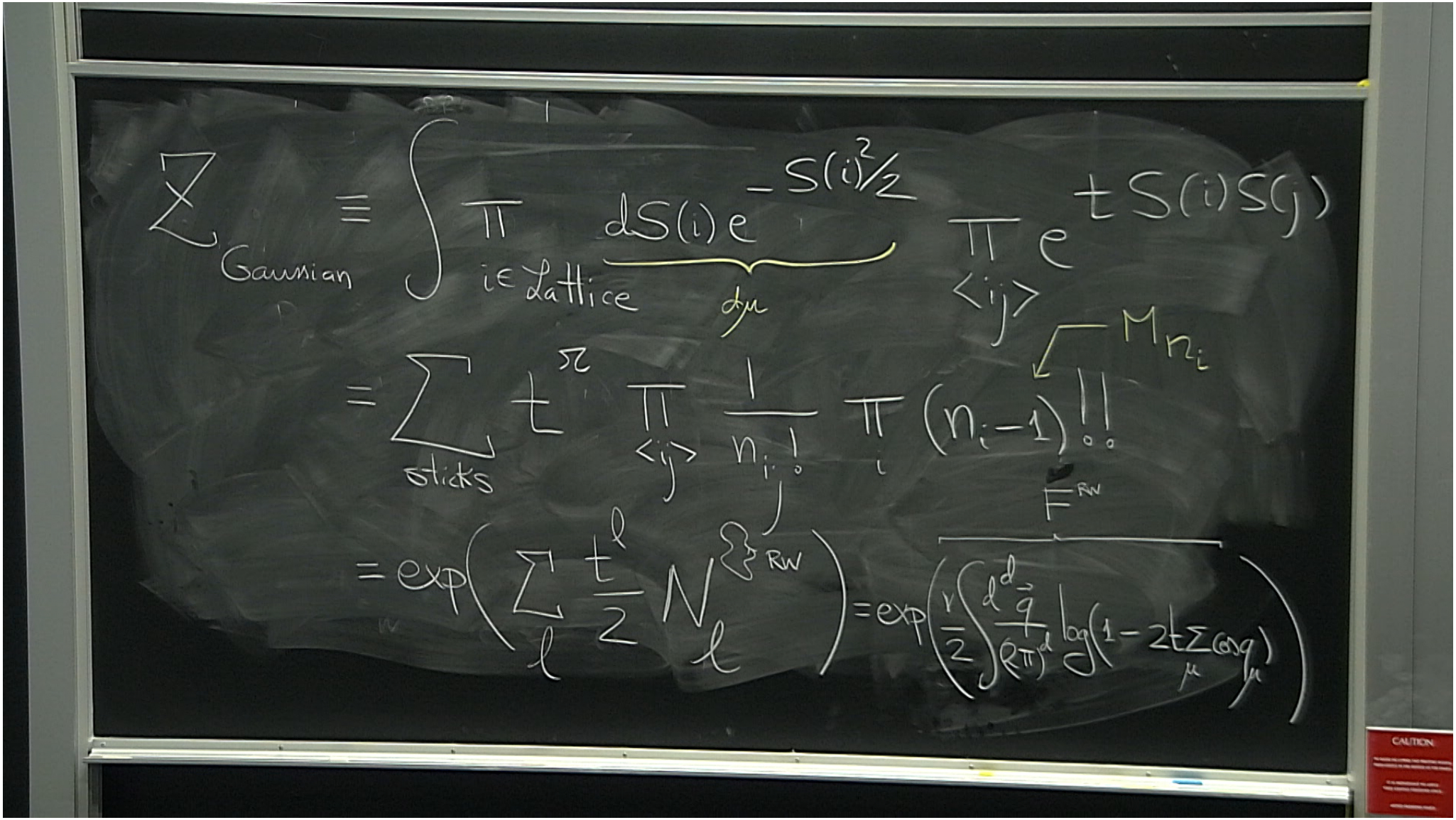
Title: PSI 2019/2020 - Statistical Mechanics (Vieira) - Lecture 8

