Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 11

Speakers: Dan Wohns, Gang Xu

Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Date: October 25, 2019 - 9:00 AM

URL: http://pirsa.org/19100023

Yesterday

we write down all Lorentz covariant fermion bilinears.

Classified with Parity (Mirror symmetry)

Global symmetry.

$$\psi \to e^{-i\alpha}\psi$$

Noether theorem

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

$$N_f = \int \psi^\dagger \psi \, d^3x$$

N_f: fermion #

symmetry.

$$\mathcal{L}_{int} = \bar{\psi}\gamma^\mu \psi$$

$e^{-i\alpha}\psi$

eorem

$\gamma^\mu \psi$

$\int \psi^\dagger \psi \, d^3x$

symmetry.

$e^{-i\alpha}\psi$

eorem

$\gamma^\mu\psi$

$\int \psi^\dagger\psi \, d^3x$.

$\mathcal{L}_{int} = \bar{\psi}\gamma^\mu A_\mu \psi$

remember $A_\mu \to \partial_\mu\alpha + A_\mu$.

gauge symmetry. $\mathcal{L}_{int}$ extra term.

$\bar{\psi}\gamma^\mu \partial_\mu \alpha \psi$

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

A_μ.

gauge the global symmetry: local the α parameter

$$\psi \to e^{-i\alpha(x)}\psi$$

$$\mathcal{L}_{int} = \bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

remember $A_\mu \to \partial_\mu\alpha + A_\mu$.

when α is constant

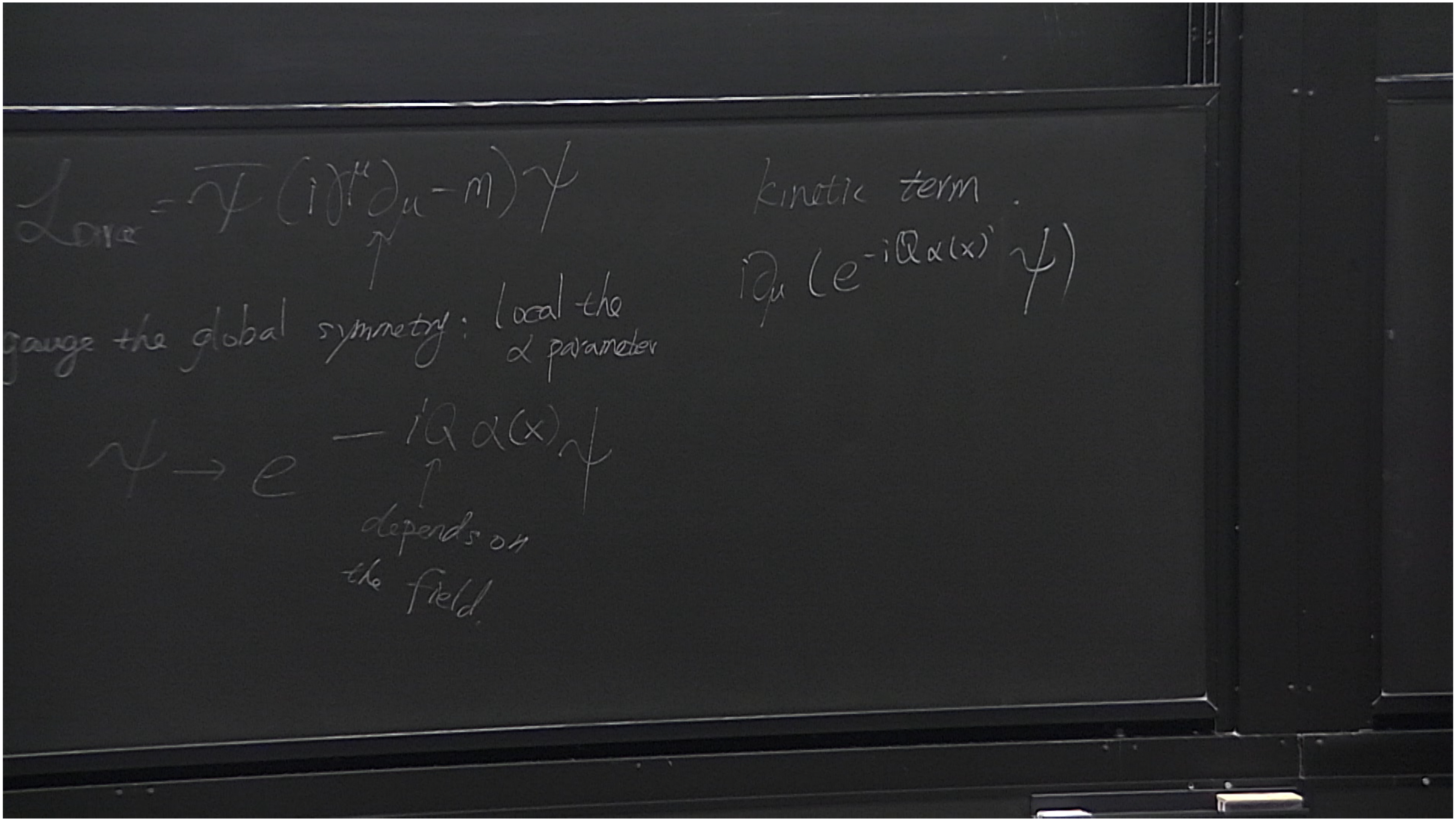
gauge symmetry $\mathcal{L}_{int}$ extra term:

$$\bar{\psi}\gamma^{\mu}\partial_\mu\alpha\psi$$

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^{\mu}\partial_\mu - m)\psi$$

gauge the global symmetry: local the α parameter

$$\psi \to e^{-i\alpha(x)}\psi$$


$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$
gauge the global symmetry: local the α parameter
$\psi \to e^{-iQ\alpha(x)}\psi$
depends on the field
kinetic term:
$i\partial_\mu(e^{-iQ\alpha(x)}\psi)$

$$\mathcal{L}_{Dirac} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$$

gauge the global symmetry: local the α parameter

$$\psi \to e^{-iQ\alpha(x)}\psi$$

depends on the field

kinetic term:

$$i\partial_\mu \left(e^{-iQ\alpha(x)}\psi\right)$$

$$= Q\,\partial_\mu \alpha(x)\, e^{-iQ\alpha(x)}\psi + e^{-iQ\alpha(x)}\, i\partial_\mu \psi$$

