

Title: PSI 2019/2020 - Statistical Mechanics (Vieira) - Lecture 7

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Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

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URL: <http://pirsa.org/19100032>

$$\frac{1}{\sqrt{2\pi}} e^{-S_i^2/2}$$

$$du(S)$$

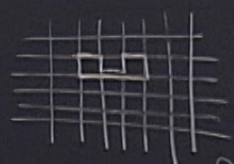
$$\exp\left(-t \sum_{\langle ij \rangle} S_i S_j\right)$$

sticks

$$t^{\sum}$$

$$\prod_{\langle ij \rangle} \frac{1}{n_{ij}!}$$

$$\prod_i (n_i - 1)$$



(and)
example

$\sum = \# \text{ sticks}$

$n_{ij} = \# \text{ stick in the link } i \longleftrightarrow j \text{ (} n_{ij} = 2 \text{)}$

$n_i = \# \text{ sticks leaving site } i \text{ } i \text{ --- } \cdot \text{ (} n_i = 4 \text{)}$

number of closed paths of length l .

N_l

$$L^d = L^2$$

$$(2V) + t^4 \frac{1}{2} \left(2V + \begin{array}{c} \square \\ \equiv \end{array} + \begin{array}{c} \sqcup \\ \equiv \end{array} + \dots \right) + t^6 \left(\begin{array}{c} \square \\ 2V \end{array} + \begin{array}{c} \sqcup \\ V \end{array} + \dots \right) + \dots$$

$$\sum_{\text{sticks}} t^{\sum} \prod_{\langle ij \rangle} \frac{1}{n_{ij}!} \prod_i \underbrace{(n_i - 1)!!}_{\langle S^{n_i} \rangle_{\mu}} \sum_{n_i \text{ even}}$$

$\sum = \# \text{ sticks}$

$n_{ij} = \# \text{ stick in the link } i \rightleftharpoons j \text{ } (n_{ij}=2)$

$n_i = \# \text{ sticks leaving site } i \text{ } i \equiv \cdot \text{ } (n_i=4) \leftarrow \text{needs to be even } (0, 2, 4, \dots)$

