Title: PSI 2019/2020 - Statistical Mechanics (Vieira) - Lecture 10

Speakers: Pedro Vieira

Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

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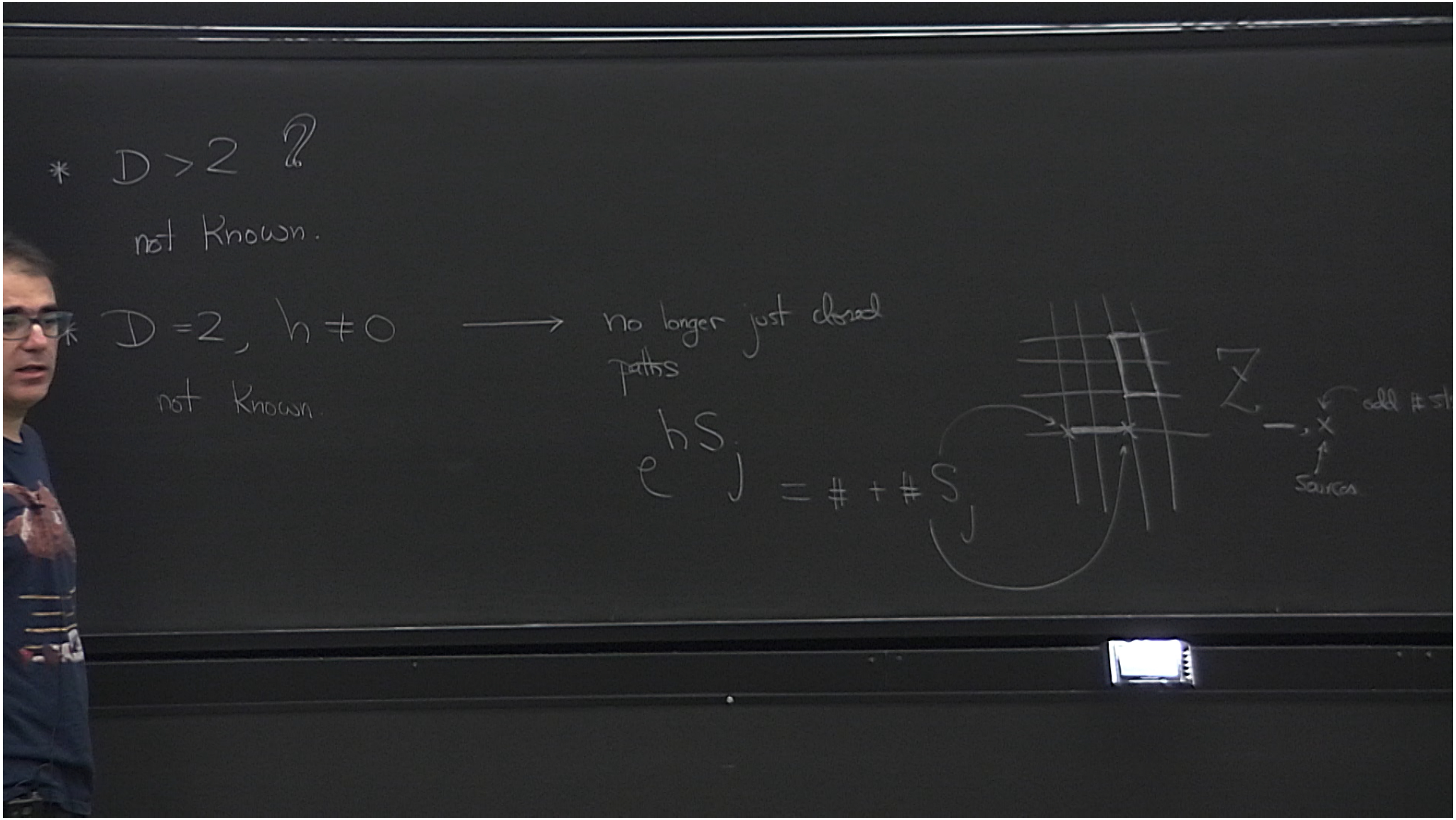
$$Z_{\text{2D Ising on Square lattice}} = 2^{L^2}\left(1-v^2\right)^{-L^2/2} \exp\left(-\frac{L^2}{2}\int \frac{d^2\vec{q}}{(2\pi)^2} \log\Big(\underbrace{(1+v^2)^2 - 2v(1-v^2)\sum_{\mu=1}^{2}\cos q_\mu}_{\det(1-vT_{ij})}\Big)\right)$$

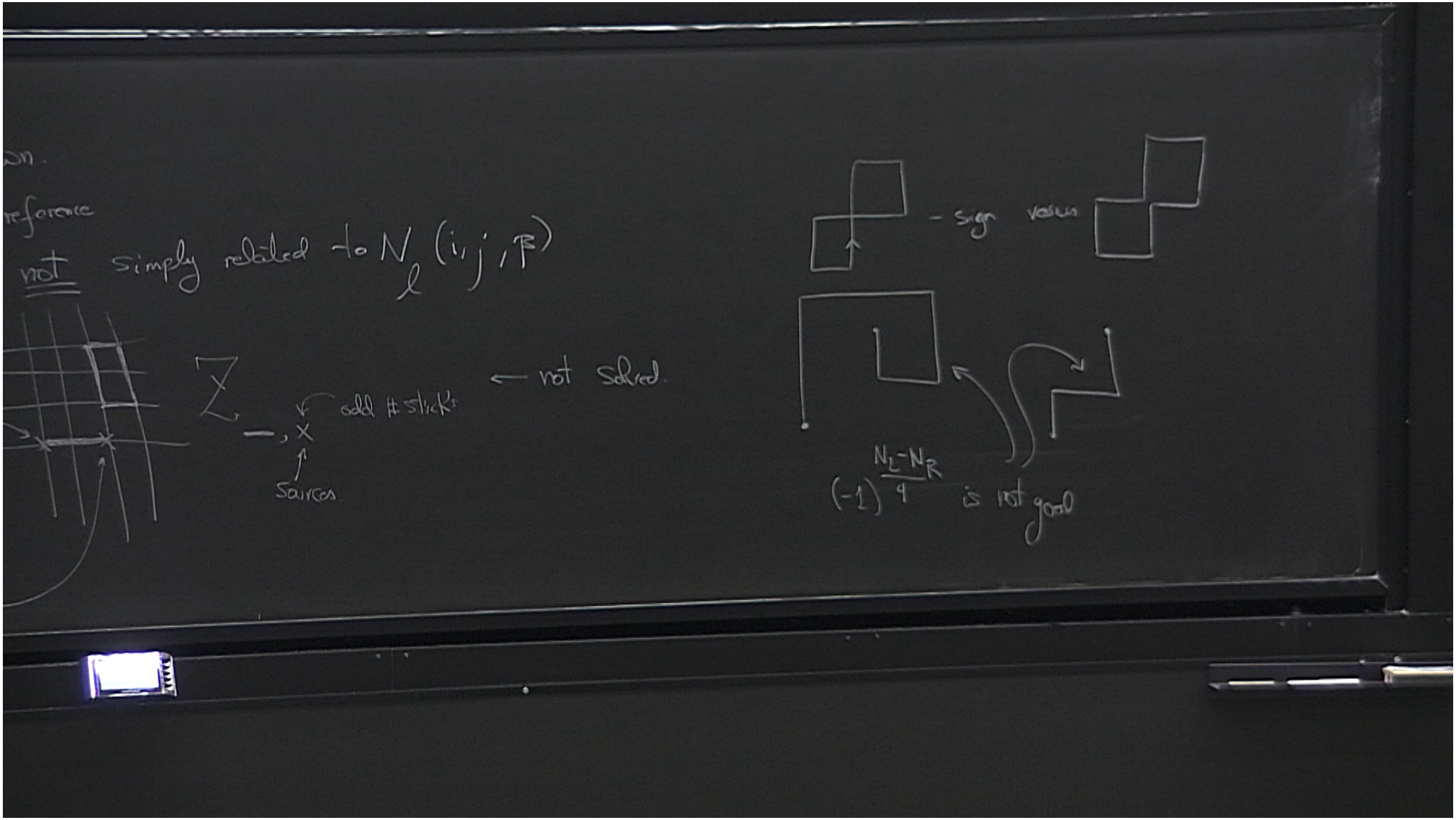
$$e^{\beta S_i S_j} = \#\left(1 + v S_i S_j\right)$$

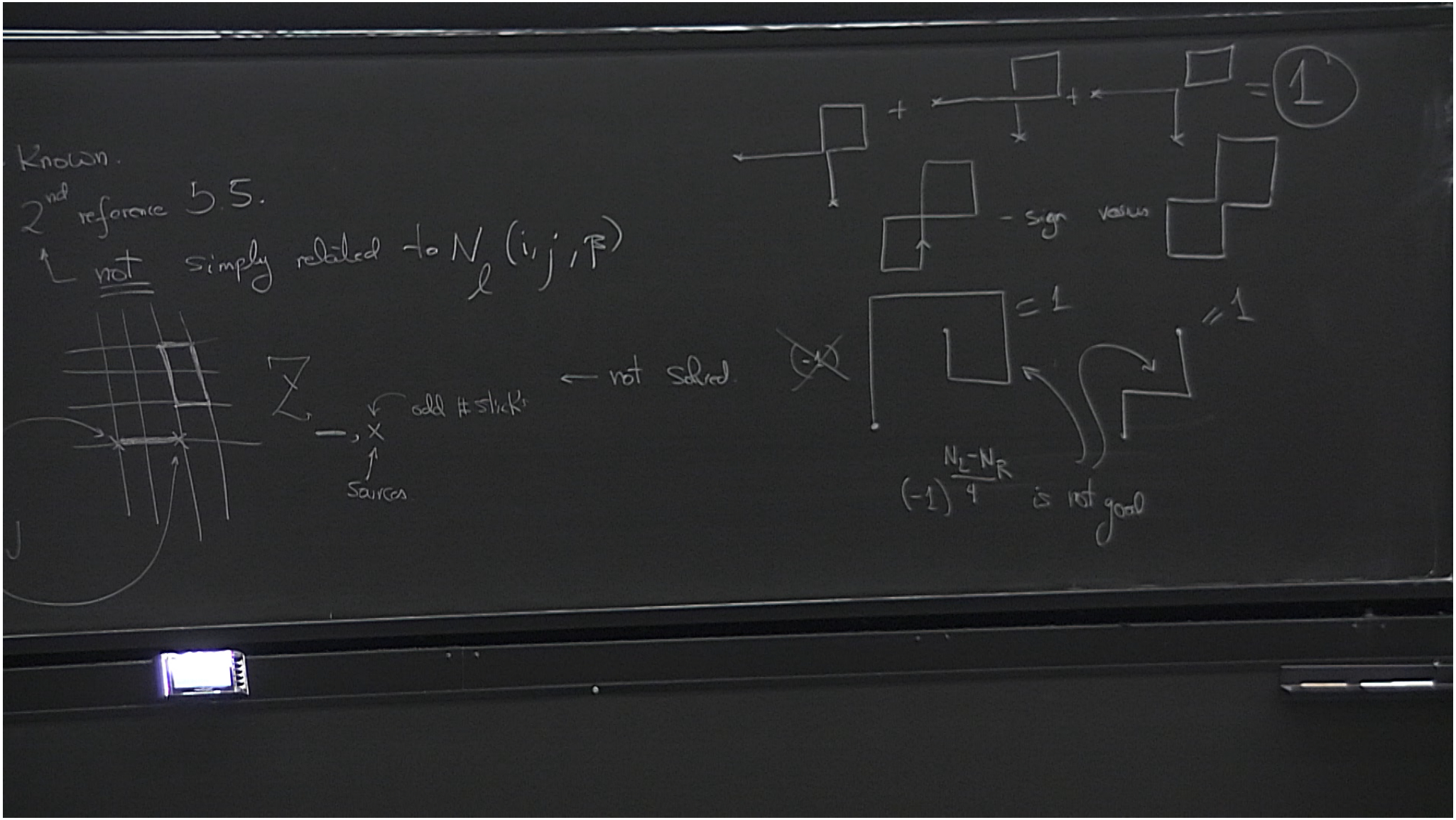
$$\sim \log\left(\#(v-v_c)^2 + \#\vec{q}^{\,2}\right)$$

