Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 10

Speakers: Dan Wohns, Gang Xu

Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Date: October 24, 2019 - 9:00 AM

URL: http://pirsa.org/19100022

Quiz 8 (due tonight) No upgraded feedback

Quiz 9 (due Sunday night) has some derivation

will send upgraded feedback

lec note 12 available in quiz feedback

Homework 4 problem 3: just speculate.

strong constraint

$$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$$

if unitary similar transformation

$M^{-1} \gamma^\mu M$ similar transformation

not unitary

Weyl Spinor

lecture: $\psi_{\pm} = P_{\pm}\psi$

$$P_{\pm} = \frac{1 \pm \gamma^5}{2}$$

tutorial: a good Weyl spinor $\Rightarrow$ massless Dirac spinor

have definite handedness

is conserved over time

Lorentz invariant

$\psi_{-} \longrightarrow (\psi_{-})^{(c)}$

right-handed $\qquad$ left-handed

$\psi = \psi_{1,-} + (\psi_{2,-})^{(c)}$

Dirac spinor $\qquad$ 2 Weyl spinors

$\mathcal{L} = \bar{\psi}(i\gamma - m$

Weyl Spinor

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$\psi_- \longrightarrow (\psi_-)^{(c)}$

right-handed — left-handed

$\psi = \psi_{1,-} + (\psi_{2,-})^{(c)}$

Dirac spinor — 2 Weyl spinors

$\mathcal{L} = \bar{\psi}(i\gamma - m)\psi$

mass term

Puzzle: How to make a massive Dirac spinor out of 2 massless Weyl spinors?

$\bar{\psi}(i\not{\partial}-m)\psi$

mass term

How to make a massive Dirac spinor of 2 massless Weyl spinor?