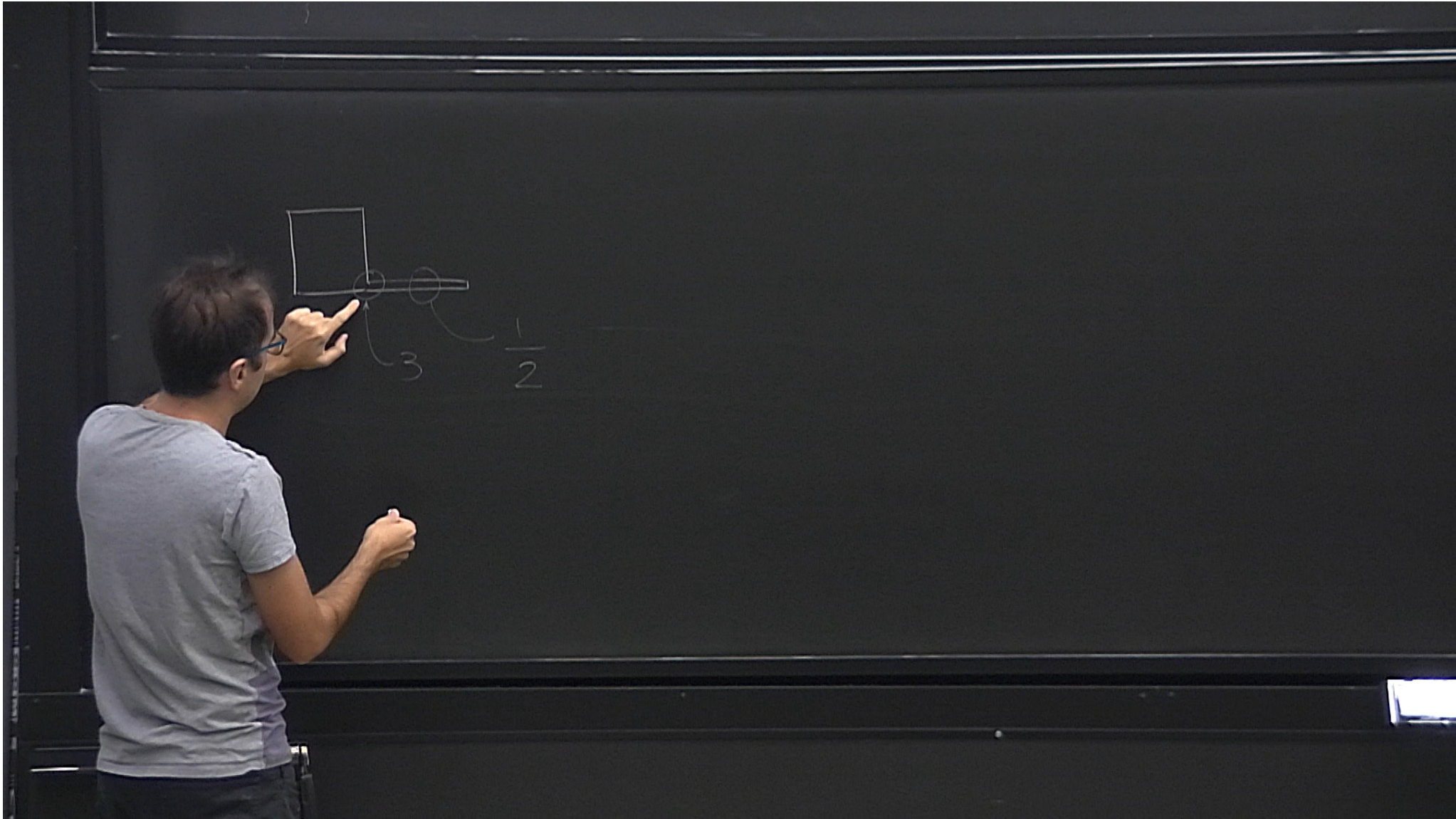
$$t^{\frac{\delta}{2}} \left(\begin{array}{c} \square \\ 2V + \Gamma + \dots \end{array} \right) + \dots$$