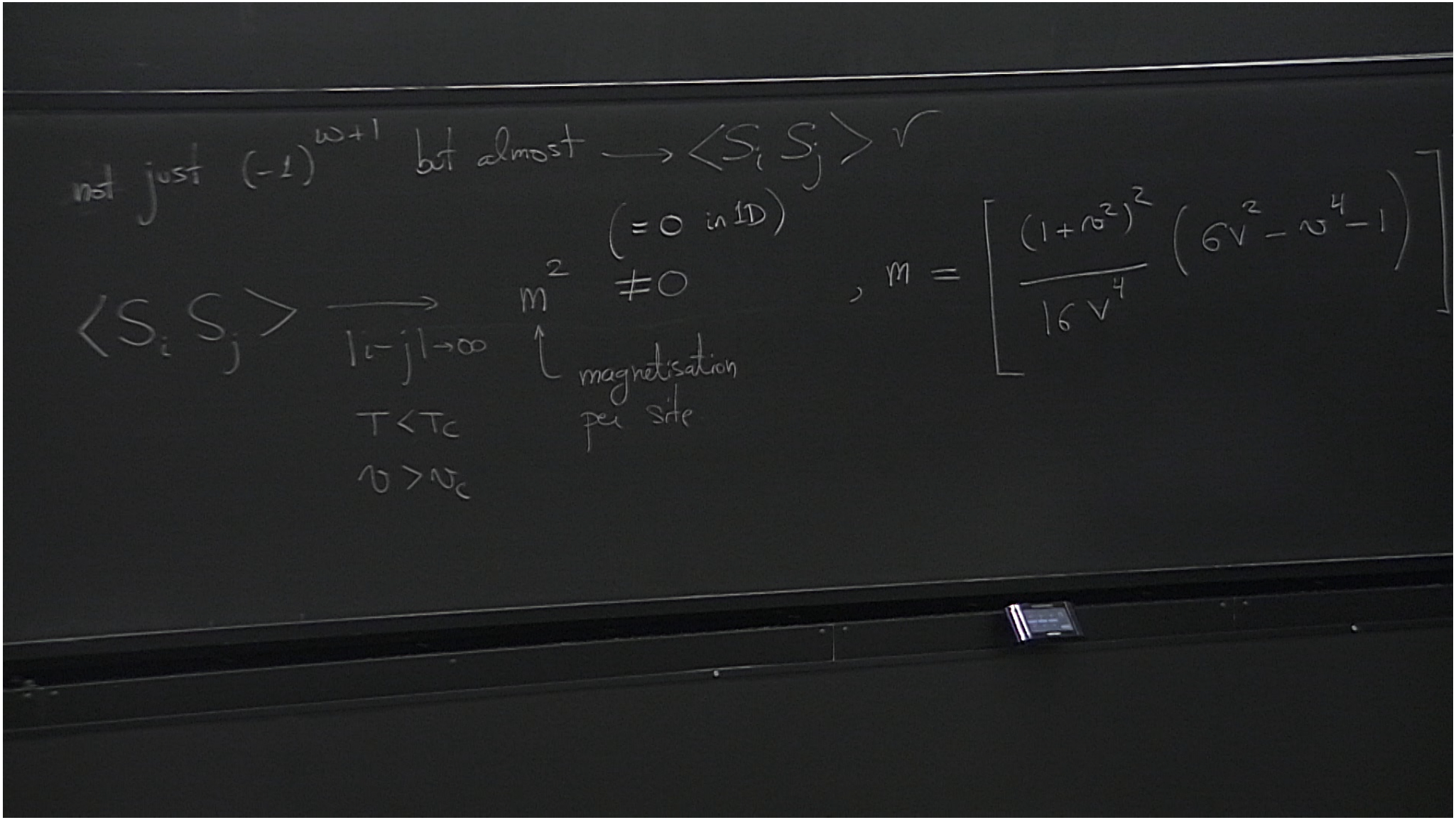
$$\sim \exp\left(\text{regular} + \#(T-T_c)^2 \log(T-T_c)^2\right) \qquad v \simeq 1-\sqrt{2} \equiv v_c\,,\ \vec{q} \simeq 0$$

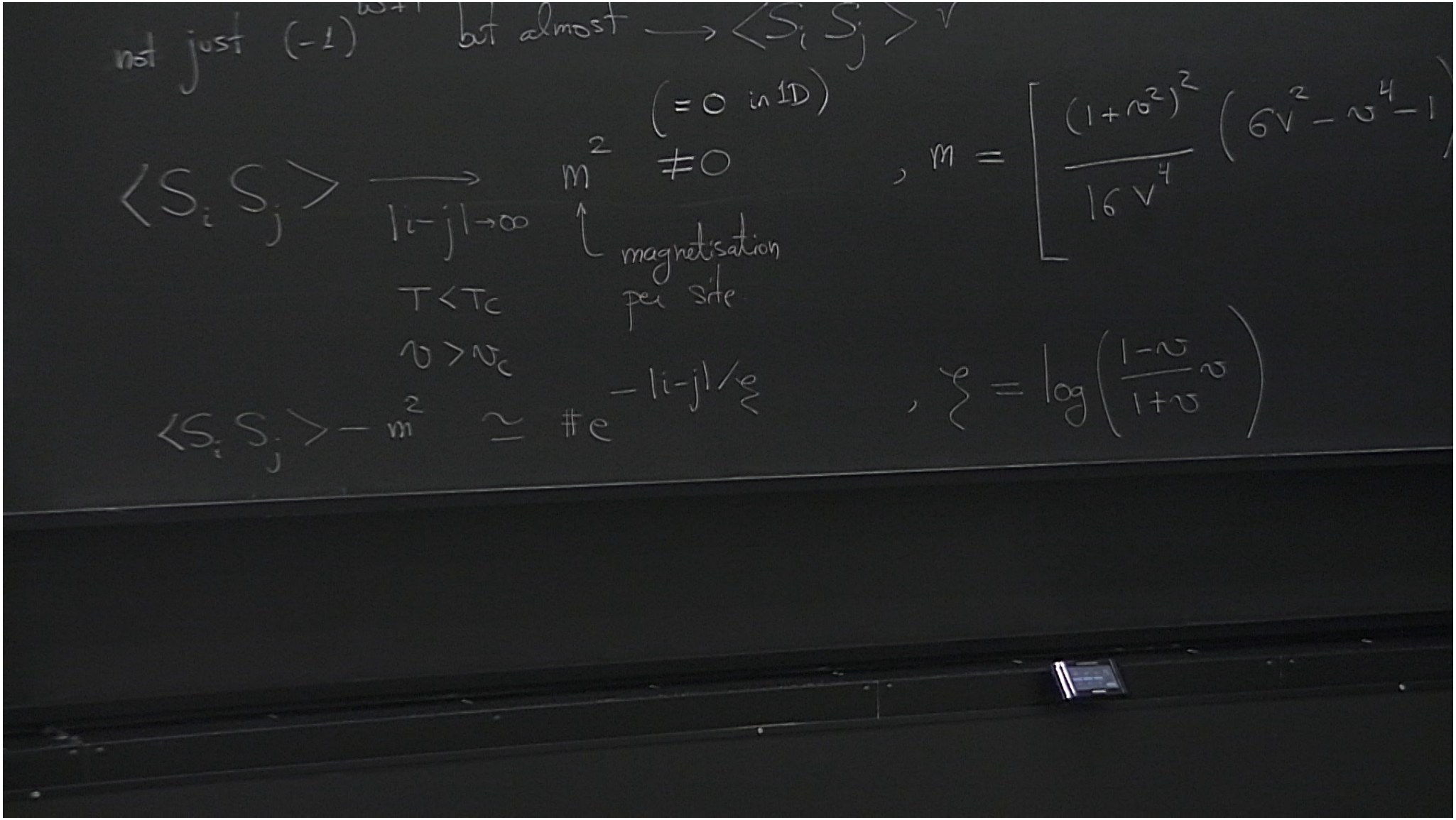
$$\Longrightarrow C = \frac{dE}{dT} \propto \frac{d^2}{d\beta^2}\log Z \propto \log(T-T_c) + \text{regular}$$