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

gauge the global symmetry: local the α parameter

$$\psi \to e^{-iQ\alpha(x)}\psi$$

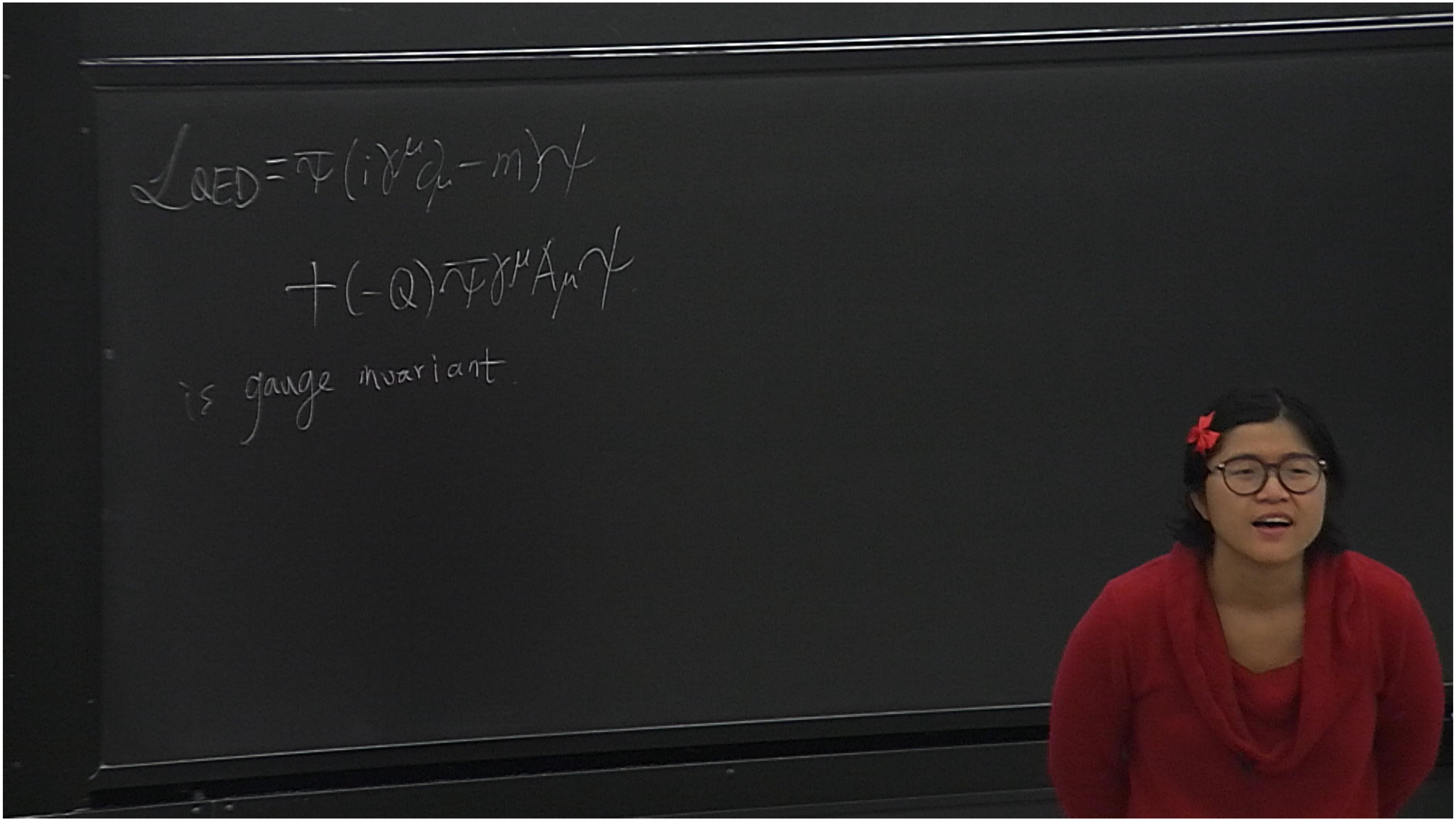
depends on the field

kinetic term.

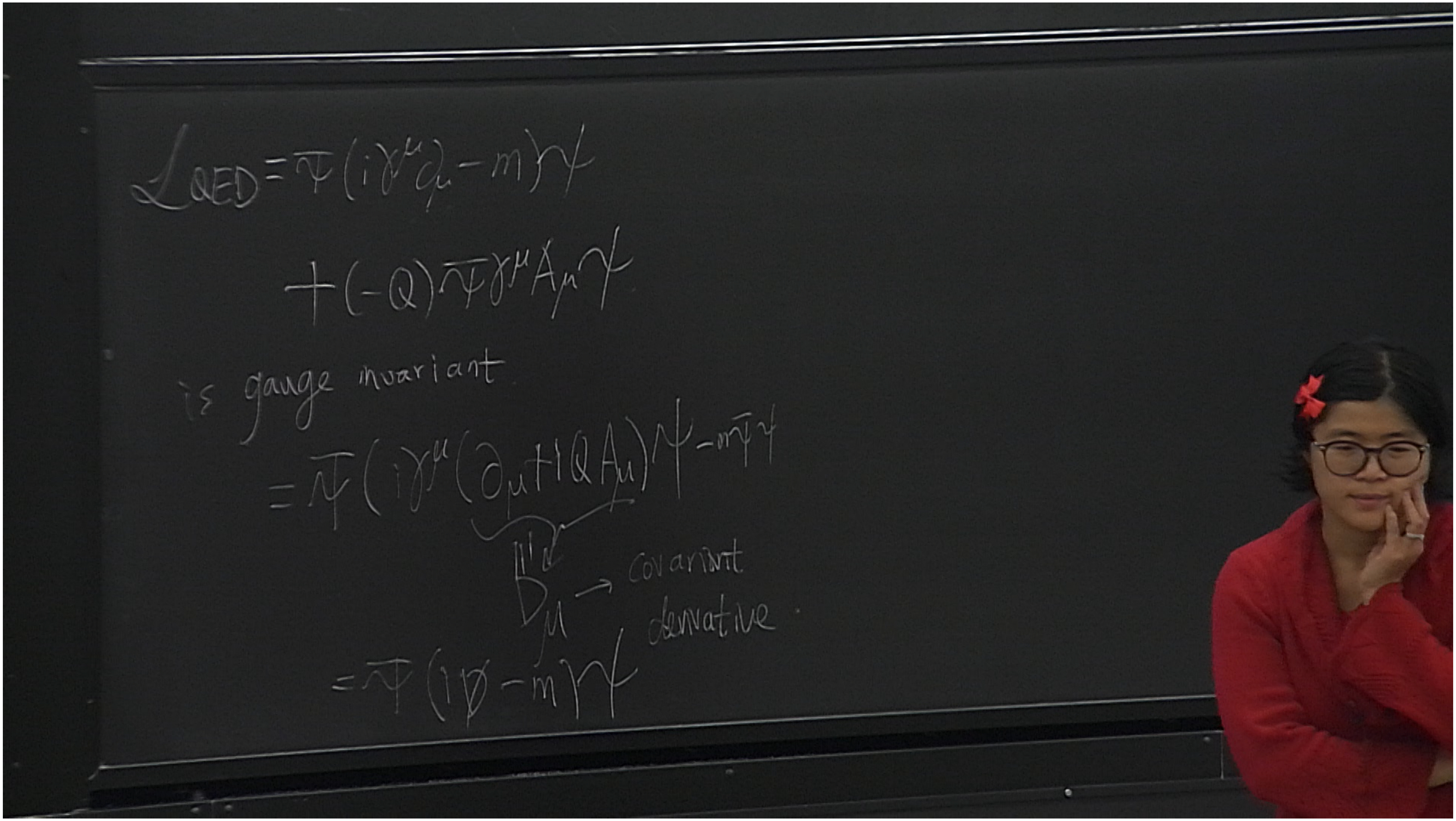
$$i\partial_\mu \left(e^{-iQ\alpha(x)}\psi\right)$$