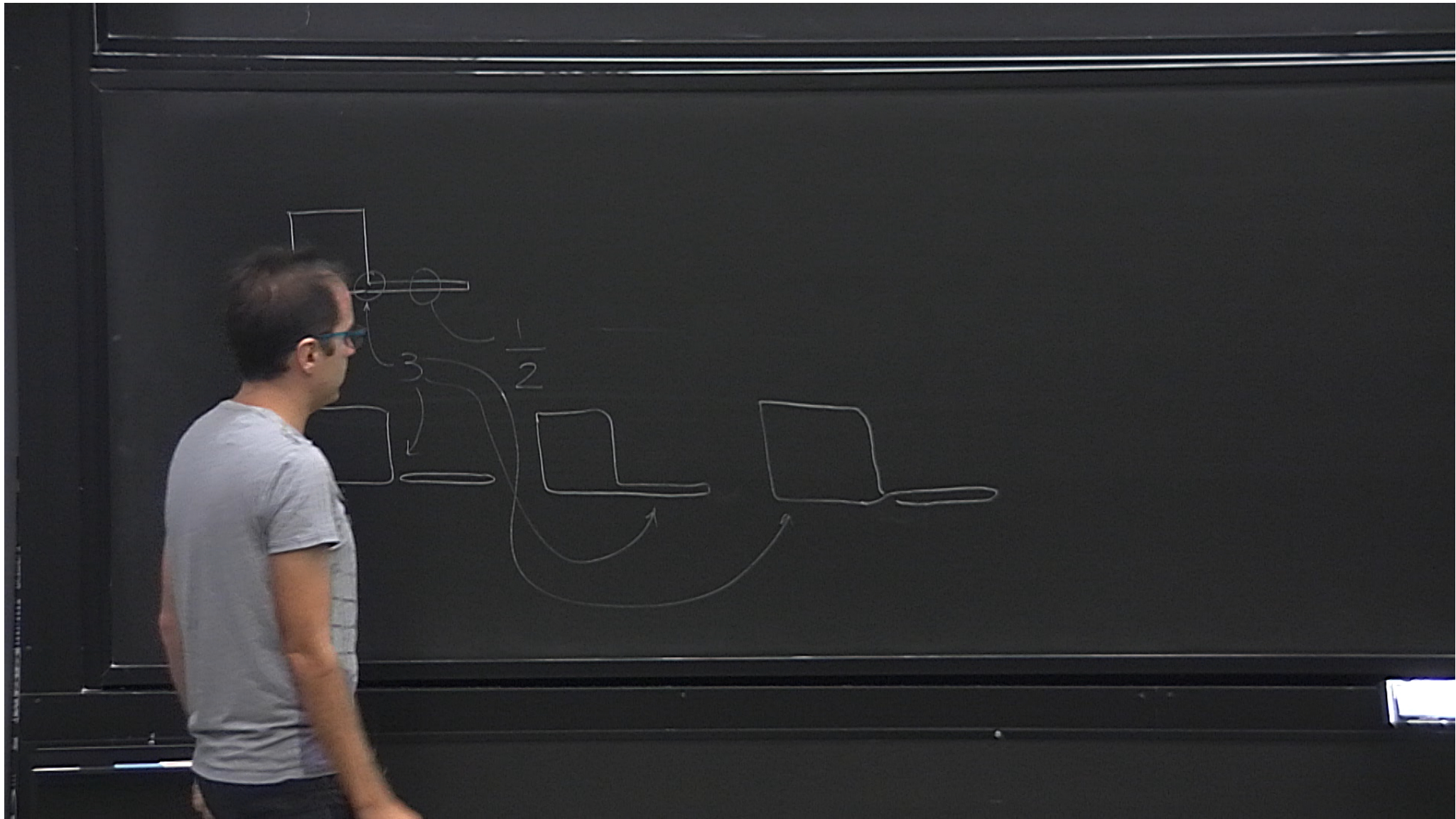
Tomorrow Stat. Ph
QFT

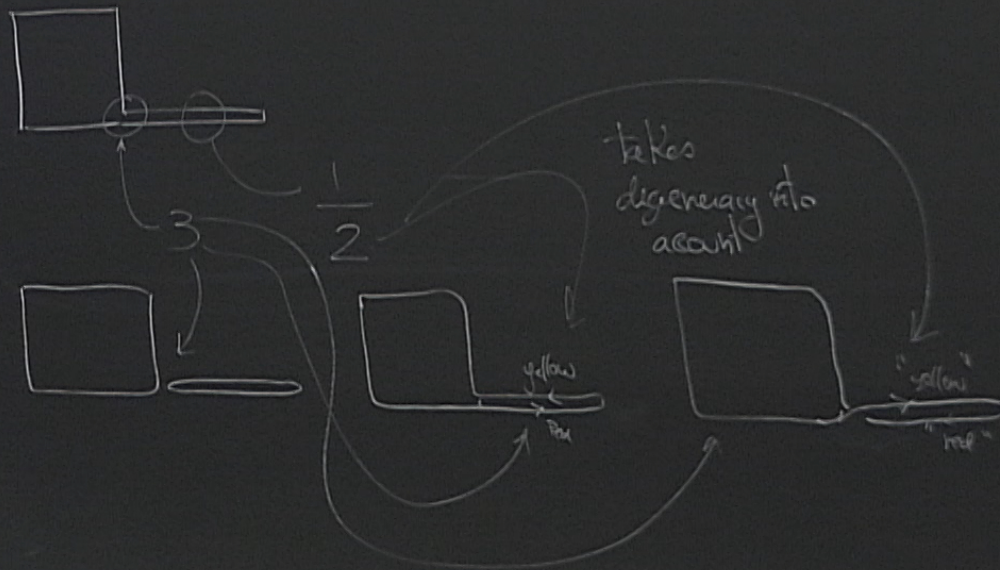
$$\frac{1}{1!} = 1$$

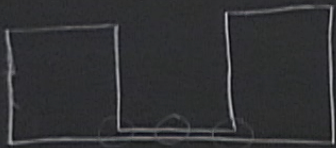
but

$$(2-1)!! = 1$$



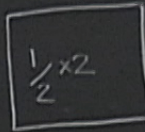




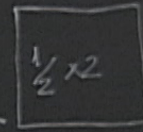


Weight: $\frac{9}{2}$

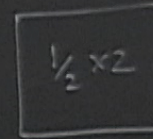
$\frac{1 \times 2}{2!}$



$\frac{1}{2}$

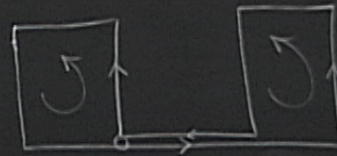


$\frac{1}{2}$

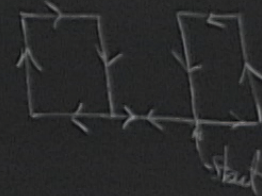
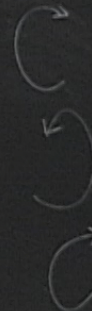


plus of

2



$\frac{4}{2}$



