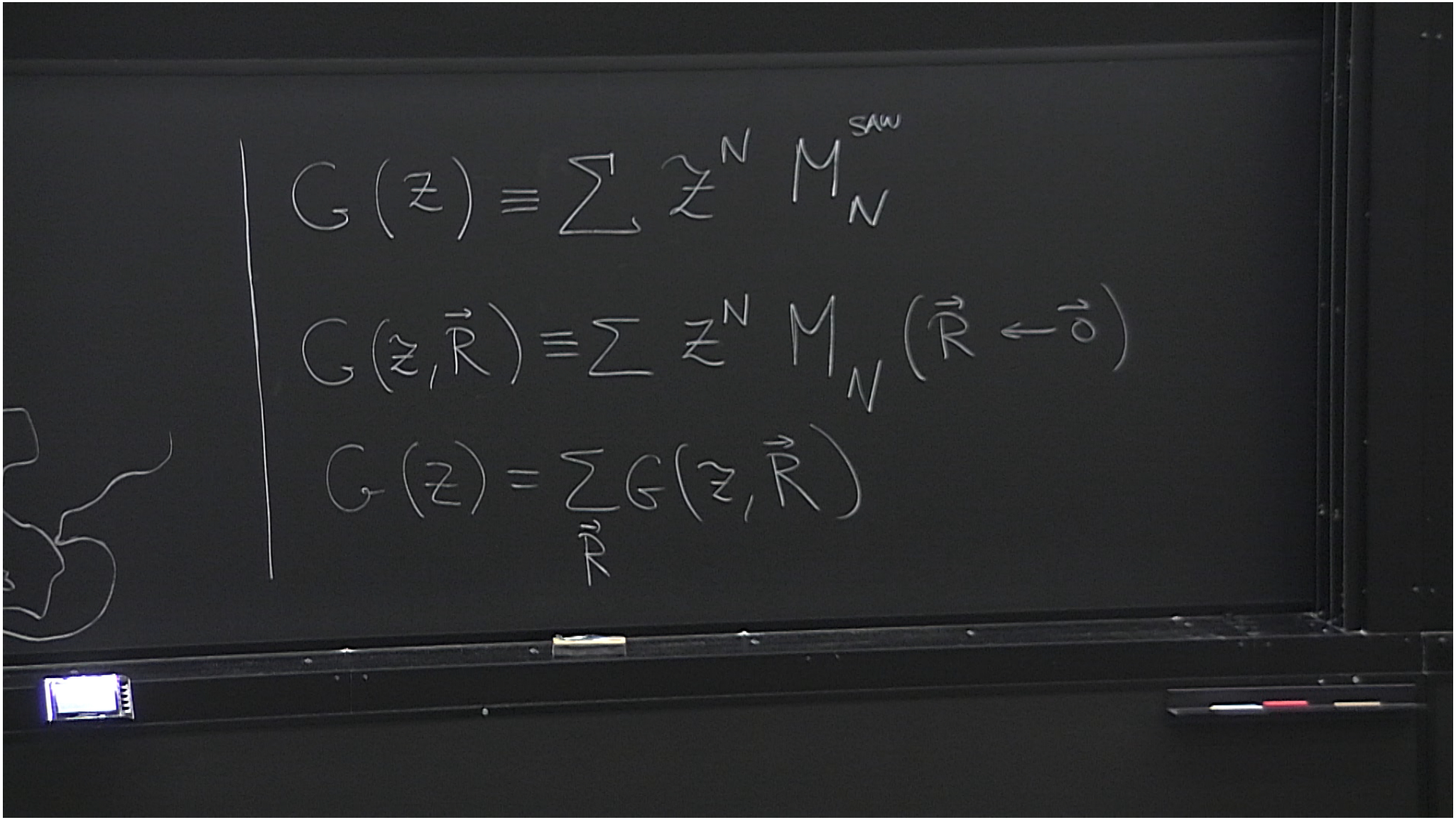
Idea

* $G_{SAW} \sim$ close to $Z_c \sim \equiv$

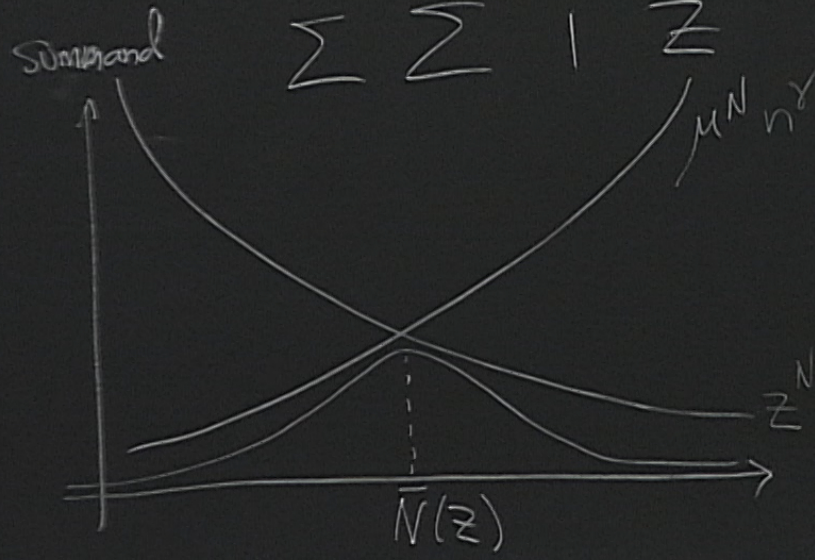
* approximate RG (as above)

