Title: PSI 2019/2020 - Statistical Mechanics (Vieira) - Lecture 11

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Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

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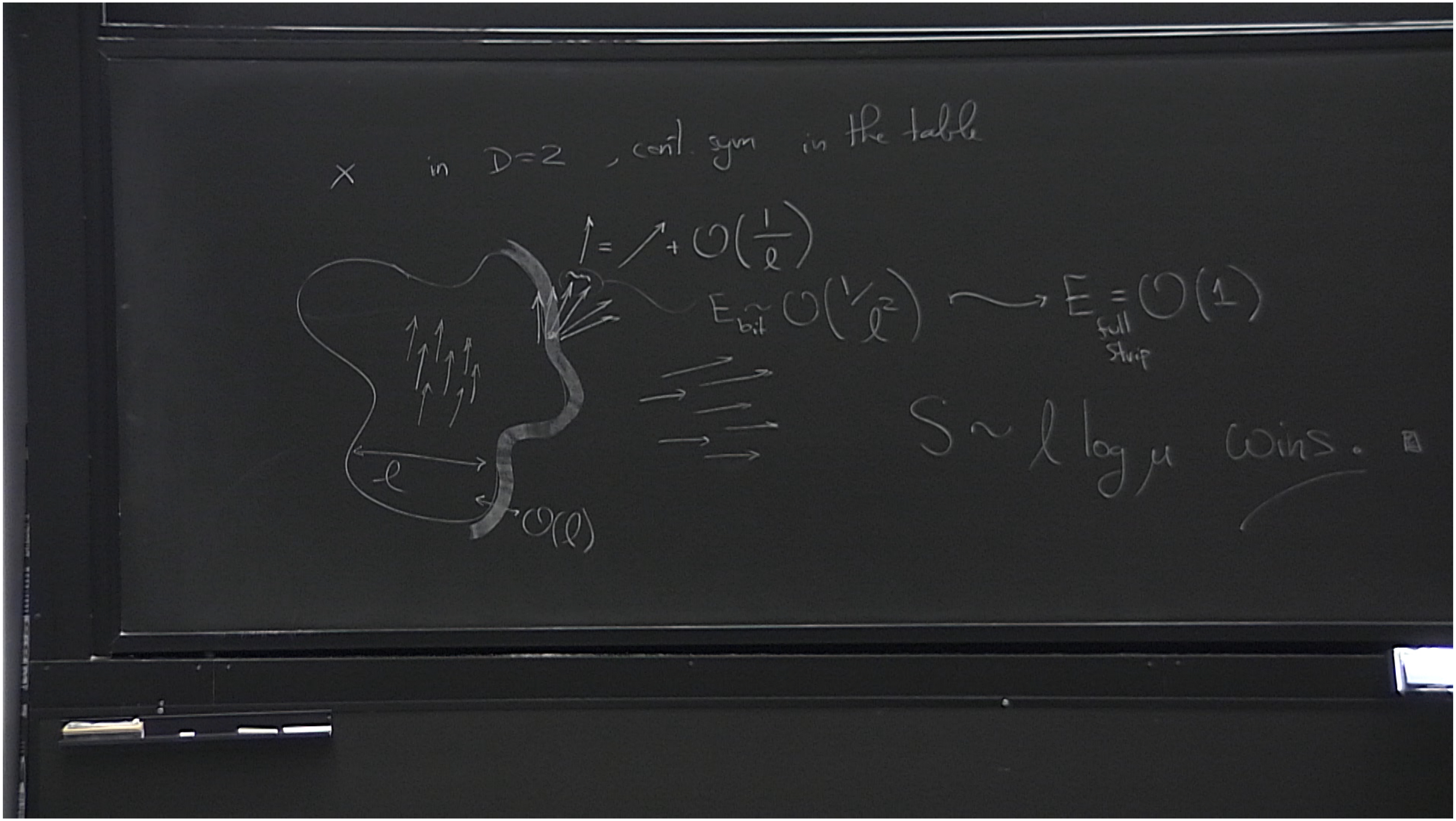
URL: http://pirsa.org/19100035


× in D=2 , cont. sym in the table
$\uparrow = \nearrow + O(\frac{1}{\ell})$
$E_{bit} \sim O(1/\ell^2) \longrightarrow E_{full\ strip} = O(1)$
$S \sim \ell \log \mu$ wins.
ℓ
$O(\ell)$