$\frac{9}{2}$ ✓

$$1) \quad \sum_{\text{Gaussian}} \stackrel{\checkmark}{=} \sum_{\text{RW}}$$

$$2) \quad \langle S_a S_b \rangle_{\text{Gaussian}} \stackrel{\text{next}}{=} \sum_l t^l N(a \rightarrow b)_l$$

$$\stackrel{||}{=} \frac{1}{Z} \int \prod_i d\mu(S(i)) \exp\left(\pm \sum_{\langle i,j \rangle} S_i S_j\right) S_a S_b$$

$$1) \quad Z_{\text{Gaussian}} \stackrel{\checkmark}{=} Z_{\text{RW}}$$

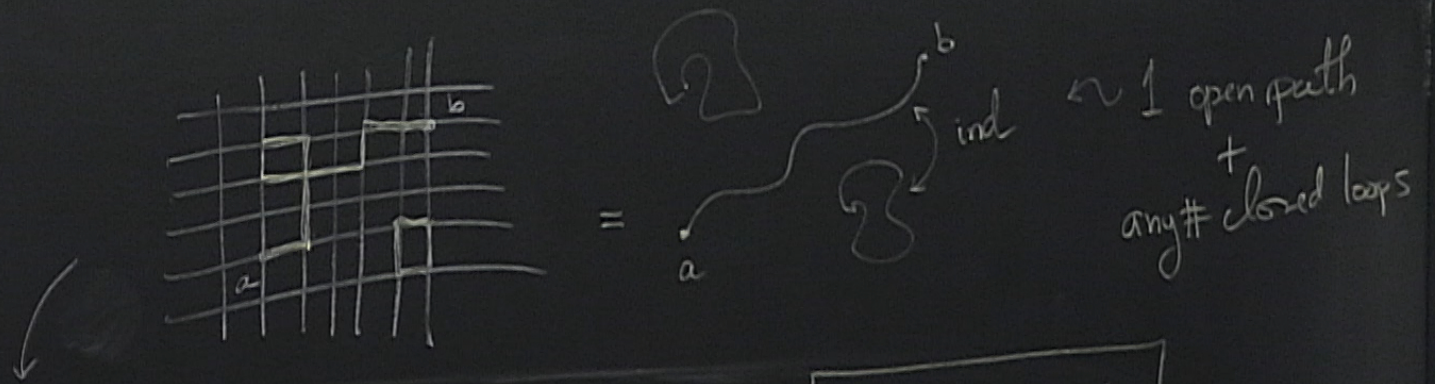
$$2) \quad \langle S_a S_b \rangle_{\text{Gaussian}} \stackrel{\text{next}}{=} \sum_l t^l N(a \rightarrow b)_l$$