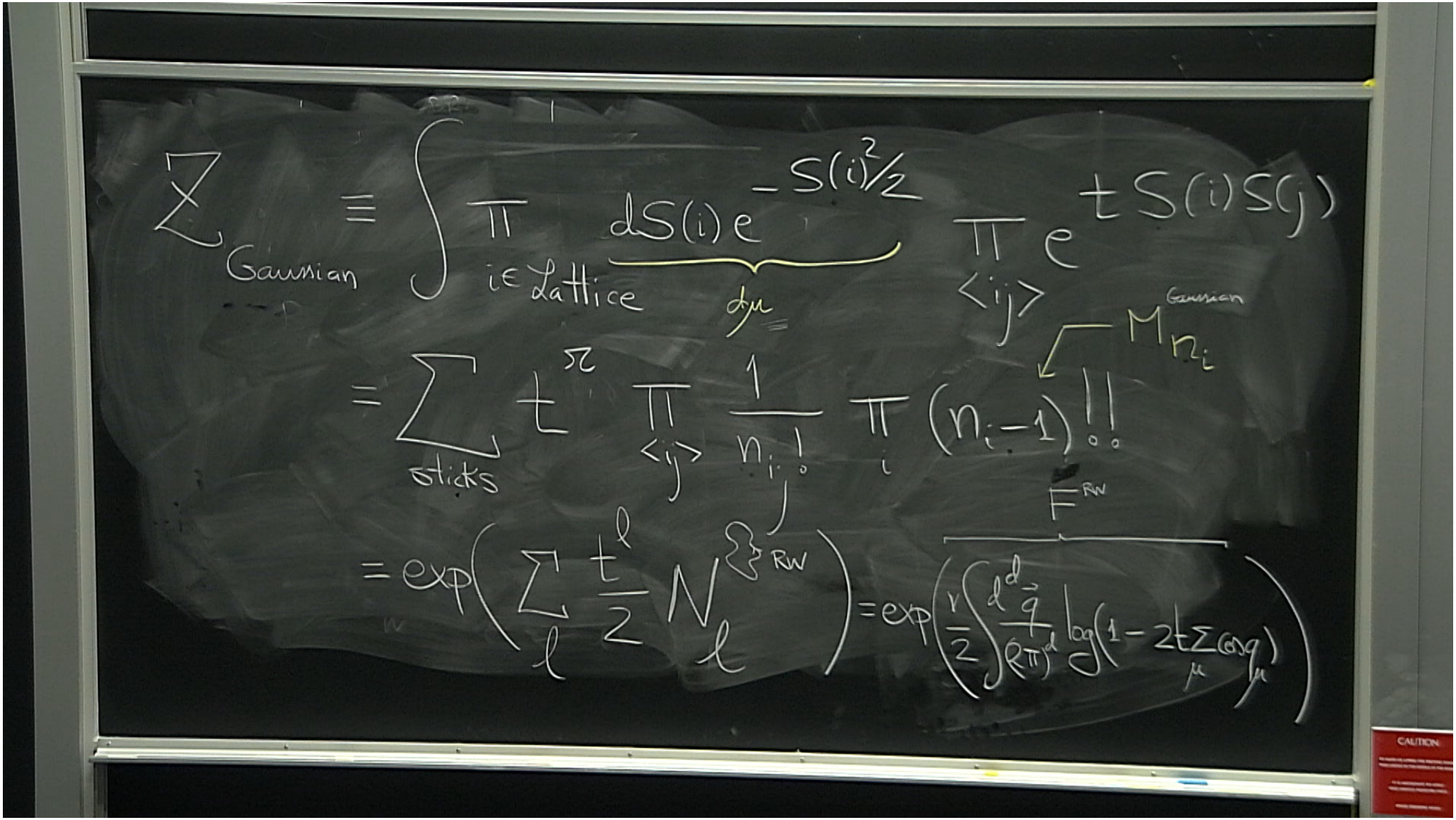
Speakers: Pedro Vieira

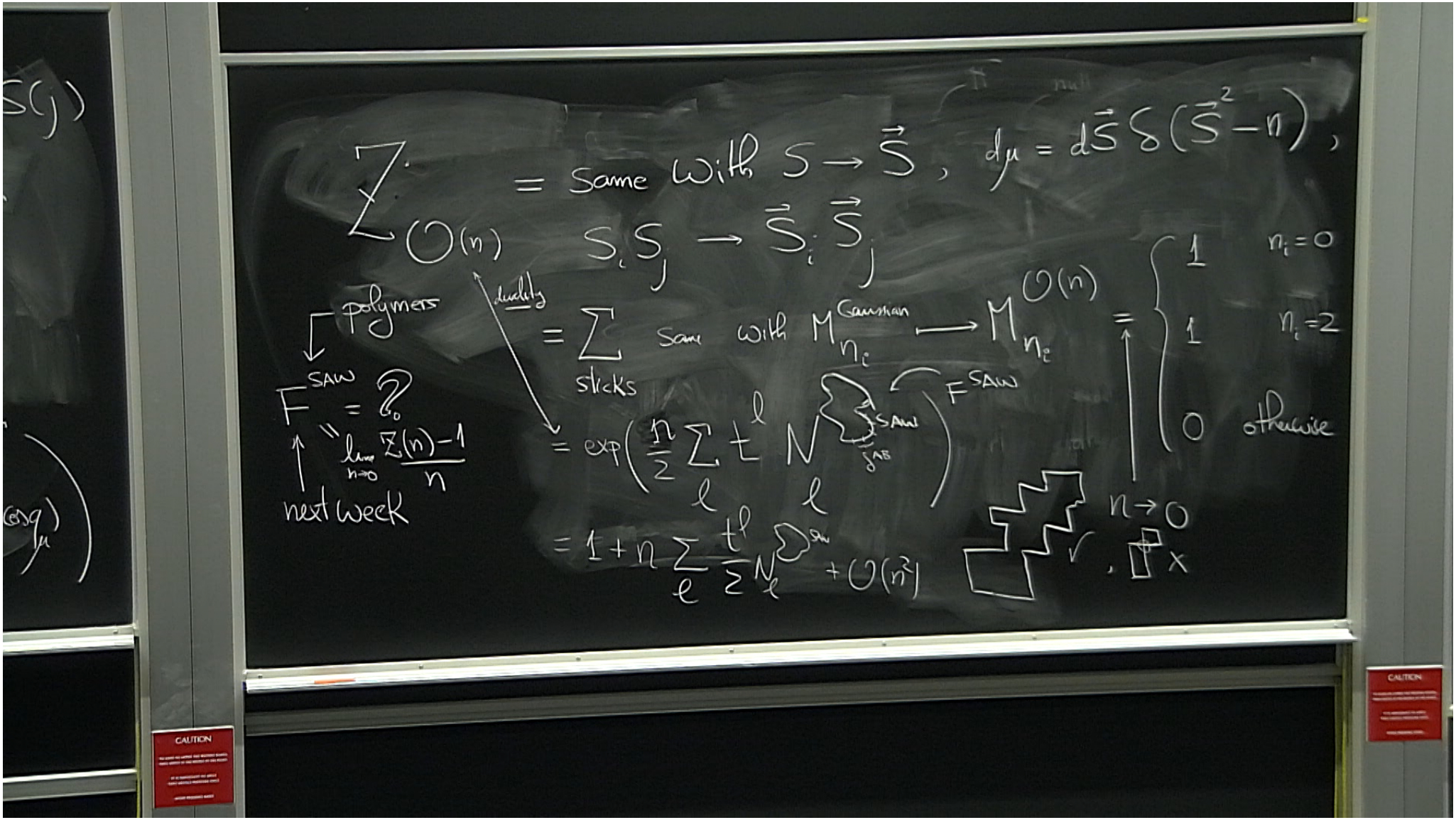
Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