Mermin-Wigner Theorem

no spontaneous sym breaking in D=2

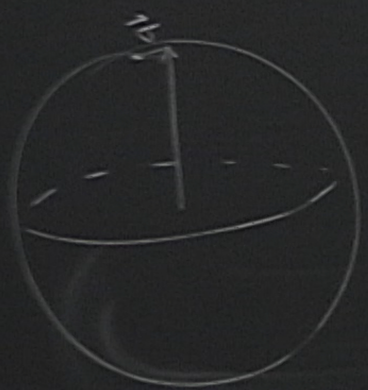
continuous

$\uparrow\uparrow\uparrow\uparrow \rightsquigarrow$ destroyed

Suppose it was ordered

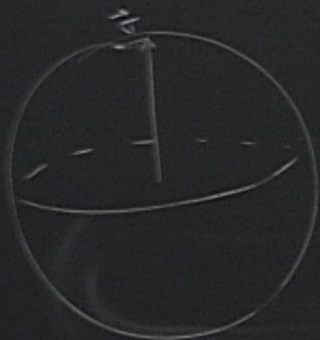
$\vec{S} = \uparrow +$ fluctuations

$\vec{S}\Big|_{O(n),\ \vec{S}^2=1} = \left(\sqrt{1-\vec{\pi}^2}, \vec{\pi}\right)$, with $\vec{\pi}$ small



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$\simeq \text{const} + J \int d^D\vec{x}\, (\partial \vec{S})^2 \simeq \text{const} + J \int d^D\vec{x}\, (\partial \vec{\sigma})^2 + O(\vec{\sigma}^4)$$



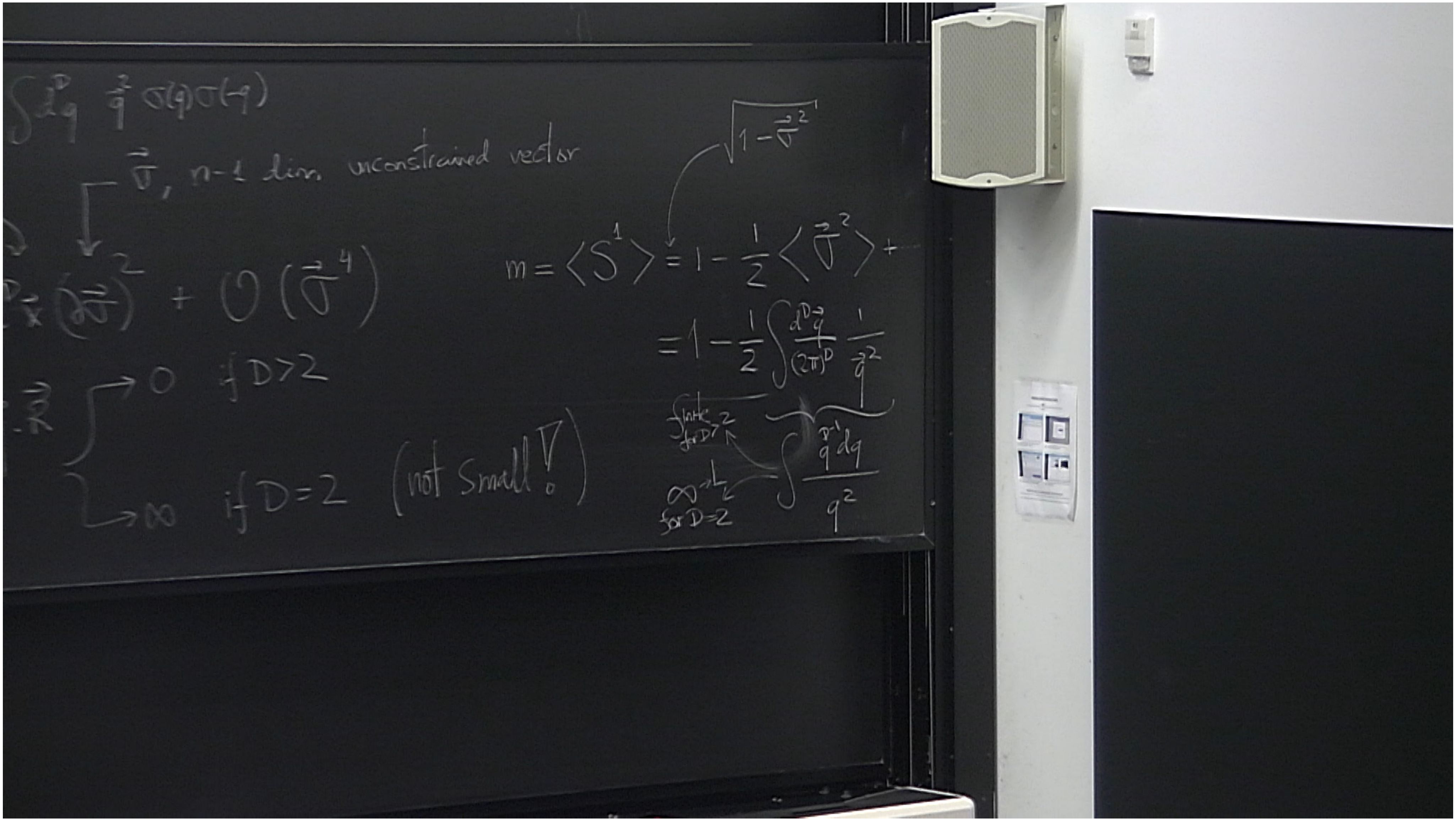
$$H = -J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j \qquad \vec{S}^2 = 1$$