$$\frac{1}{Z} \int \prod_i d\mu(S(i)) \exp\left(\pm \sum_{\langle i,j \rangle} S_i S_j\right) S_a S_b = \frac{1}{Z} \sum_{\text{Stic}}$$

$$N(a \rightarrow b)$$

$$l$$

$$S_i S_j) S_a S_b = \frac{1}{\sum} \sum_{\text{sticks}} t^{\sum} \prod_{\langle ij \rangle} \frac{1}{n_{ij}!} \prod_{i \neq a, b} (n_i - 1)!!$$



$$\sum_{\text{sticks}} \pm^{\Sigma} \prod_{\langle ij \rangle} \frac{1}{n_{ij}!} \prod_{i \neq a, b} (n_i - 1)!! \boxed{n_a!! \cdot n_b!!} \delta_{\substack{n_i \text{ even} \\ \text{except} \\ n_a, n_b \leftarrow \text{odd}}}$$

Universality in RW, in Gaussian

Other high T expansion for other models $\longleftrightarrow$ new dualities.

2.1) $\mathcal{O}(n)$ model $\longleftrightarrow$ SAWs $\xleftarrow{!}$ (McGreery & Cardy)

2.2) Ising Model $\longleftrightarrow$ Winding Walks $\Rightarrow$ Solve 2D

$\rightarrow$ new dualities. } Parisi
 SAWs $\leftarrow$ (McGreery & Cardy) }
 Dimer Walks $\Rightarrow$ Solve 2D Ising model. } 2nd ref, Parisi, Feynman.
 start by mid in afternoon, finish tomorrow

Universality

RW: start with prob with $C_2, C_3, C_4, \dots \rightarrow$ Gaussian

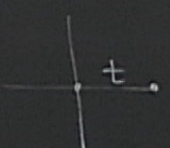
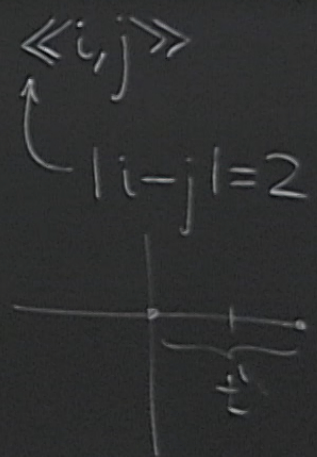
Gaussian: $\langle S(i) S(j) \rangle = \int \frac{d^d \vec{q}}{(2\pi)^d} \frac{e^{i\vec{q} \cdot \vec{R}}}{1 - 2t \sum \cos q_\mu} \approx$

$\underbrace{\vec{R}}_{\text{apart}}$

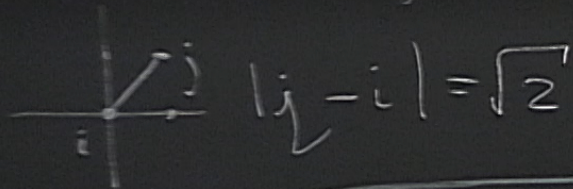
$\vec{R} C_2, C_3, C_4, \dots \longleftrightarrow \text{Gaussian } C_2 \neq 0, C_{n>2} = 0$

$$> = \int \frac{d^d \vec{q}}{(2\pi)^d} \frac{e^{i\vec{q} \cdot \vec{R}}}{1 - 2\pm \sum \cos q_\mu} \simeq m^2 + \vec{q}^2 \quad @ \text{ small } \vec{q} \leftrightarrow \text{ large } \vec{R}$$