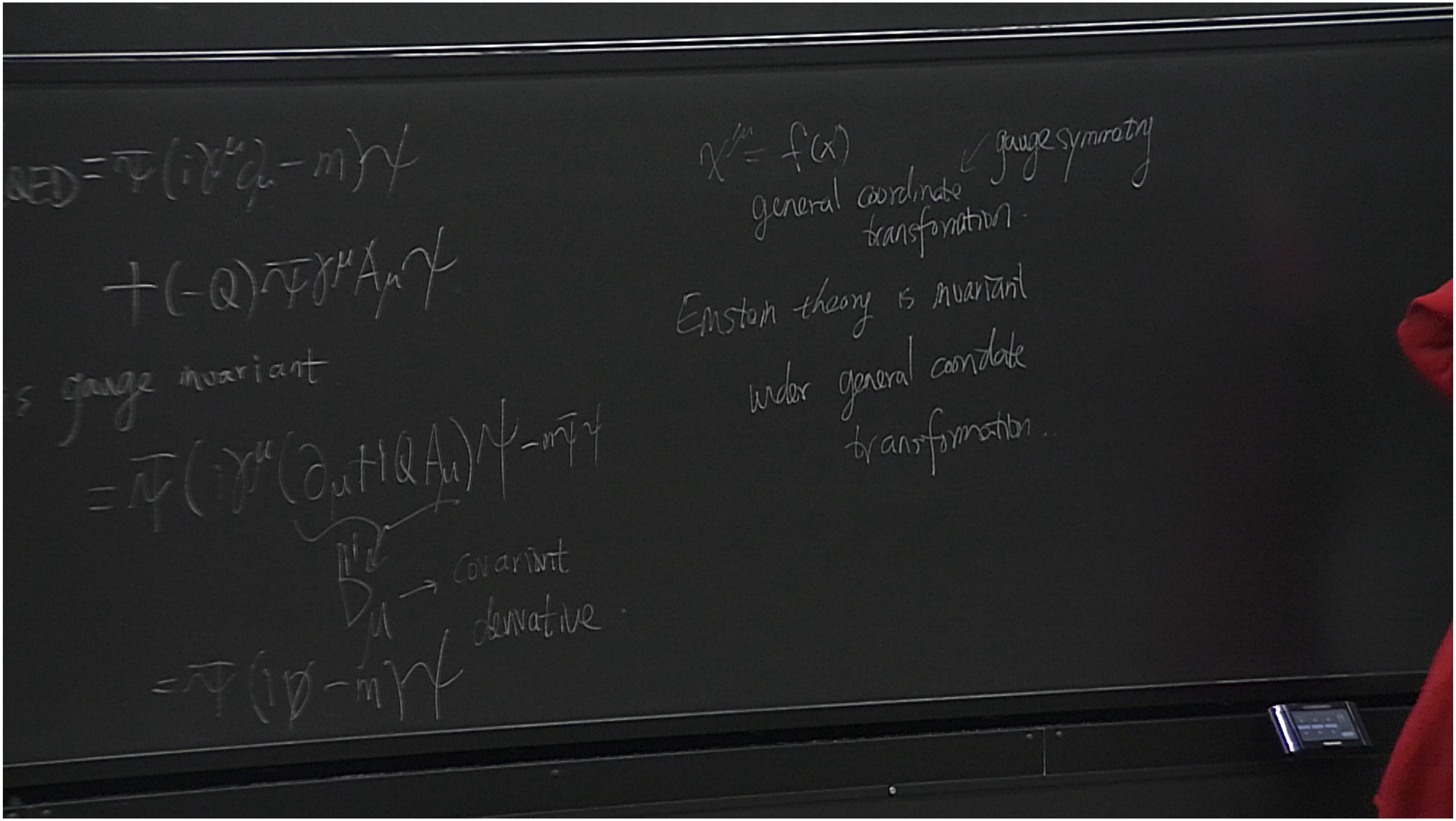
$$= Q\,\partial_\mu \alpha(x)\, e^{-iQ\alpha(x)}\psi + e^{-iQ\alpha(x)}\, i\partial_\mu \psi$$

new term $\bar{\psi}(\gamma^\mu Q\, \partial_\mu \alpha(x))\psi$



$\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$
$+ (-Q)\bar{\Psi}\gamma^\mu A_\mu \Psi$
is gauge invariant.


$\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\psi$
$+ (-Q)\bar{\Psi}\gamma^\mu A_\mu \psi$
is gauge invariant
$= \bar{\Psi}(i\gamma^\mu(\partial_\mu + iQA_\mu))\psi - m\bar{\Psi}\psi$
$D_\mu \rightarrow$ covariant derivative
$= \bar{\Psi}(i\not{D} - m)\psi$