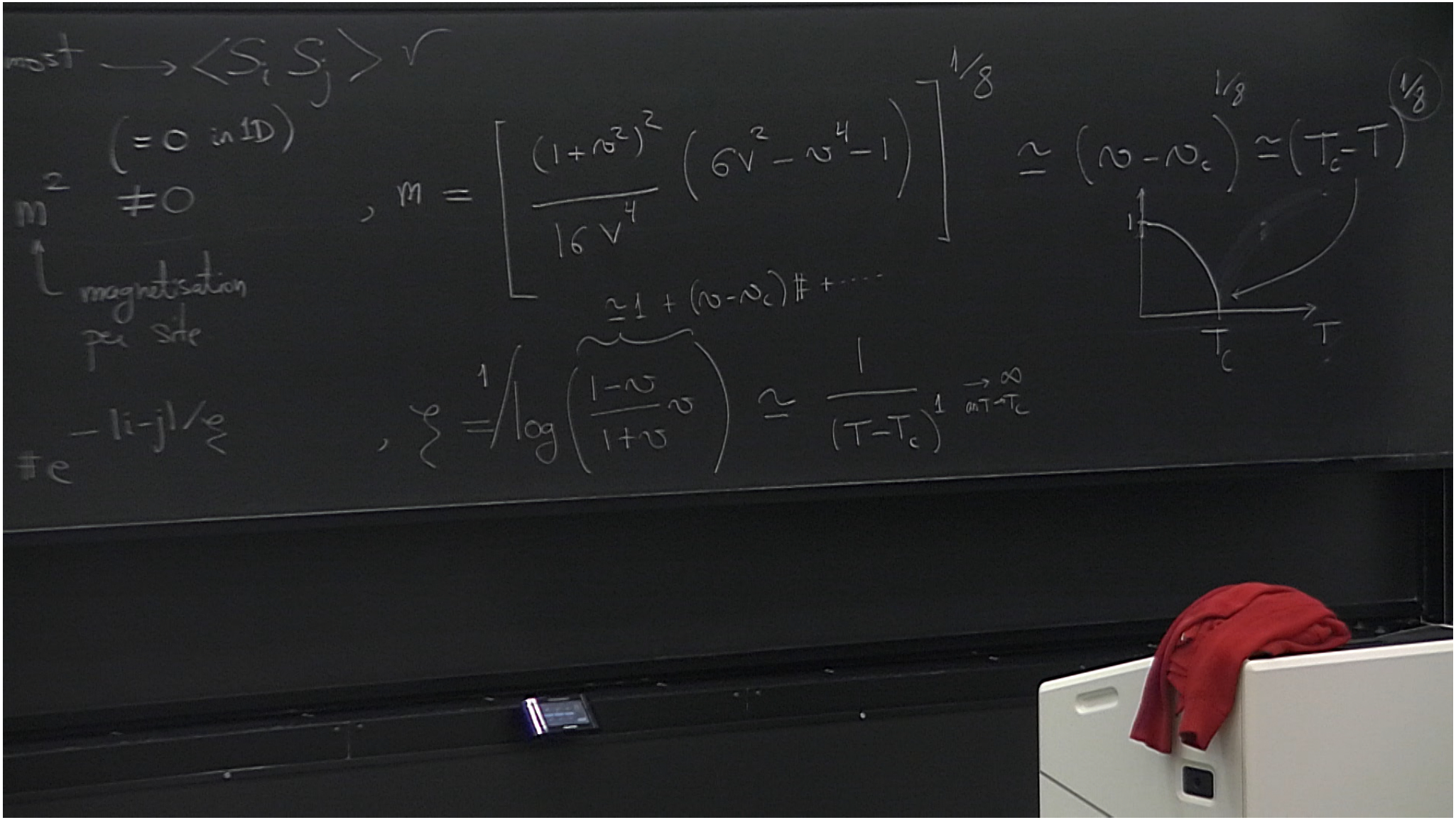

* D > 2 ?
not Known.
* D = 2, h ≠ 0 ⟶ no longer just closed paths
not Known.
$e^{hS_j} = \# + \# S_j$
$Z_{_}$
X
sources
odd # st

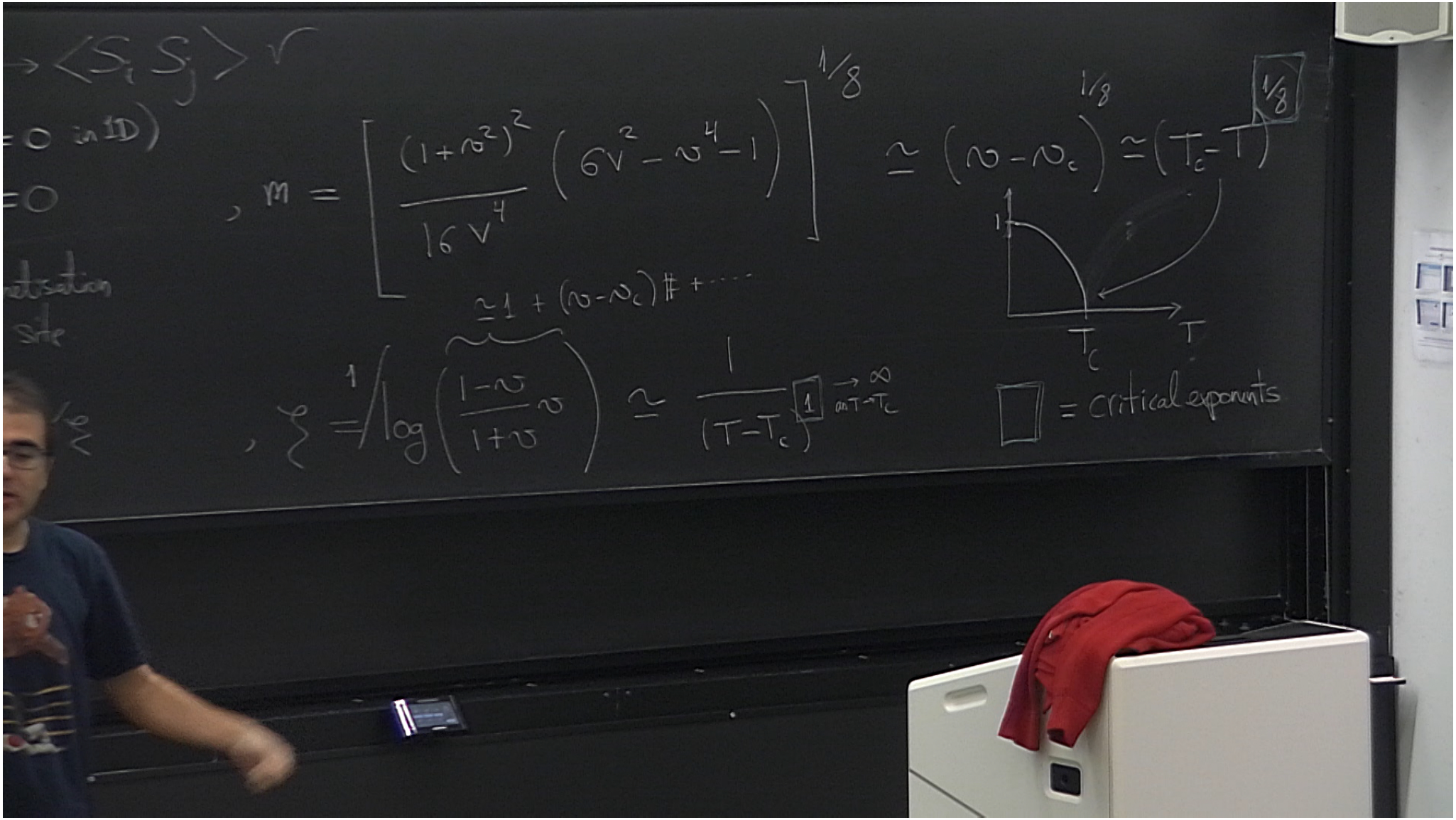

not simply related to $N_\ell(i,j,\mathbb{F})$
← not solved.
odd # sticks
sources
- sign rules
$(-1)^{\frac{N_L - N_R}{4}}$ is not good

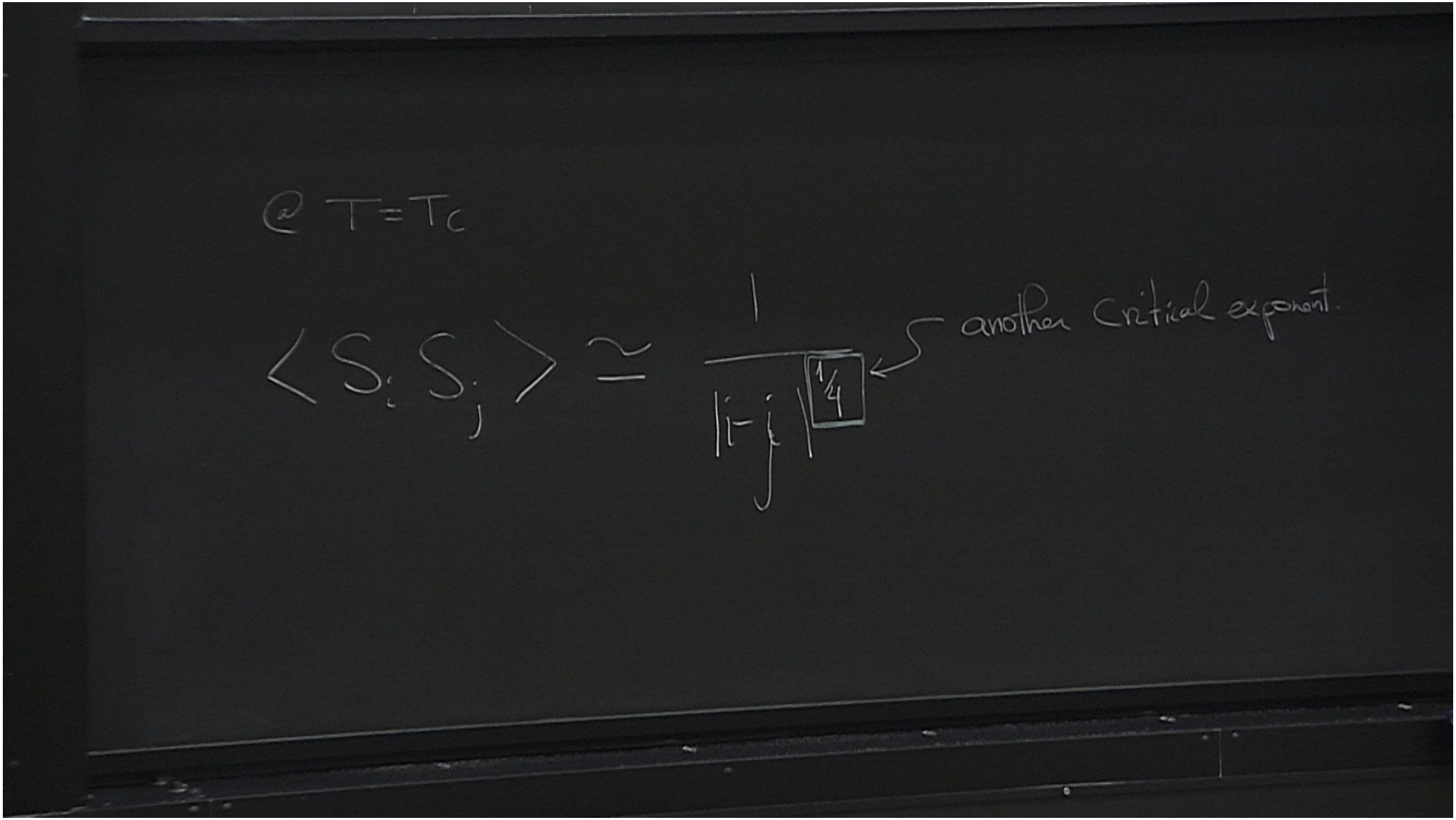

Known.
2nd reference 5.5.
not simply related to $N_\ell(i,j,\mathbb{F})$
Z_2
odd # sticks
sources
← not solved.
+
+
= 1
- sign varies
= 1
(-1)
= 1
$(-1)^{\frac{N_L - N_R}{4}}$
is not good