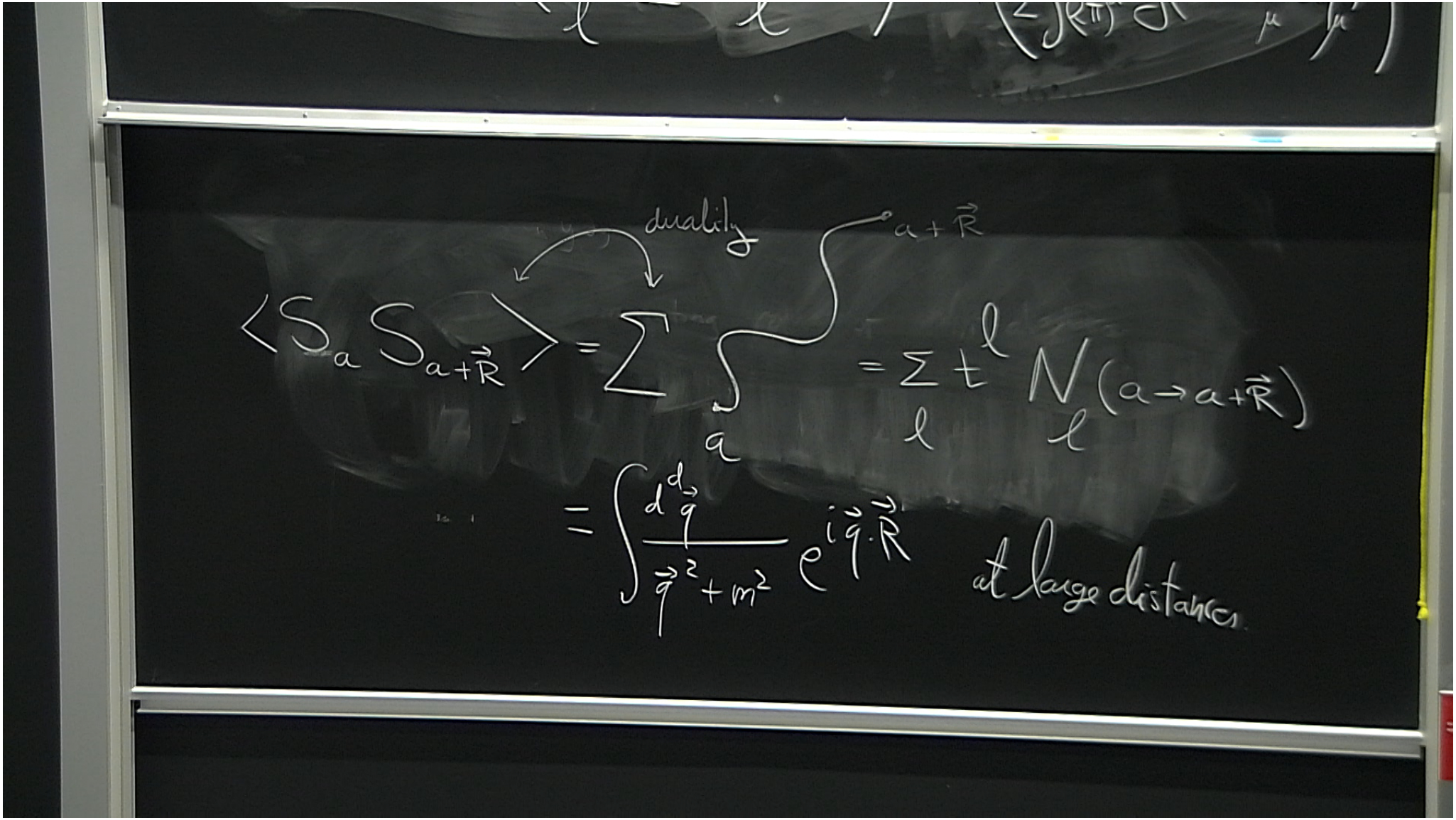
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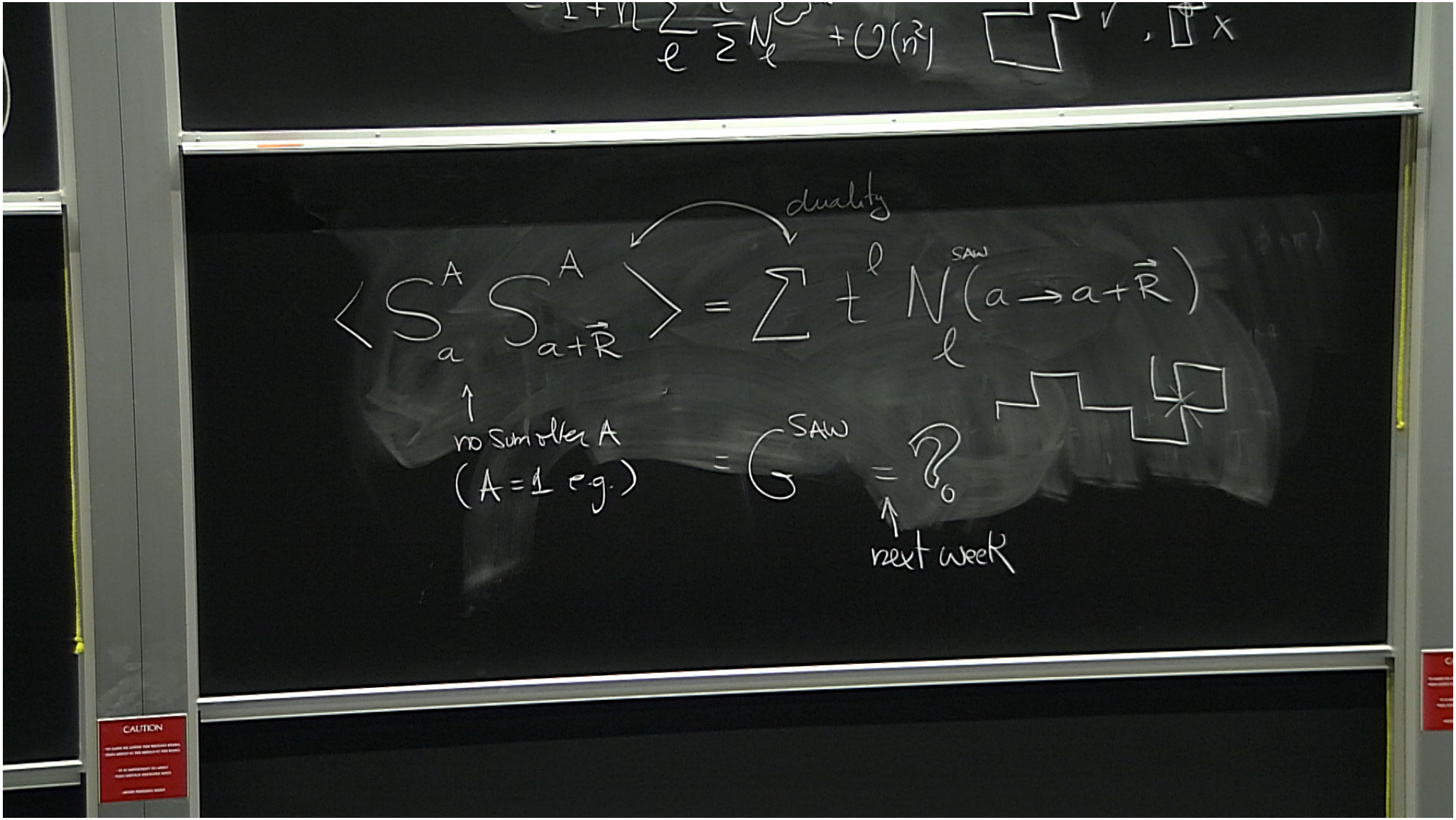
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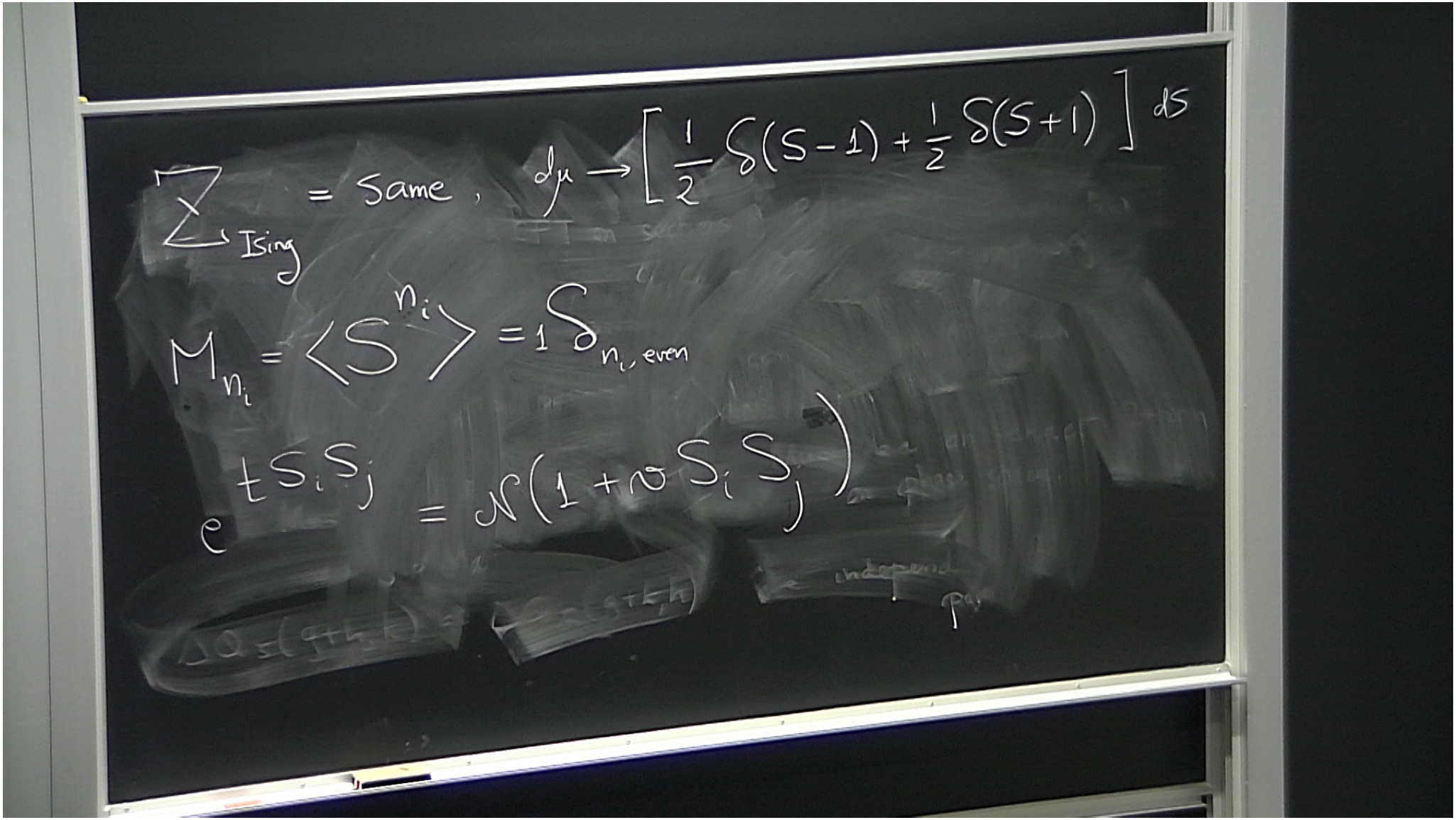

$Z_{\text{Gaussian}} \equiv \int \prod_{i \in \text{Lattice}} dS(i)\, e^{-S(i)^2/2} \prod_{\langle ij \rangle} e^{t S(i) S(j)}$
$dS(i)\, e^{-S(i)^2/2} = du$
$= \sum_{\text{sticks}} t^{\pi} \prod_{\langle ij \rangle} \frac{1}{n_{ij}!} \prod_i (n_i - 1)!!$
M_{n_i}
F^{RW}
$= \exp\left(\sum_l \frac{t^l}{2} N_l^{\{\}^{RW}} \right) = \exp\left(\frac{r}{2} \int \frac{d^d \vec{q}}{(2\pi)^d} \log\left(1 - 2t \sum_\mu \cos q_\mu\right) \right)$
CAUTION