* try to do better ⟶ continuum SAW

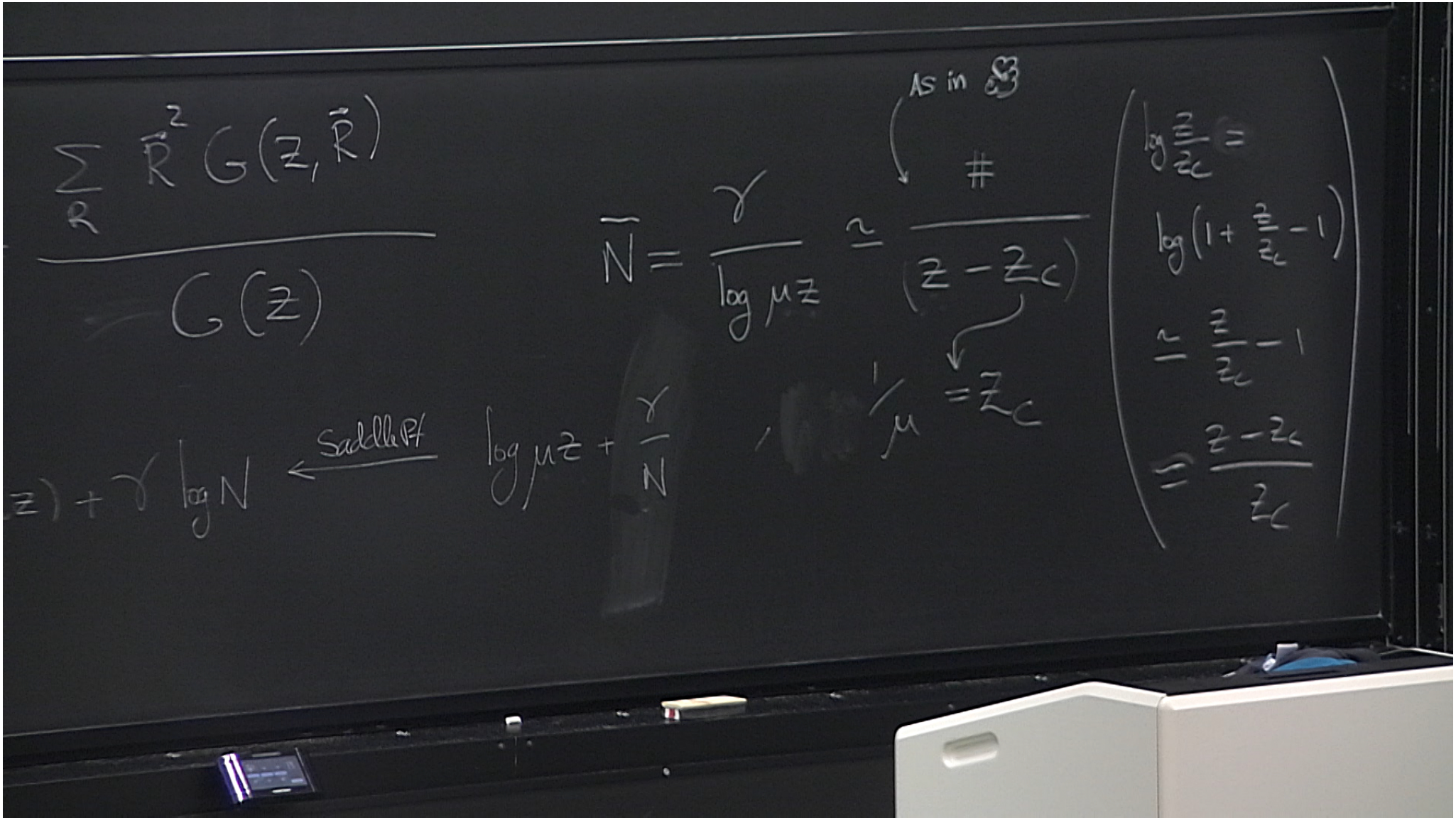



$G(z) \equiv \sum z^N M_N^{SAW}$
$G(z,\vec{R}) \equiv \sum z^N M_N(\vec{R} \leftarrow \vec{0})$
$G(z) = \sum_{\vec{R}} G(z,\vec{R})$

$$\xi^2(z) \equiv \frac{\sum_N \sum_{\vec{R}} \vec{R}^2 z^N M_N(\vec{R})}{\sum_N \sum_{\vec{R}} 1 \, z^N M_N(\vec{R})} \sim \mu^N N^\gamma = \frac{\sum_R \vec{R}^2 G(z, \vec{R})}{G(z)}$$



$$e^{N \log(\mu z) + \gamma \log N}$$ ← Saddle Pt


$\frac{\sum_R \vec{R}^2 G(z,\vec{R})}{G(z)}$
$\tilde{N} = \frac{\gamma}{\log \mu z} \simeq \frac{\#}{(z - z_c)}$
As in
$\log \frac{z}{z_c} =$
$\log\left(1 + \frac{z}{z_c} - 1\right)$
$\simeq \frac{z}{z_c} - 1$
$= \frac{z - z_c}{z_c}$
$\frac{1}{\mu} = z_c$
$\log \mu z + \frac{\gamma}{N}$
Saddle Pt
$+ \gamma \log N$