$$\int d^D q \; \vec{q}^{\,2}\, \sigma(q)\,\sigma(-q)$$

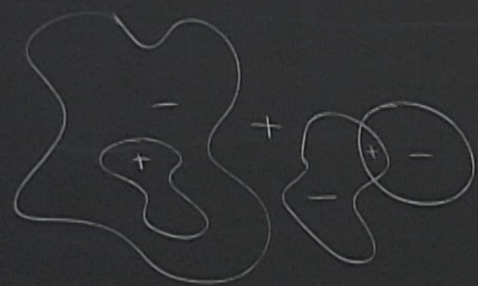
$\vec{\sigma}$, n-1 dim. unconstrained vector

$$\simeq \text{const} + J\int d^D\vec{x}\,(\partial\vec{S})^2 \simeq \text{const} + J\int d^D\vec{x}\,(\partial\vec{\sigma})^2 + O(\vec{\sigma}^4)$$

$$\langle \sigma^A(x)\,\sigma^B(y)\rangle \simeq \frac{1}{J}\int \frac{d^D q}{(2\pi)^D}\,\frac{\delta^{AB}}{\vec{q}^{\,2}}\, e^{i\vec{q}\cdot\vec{R}} \begin{cases} \to 0 & \text{if } D>2 \\ \to \infty & \text{if } D=2 \end{cases} \quad \text{(not small!)}$$

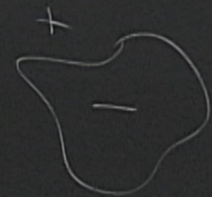

n-1 dim. unconstrained vector
if D>2
if D=2
(not small!)
finite for D>2
for D=2

* $h \neq 0$



$$E_{\uparrow\uparrow\uparrow\uparrow} = 2JV + hV$$

$$H = E_{\uparrow\uparrow\ldots\uparrow} + \underbrace{\left(E - E_{\uparrow\uparrow\ldots\uparrow}\right)}_{2J \times [\text{length of loop}] \; + 2h \times [\text{area of loop}]}$$