$m\bar{\psi}\psi$

$= m\psi^\dagger(P_+ + P_-)\gamma^0(P_+ + P_-)\psi$

naively 4 terms

$P_+\gamma^0 P_+$

$= \frac{1+\gamma^5}{2}\gamma^0 P_+$

$= \frac{\gamma^0+\gamma^5\gamma^0}{2} P_+$

$= \frac{\gamma^0-\gamma^0\gamma^5}{2} P_+$

$= \gamma^0 P_- P_+ = 0$

$$m\psi^\dagger P_+\gamma^0 P_-\psi + m\psi^\dagger P_-\gamma^0 P_+\psi$$

$$= m\psi_+^\dagger\gamma^0\psi_- + m\psi_-^\dagger\gamma^0\psi_+$$

$$= m\bar{\psi}_+\psi_- + m\bar{\psi}_-\psi_+$$

interaction terms

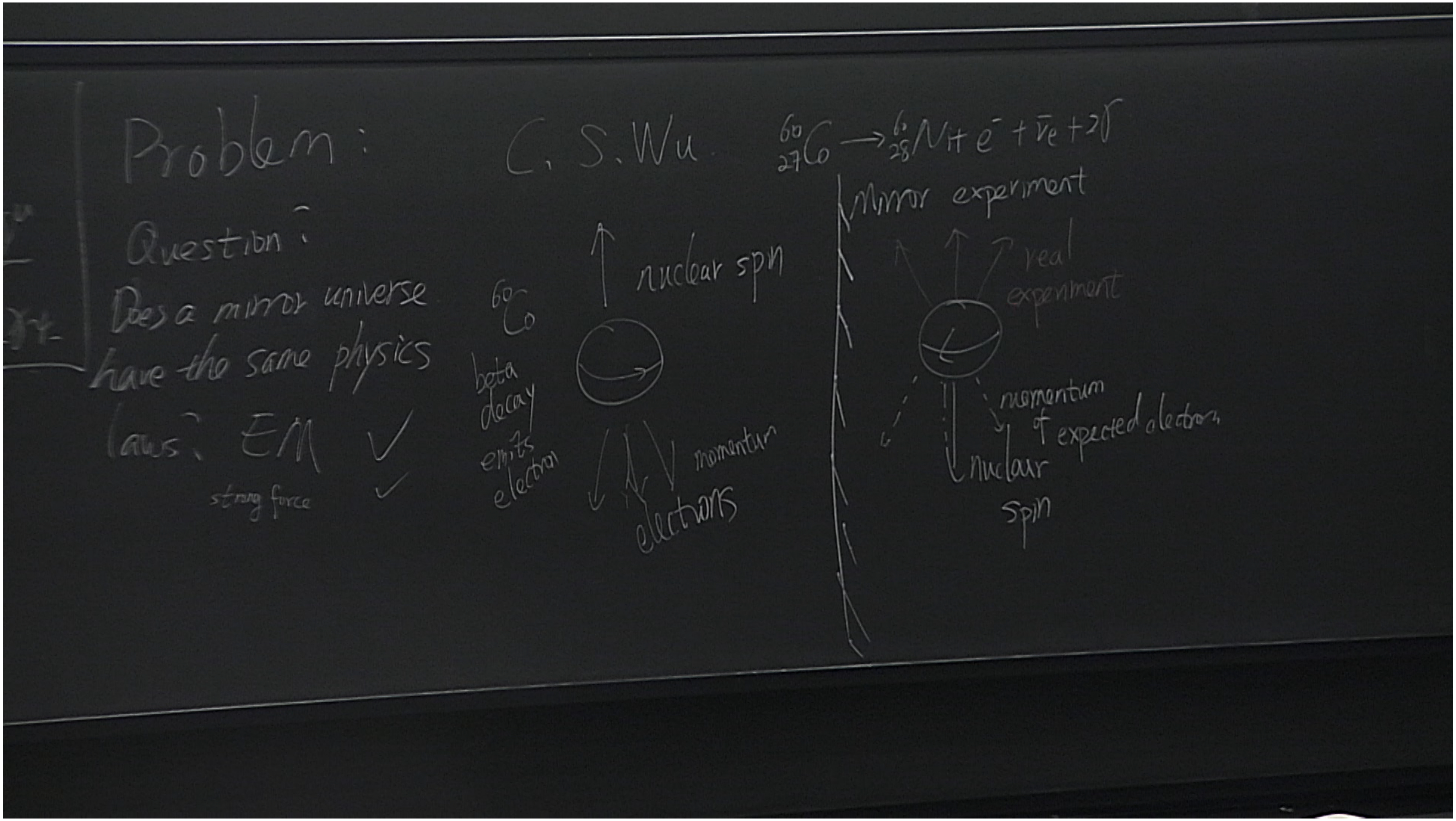
coefficient $\langle\varphi\rangle$

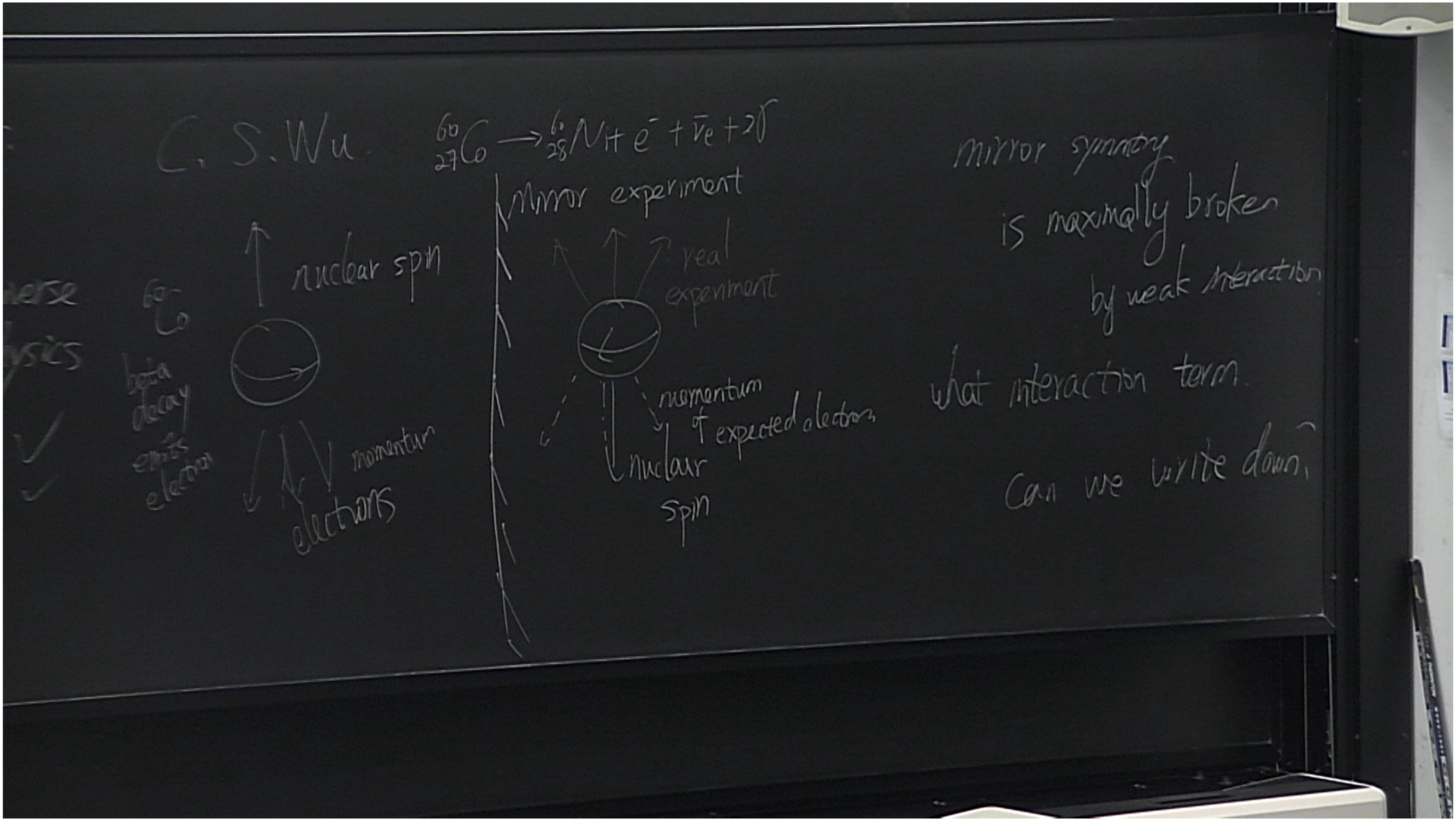
shared by all particle

ex

kinetic term γ^μ

$$i\bar{\psi}_+\gamma\psi_+ + i\bar{\psi}_-\gamma\psi_-$$


Problem:
C. S. Wu
$^{60}_{27}Co \to ^{60}_{28}Ni + e^- + \bar{\nu}_e + 2\gamma$
Question:
Does a mirror universe have the same physics laws? EM ✓
strong force ✓
60Co
beta decay emits electron
nuclear spin
momentum
electrons
Mirror experiment
real experiment
momentum of expected electron
nuclear spin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Mirror experiment
real experiment
momentum of expected electron
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mirror symmetry
is maximally broken
by weak interaction
what interaction term
Can we write down?

Discrete symmetry.