$$\bar{N} \propto \frac{1}{z - z_c}$$

Goal using RG ideas

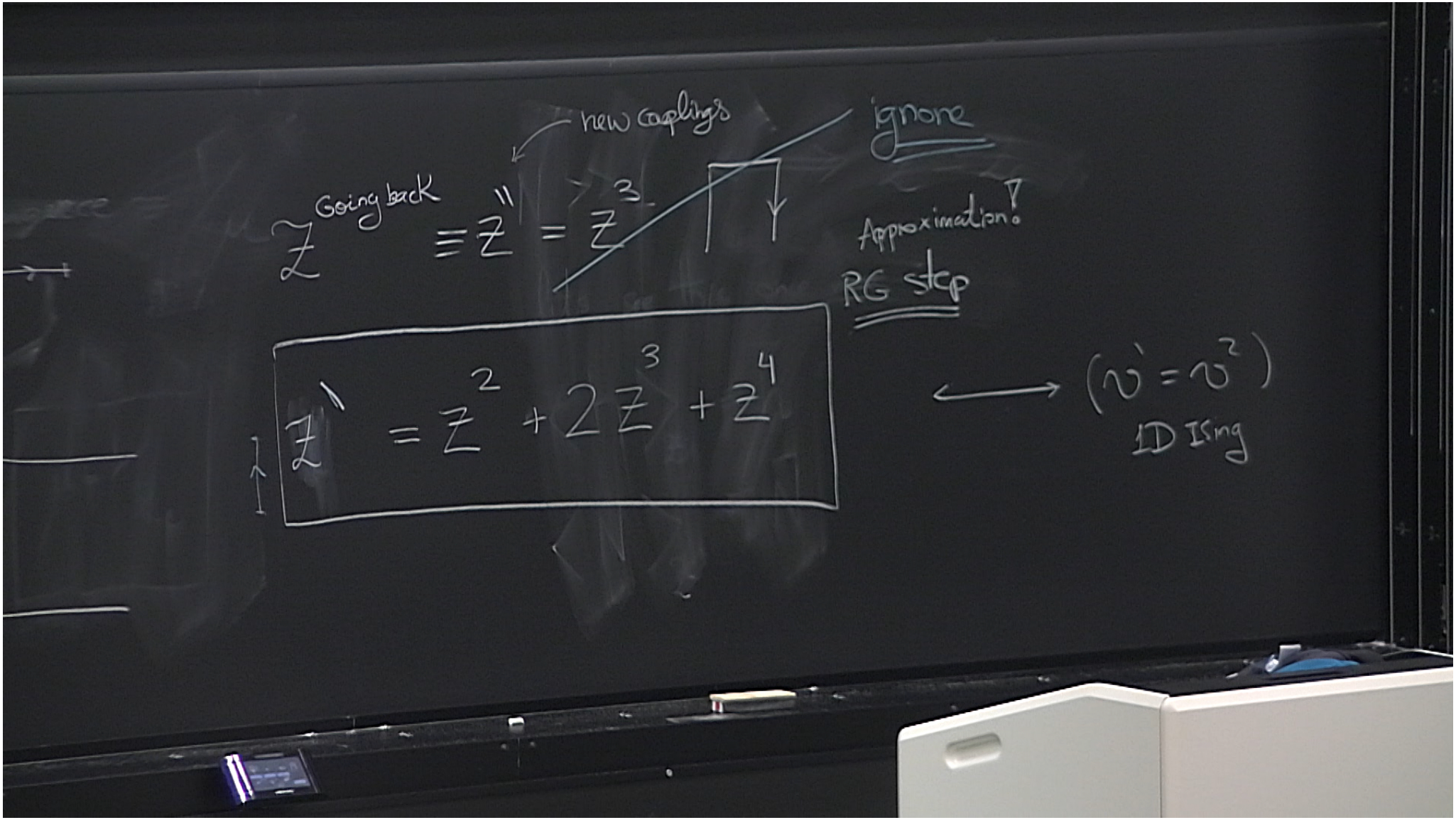
z fixed