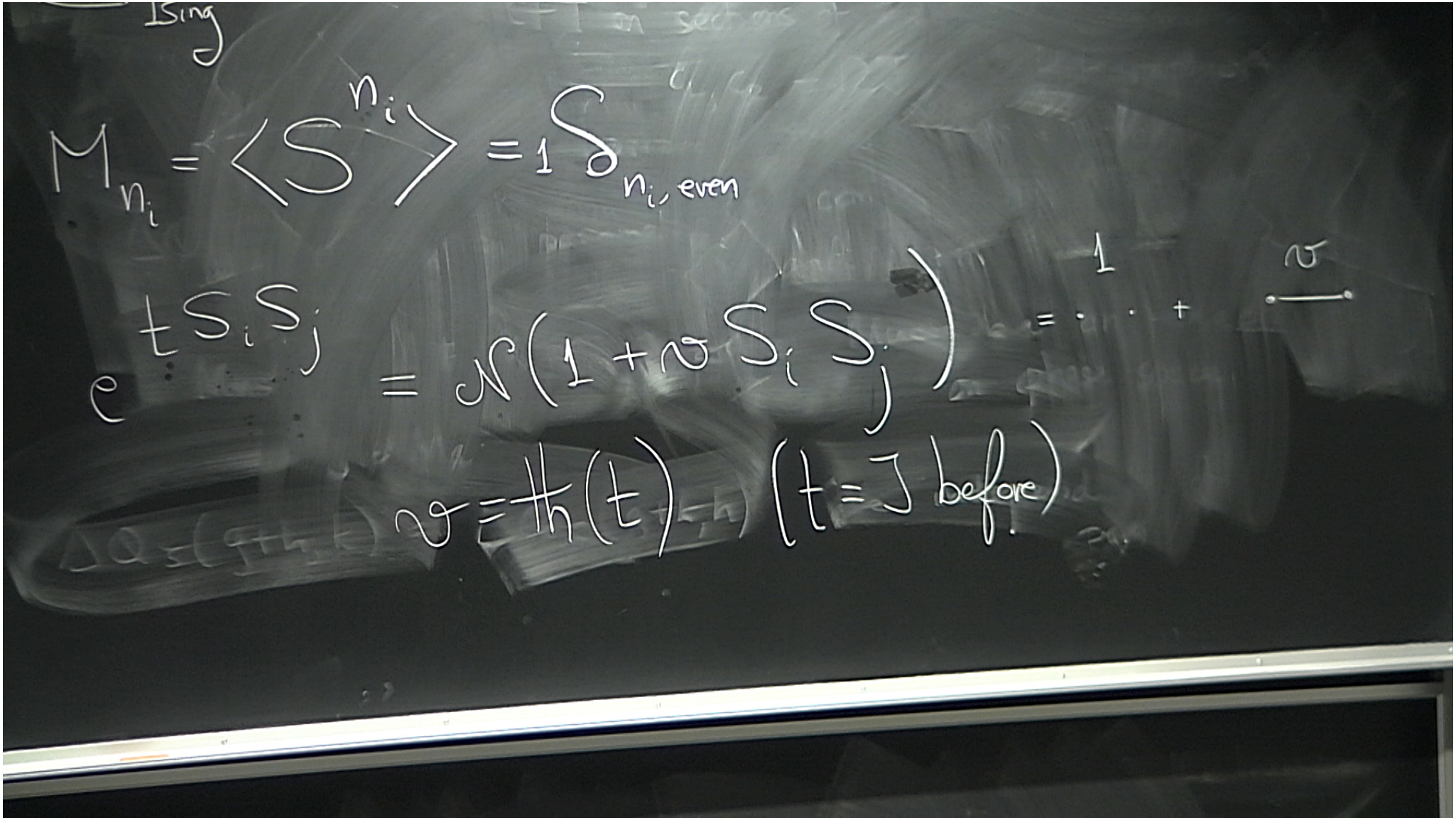

$Z_{\text{Gaussian}} \equiv \int \prod_{i \in \text{Lattice}} dS(i)\, e^{-S(i)^2/2}}_{d\mu} \prod_{\langle ij \rangle} e^{t S(i) S(j)}$
$= \sum_{\text{sticks}} t^{\pi} \prod_{\langle ij \rangle} \frac{1}{n_{ij}!} \prod_i (n_i - 1)!!$
M_{n_i}
Gaussian
F^{RW}
$= \exp\left(\sum_l \frac{t^l}{2} N_l^{\{\}_{RW}} \right) = \exp\left(\frac{V}{2} \int \frac{d^d q}{(2\pi)^d} \log\left(1 - 2t \sum_\mu \cos q_\mu\right) \right)$

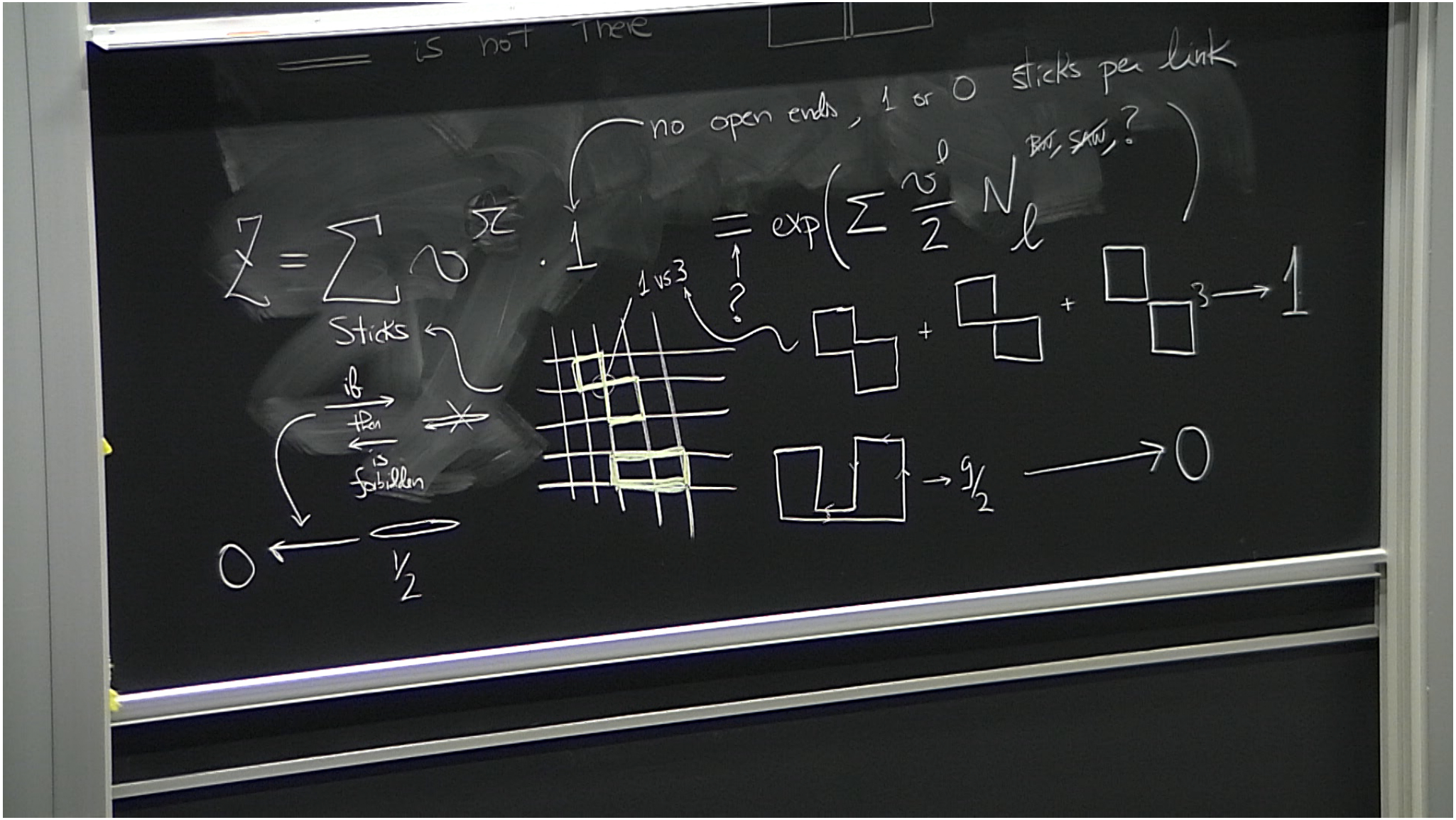

$Z_{O(n)}$ = same with $S \to \vec{S}$, $S_i S_j \to \vec{S}_i \cdot \vec{S}_j$
$d\mu = d\vec{S}\,\delta(\vec{S}^2 - n)$,
polymers
duality
$= \sum_{\text{sticks}}$ same with $M^{\text{Gaussian}}_{n_i} \longrightarrow M^{O(n)}_{n_i}$
$F^{\text{SAW}} = ?$
$= \lim_{n\to 0} \frac{Z(n)-1}{n}$
next week
$= \exp\left(\frac{n}{2} \sum_l t^l N^{\text{SAW}}_l\right)$
F^{SAW}
$= 1 + n \sum_l t^l N^{\text{SAW}}_l + O(n^2)$
$= \begin{cases} 1 & n_i = 0 \\ 1 & n_i = 2 \\ 0 & \text{otherwise} \end{cases}$
$n \to 0$
r
x