$$t \sum_{\langle ij \rangle} S_i S_j \longrightarrow t \sum_{\langle ij \rangle} S_i S_j + t' \sum_{\langle\langle ij \rangle\rangle} S_i S_j + t'' \sum_{\langle\langle\langle ij \rangle\rangle\rangle} S_i S_j$$

$$\sum S_i S_j + \dots$$



$$|j-i| = \sqrt{2}$$

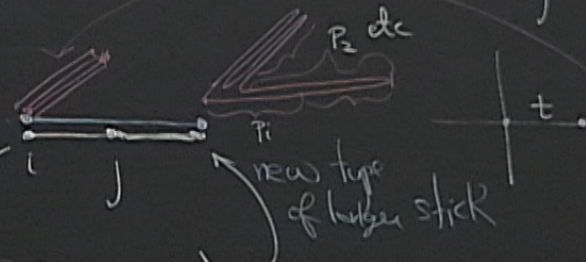
Same type $\frac{1}{m^2 + \vec{q}^2}$ prop at large dist $\leftarrow$
 $\# + \# \vec{q} + \dots$
 Universality

$$\rightarrow 1 - 2t \sum_{\mu=1}^d \cos q_{\mu} - 2t^2 \sum \cos 2q_{\mu} - 2t^2 (\cos(q_x + q_y) + \cos(-q_x + q_y) + \dots)$$

$$t \sum_{\pm, \mu} e^{\pm q_{\mu}} = t \sum_{\pm, \mu} e^{\pm \vec{q} \cdot \hat{e}_{\mu}}$$

$$1) \quad t \sum_{\langle ij \rangle} S_i S_j \longrightarrow t \sum_{\langle ij \rangle} S_i S_j + t' \sum_{\langle\langle ij \rangle\rangle} S_i S_j +$$

2) $\uparrow$
sticks



$\uparrow$
 $\langle\langle ij \rangle\rangle$
 $|i-j|=2$

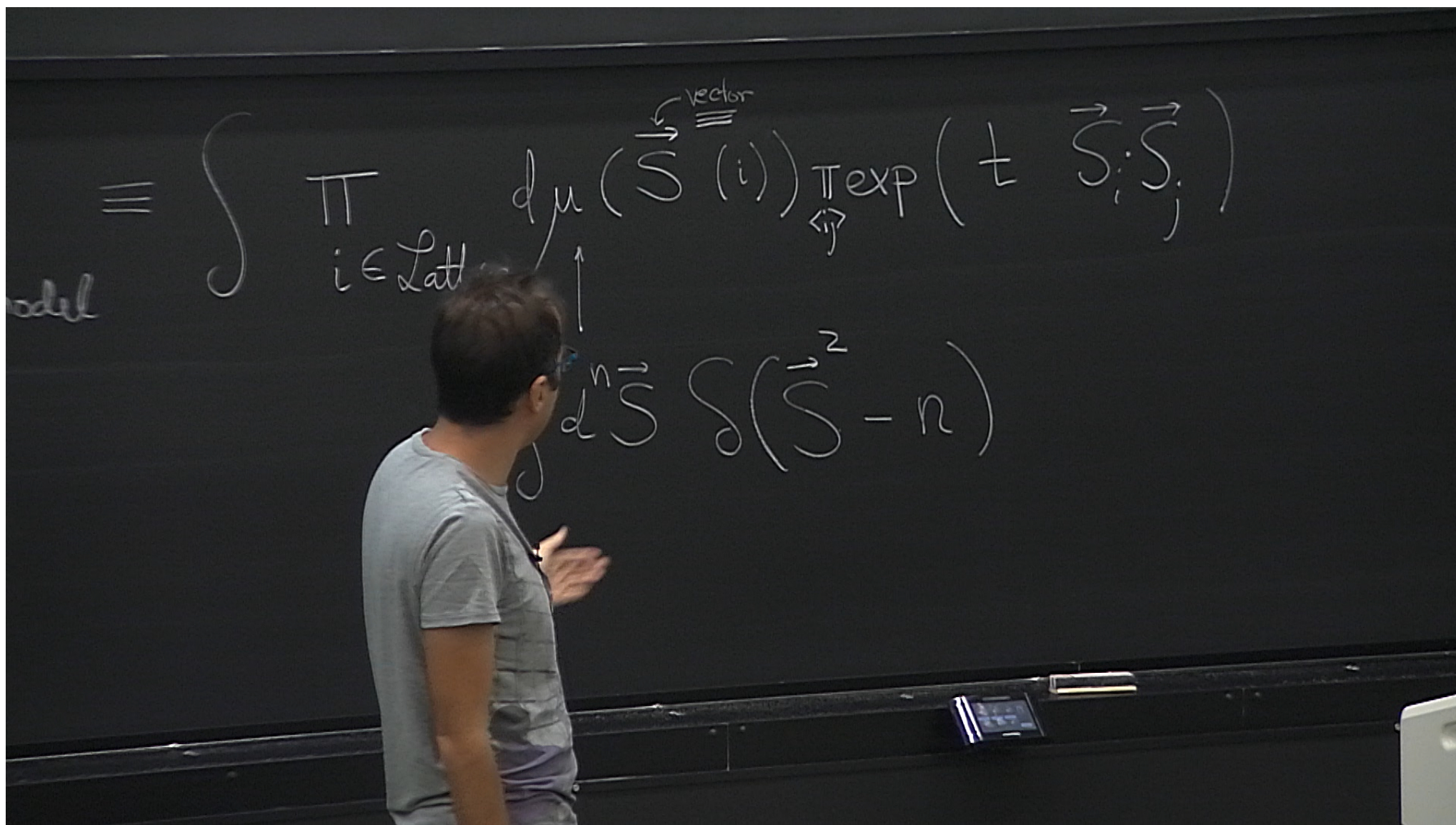
$$\pi e \sum_{\langle ij \rangle} t S_i S_j \quad \pi e \sum_{\langle\langle ij \rangle\rangle} t' S_i S_j \quad \pi e \sum_{\text{diag}} t'' S_i S_j$$