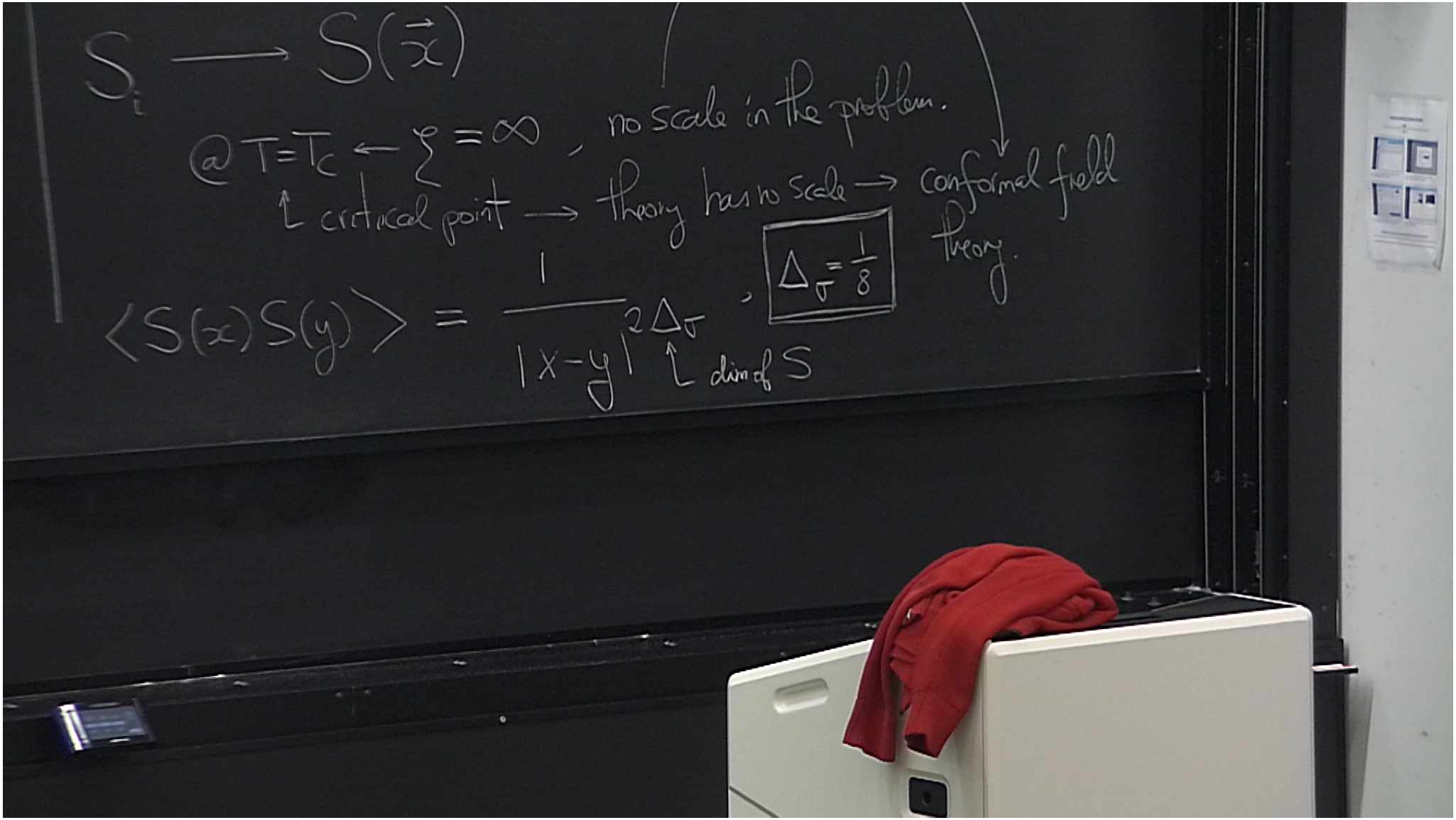

not just $(-1)^{\omega+1}$ but almost $\longrightarrow \langle S_i S_j \rangle$
$(=0$ in 1D$)$
$\langle S_i S_j \rangle \longrightarrow m^2 \neq 0$
$|i-j| \to \infty$
magnetisation per site
$T < T_c$
$v > v_c$
$, m = \left[\frac{(1+v^2)^2}{16 v^4} \left(6v^2 - v^4 - 1 \right) \right]$

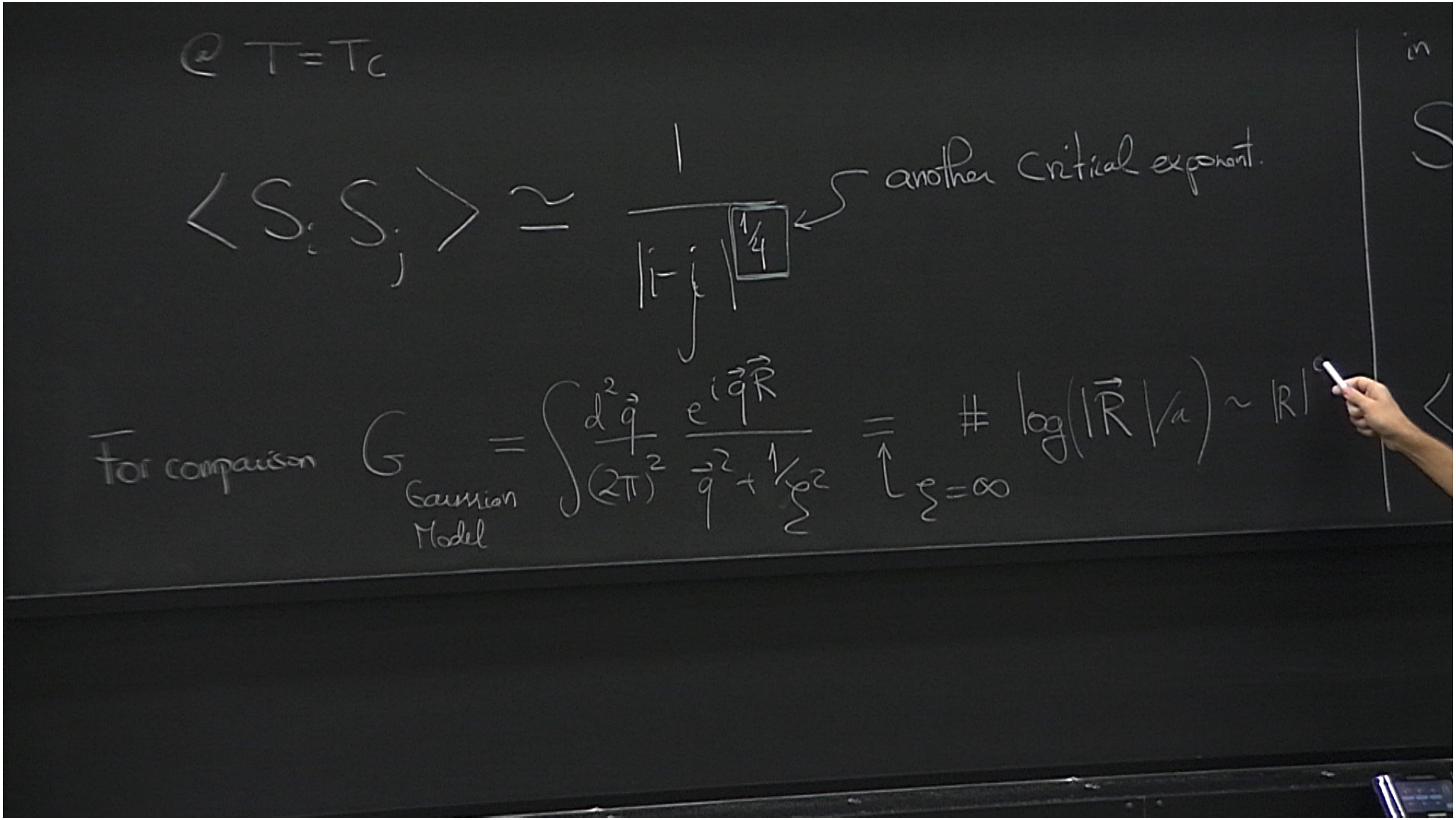

not just $(-1)^{|i-j|}$ but almost $\longrightarrow \langle S_i S_j \rangle$
$\langle S_i S_j \rangle \longrightarrow m^2 \neq 0$ $(= 0$ in 1D$)$
$|i-j| \to \infty$
magnetisation per site
$T < T_c$
$v > v_c$
$\langle S_i S_j \rangle - m^2 \simeq \# e^{-|i-j|/\xi}$
$m = \left[\frac{(1+v^2)^2}{16 v^4} (6v^2 - v^4 - 1) \right]$
$\xi = \log\left(\frac{1-v}{1+v} v \right)$