$U(n)$, $\vec{S}^2 = 1$ small

$$E_{\uparrow\uparrow\uparrow\uparrow} = 2JV + hV$$

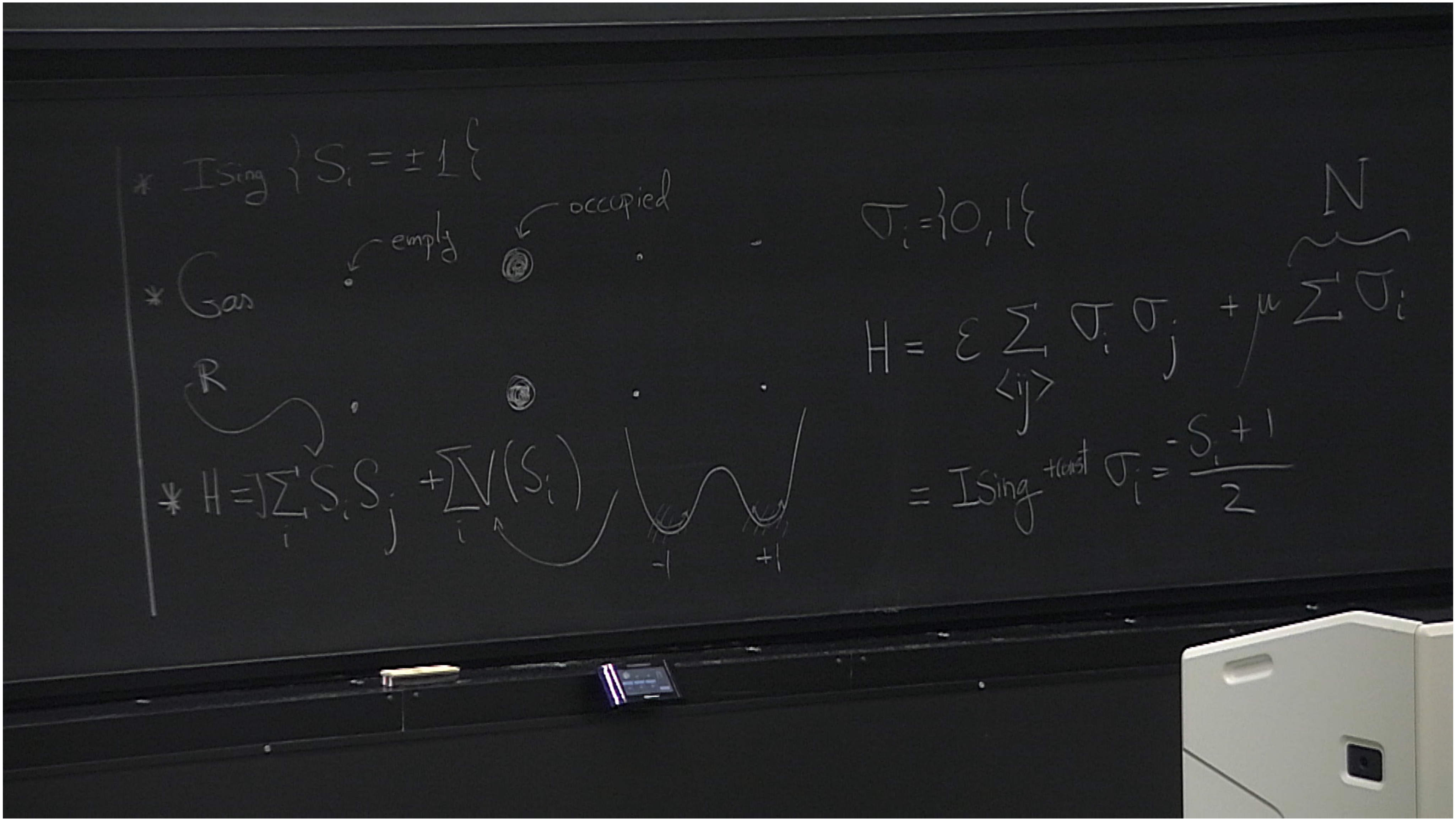
$$H = E_{\uparrow\uparrow\ldots\uparrow} + \underbrace{\left(E - E_{\uparrow\uparrow\ldots\uparrow}\right)}_{2J \times [\text{length of loop}] + 2h \times [\text{area of loop}]}$$