$$Z_{O(n) \text{ model}} \equiv \int \prod_{i \in \text{Lattice}} d\mu(\vec{S}(i)) \exp$$

$\vec{S} \equiv \text{vector}$

$$\begin{aligned}
 &\equiv \int \prod_{i \in \text{Lattice}} d\mu(\vec{S}(i)) \prod_{\langle ij \rangle} \exp\left(-\frac{1}{2} \vec{S}_i \cdot \vec{S}_j\right) \\
 &\quad \int d^n \vec{S} \, \delta(\vec{S}^2 - n)
 \end{aligned}$$

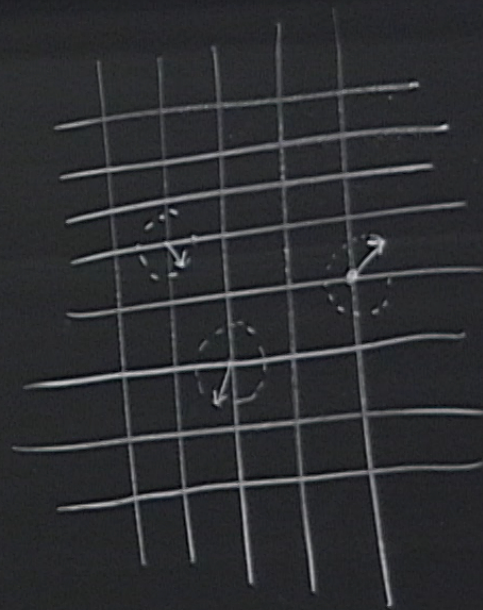
(Note: In the original image, there is an arrow pointing from the word "vector" to the vector $\vec{S}$ in the first integral, and an arrow pointing from the $d\mu$ measure to the $d^n \vec{S}$ measure in the second integral.)