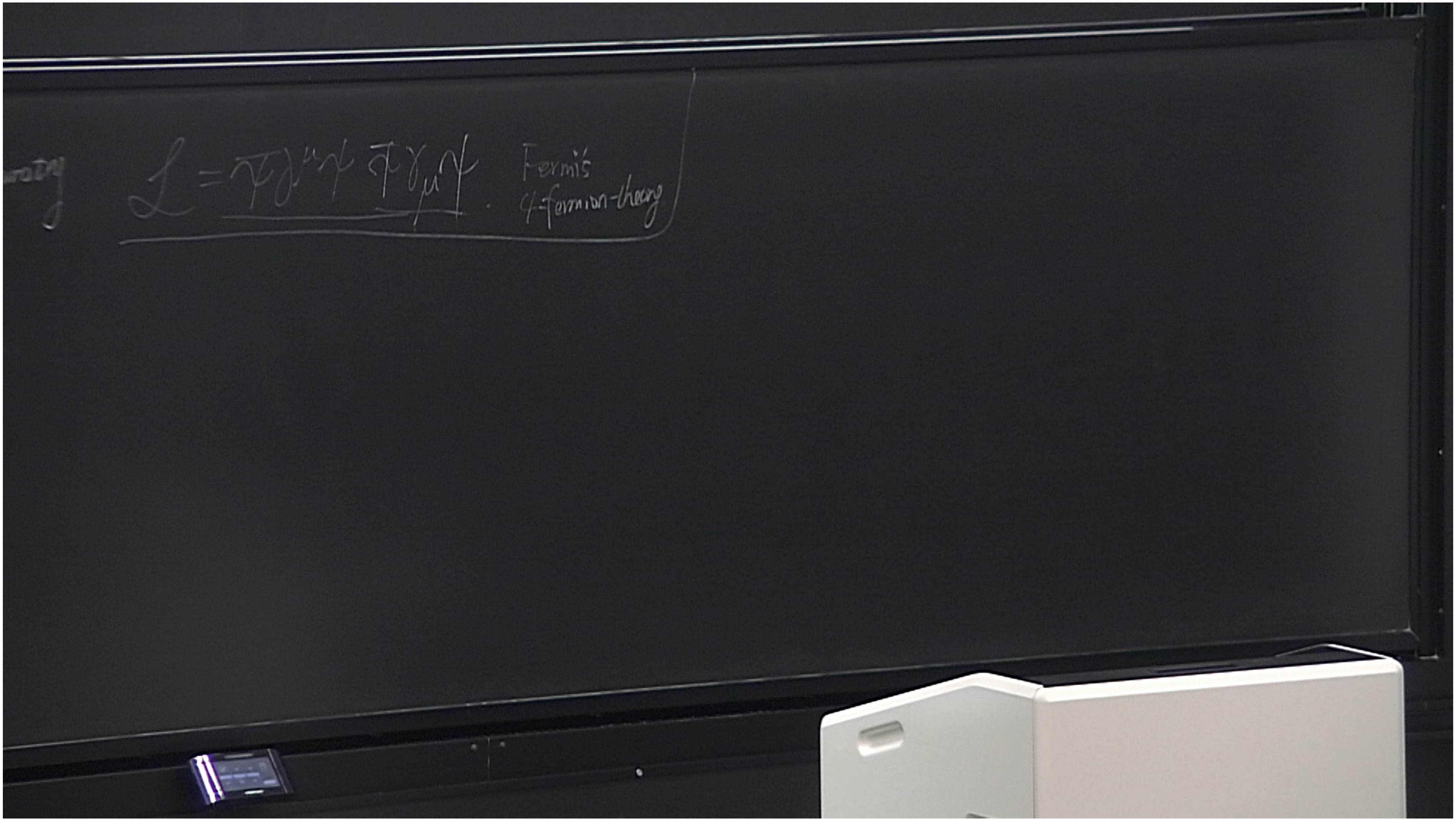

$\mathcal{L}_{QED} = \bar\Psi(i\gamma^\mu\partial_\mu - m)\Psi$
$+(-Q)\bar\Psi\gamma^\mu A_\mu\Psi$
is gauge invariant
$= \bar\Psi(i\gamma^\mu(\partial_\mu + iQA_\mu))\Psi - m\bar\Psi\Psi$
D_μ → covariant derivative
$= \bar\Psi(i\not{D} - m)\Psi$
$x'^\mu = f(x)$
general coordinate transformation
gauge symmetry
Einstein theory is invariant under general coordinate transformation.

$x'^{\mu} = f(x)$ ← gauge symmetry

general coordinate transformation.

Einstein theory is invariant under general coordinate transformation..

covariant derivative


$\mathcal{L} = \bar{\psi}\gamma^{\mu}\psi\,\bar{\psi}\gamma_{\mu}\psi$.
Fermi's
4-fermion-theory

$$\boxed{\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi + (-Q)\bar{\Psi}\gamma^\mu A_\mu \Psi}$$

is gauge invariant

$$= \bar{\Psi}(i\gamma^\mu(\underbrace{\partial_\mu + iQA_\mu}_{D_\mu}))\Psi - m\bar{\Psi}\Psi$$

$D_\mu \rightarrow$ covariant derivative.

$$= \boxed{\bar{\Psi}(i\not{D} - m)\Psi}$$

$x'^\mu = f(x)$ ← gauge symmetry

general coordinate transformation.

Einstein theory is invariant under general coordinate transformation.

covariant derivative

ymmetry

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}\psi\,\bar{\psi}\gamma_{\mu}\psi$$

Fermi's 4-fermion-theory

Quantization of all fields:

① LI Lagrangian

② momentum π. Hamiltonian

③ impose commutator relationship

④ normal ordering

$$\boxed{\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi + (-Q)\bar{\Psi}\gamma^\mu A_\mu \Psi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

is gauge invariant

$$= \bar{\Psi}(i\gamma^\mu \underbrace{(\partial_\mu + iQA_\mu)}_{D_\mu}\Psi - m\bar{\Psi}\Psi$$

$D_\mu \rightarrow$ covariant derivative

$$= \boxed{\bar{\Psi}(i\not{D} - m)\Psi}$$

$x'^\mu = f(x)$ ← gauge symmetry

general coordinate transformation.

Einstein theory is invariant under general coordinate transformation.