$+$ $-$

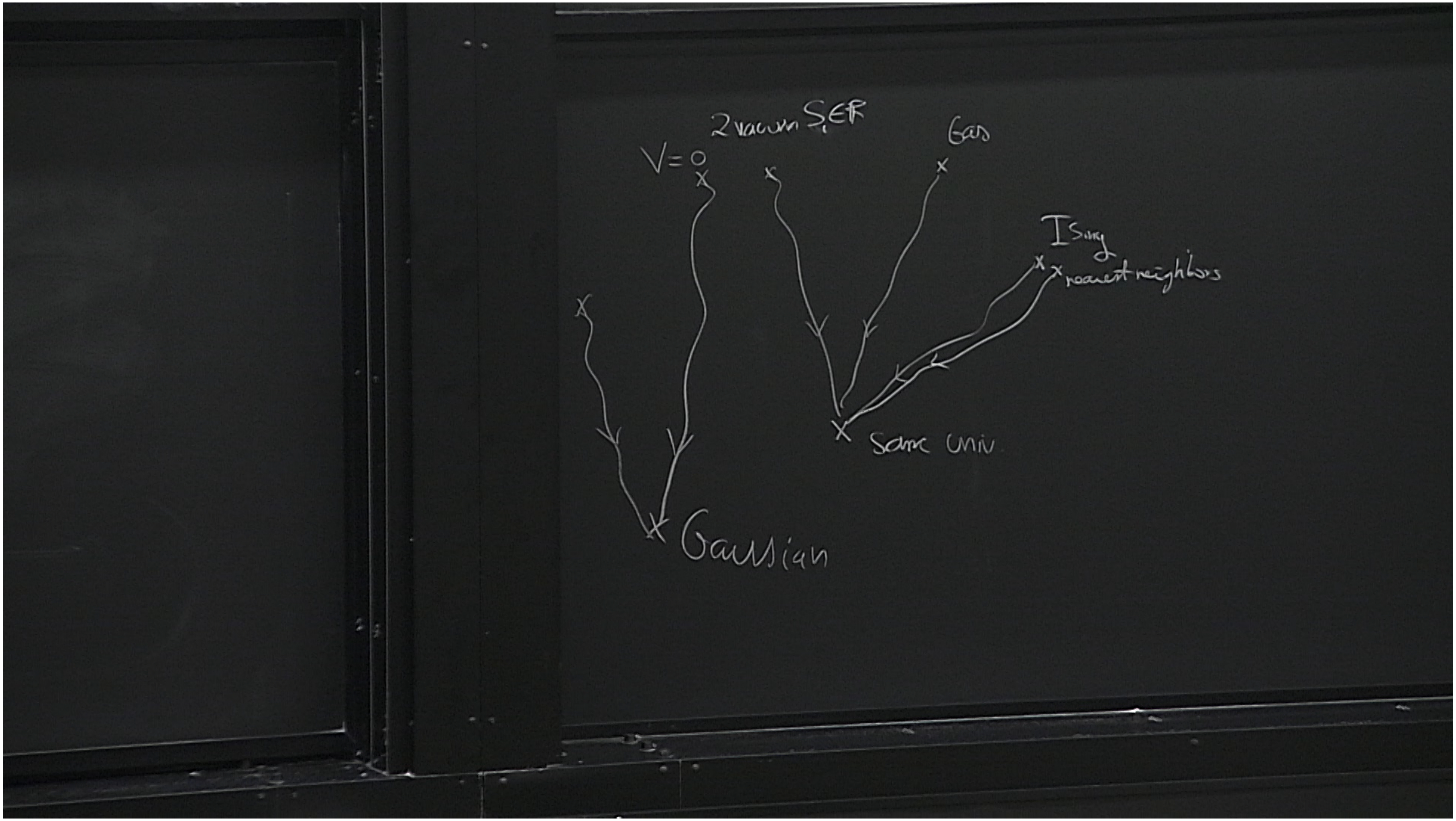
N_ℓ ✓ ← add area to this construction (need)

not known

$$\sum_{C_\ell} (-1)^{w_C+1} v^{\ell(C)} \mu^{\text{area}(C)} = \log Z^{h\neq 0}_{\text{Ising}}$$

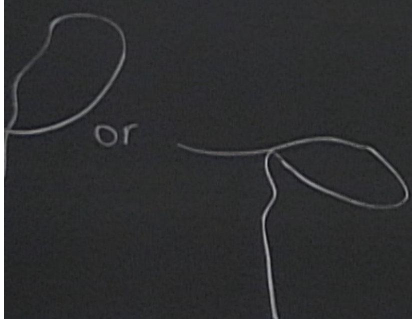


* Ising }S_i = ± 1{
occupied
empty
* Gas
R
* H = J Σ' S_i S_j + Σ V(S_i)
-1
+1
σ_i = }0, 1{
N
H = ε Σ_⟨ij⟩ σ_i σ_j + μ Σ σ_i
= Ising +const σ_i = (-S_i + 1)/2



2 vacuum $S_i \in \mathbb{R}$
$V = 0$
Gas
Ising
nearest neighbors
Same univ.
Gaussian

olymer


or
is forbidden

* Counting problems
* Solve some simple subclass of SAW
* Estimate large dist statistics by RG
* Flory's argument ($\sim$RG) for

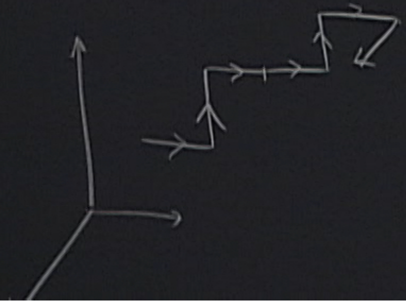
$N \doteq 4$

$2 < \mu < 3$ in $d=2$

$d < \mu < 2d-1$

$d^N < M_N < 2d(2d-1)^N$

all paths that do not do this



$$N=1 \qquad N=2 \qquad N=3$$

$$M_N \equiv \left\{ 4,\ 4\cdot 3,\ 4\cdot 3\cdot 3,\ \text{hard} \right\}$$

$N \geq 4$

$D=2$

number of SAWs
with N steps

$$M_N \sim \mu^N$$

$\mu = ?$

$2 <$

$d <$

$d^N <$