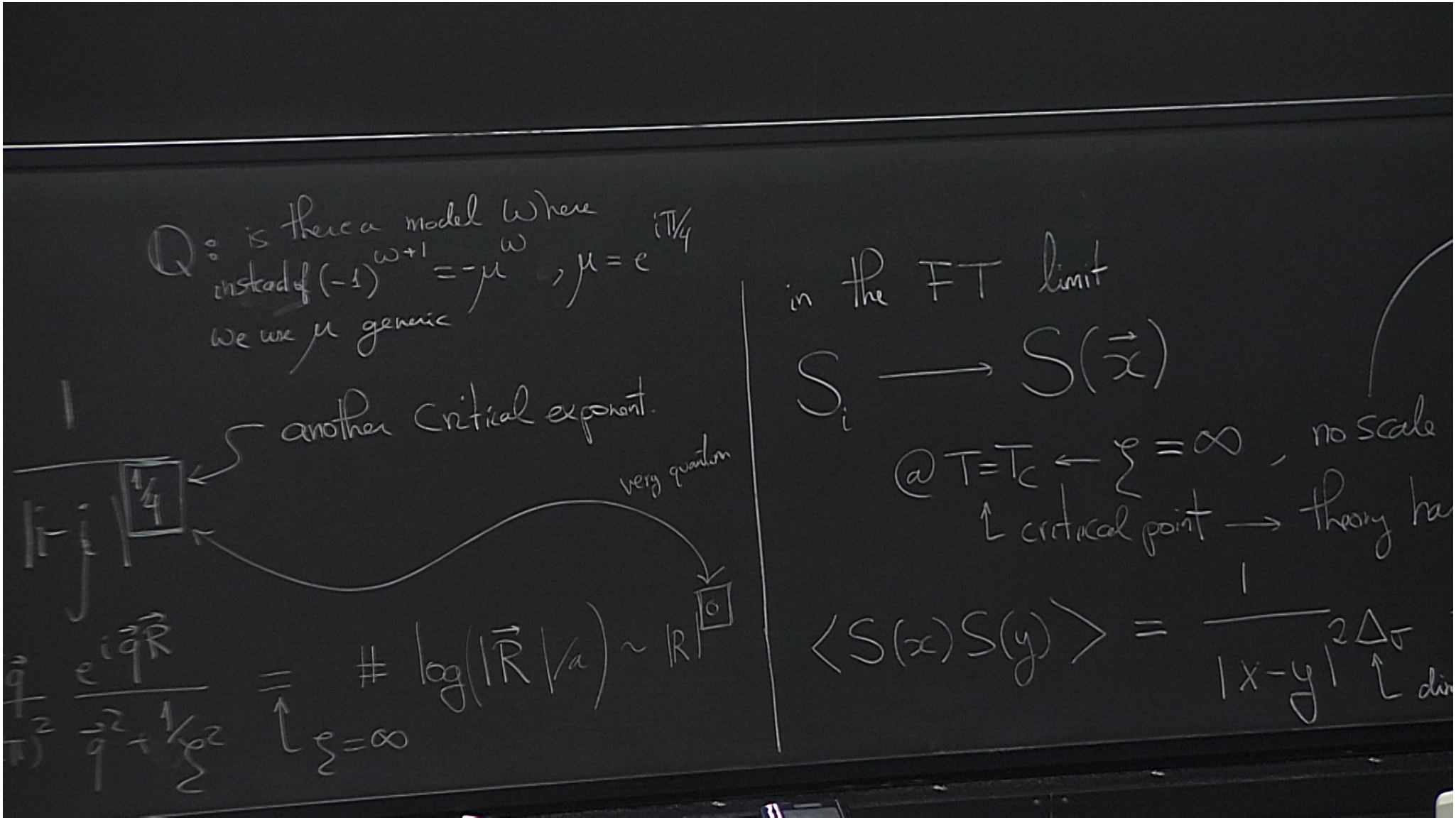

most → ⟨S_i S_j⟩ ↘
(= 0 in 1D)
m² ≠ 0
↑ magnetisation per site
, m = [(1+v²)² / 16 v⁴ (6v² − v⁴ − 1)]^{1/8}
≃ 1 + (v − v_c) # + ⋯
≃ (v − v_c)^{1/8} ≃ (T_c − T)^{1/8}
1
T_c
T
, ξ = 1/log((1−v)/(1+v) v) ≃ 1/(T−T_c)^1 → ∞ as T → T_c
e^{−|i−j|/ξ}


$\langle S_i S_j \rangle$
$m = \left[\frac{(1+v^2)^2}{16 v^4} (6v^2 - v^4 - 1) \right]^{1/8} \simeq (v - v_c)^{1/8} \simeq (T_c - T)^{1/8}$
$\simeq 1 + (v - v_c) \# + \cdots$
$\xi = \frac{1}{\log\left(\frac{1-v}{1+v} v\right)} \simeq \frac{1}{(T - T_c)^{1}} \xrightarrow[\text{as } T \to T_c]{} \infty$
T_c
T
= critical exponents

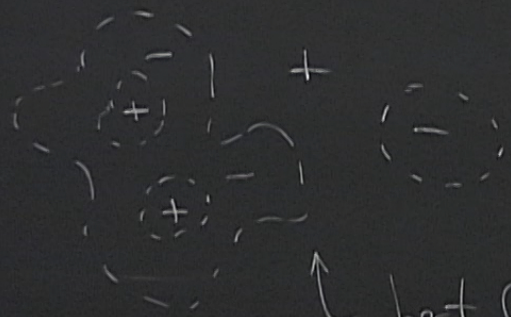

@ T=Tc
⟨Si Sj⟩ ≃ 1 / |i−j|^(1/4)
another critical exponent.


$S_i \longrightarrow S(\vec{x})$
@ $T=T_c \leftarrow \xi = \infty$, no scale in the problem.
↕ critical point → theory has no scale → conformal field theory.
$\langle S(x)S(y)\rangle = \frac{1}{|x-y|^{2\Delta_\sigma}}$. $\Delta_\sigma = \frac{1}{8}$
↕ dim of S


@ T=Tc
$\langle S_i S_j \rangle \simeq \frac{1}{|i-j|^{1/4}}$
another Critical exponent.
For comparison
Gaussian Model
$G = \int \frac{d^2\vec{q}}{(2\pi)^2} \frac{e^{i\vec{q}\vec{R}}}{\vec{q}^2 + 1/\xi^2} = \xi=\infty \; \# \log(|\vec{R}|/a) \sim |R|$
in
S


Q: is there a model where
instead of $(-1)^{\omega+1} = -\mu^{\omega}$, $\mu = e^{i\pi/4}$
we use μ generic
another critical exponent
very quantum
$\frac{1}{q^2} \frac{e^{i\vec{q}\vec{R}}}{\vec{q}^2 + 1/\xi^2} = \# \log(|\vec{R}|/a) \sim |R|^0$
$\xi = \infty$
in the FT limit
$S_i \longrightarrow S(\vec{x})$
@ $T = T_c \leftarrow \xi = \infty$, no scale
critical point → theory
$\langle S(x) S(y) \rangle = \frac{1}{|x-y|^{2\Delta_\sigma}}$

