Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 9

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Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Date: October 22, 2019 - 9:00 AM

URL: http://pirsa.org/19100020

8 (3 clickers not on spinor)
Quiz (short, review,) due 10pm Thursday

Homework 4 H for ψ due 9am Friday
shorter than
other QFT Hw

notes 9. 10 are up.

Next week Friday → Tutorial Room

The Dirac equation

$$(i\not\partial - m)\psi = 0$$

$$(i(\gamma^\mu)_{ab}\partial_\mu - m\mathbb{1}_{ab})\psi_b = 0$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$

$p^2 = m^2$

tutorial on Lorentz group

$$\psi(x) \to \psi'(x')_a = S(\Lambda)_{ab}\psi(x)_b$$

$$S(\Lambda)_{ab} = e^{\frac{1}{2}\varepsilon_{\mu\nu}(S^{\mu\nu})_{ab}}$$

convention

$\varepsilon_{12}S^{12} + \varepsilon_{21}S^{21}$

$$S^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$$

Chir

$\gamma^0 =$

Chiral/Weyl basis

$$\gamma^0 = \begin{pmatrix} & \mathbb{1}_{2\times2} \\ \mathbb{1}_{2\times2} & \end{pmatrix} \quad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

unique - all others equivalent to this

unitary

$$\begin{cases} U \cdot U^\dagger = \mathbb{1} \\ U^\dagger = U^{-1} \end{cases}$$

$$S(\Lambda_{rot}) = e^{\begin{pmatrix} \frac{i}{2}\vec{\sigma}\cdot\vec{\theta} & \\ & \frac{i}{2}\vec{\sigma}\cdot\vec{\theta} \end{pmatrix}}$$ is unitary

only block diagonal in Weyl basis

$$S(\Lambda_{boost}) = e^{\begin{pmatrix} \frac{1}{2}\vec{\chi}\cdot\vec{\sigma} & \\ & -\frac{1}{2}\vec{\chi}\cdot\vec{\sigma} \end{pmatrix}}$$ not unitary

unitary

$$\begin{cases} U \cdot U^{\dagger} = \underline{1} \\ U^{\dagger} = U^{-1} \end{cases}$$

to this

$\vec{\sigma}\cdot\vec{\theta}$ is unitary

$\vec{\chi}\cdot\vec{\sigma}$ not unitary

only block diagonal in Weyl basis

Dirac spinor is actually reducible!

$$\psi = \begin{pmatrix} w_+ \\ w_- \end{pmatrix}$$ 2 Weyl spinor (2-components)

rotation $w_\pm \longrightarrow e^{\frac{i}{2}\vec{\sigma}\cdot\vec{\theta}}\, w_\pm$

boost $w_\pm \longrightarrow e^{\pm\frac{1}{2}\vec{\chi}\cdot\vec{\theta}}\, w_\pm$

Chiral/Weyl basis

$$\gamma^0 = \begin{pmatrix} & \mathbb{1}_{2\times2} \\ \mathbb{1}_{2\times2} & \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

unique - all others equivalent to this

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$$\underbrace{S(\Lambda_{rot}) = e^{\begin{pmatrix} \frac{i}{2}\vec{\sigma}\cdot\vec{\theta} & \\ & \frac{i}{2}\vec{\sigma}\cdot\vec{\theta} \end{pmatrix}}}\begin{pmatrix} w_+ \\ w_- \end{pmatrix}$$ is unitary

$$S(\Lambda_{boost}) = e^{\begin{pmatrix} \frac{1}{2}\vec{\chi}\cdot\vec{\sigma} & \\ & -\frac{1}{2}\vec{\chi}\cdot\vec{\sigma} \end{pmatrix}}\begin{pmatrix} w_+ \\ w_- \end{pmatrix}$$ not unitary

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boost $w_\pm \longrightarrow e^{\pm\frac{1}{2}\vec{\chi}\cdot\vec{\theta}}\, w_\pm$

You should be suspicious

general basis

How to get Weyl spinor?