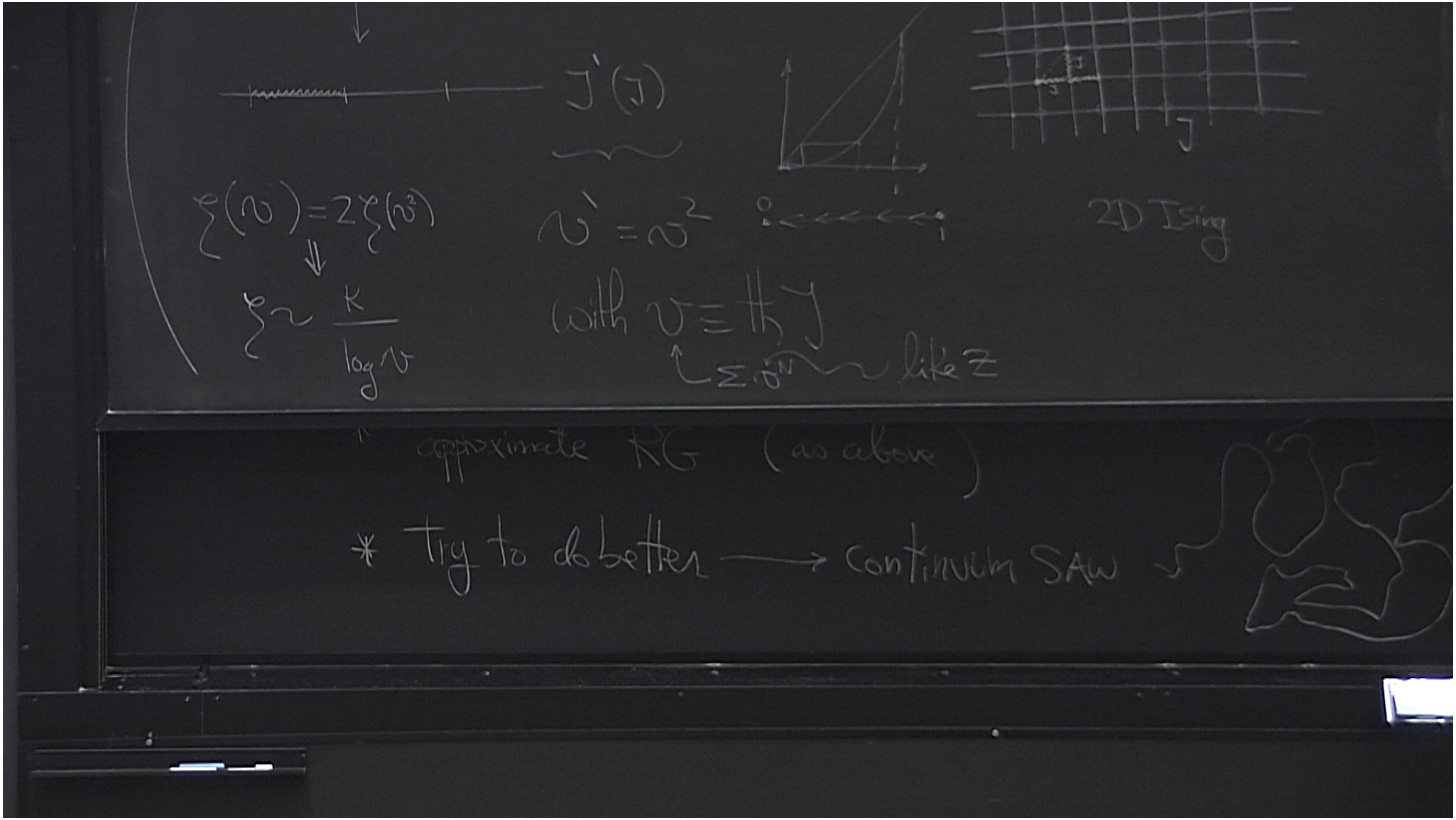
N fixed

if $\xi^2_{(2)} = \bar{R}^2 \sim \frac{1}{(z - z_c)^{2\nu}}$

then ν is the critical