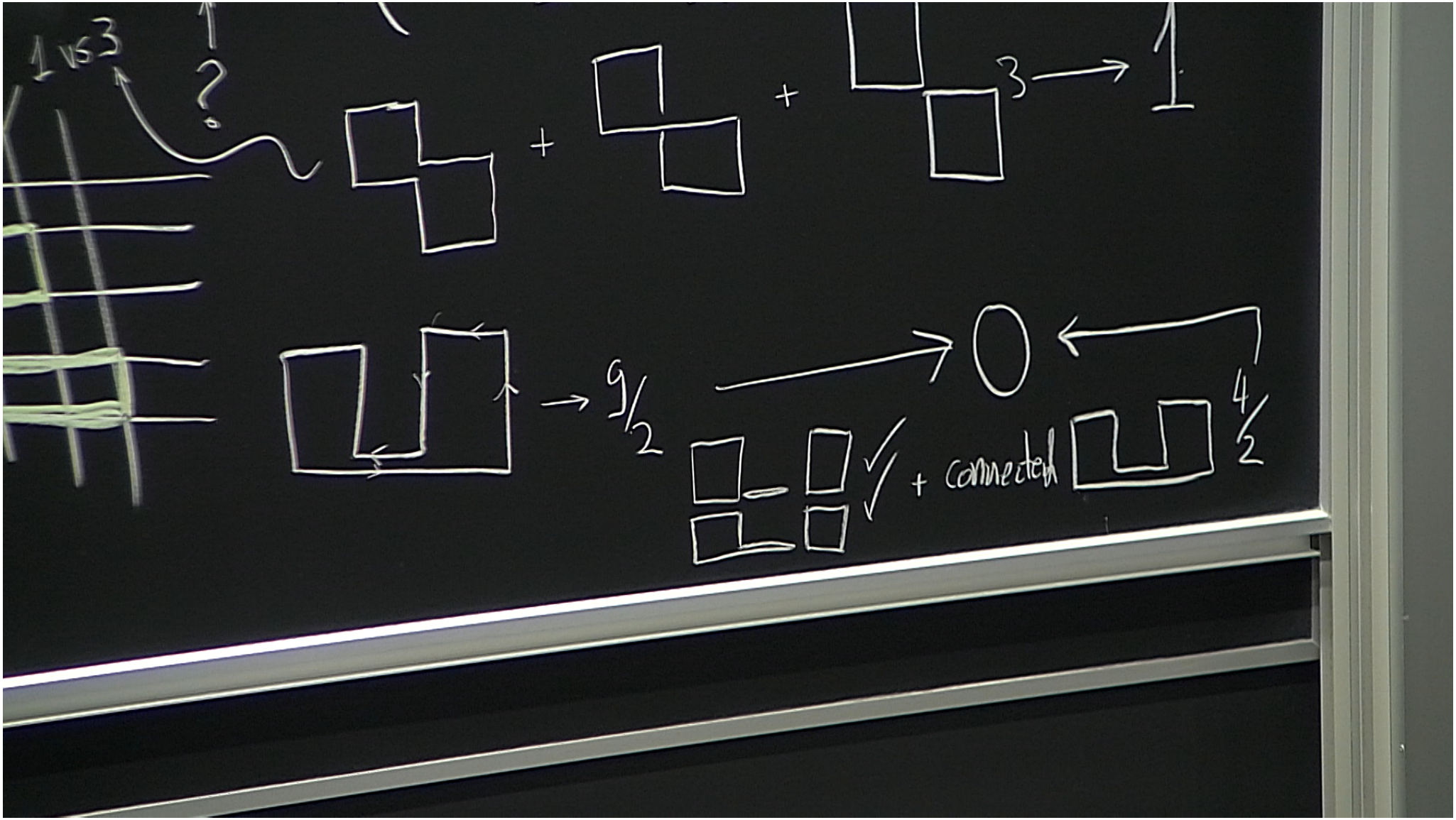

duality
$a+\vec{R}$
$\langle S_a S_{a+\vec{R}} \rangle = \sum_{\ell} \int_a = \sum_{\ell} t^{\ell} N_{\ell}(a \to a+\vec{R})$
$= \int \frac{d^d\vec{q}}{\vec{q}^2+m^2} e^{i\vec{q}\cdot\vec{R}}$ at large distances

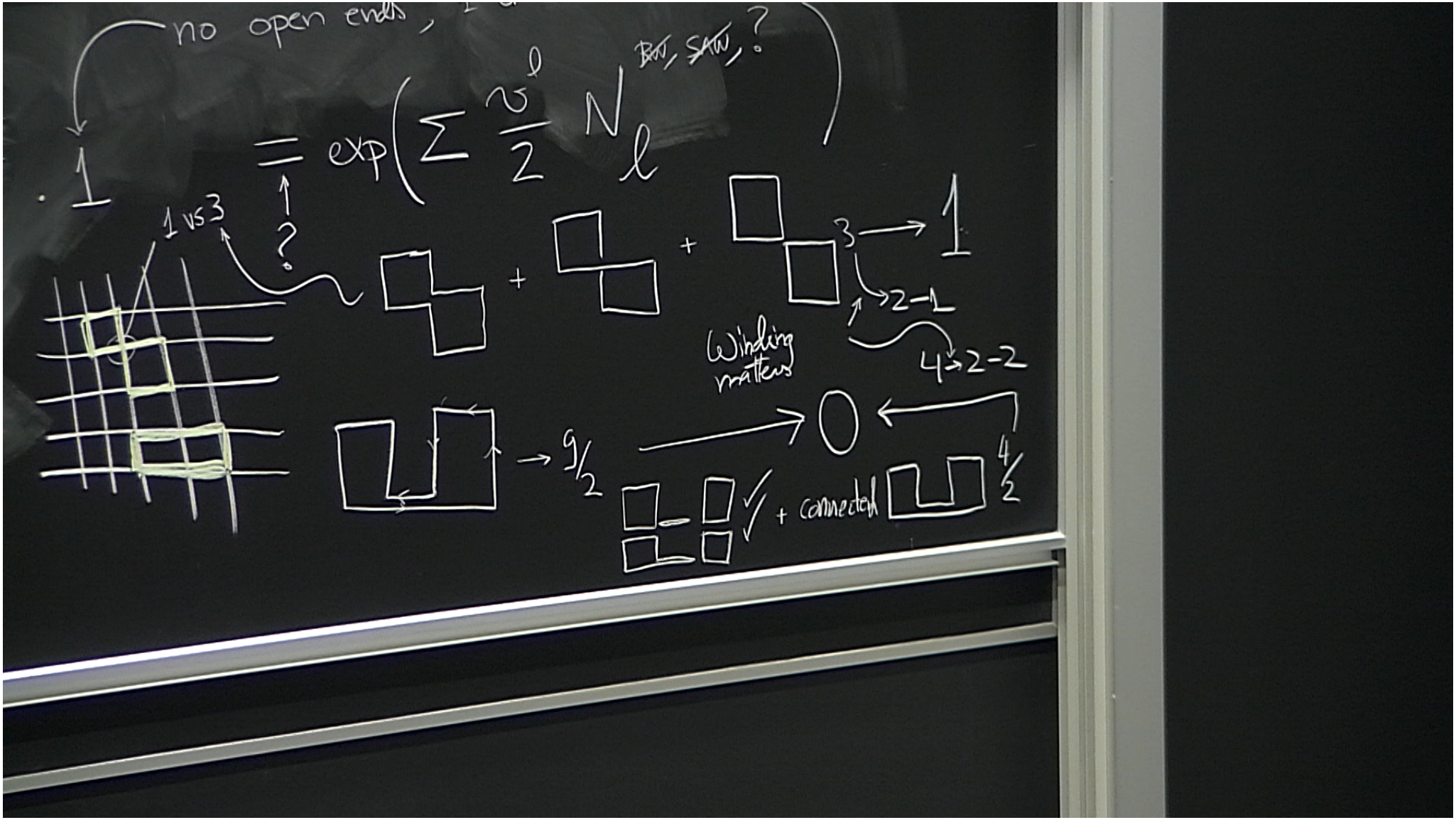

duality
$\langle S_a^A S_{a+\vec{R}}^A \rangle = \sum_\ell t^\ell N_\ell^{SAW}(a \to a+\vec{R})$
no sum over A
(A = 1 e.g.)
$= G^{SAW} = ?$
next week