covariant derivative

free Dirac field

step1 $\bar{\psi}(i\partial\!\!\!/-m)\psi$

step2 $\pi = i\psi^\dagger$

$\mathcal{H} = \psi^\dagger(-i\gamma^0\gamma^i\partial_i - \gamma^0 m)\psi$

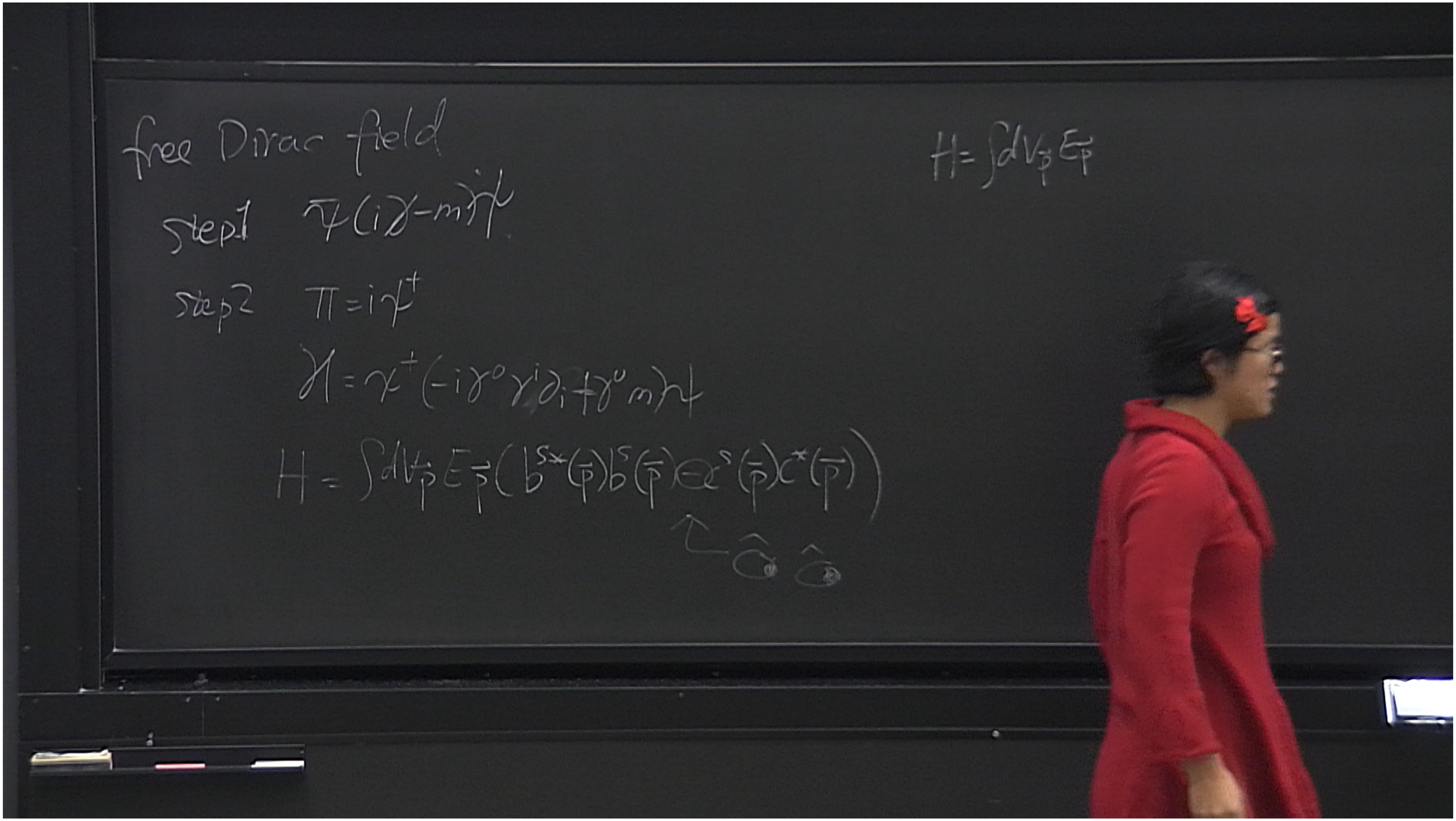
free Dirac field

step1 $\bar{\psi}(i\not{\partial}-m)\psi$

step2 $\pi = i\psi^{\dagger}$

$$\mathcal{H} = \psi^{\dagger}(-i\gamma^0\gamma^i\partial_i + \gamma^0 m)\psi$$

$$H = \int d^3p\, E_{\vec{p}}\left(b^{s*}(\vec{p})\,b^{s}(\vec{p}) - c^{s}(\vec{p})\,c^{*}(\vec{p})\right)$$


free Dirac field
step1 $\bar\psi(i\partial\!\!\!/-m)\psi$
step2 $\pi = i\psi^\dagger$
$\mathcal{H} = \psi^\dagger(-i\gamma^0\gamma^i\partial_i + \gamma^0 m)\psi$
$H = \int dV_{\vec p}\, E_{\vec p}\left(b^{s*}(\vec p)\, b^s(\vec p) - c^s(\vec p)\, c^*(\vec p)\right)$
$\hat c \quad \hat c$
$H = \int dV_{\vec p}\, E_{\vec p}$

$$H = \int dV_{\vec p}\, E_{\vec p} \left(\underbrace{b^{s+}_{\vec p}\, b^{s}_{\vec p}}_{N_b} - c^{s}_{\vec p}\, c^{s+}_{\vec p} \right) = \int dV_{\vec p}\, E_{\vec p} \left(b^+ b - [c, c^+] - \underbrace{c^+ c}_{N_c} \right)$$

$$c c^+ = c c^+ - c^+ c + c^+ c$$

$$= [c, c^+] + c^+ c$$

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(\underbrace{b^{s\dagger}_{\vec{p}}\, b^{s}_{\vec{p}}}_{N_b} - c^{s}_{\vec{p}}\, c^{s\dagger}_{\vec{p}} \right) = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{\dagger} b - [c, c^{\dagger}] - \underbrace{c^{\dagger} c}_{N_c} \right)$$

minus sign a problem

$$c c^{\dagger} = c c^{\dagger} - c^{\dagger} c + c^{\dagger} c$$

$$= [c, c^{\dagger}] + c^{\dagger} c$$

energy should not be unbounded from below

$$= \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{\dagger} b - [c, c^{\dagger}] - \underbrace{c^{\dagger} c}_{N_c} \right)$$

minus sign a problem → I want this to be plus.

energy should not be unbounded from below

the field.

$$H = \int dV_{\vec p}\, E_{\vec p} \left(\underbrace{b^{s\dagger}_{\vec p}\, b^{s}_{\vec p}}_{N_b} - c^{s}_{\vec p}\, c^{s\dagger}_{\vec p} \right) = \int dV_{\vec p}\, E_{\vec p} \left(b^\dagger b - [c, c^\dagger] - \underbrace{c^\dagger c}_{N_c} \right)$$

minus sign a problem → I want this to be plus

$$c c^\dagger = c c^\dagger \pm c^\dagger c \mp c^\dagger c$$

$$= [c, c^\dagger] + c^\dagger c$$

I want this to be minus

energy should not be unbounded from below

field

$$H = \int dV_{\vec p}\, E_{\vec p} \left(\underbrace{b^{s\dagger}_{\vec p} b^{s}_{\vec p}}_{N_b} - c^{s}_{\vec p} c^{s\dagger}_{\vec p} \right) = \int dV_{\vec p}\, E_{\vec p} \left(b^\dagger b - [c, c^\dagger] - \underbrace{c^\dagger c}_{N_c} \right)$$

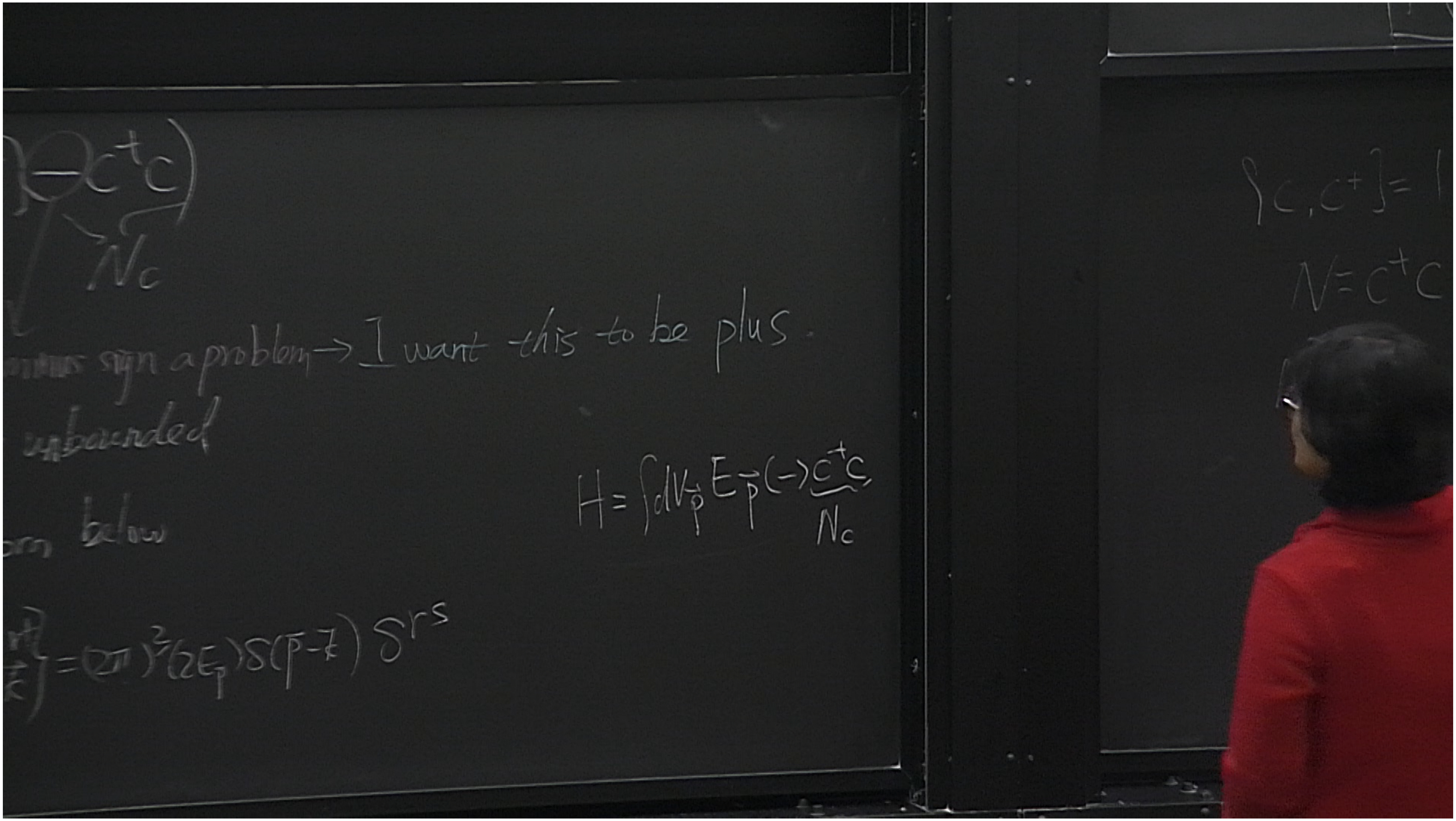
minus sign a problem → I want this to be plus

$$c c^\dagger = c c^\dagger \pm c^\dagger c \mp c^\dagger c = [c, c^\dagger] + c^\dagger c$$