$$e \int d\mu(\vec{S}(i)) \prod_{\langle ij \rangle} \exp\left(-\frac{1}{2} \vec{S}_i \cdot \vec{S}_j\right)$$

$\xrightarrow{\text{vector}}$
 $\xrightarrow{\text{vector}}$

$$\int d^n \vec{S} \delta(\vec{S}^2 - n) \quad \left(\text{ising if } n=1 \right)$$



$$(n \neq d)$$

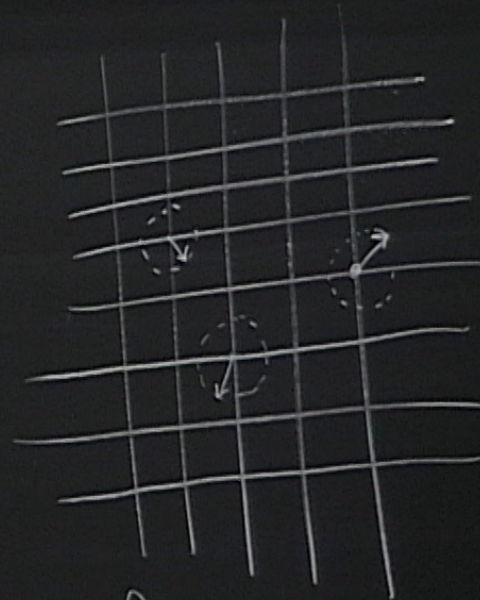
$$\prod_{i \in \text{Lattice}} d\mu(\vec{S}(i)) \prod_{\langle ij \rangle} \exp\left(\pm \vec{S}_i \cdot \vec{S}_j\right)$$

$i \in \text{Lattice}$

$d \nearrow$
L-points

$$\int d^n \vec{S} \delta(\vec{S}^2 - n) \quad \left(\text{ising if } n=1 \right)$$

$\longrightarrow \text{sticks + Spins} \longrightarrow \text{sticks} \longrightarrow \text{paths}$



$$\langle S^n \rangle = \sum_{n, \text{even}} \quad \text{Ising model}$$

$$= (n-1)!! \sum_{n, \text{even}}$$

Gaussian Model

$$\langle S^{a_1} S^{a_2} \dots S^{a_n} \rangle$$

$\swarrow$ component a_1 of $\vec{S}$

$$= \sum_{n, \text{even}}$$

$a_1 \dots a_n$
Tensor

$$\langle 1 \rangle = 1$$

$$\langle S^a S^b \rangle = \# \int^{ab}, \quad \# = 1$$

components, not positions
 $b=a, \sum_a$

$$\langle \vec{S}^2 \rangle = n$$

$$\langle 1 \rangle = 1$$

$$\langle S^a S^b \rangle = \# \delta^{ab}$$

components, not positions
 $b=a, \sum_a$

$$\langle S^2 = n \rangle = n$$

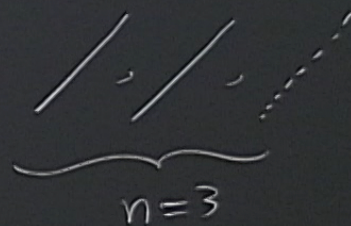
$\# = 1 \leftarrow \frac{1}{n}$

$\delta_{AB} = \frac{1}{A} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1}$

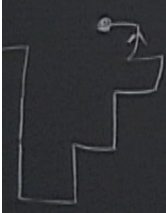
$\sum_c S^a \cdot S^c = 1$ x path A → B

positions!

other configs
with $n_j > 1$
and/or $n_i > 2$



$$= 0$$



$$\langle S^a S^b \rangle_\mu = 1 \delta^{ab}$$

$\vdash$ good candidate $\# \text{blue}, \# \text{white is even}$

$$\langle S^a S^b S^c S^d \rangle = \# \left(\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc} \right)$$

$$n^2 =$$

$$\uparrow$$

$$a \leftrightarrow b$$

$$c \leftrightarrow d$$