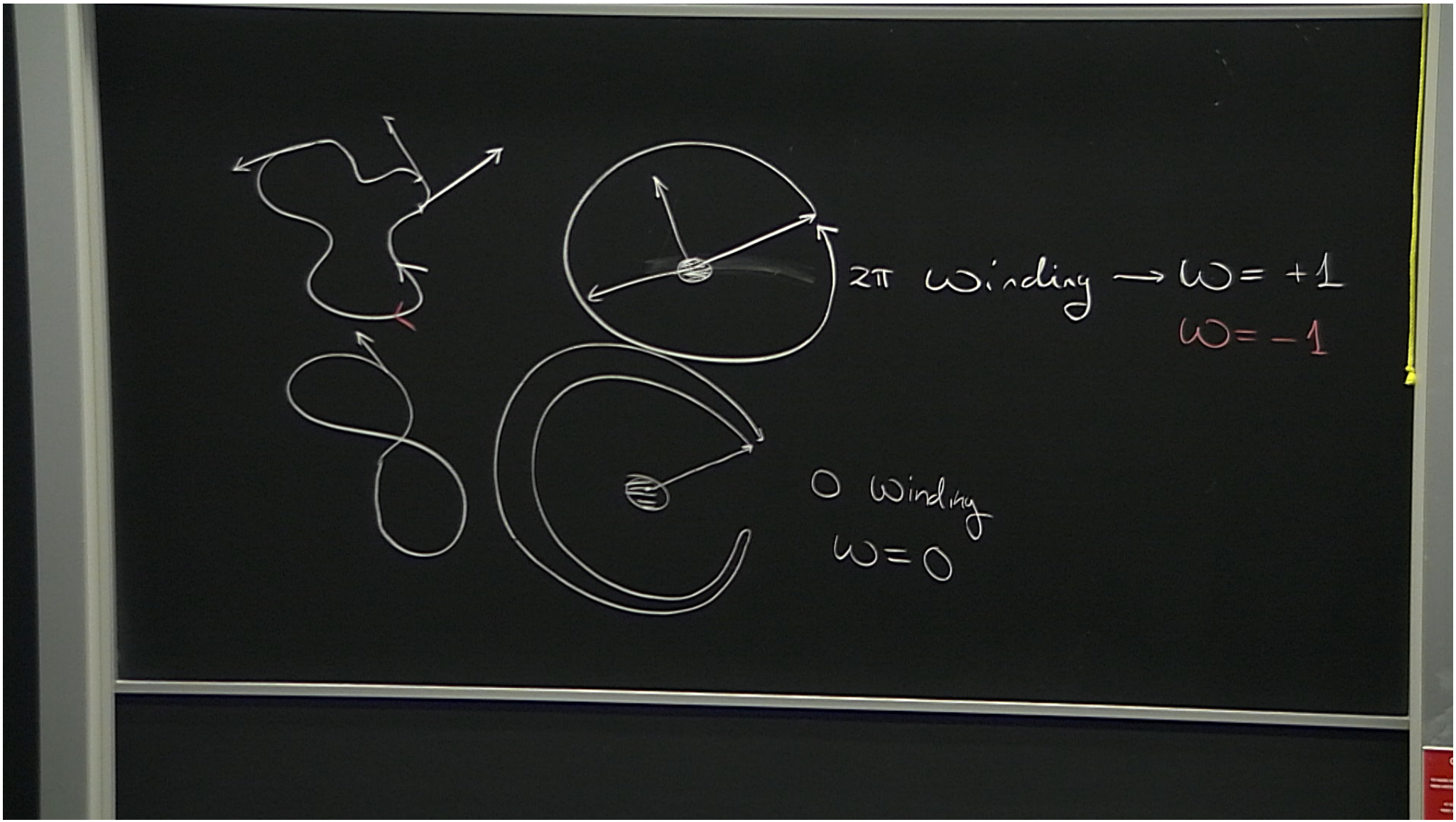

$\sum_{\text{Ising}} = \text{same}, \quad d\mu \to \left[\frac{1}{2}\delta(S-1) + \frac{1}{2}\delta(S+1)\right] dS$
$M_{n_i} = \langle S^{n_i} \rangle = 1 \quad n_i \text{ even}$
$e^{tS_iS_j} = r(1 + \sigma S_i S_j)$

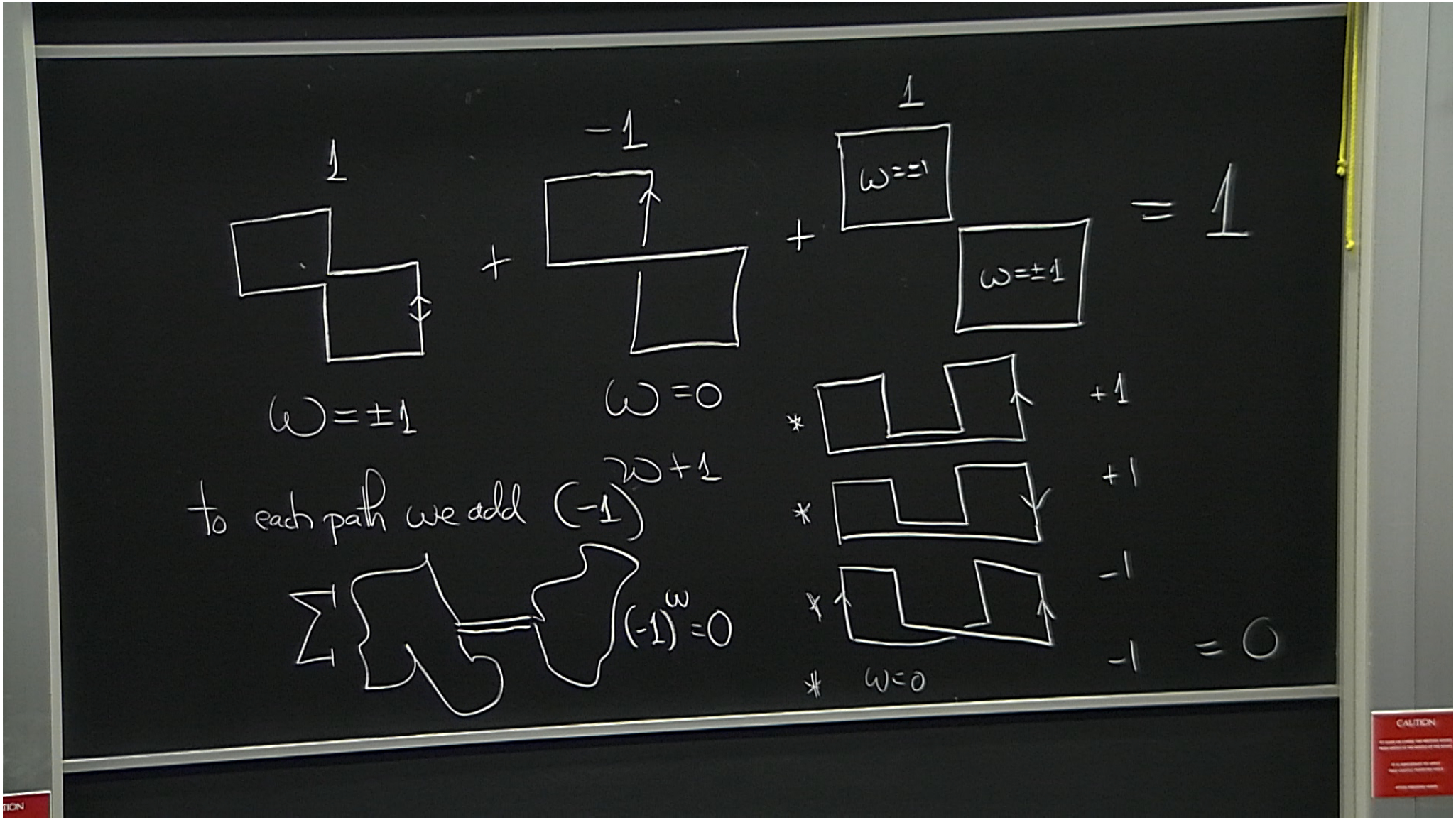

Ising
$M_{n_i} = \langle S^{n_i} \rangle = 1 \delta_{n_i, \text{even}}$
$e^{t S_i S_j} = r(1 + v S_i S_j)$
$v = \tanh(t)$ $(t = J \text{ before})$
1
v