$e^{\frac{1}{2}\xi_{\mu\nu}(S^{\mu\nu})_{ab}}$ ait work

Parity. flipping x. y. z axis

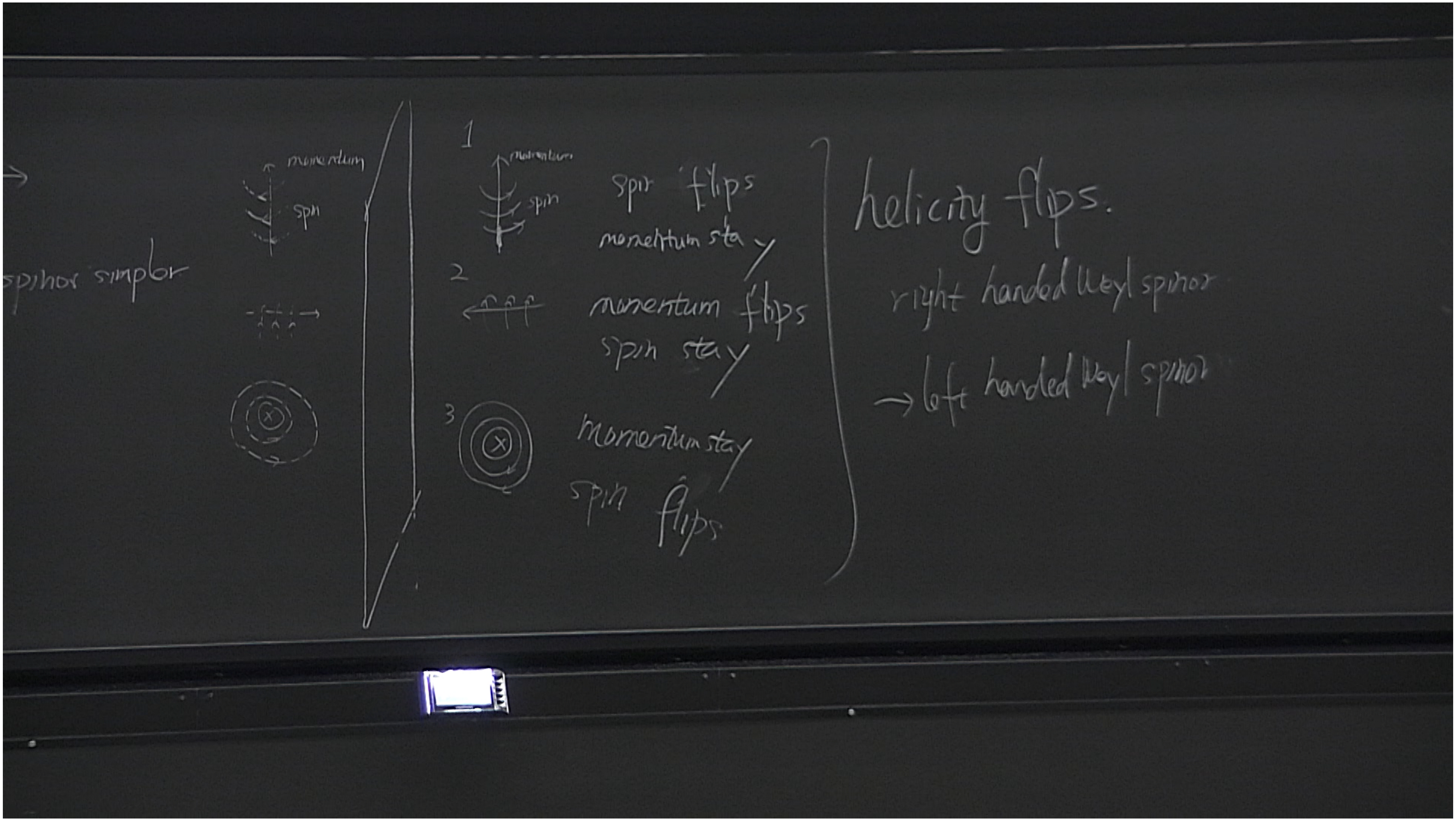
180° rotation around some axis

$\longrightarrow$ mirror symmetry

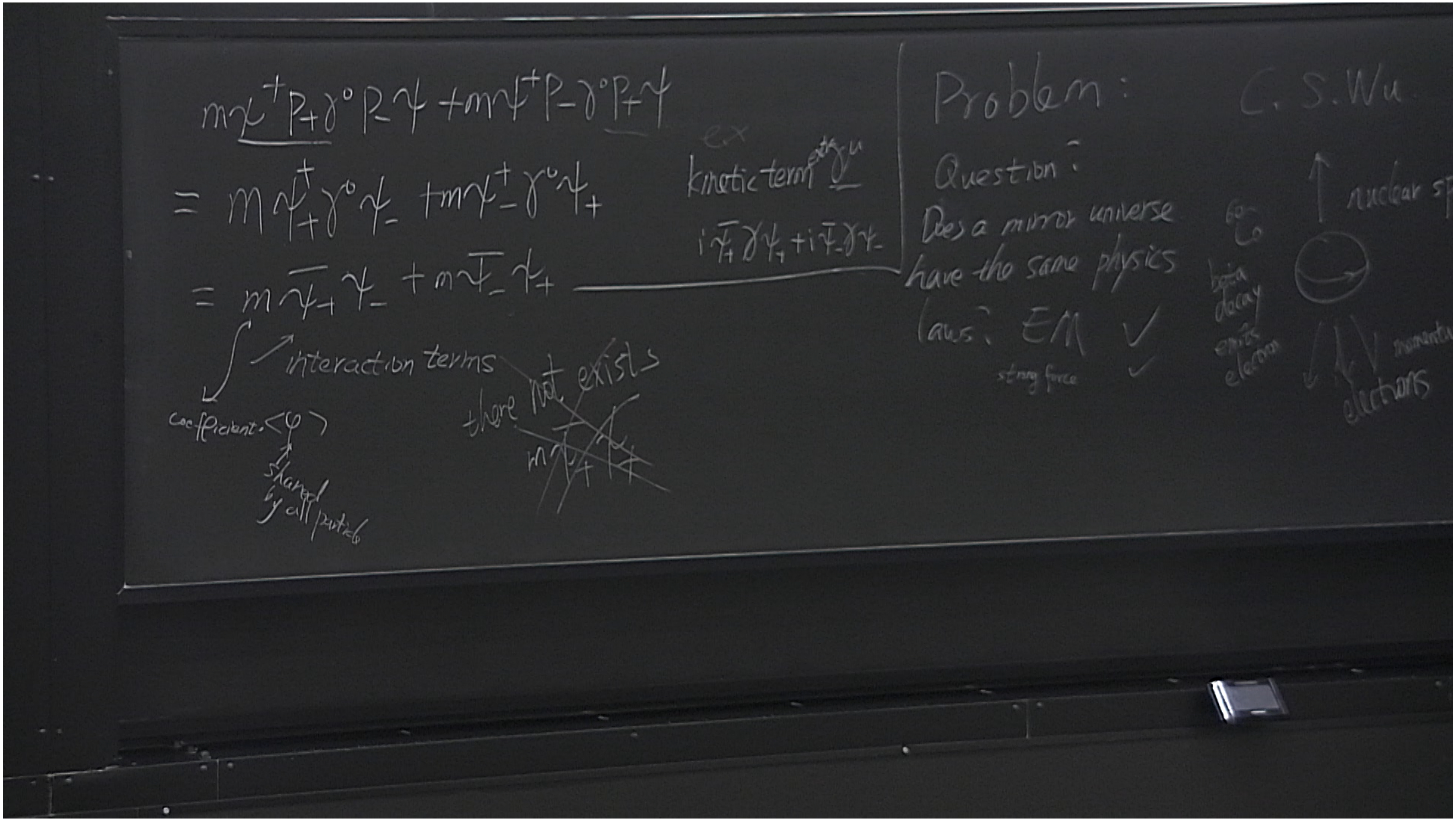
P $\psi \longrightarrow$

parity

Weyl spinor simpler



spinor simpler
momentum
spin
1
momentum
spin
spin flips
momentum stay
2
momentum flips
spin stay
3
momentum stay
spin flips
helicity flips.
right handed Weyl spinor
→ left handed Weyl spinor