I want this to be minus

energy should not be unbounded from below

$$\{ c^{s}_{\vec p}, c^{r\dagger}_{\vec k} \} = (2\pi)^3 (2E_p)\, \delta(\vec p - \vec k)\, \delta^{rs}$$


sign a problem → I want this to be plus.
unbounded
below
{c, c+} = 1
N = c+c

$\{c, c^+\} = 1$

$N = c^+ c$

$N^2 = N$

c^+ creation operator

c annihilation operator

$c^+|0\rangle$

$\{c, c^+\} = 1$

$N = c^+ c$ $\quad N = 0$ or 1

$N^2 = N$

fermion

c^+ creation operator

c anniliation operator

$c^+|0\rangle$

$$\int dV_{\vec{p}}\, E_{\vec{p}} \left(b^\dagger b - [c, c^\dagger] - \underbrace{c^\dagger c}_{N_c} \right)$$

minus sign a problem → I want this to be plus.

energy should not be unbounded from below

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} (-) \underbrace{c^\dagger c}_{N_c}$$

$$\{ c^s_{\vec{p}}, c^{r\dagger}_{\vec{k}} \} = (2\pi)^3 (2E_p) \delta(\vec{p} - \vec{k}) \delta^{rs} \Leftrightarrow \{\psi, \pi\}$$

$$\int dV_{\vec{p}}\, E_{\vec{p}} \left(b^\dagger b - [c, c^\dagger] - \underbrace{c^\dagger c}_{N_c} \right)$$

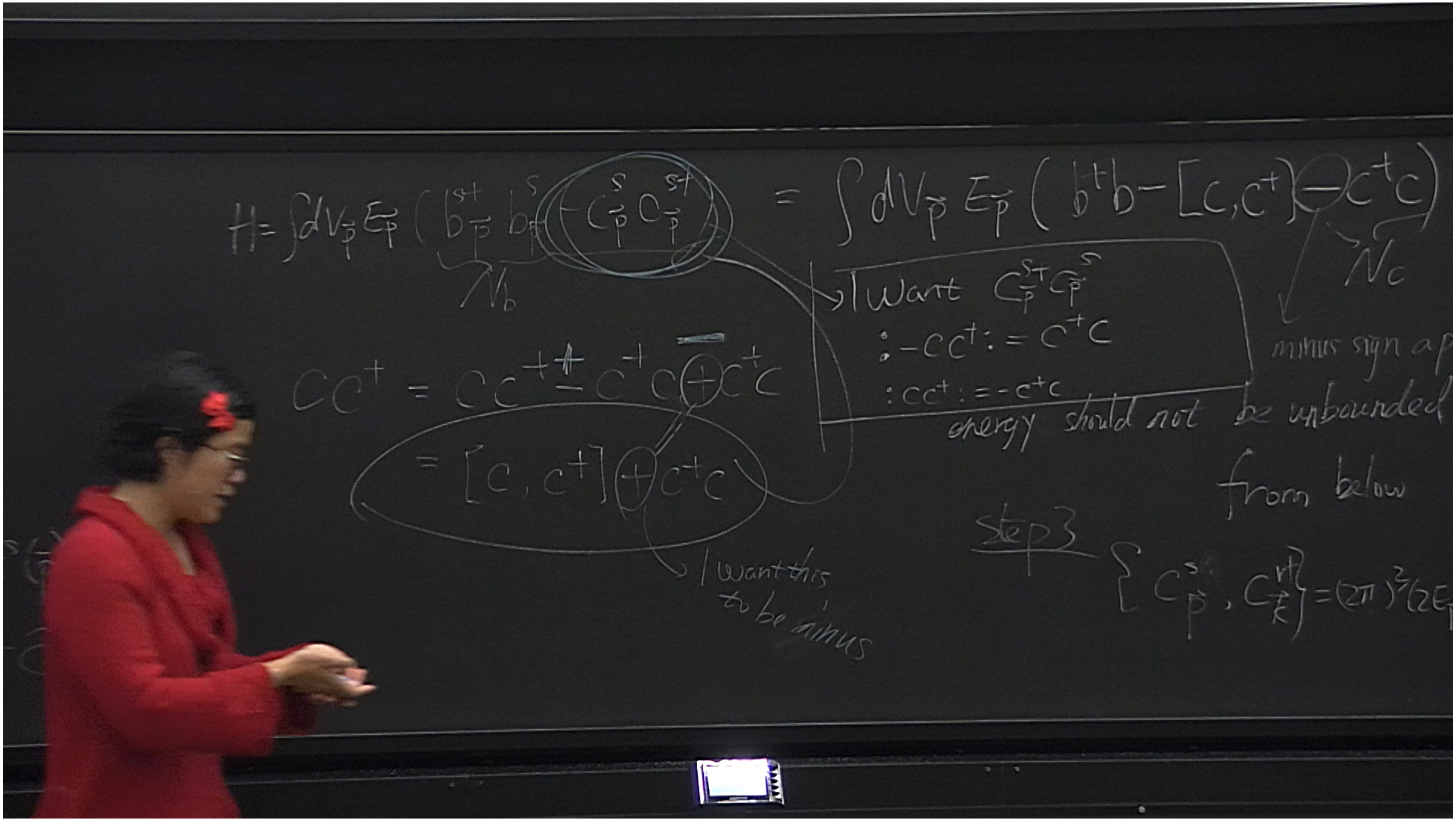
minus sign a problem → I want this to be plus.

energy should not be unbounded from below

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} (-) \underbrace{c^\dagger c}_{N_c}$$

Step 3

$$\{ C^{s}_{\vec{p}}, C^{r\dagger}_{\vec{k}} \} = (2\pi)^3 (2E_p)\, \delta(\vec{p} - \vec{k})\, \delta^{rs} \iff \boxed{\begin{array}{l} \{\psi(\vec{x}), \pi(\vec{y})\} = i\delta(\vec{x} - \vec{y}) \\ \{\psi, \psi\} = 0 \\ \{\pi, \pi\} = 0 \end{array}}$$