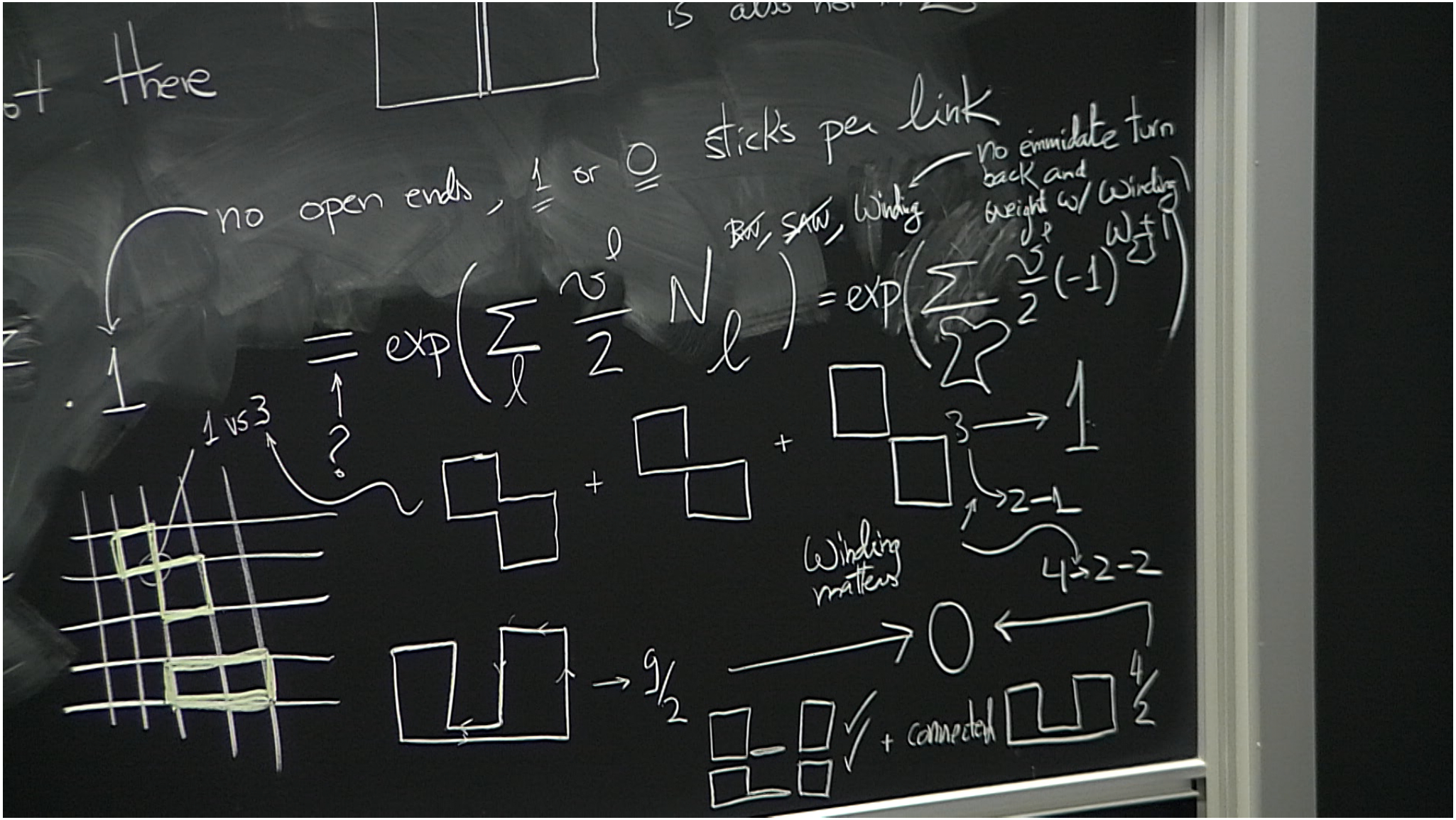

is not There
no open ends, 1 or 0 sticks per link
Sticks
1 vs 3
?
if
then
is
forbidden
0
1/2
3 → 1
→ 0


1 vs 3
?
+
+
3 → 1
→ 9/2
0
+ connected
4/2


no open ends
1
= exp(Σ v^l/2 N_l
RW, SAW, ?
1 vs 3
?
+
+
3 → 1
2-1
Winding matters
4 → 2-2
→ 9/2
0
4/2
+ connected

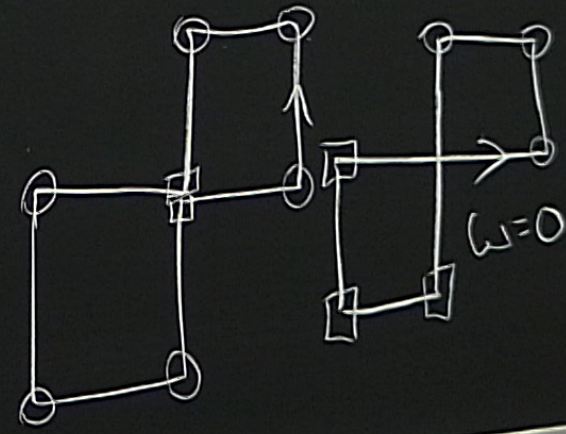

2π winding → ω = +1
ω = −1
0 winding
ω = 0

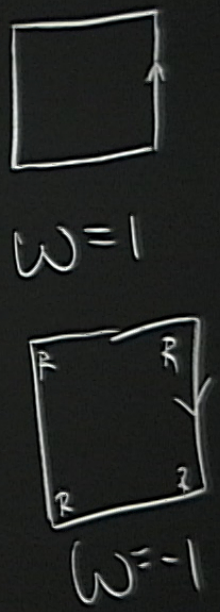

1
-1
1
ω=±1
ω=±1
= 1
ω=±1
ω=0
+1
+1
-1
-1
to each path we add $(-1)^{\omega+1}$
Σ
$(-1)^{\omega}=0$
ω=0

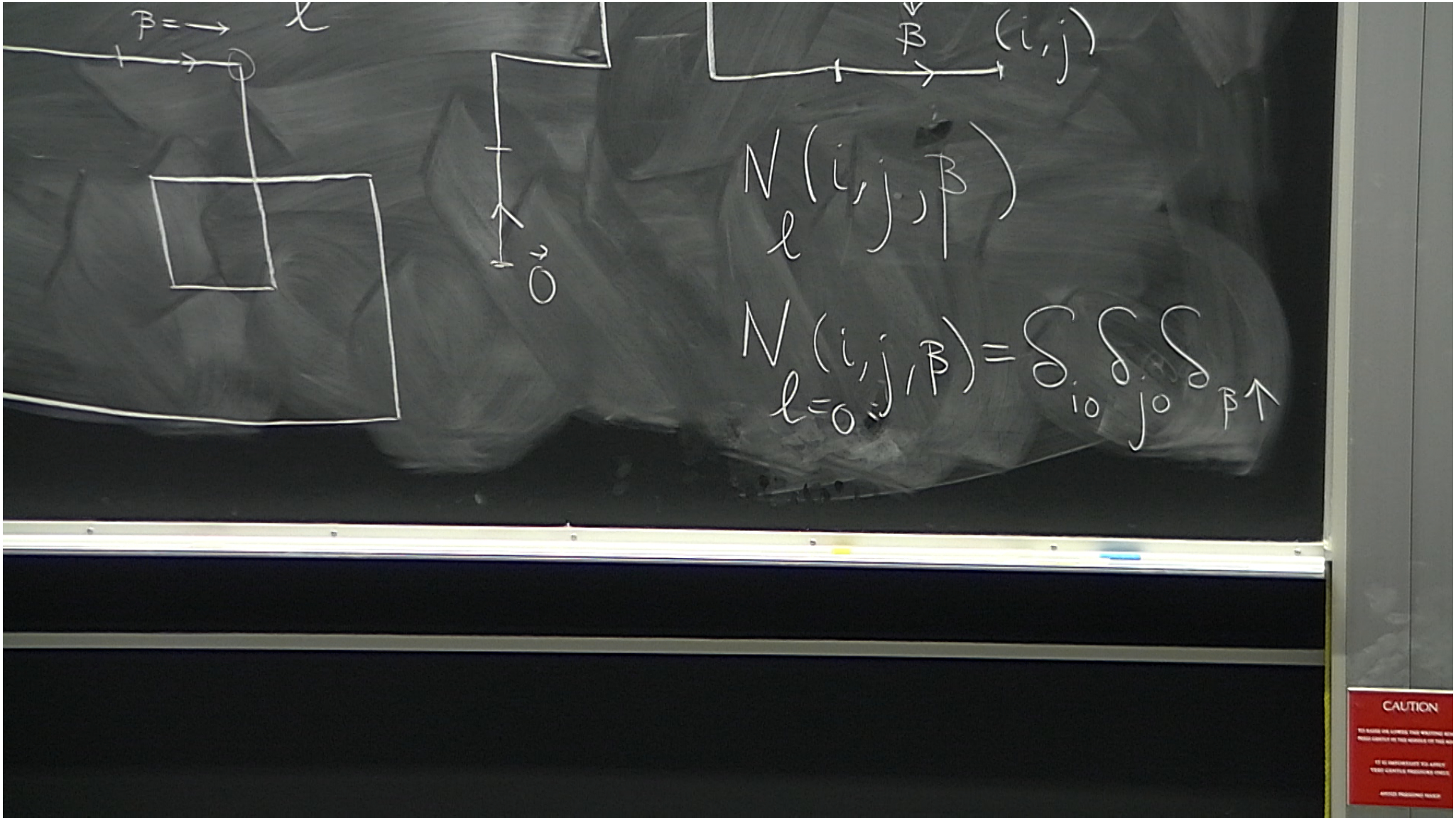

there
no open ends, 1 or 2
sticks per link
no immidate turn back and overlight w/ winding
BW, SAW, Winding
1 vs 3
?
3 → 1
2 → 1
Winding matters
4 → 2 → 2
→ 9/2
0
+ connected