$m\psi^\dagger P_+ \gamma^0 P_- \psi + m\psi^\dagger P_- \gamma^0 P_+ \psi$
$= m\psi_+^\dagger \gamma^0 \psi_- + m\psi_-^\dagger \gamma^0 \psi_+$
$= m\bar{\psi}_+ \psi_- + m\bar{\psi}_- \psi_+$
interaction terms
coefficient $\cdot \langle \varphi \rangle$
shared by all particle
there not exists
$m\bar{\psi}_+ \psi_+$
kinetic term
$i\bar{\psi}_+ \gamma \psi_+ + i\bar{\psi}_- \gamma \psi_-$
Problem:
Question:
Does a mirror universe have the same physics laws? EM ✓
strong force ✓
C. S. Wu
nuclear sp
beta decay
emits electron
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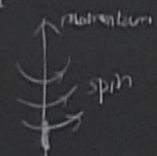
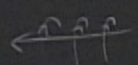
$= \gamma^0 P_- P_+ = 0$

1


spin flips
momentum stay

2


momentum flips
spin stay

3


momentum stay
spin flips

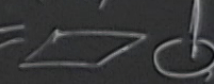
helicity flips.

→ right handed Weyl spinor

→ left handed Weyl spinor

Weyl basis massless

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} w_+ \\ w_- \end{pmatrix} \xrightarrow[\text{Parity}]{\text{mirror symmetry}} \begin{pmatrix} w_- \\ w_+ \end{pmatrix}$$

$=$ 

$$P\psi = \gamma^0 \psi$$

$$\psi(t, -\vec{x}) = \gamma^0 \psi(t, \vec{x})$$

$$\psi \xrightarrow{P} \gamma^0 \psi$$

$$\psi^\dagger \xrightarrow{P} \psi^\dagger \gamma^0$$

$$\bar{\psi} \xrightarrow{P} \psi^\dagger \gamma^0 \gamma^0 = \psi^\dagger$$

$$\underline{\underline{\bar{\psi} \xrightarrow{P} \psi^\dagger}}$$

$$\bar{\psi}\psi \xrightarrow{P} \psi^\dagger \gamma^0 \psi = \bar{\psi}\psi$$

$$\underline{\bar{\psi}\gamma^\mu\psi} \xrightarrow{P} \begin{cases} \psi^\dagger \gamma^0 \gamma^0 \gamma^0 \psi = \bar{\psi}\gamma^0\psi & \mu=0 \\ \psi^\dagger \gamma^i \gamma^0 \psi = -\psi^\dagger \gamma^0 \gamma^i \psi = -\bar{\psi}\gamma^i\psi & \mu=i \end{cases}$$

$$\bar\psi\psi \longrightarrow 1$$

$$\bar\psi\gamma^\mu\psi \quad \text{vector QED (QCD)} \longrightarrow 4$$

$$\bar\psi\gamma^\mu\gamma^\nu\psi \longrightarrow \bar\psi S^{\mu\nu}\psi \longrightarrow 6$$

$$\bar\psi\gamma^\rho\gamma^\mu\gamma^\nu\psi \longrightarrow \bar\psi\gamma^5\gamma^\mu\psi \longrightarrow 4$$

$$\bar\psi\gamma^\rho\gamma^\sigma\gamma^\mu\gamma^\nu\psi \longrightarrow \bar\psi\gamma^5\psi \longrightarrow 1$$

$$\cancel{\bar\psi\gamma^\alpha\gamma^\rho\gamma^\sigma\gamma^\mu\gamma^\nu\psi}$$

$$\cancel{\bar\psi\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\sigma\gamma^\mu\gamma^\nu\psi}$$

$$16$$

$\bar\psi \psi \rightarrow 1$

$\bar\psi \gamma^\mu \psi$ vector QED (QED) $\rightarrow 4$

$\bar\psi \gamma^\mu \gamma^\nu \psi \rightarrow \bar\psi S^{\mu\nu} \psi F_{\mu\nu} \rightarrow 6$

weak interaction

$\bar\psi \gamma^\rho \gamma^\mu \gamma^\nu \psi \rightarrow \boxed{\bar\psi \gamma^5 \gamma^\mu \psi} \rightarrow 4$ axial vector