exponent we are after: $R^2 \sim N^{2\nu}$

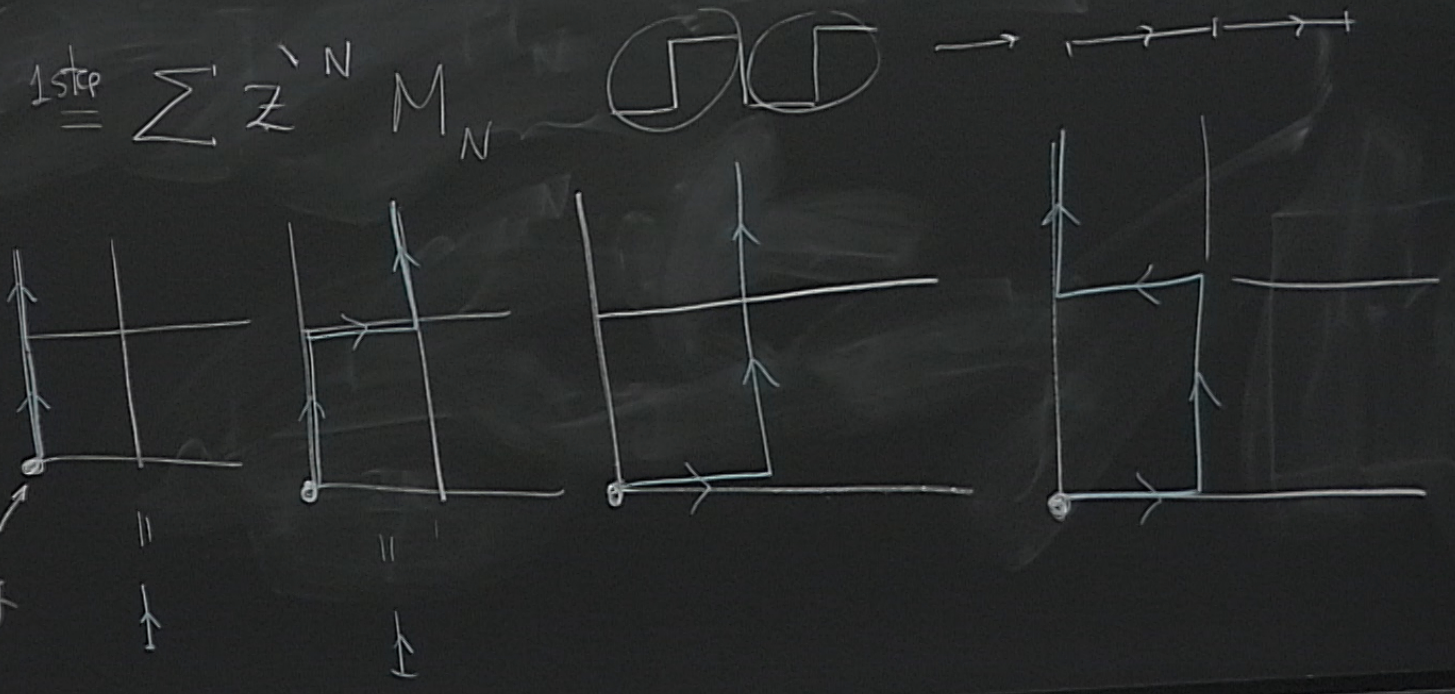


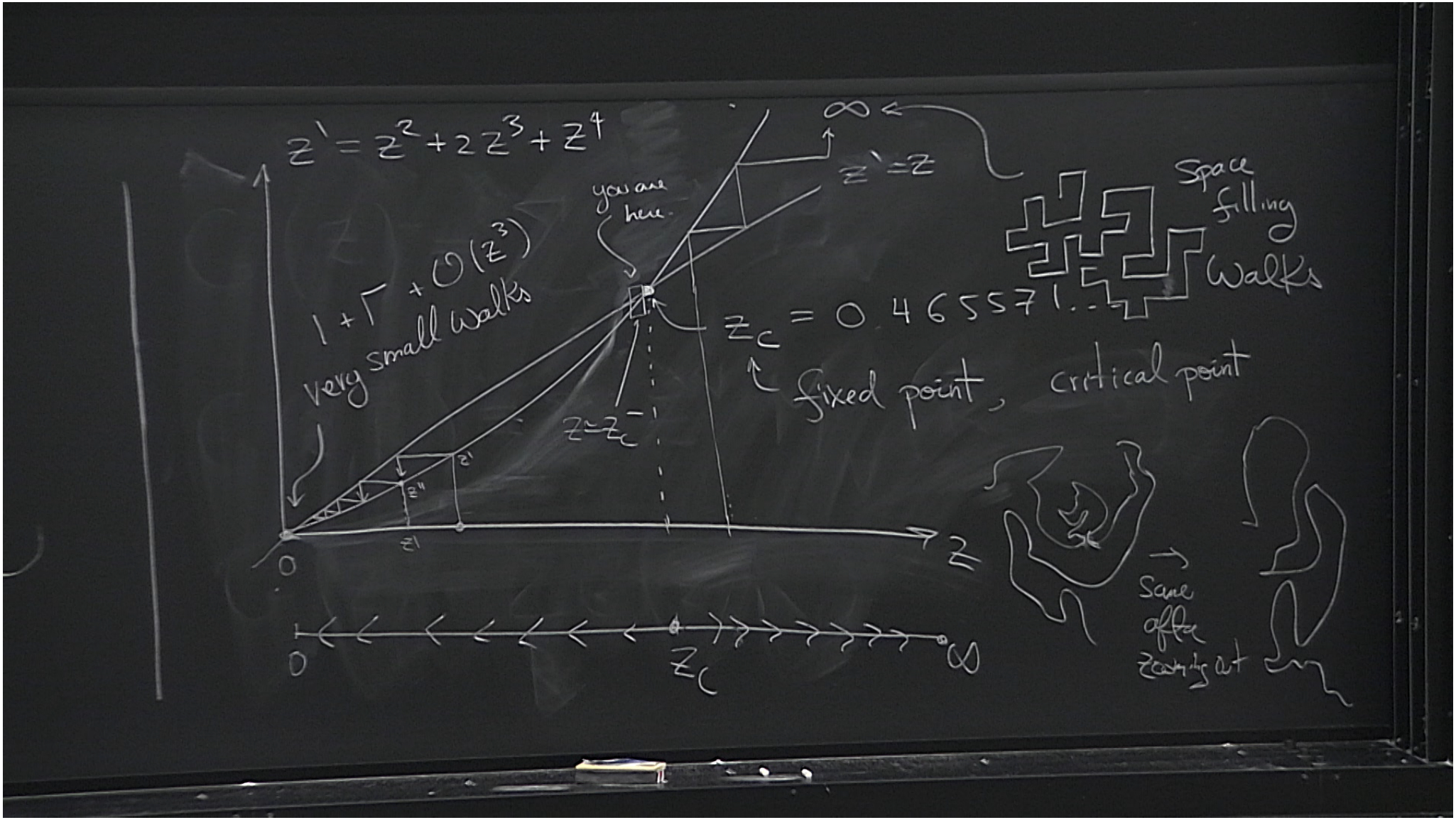

new couplings
ignore
Going back
$Z \equiv Z' = Z''$
$Z' = Z^2 + 2Z^3 + Z^4$
Approximation!
RG step
$(v' = v^2)$
1D Ising