$$v = \tanh J = \frac{1 - e^{-2J}}{1 + e^{-2J}}$$

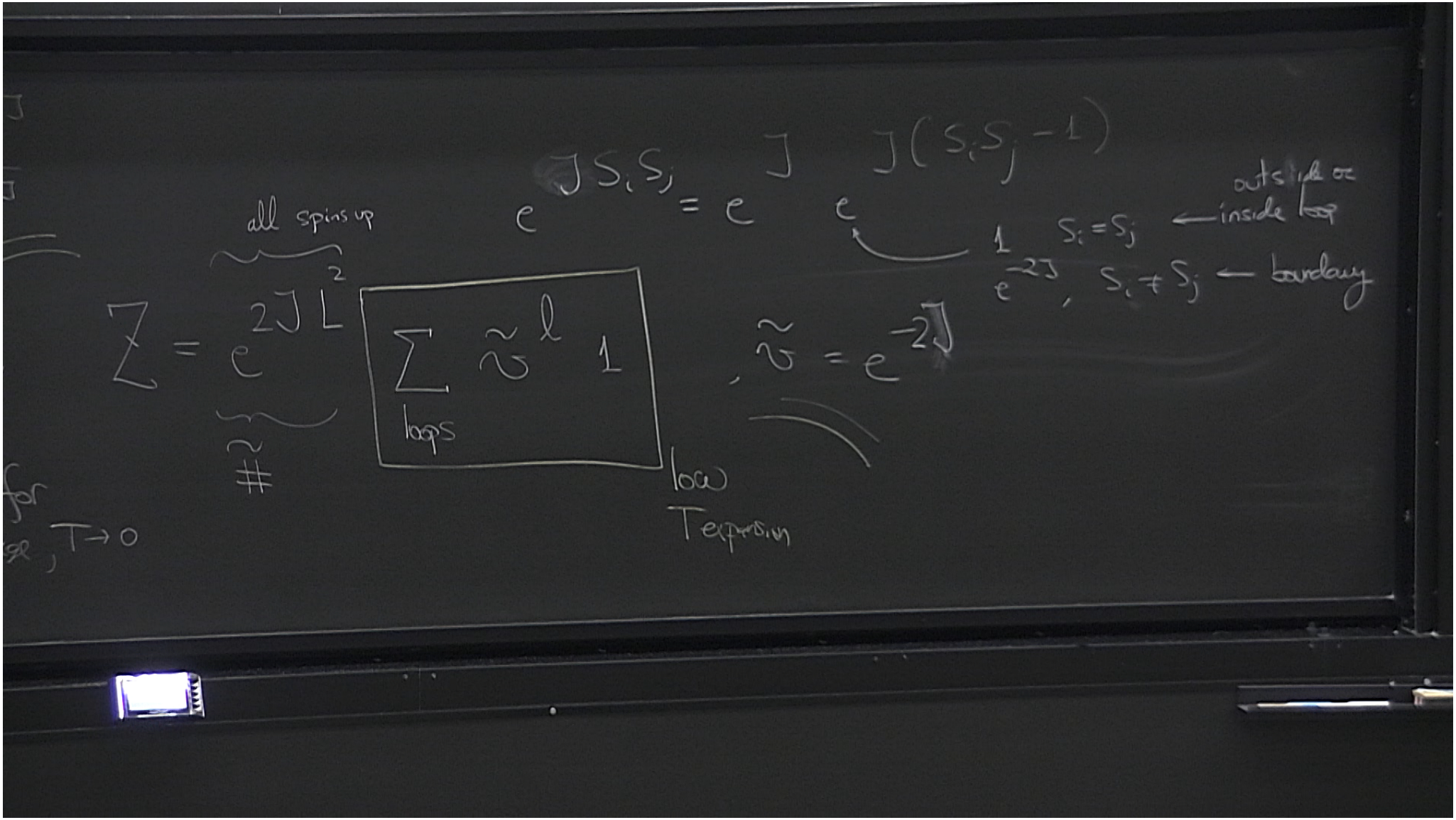


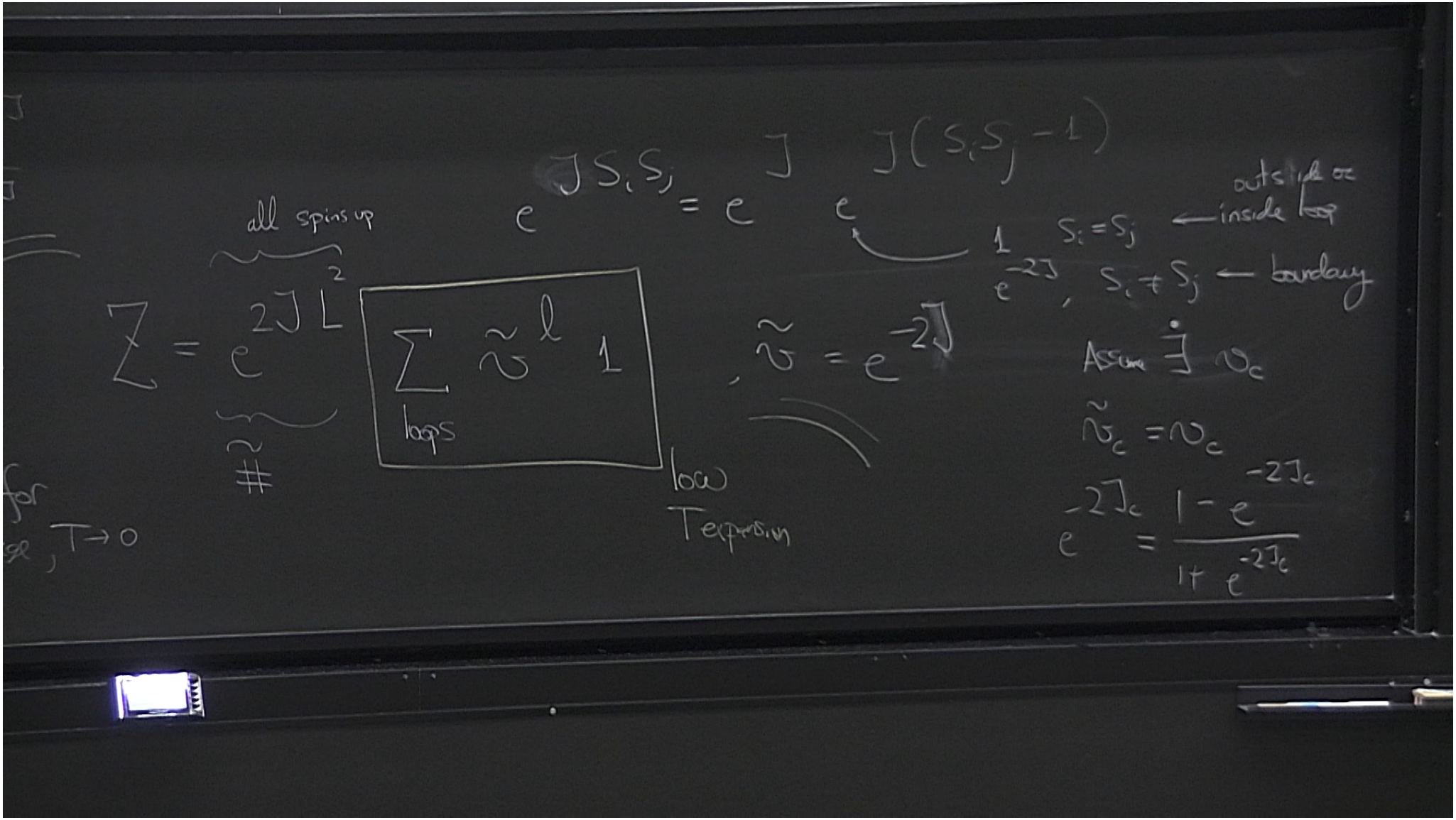
best for β large, $T \to 0$

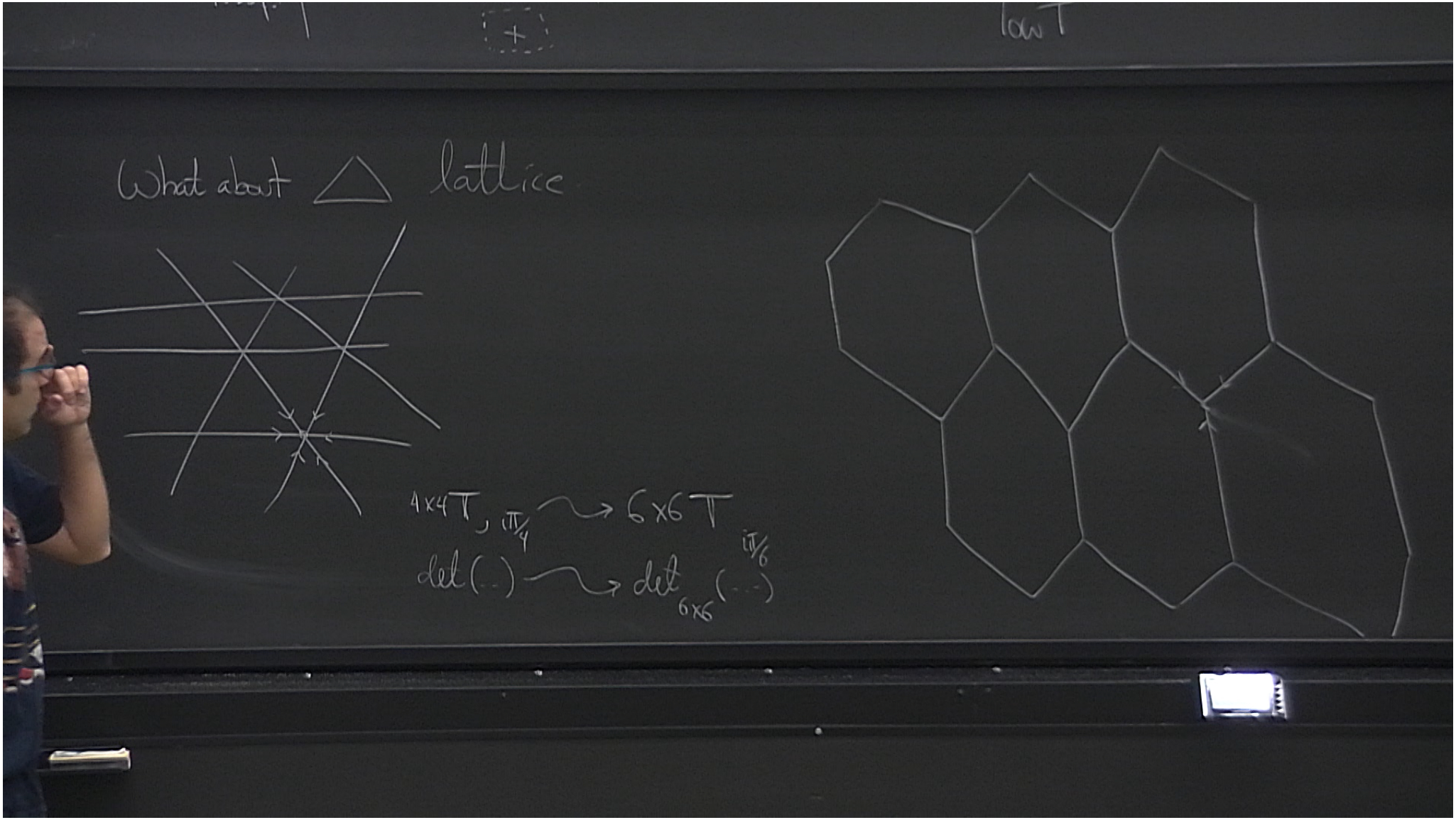
low T

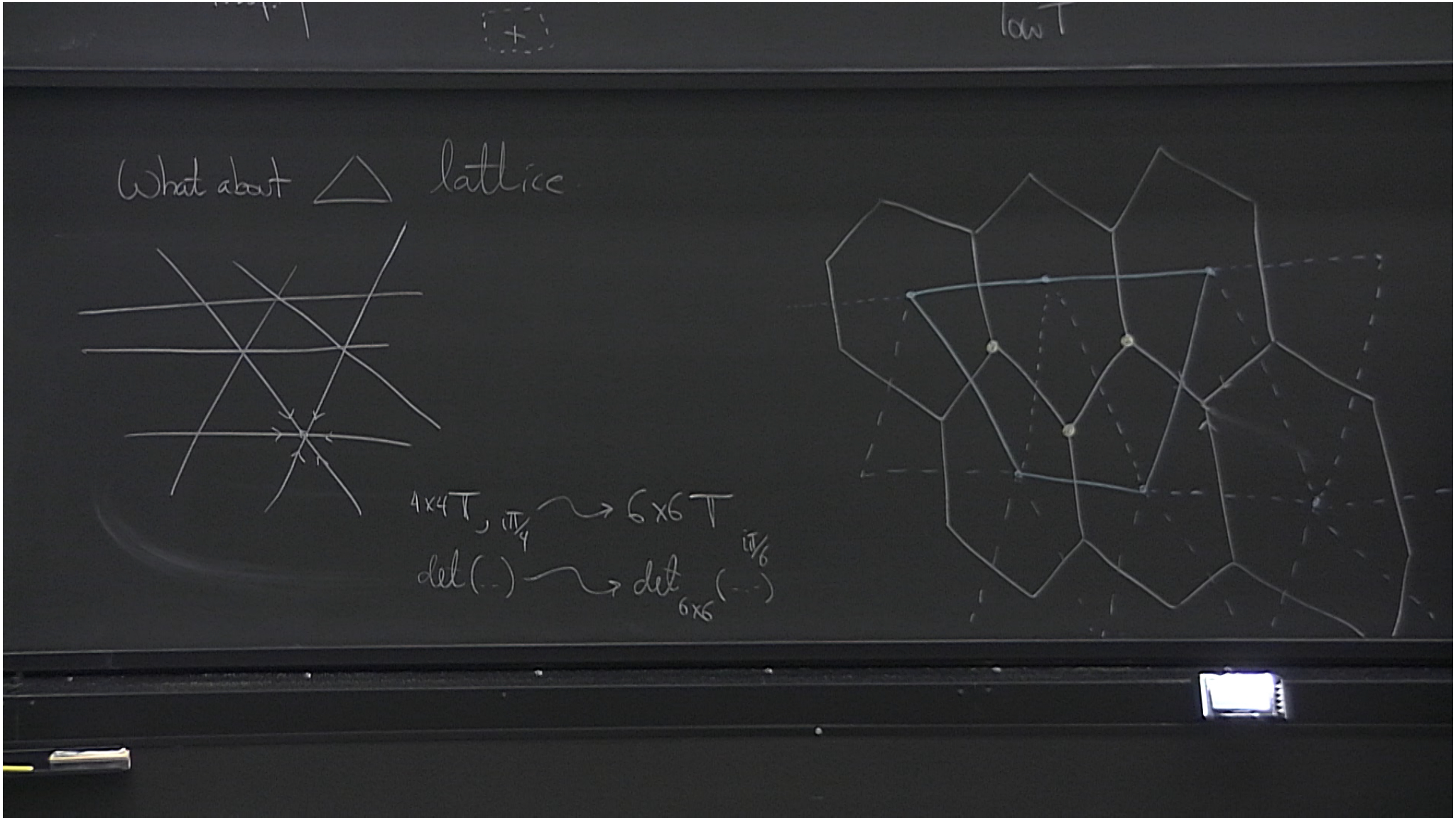
all spins up

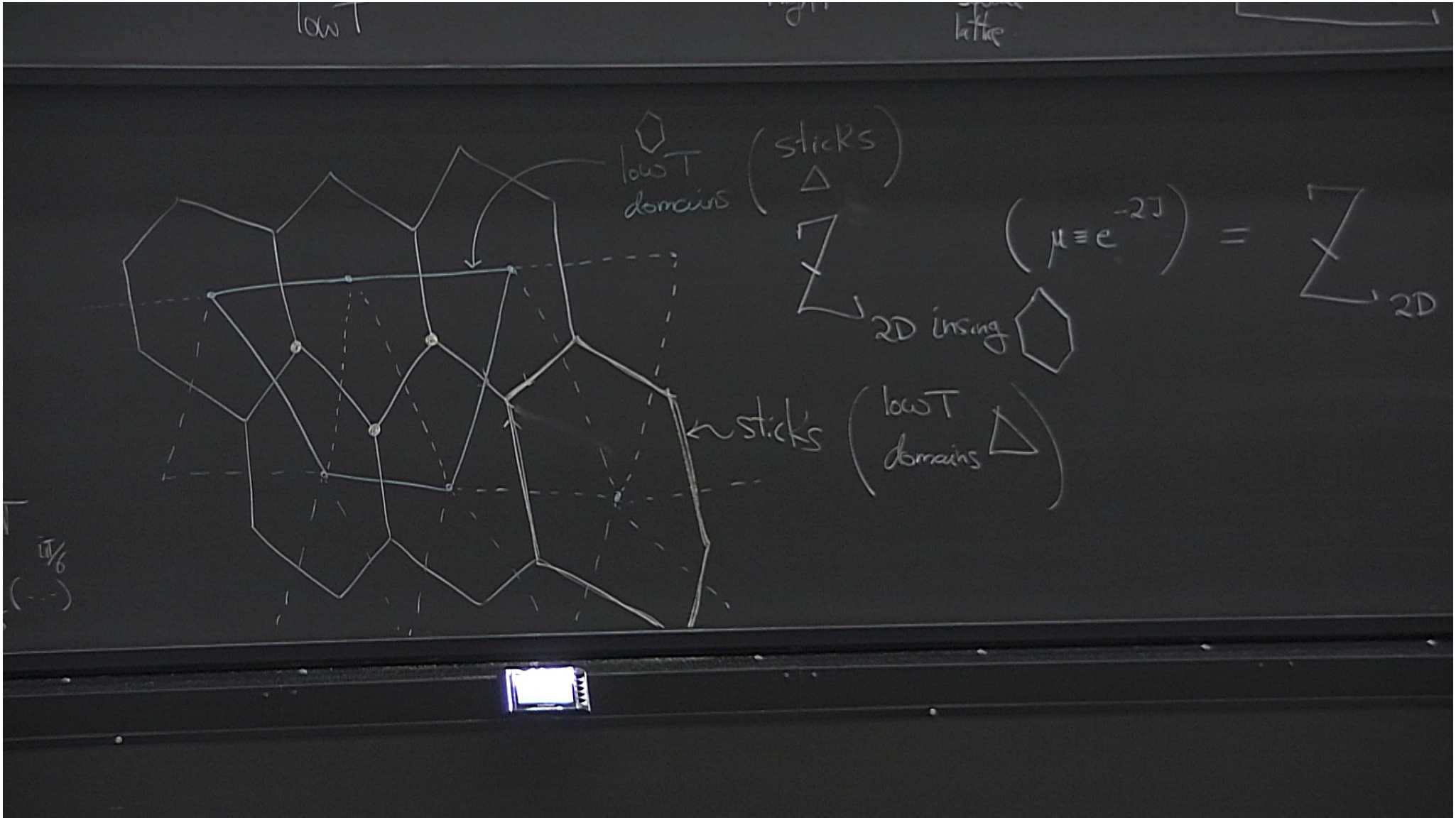
$$Z = e^{2JL^2} \sum_{\text{loops}} \tilde{v}^{l} \, 1 \qquad \tilde{v} = e^{-2J}$$

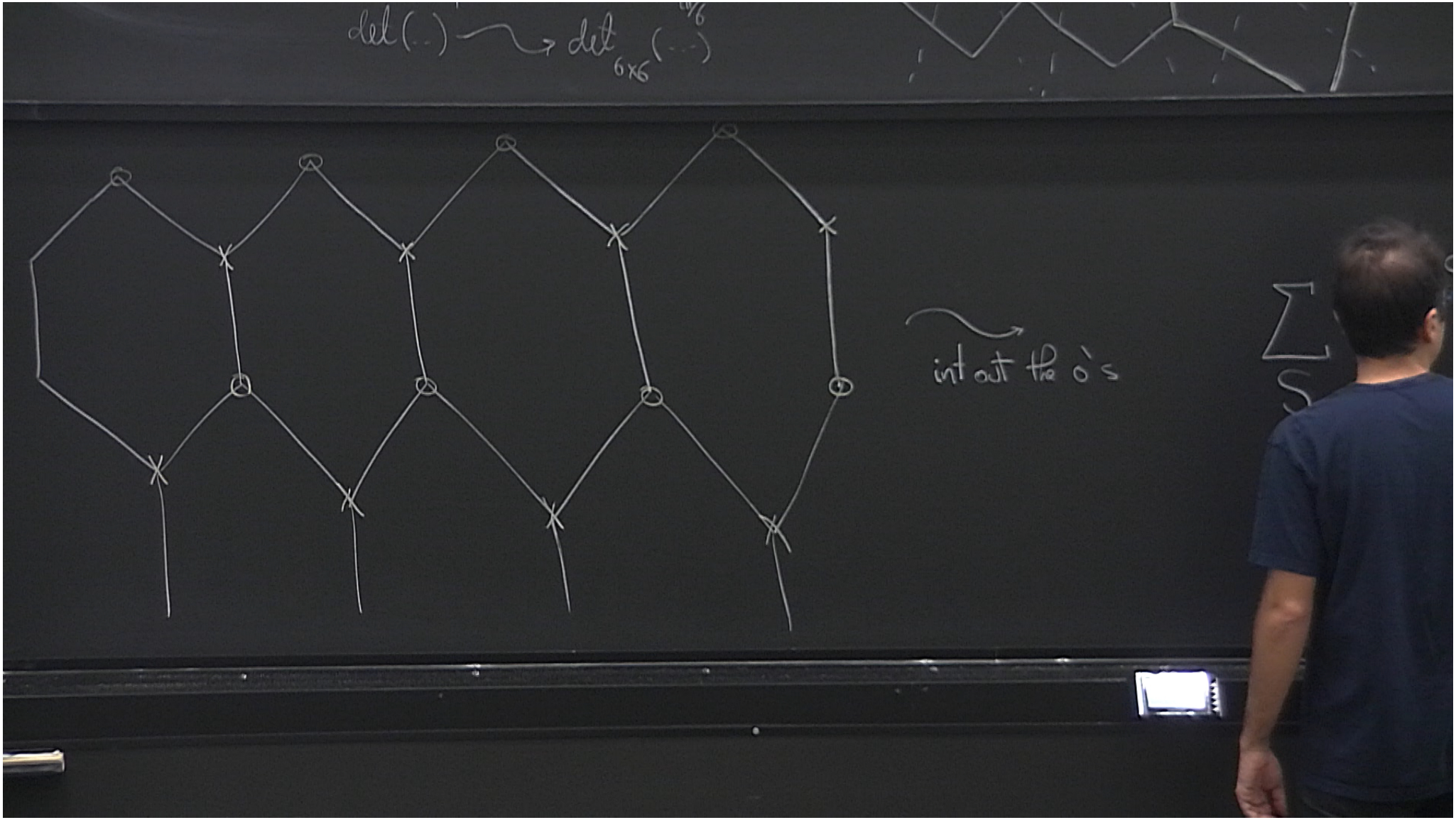

all spins up
$e^{JS_iS_j} = e^{J} e^{J(S_iS_j - 1)}$