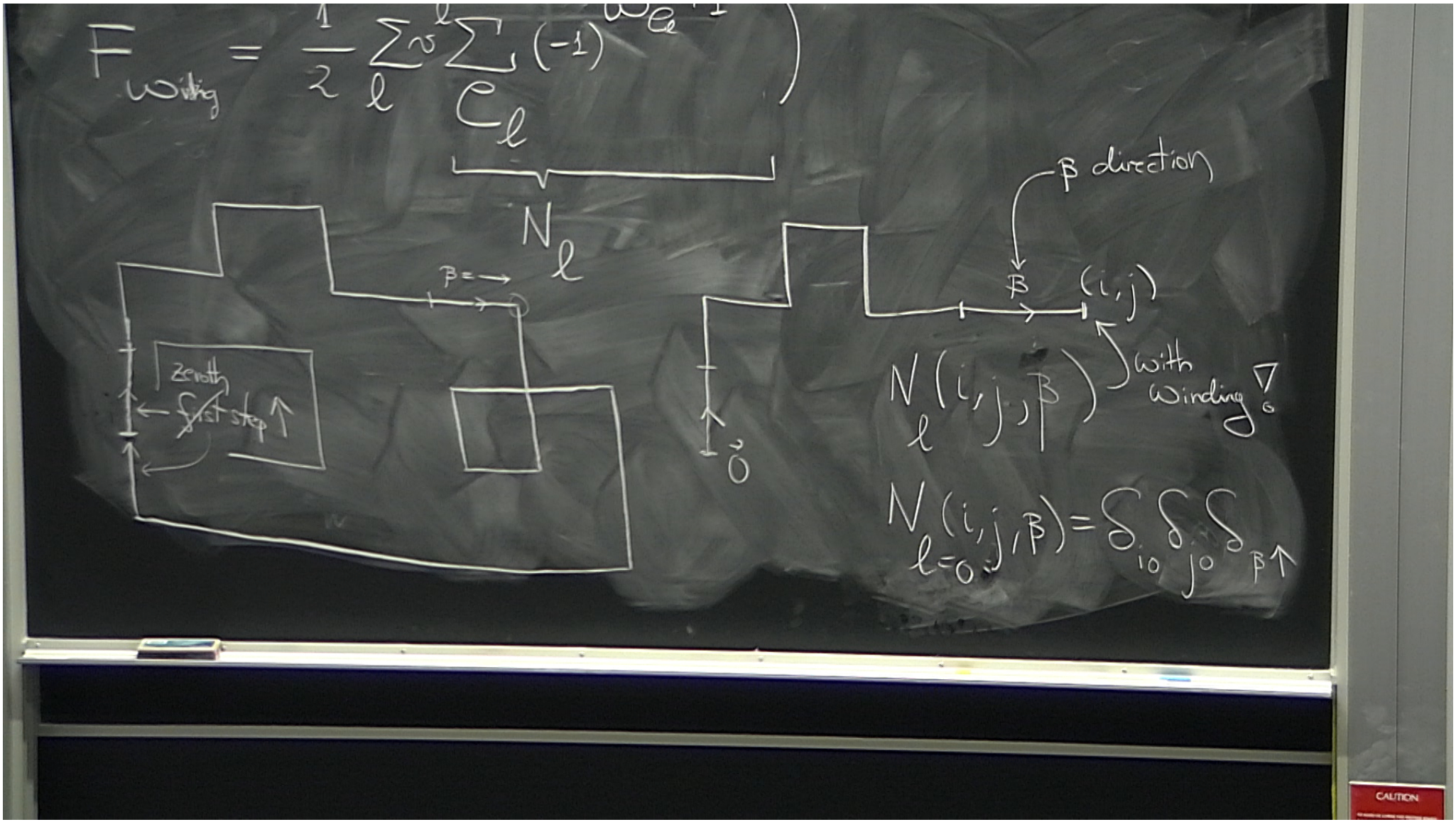
Winding is local

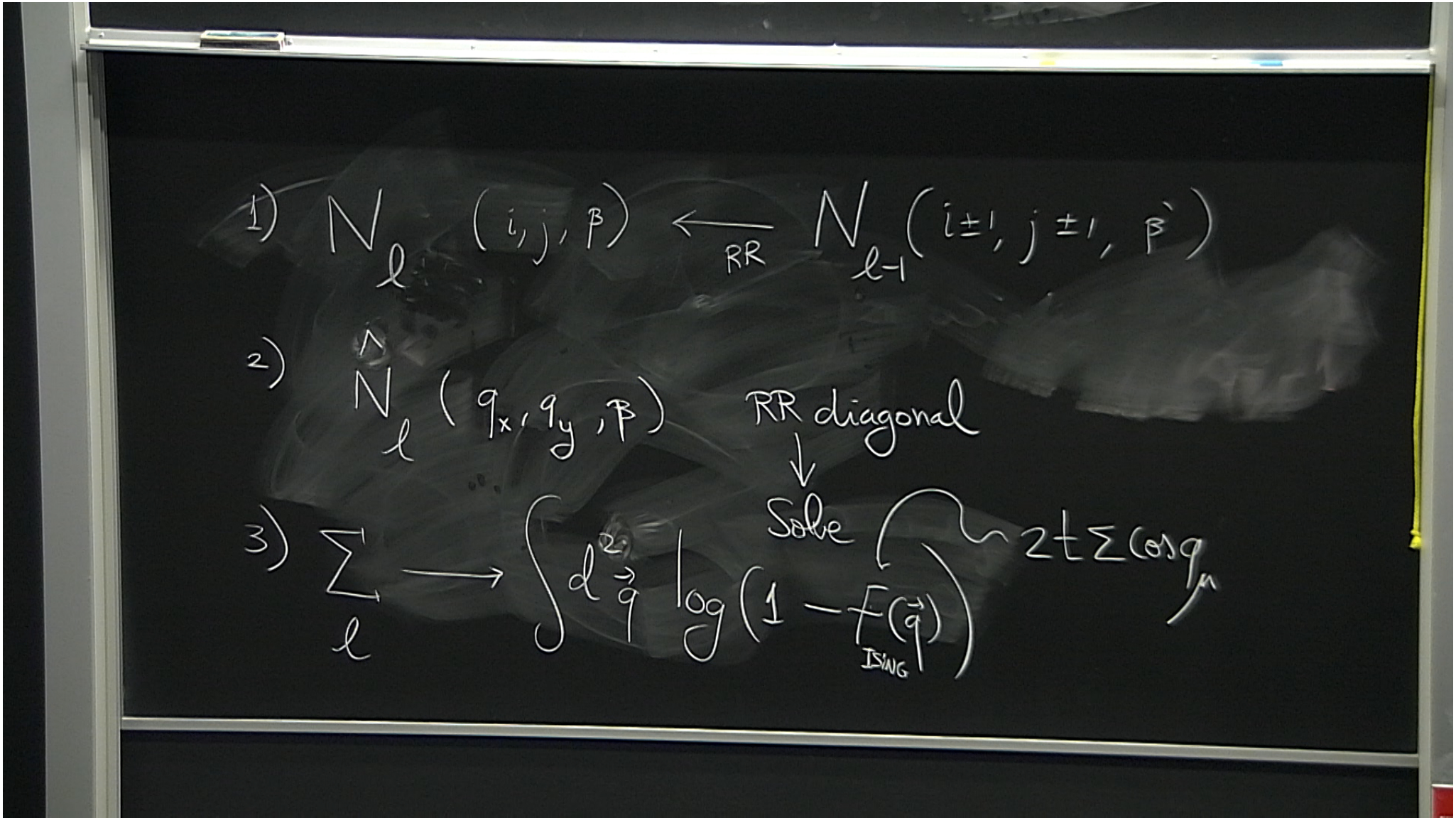
$$w = \frac{1}{4}\left(N_L - N_R\right)$$






β
ℓ
0
β
(i, j)
$N_\ell(i,j,\beta)$
$N_{\ell=0}(i,j,\beta) = \delta_{i0}\,\delta_{j0}\,\delta_{\beta 1}$
CAUTION


$F_{winding} = \frac{1}{2} \sum_{\ell} \sum_{C_\ell} (-1)^{\omega_{C_\ell}}$
N_ℓ
β direction
β
(i,j)
$N_\ell(i,j,\beta)$
with winding!
$N_{\ell=0}(i,j,\beta) = \delta_{i0}\,\delta_{j0}\,\delta_{\beta\uparrow}$
Zeroth
first step
$\vec{0}$

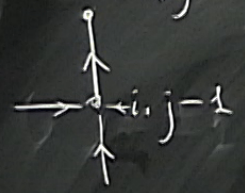

1) $N_\ell(i,j,\beta) \xleftarrow{RR} N_{\ell-1}(i\pm1, j\pm1, \beta')$
2) $\hat{N}_\ell(q_x, q_y, \beta)$
RR diagonal
Solve
3) $\sum_\ell \longrightarrow \int d^2\vec{q} \log\left(1 - \underset{ISING}{F(\vec{q})}\right)$
$2t\Sigma\cos q_\mu$

$N_\ell(i,j,\uparrow) = e^{\frac{i\pi}{4}} N_{\ell-1}(i,j-1,\rightarrow) +$ $\curvearrowleft (-1)^{1/4}$

$1\ N_{\ell-1}(i,j-1,\uparrow) +$

$e^{-i\pi/4} N_{\ell-1}(i,j-1,\leftarrow) +$

$\circ\ N_{\ell-1}(i,j-1,\downarrow)$



$\ell, i, j \leftarrow \ell-1, i\pm1, j\pm1$

$\ell, \vec{q}, \leftarrow \ell-1, \vec{q}$