$H = \int dV_{\vec p} E_{\vec p} (b^{s\dagger}_{\vec p} b^{s}_{\vec p} - c^{s}_{\vec p} c^{s\dagger}_{\vec p})$
$= \int dV_{\vec p} E_{\vec p} (b^\dagger b - [c, c^\dagger] - c^\dagger c)$
N_b
N_c
I want $c^{s\dagger}_{\vec p} c^{s}_{\vec p}$
$:-cc^\dagger: = c^\dagger c$
$:cc^\dagger: = -c^\dagger c$
minus sign
energy should not be unbounded from below
$cc^\dagger = cc^\dagger \pm c^\dagger c \mp c^\dagger c$
$= [c, c^\dagger] + c^\dagger c$
I want this to be minus
Step 3
$\{ c^{s\dagger}_{\vec p}, c^{r\dagger}_{\vec k} \} = (2\pi)^3 (2E$

Step 4: For normal ordering

in fermion case.

want anilation operator right

except swapping operators

pick up a minus sign

Step 4: For normal ordering

in fermion case.

want annilation operator right

except swapping operators

pick up a minus sign

$ab + ba \overset{\text{most of time}}{=} 0$

$ab = -ba$

states

$b^r_{\vec{p}}|0\rangle = 0$

$c^s_{\vec{p}}|0\rangle = 0$

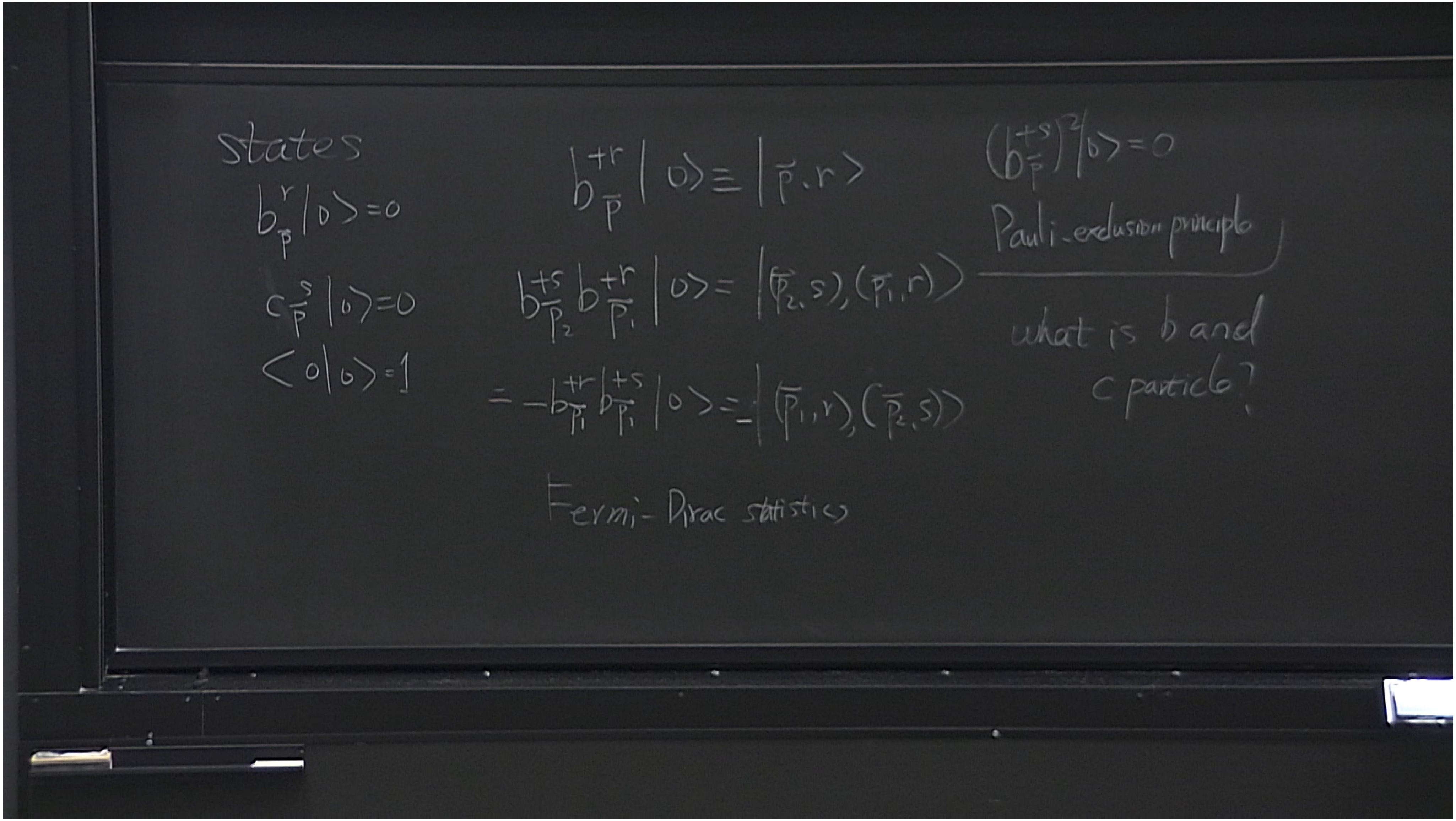
$\langle 0|0\rangle = 1$

$b^{\dagger r}_{\vec{p}}|0\rangle \equiv |\vec{p}, r\rangle$

$b^{\dagger s}_{\vec{p}_2} b^{\dagger r}_{\vec{p}_1}|0\rangle = |(\vec{p}_2, s), (\vec{p}_1, r)\rangle$

$= -b^{\dagger r}_{\vec{p}_1} b^{\dagger s}_{\vec{p}_1}|0\rangle = -|(\vec{p}_1, r), (\vec{p}_2, s)\rangle$

Fermi-Dirac statistics


states
$b^{r}_{\vec{p}}|0\rangle=0$
$c^{s}_{\vec{p}}|0\rangle=0$
$\langle 0|0\rangle=1$
$b^{+r}_{\vec{p}}|0\rangle\equiv|\vec{p},r\rangle$
$b^{+s}_{\vec{p}_2}b^{+r}_{\vec{p}_1}|0\rangle=|(\vec{p}_2,s),(\vec{p}_1,r)\rangle$
$=-b^{+r}_{\vec{p}_1}b^{+s}_{\vec{p}_1}|0\rangle=-|(\vec{p}_1,r),(\vec{p}_2,s)\rangle$
Fermi-Dirac statistics
$(b^{+s}_{\vec{p}})^2|0\rangle=0$
Pauli-exclusion principle
what is b and c particle?