$S_i = S_j$ ← inside loop
$S_i \neq S_j$ ← boundary
$Z = e^{2JL^2} \sum_{\text{loops}} \tilde{v}^{l} \cdot 1$
$\tilde{v} = e^{-2J}$
low Temperature
for T → 0

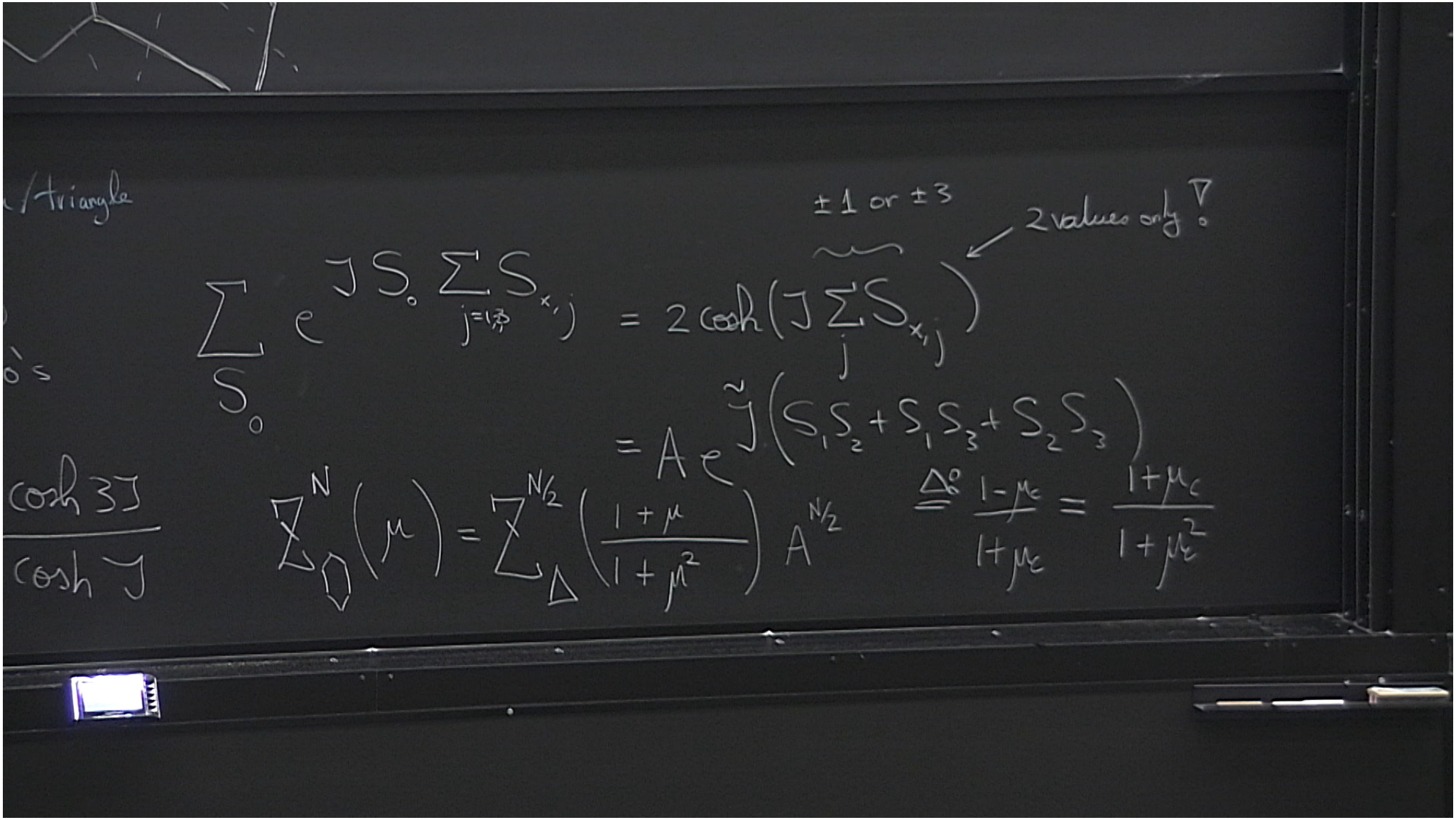

all spins up
$e^{JS_iS_j} = e^{J} e^{J(S_iS_j - 1)}$
outside or inside loop
$S_i = S_j$ ← inside loop
$S_i \neq S_j$ ← boundary
$Z = e^{2JL^2} \sum_{\text{loops}} \tilde{v}^{l} 1$
$\tilde{v} = e^{-2J}$
Assume $\exists\, v_c$
$\tilde{v}_c = v_c$
low Temperature
$e^{-2J_c} = \frac{1 - e^{-2J_c}}{1 + e^{-2J_c}}$