Projection operator

$\sum P_i = 1 \quad P_i^2 = P_i \quad (P_i^\dagger = P_i \; , P_i P_j \overset{i \neq j}{=} 0)$

$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$

$(\gamma^5)^2 = 1 \quad \gamma^{5\dagger} = \gamma^5 \quad \{\gamma^5, \gamma^\mu\} = 0$

$\Rightarrow [\gamma^5, \underbrace{[\gamma^\mu, \gamma^\nu]}_{S^{\mu\nu}}] = 0$

$P_\pm = \frac{1 \pm \gamma^5}{2}$

$P_+^2 = \frac{(1+\gamma^5)^2}{4} = \frac{1+2\gamma^5+1}{4} = P_+$

$P_+ P_- = 0$

$\psi_\pm = P_\pm \psi$

$$P_\pm = \frac{1 \pm \gamma^5}{2}$$

$$P_+^2 = \frac{(1+\gamma^5)^2}{4} = \frac{1+2\gamma^5+1}{4} = P_+$$

$$P_+ P_- = 0$$

$$\psi_\pm = P_\pm \psi$$

$$= P_i \,,\; P_i P_j^{(i \neq j)} = 0)$$

$$\gamma^5 \quad \{\gamma^5, \gamma^\mu\} = 0 \Rightarrow [\gamma^5, \underbrace{[\gamma^\mu, \gamma^\nu]}_{S^{\mu\nu}}] = 0$$

Weyl: $\gamma^5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\psi_+ = \begin{pmatrix} w_+ \\ 0 \end{pmatrix}$$

$$\psi_- = \begin{pmatrix} 0 \\ w_- \end{pmatrix}$$

$= \frac{1\pm\gamma^5}{2}$

$= \frac{(1+\gamma^5)^2}{4} = \frac{1+2\gamma^5+1}{4} = P_+$

$P_+P_- = 0$

$\psi_\pm = P_\pm \psi$

$[\gamma^\mu, \gamma^\nu]) = 0$

$S^{\mu\nu}$

Weyl

$\gamma^5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\psi_+ = \begin{pmatrix} w_+ \\ 0 \end{pmatrix}$

$\psi_- = \begin{pmatrix} 0 \\ w_- \end{pmatrix}$

Majorana spinors

complex Φ . $\Phi^* = \Phi \to \varphi$ real scalar

Can I have $\psi^* = \psi$? "reality" condition

$(S^{\mu\nu})^* = S^{\mu\nu}$

$[\gamma^\mu, \gamma^\nu]$

$\gamma^{\mu *} = \pm \gamma^\mu$

pure imaginary

$\gamma_m^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$ $\gamma_m^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}$

$\gamma_m^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$ $\gamma_m^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}$

Majorana spinors

complex Φ . $\Phi^* = \Phi \to \varphi$ real scalar

Can I have $\psi^* = \psi$? "reality" condition

$$(S^{\mu\nu})^* = S^{\mu\nu}$$

$[\gamma^\mu, \gamma^\nu]$

$$\gamma^{\mu *} = \pm \gamma^\mu$$

pure imaginary

$$\gamma_m^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \quad \gamma_m^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}$$

$$\gamma_m^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \quad \gamma_m^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}$$