$\{\psi, \psi\} = 0$

$\{\pi, \pi\} = 0$

$$J^{\mu}_{EM} = Q \underbrace{\int d^3x\, \psi^\dagger \psi}_{N_f}$$

be minus

$\Leftrightarrow \{\psi, \psi\} = 0$

$\{\pi, \pi\} = 0$

$$Q_{tot} = Q \underbrace{\int d^3x\, \psi^\dagger \psi}_{N_f}$$

Some calculation

$$= Q \int dV_p \left(b^{\dagger s}_p b^s_p + c^s_p c^{\dagger s}_p \right)$$

be minus

$\{\psi, \psi\} = 0$

$\{\pi, \pi\} = 0$

$$Q_{tot.} = Q \underbrace{\int d^3x\, \psi^\dagger \psi}_{N_f}$$

some calculation

$$= Q \int dV_{\vec{p}} \left(b^{\dagger s}_{\vec{p}}\, b^{s}_{\vec{p}} + c^{s}_{\vec{p}}\, c^{\dagger s}_{\vec{p}} \right)$$

normal ordering

$$:Q_{total}: = Q \int dV_{\vec{p}} \left(\underbrace{b^{\dagger s}_{\vec{p}}\, b^{s}_{\vec{p}}}_{N_b} - \underbrace{c^{\dagger s}_{\vec{p}}\, c^{s}_{\vec{p}}}_{N_c} \right)$$

$\{\psi, \psi\} = 0$

$\{\pi, \pi\} = 0$

$$Q_{tot.} = Q \underbrace{\int d^3x \, \psi^\dagger \psi}_{N_f}$$

some calculation

$$= Q \int dV_{\vec{p}} \left(b^{\dagger s}_{\vec{p}} b^s_{\vec{p}} + c^s_{\vec{p}} c^{\dagger s}_{\vec{p}} \right)$$

normal ordering

$$:Q_{total}: = Q \int dV_{\vec{p}} \Big(\underbrace{b^{\dagger s}_{\vec{p}} b^s_{\vec{p}}}_{N_b} - \underbrace{c^{\dagger s}_{\vec{p}} c^s_{\vec{p}}}_{N_c} \Big) = \int dV_{\vec{p}} \, Q N_b + \int dV_{\vec{p}} (-Q) N_c$$

be minus

$\{\psi, \psi\} = 0$

$\{\pi, \pi\} = 0$

anti-particle

$$Q_{tot.} = Q \underbrace{\int d^3x\, \psi^\dagger \psi}_{N_f}$$

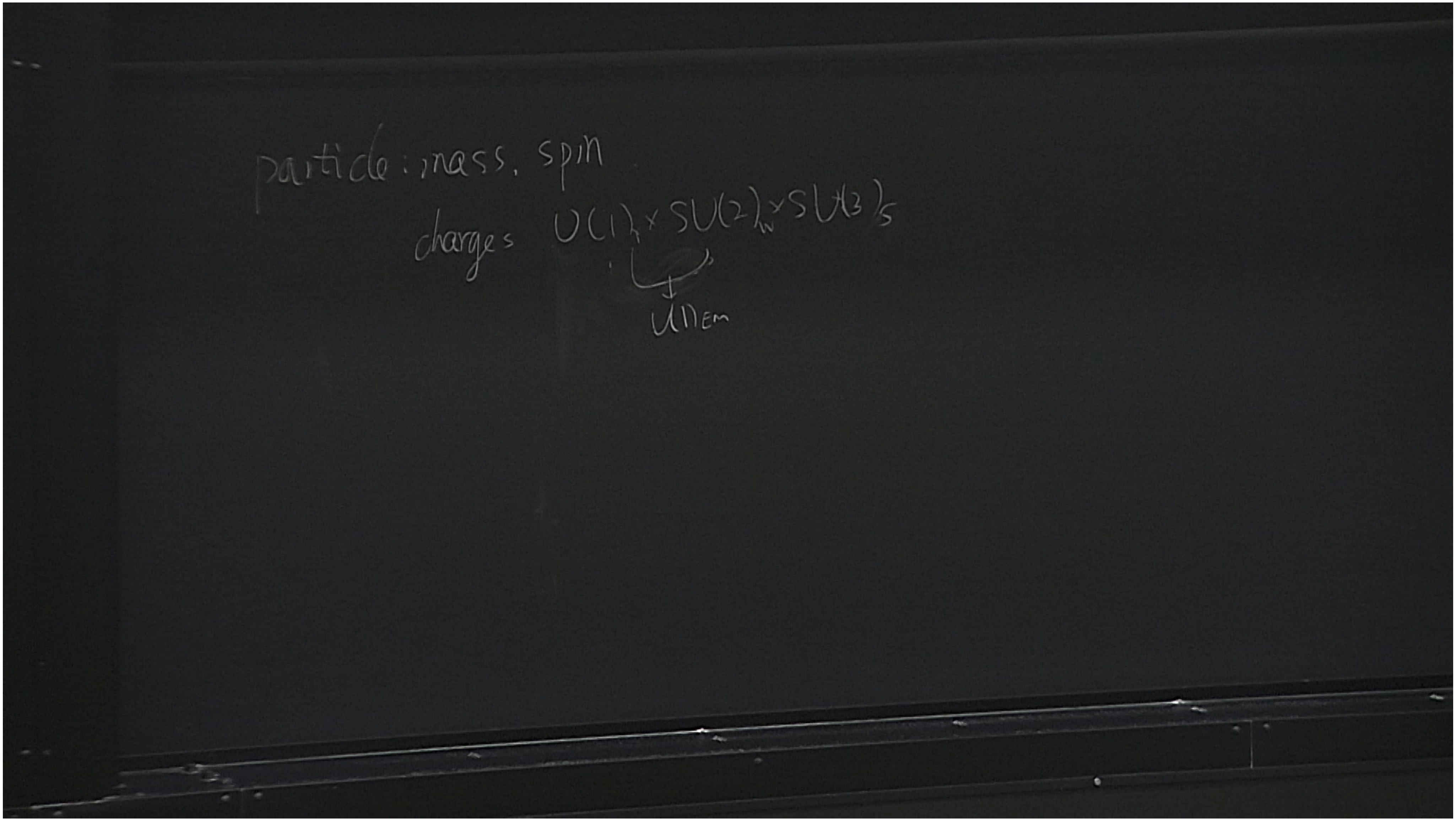
some calculation

$$= Q \int dV_{\vec{p}} \left(b^{\dagger s}_{\vec{p}} b^{s}_{\vec{p}} + c^{s}_{\vec{p}} c^{\dagger s}_{\vec{p}} \right)$$

normal ordering

$$:Q_{total}: = Q \int dV_{\vec{p}} \left(\underbrace{b^{\dagger s}_{\vec{p}} b^{s}_{\vec{p}}}_{N_b} - \underbrace{c^{\dagger s}_{\vec{p}} c^{s}_{\vec{p}}}_{N_c} \right) = \int dV_{\vec{p}}\, Q N_b + \int dV_{\vec{p}} (-Q) N_c$$

c is b's anti-particle


particle: mass, spin
charges U(1)Y × SU(2)W × SU(3)S
U(1)EM

particle: mass, spin

charges $U(1)_Y \times SU(2)_W \times SU(3)_S$

$U(1)_{Em}$

SU(5)

spin $= \frac{1}{2}$

spin $= \frac{1}{2}$

① consider a small rotation in xy-plane.

② find $\delta\psi$ calculate Noether current

$$J_z = \int j^0 d^3x = \underbrace{\boxed{\text{spin}}}_{\text{rest frame}} + \boxed{\text{orbital angular}}$$

$\vec{x} \times \vec{p}$

$$J_z(\text{rest})\, b_{\vec{p}}^{r\dagger}|0\rangle = \begin{matrix} \frac{1}{2} \text{ for } r=1 \\ -\frac{1}{2} \text{ for } r=2 \end{matrix} \; b_{\vec{p}}^{r\dagger}|0\rangle$$