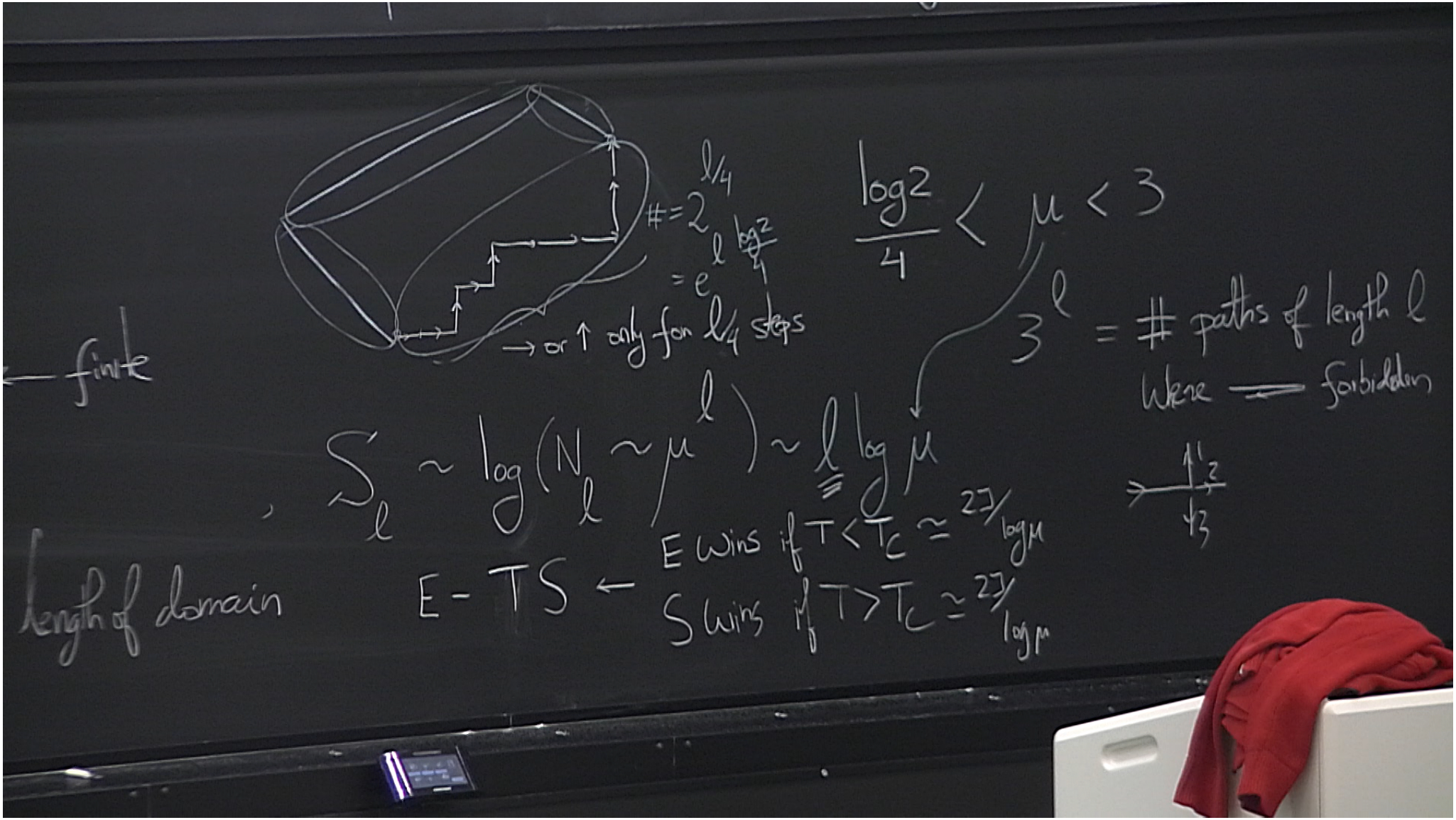

low T
What about △ lattice
4x4 T, iπ/4 → 6x6 T
det(..) → det 6x6 (..) iπ/6

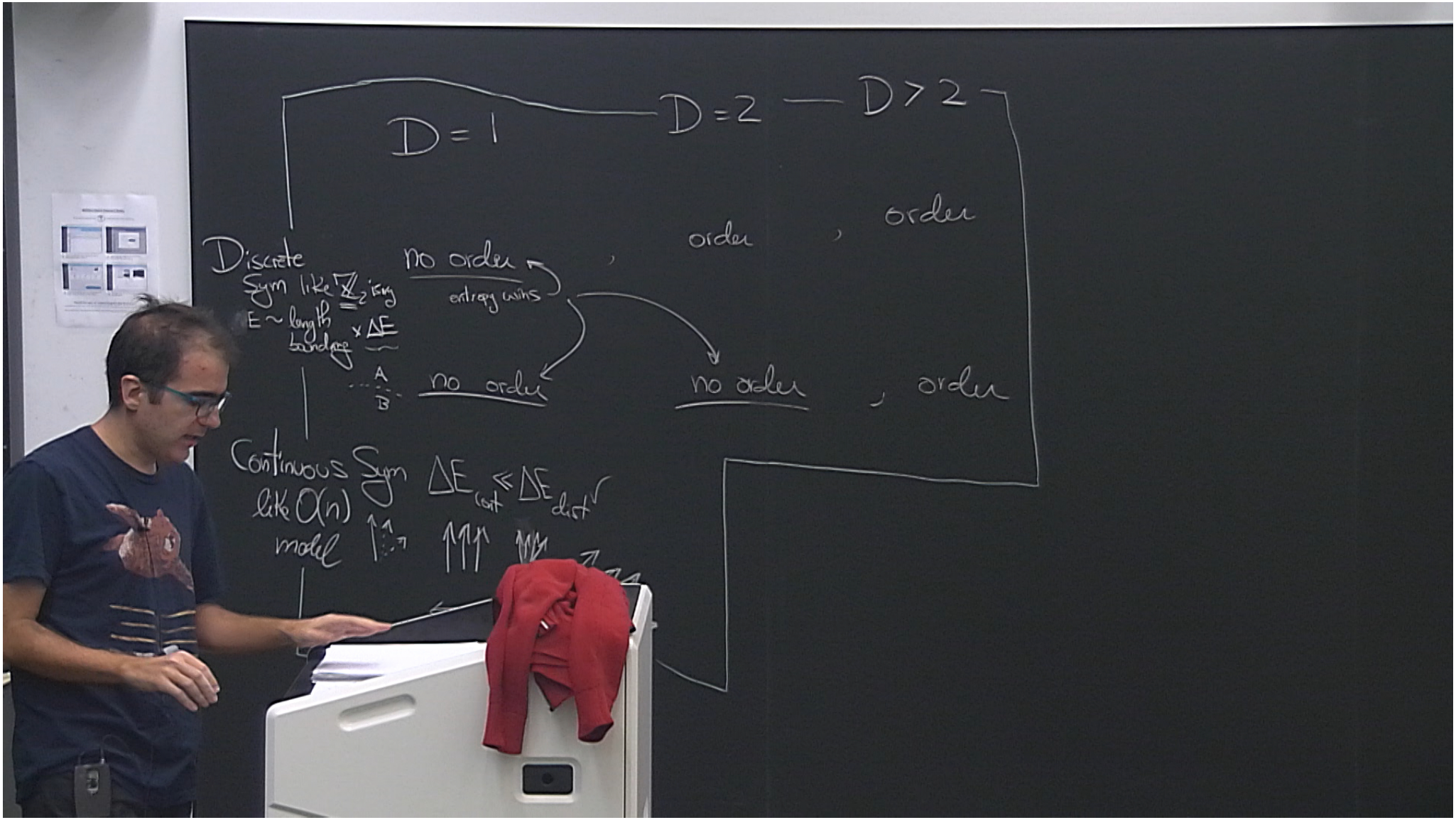

low T
What about △ lattice
4×4 T, iπ/4 → 6×6 T
det(..) → det 6×6 (..) iπ/6


low T
lattice
low T domains
(sticks)
$Z_{2D\ ising}$
$(\mu \equiv e^{-2J})$
$= Z_{2D}$
sticks
(low T domains)


det(..) ⇝ det 6×6 (...)
int out the o's


/triangle
±1 or ±3
2 values only!
$\sum_{S_0} e^{JS_0 \sum_{j=1,3} S_{x_j}} = 2\cosh\left(J \sum_j S_{x_j}\right)$
$= A e^{\tilde{J}(S_1S_2 + S_1S_3 + S_2S_3)}$
$\frac{\cosh 3J}{\cosh J}$
$Z_0^N(\mu) = Z_\Delta^{N/2}\left(\frac{1+\mu}{1+\mu^2}\right) A^{N/2}$
$\Delta^0 \frac{1-\mu_c}{1+\mu_c} = \frac{1+\mu_c}{1+\mu_c^2}$


finite
length of domain
→ or ↑ only for $l/4$ steps
$\# = 2^{l/4} = e^{l \frac{\log 2}{4}}$
$\frac{\log 2}{4} < \mu < 3$
3^l = # paths of length l
Where ⟹ forbidden
$S_l \sim \log(N_l \sim \mu^l) \sim l \log \mu$
E wins if $T < T_c \simeq 2J/\log\mu$
S wins if $T > T_c \simeq 2J/\log\mu$
E − TS ←


D = 1
D = 2
D > 2
order
order
Discrete
Sym like
E ~ length
no order
entropy wins
no order
no order
order
Continuous Sym
like O(n)
model
ΔE
ΔE