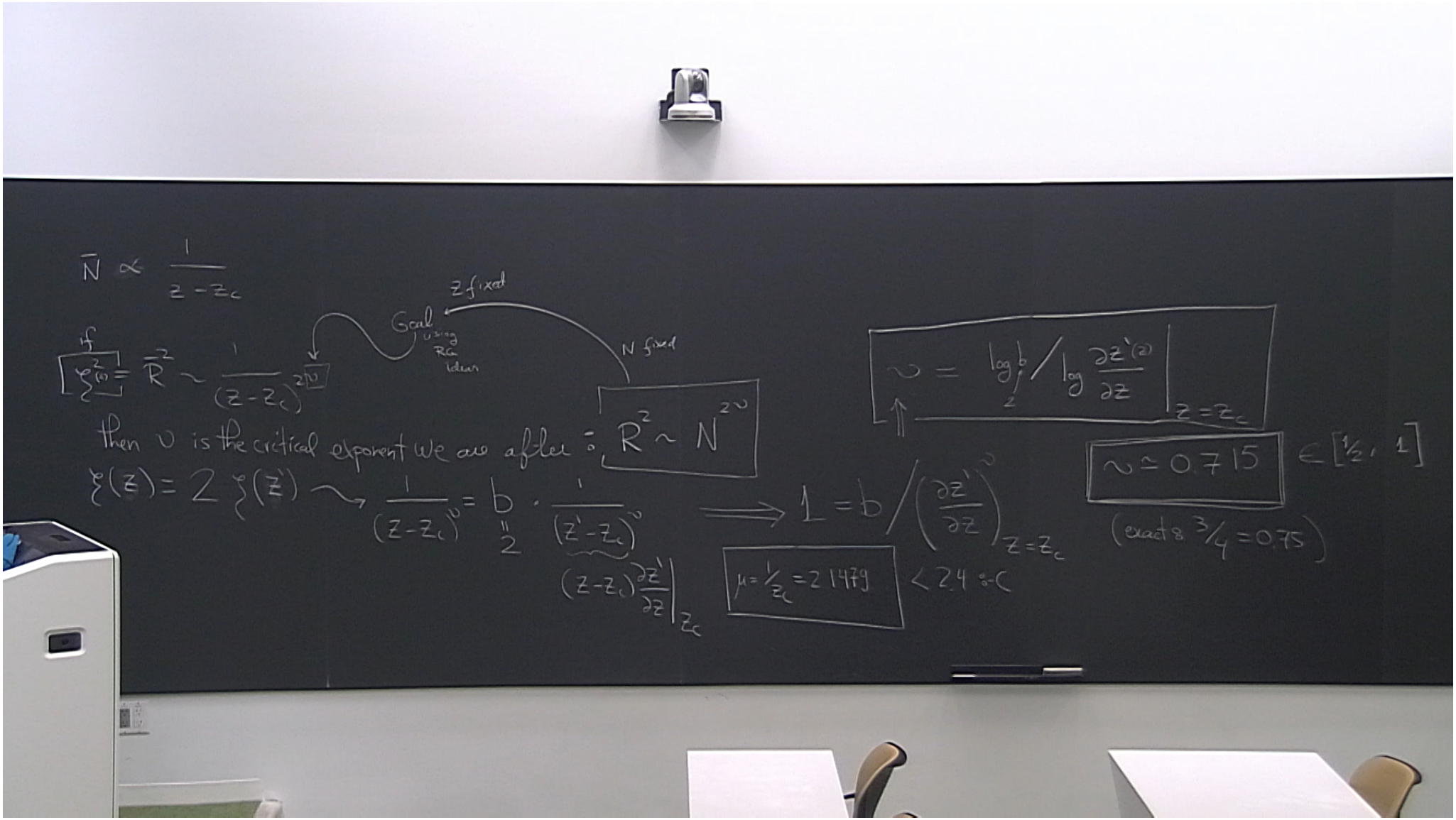

J'(J)
ξ(v) = 2ξ(v²)
ξ ~ K / log v
v' = v²
with v = tanh J
~ Σ σ^N ~ like Z
2D Ising
J
approximate RG (as above)
* try to do better → continuum SAW

$$G(z) = \sum z^N M_N$$

$$\overset{1\text{ step}}{=} \sum z^{N} M_N$$




$z' = z^2 + 2z^3 + z^4$
∞
$z' = z$
you are here
Space filling Walks
$1 + \Gamma + O(z^3)$
very small walks
$z_c = 0.465571\ldots$
fixed point, critical point
$z = z_c^-$
z'
z''
z_1
0
z
0
z_c
∞
Same after zooming out


$\bar{N} \propto \frac{1}{z - z_c}$
z fixed
Goal using RG ideas
N fixed
if $\xi^2(z) = \bar{R}^2 \sim \frac{1}{(z - z_c)^{2\nu}}$
$R^2 \sim N^{2\nu}$
$\nu = \log_b 2 \Big/ \log \frac{\partial z'(z)}{\partial z}\Big|_{z=z_c}$
$\nu \simeq 0.715 \in [\frac{1}{2}, 1]$
(exact $= 3/4 = 0.75$)
then ν is the critical exponent we are after :
$\xi(z) = 2\,\xi(z') \rightsquigarrow \frac{1}{(z - z_c)^{\nu}} = b \cdot \frac{1}{(z' - z_c)^{\nu}}$
$\Longrightarrow 1 = b \Big/ \left(\frac{\partial z'}{\partial z}\right)^{\nu}_{z=z_c}$
$b = 2$
$(z - z_c)\frac{\partial z'}{\partial z}\Big|_{z_c}$
$\mu = 1/z_c = 2.1479$ < 2.4 :(