$\bar\psi \gamma^\rho \gamma^\sigma \gamma^\mu \gamma^\nu \psi \rightarrow \bar\psi \gamma^5 \psi \rightarrow 1$ pseudo scalar

$\cancel{\bar\psi \gamma^\alpha \gamma^\rho \gamma^\sigma \gamma^\mu \gamma^\nu \psi}$

$\cancel{\bar\psi \gamma^\alpha \gamma^\beta \gamma^\rho \gamma^\sigma \gamma^\mu \gamma^\nu \psi}$

16

$\psi \xrightarrow{P} \gamma^0 \psi$

$\psi^\dagger \xrightarrow{P} \psi^\dagger \gamma^0$

$\bar\psi \xrightarrow{P} \psi^\dagger \gamma^0 \gamma^0 = \psi^\dagger$

$\bar\psi \xrightarrow{P} \psi^\dagger$

$$\psi \xrightarrow{P} \gamma^0\psi$$

$$\psi^\dagger \xrightarrow{P} \psi^\dagger\gamma^0$$

$$\bar{\psi} \xrightarrow{P} \psi^\dagger\gamma^0\gamma^0 = \psi^\dagger$$

axial vector

$$\bar{\psi} \xrightarrow{P} \psi^\dagger$$

pseudo scalar

$$\bar{\psi}\psi \xrightarrow{P} \psi^\dagger\gamma^0\psi = \bar{\psi}\psi$$

$$\bar{\psi}\gamma^\mu\psi \xrightarrow{P} \begin{cases} \psi^\dagger\gamma^0\gamma^0\gamma^0\psi = \bar{\psi}\gamma^0\psi & \mu=0 \\ \psi^\dagger\gamma^i\gamma^0\psi = -\psi^\dagger\gamma^0\gamma^i\psi \\ \quad = -\bar{\psi}\gamma^i\psi & \mu=i \end{cases}$$

$$\bar{\psi}\gamma^5\psi \xrightarrow{P} \psi^\dagger\gamma^5\gamma^0\psi = -\psi^\dagger\gamma^0\gamma^5\psi$$

$$= -\bar{\psi}\gamma^5\psi$$

$$\bar{\psi}\gamma^5\gamma^\mu\psi \xrightarrow{P} \begin{cases} - & \mu=0 \\ + & \mu=i \end{cases}$$

$= m\psi_+^\dagger\gamma^0\psi_- + m\psi_-^\dagger\gamma^0\psi_+$

$= m\bar\psi_+\psi_- + m\bar\psi_-\psi_+$

interaction terms

coefficient $\cdot\langle\varphi\rangle$

shared by all particle

kinetic term $\partial_\mu\gamma^\mu$

$i\bar\psi_+\gamma\psi_+ + i\bar\psi_-\gamma\psi_-$

there not exists $\cancel{m\bar\psi_+\psi_+}$

$\bar\psi_+ H\psi_-$

scalar

Question: Does a mirror universe have the same physics laws? EM ✓

strong force ✓

^{60}Co

beta decay emits electr

Continuous Symmetry

$$\delta\psi = \Upsilon$$

special case $\delta\mathcal{L} = 0$

$$J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\Upsilon = i\bar{\psi}\gamma^\mu\Upsilon = \bar{\psi}\gamma^\mu\psi$$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

$$\psi \to \psi e^{-i\alpha}$$

$$\Upsilon = \delta\psi \to (-i\alpha)\psi$$

$$j^0 = \bar{\psi}\gamma^0\psi = \psi^\dagger\gamma^0\gamma^0\psi = \psi^\dagger\psi$$

$$N_f = \int d^3x\, \psi^\dagger\psi$$

Dirac ☺