in a general basis

$$\psi = \psi^{(c)} \equiv C\psi^* \qquad \text{"reality" condition}$$

possible

constant matrix

$\psi^{(c)}, \psi$ transform the same way

$\psi^{(c)}, \psi$ both satisfy Dirac equation

$$(\psi^{(c)})^{(c)} = \psi \Rightarrow CC^* = 1$$

$$\psi = \psi^{(c)} \equiv C\psi^*$$

$$\psi \to S(\Lambda)\psi$$

$$\psi^* \to S^*(\Lambda)\psi^*$$

$$C\psi^* \to CS^*(\Lambda)\psi^*$$

We want $\underline{C\psi^*} \to S(\Lambda)\underline{C\psi^*}$

$$S(\Lambda)C = CS^*(\Lambda)$$

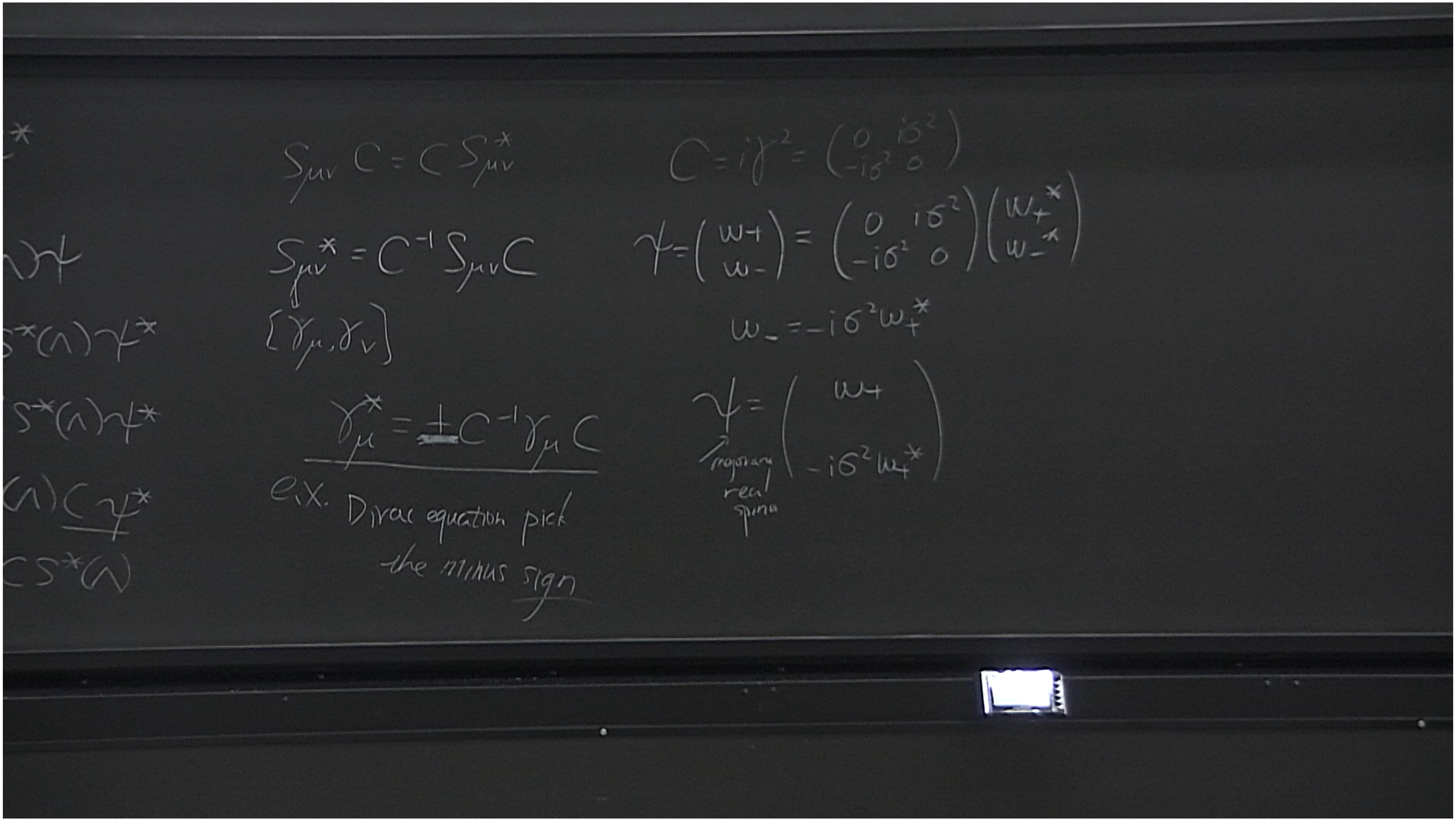
$$S_{\mu\nu}C = CS^*_{\mu\nu}$$

$$S^*_{\mu\nu} = C^{-1}S_{\mu\nu}C$$

$$[\gamma_\mu, \gamma_\nu]$$

$$\underline{\gamma^*_\mu = \pm C^{-1}\gamma_\mu C}$$

e.x. Dirac equation pick the minus sign


$S_{\mu\nu} C = C S_{\mu\nu}^*$
$S_{\mu\nu}^* = C^{-1} S_{\mu\nu} C$
$[\gamma_\mu, \gamma_\nu]$
$\gamma_\mu^* = \pm C^{-1} \gamma_\mu C$
e.x. Dirac equation pick the minus sign
$C = i\gamma^2 = \begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}$
$\psi = \begin{pmatrix} w_+ \\ w_- \end{pmatrix} = \begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix} \begin{pmatrix} w_+^* \\ w_-^* \end{pmatrix}$
$w_- = -i\sigma^2 w_+^*$
$\psi = \begin{pmatrix} w_+ \\ -i\sigma^2 w_+^* \end{pmatrix}$
majorana real spinor

$$\begin{pmatrix} W_+^* \\ W_-^* \end{pmatrix} = \begin{pmatrix} i\sigma^2 W_-^* \\ -i\sigma^2 W_+^* \end{pmatrix}$$

$$\begin{pmatrix} W_+ \\ W_- \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} W_+^* \\ W_-^* \end{pmatrix}$$

$$W_- = f(W_+^*)$$

Solve Dirac Equation

Weyl basis

$$\sigma^\mu = (1, \vec{\sigma}) \qquad \gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}$$

$$\bar{\sigma}^\mu = (1, -\vec{\sigma})$$

$$i(\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\psi_a(x) = u_a(\vec{p})\, e^{-ip\cdot x}$$

Solve Dirac Equation

Weyl basis

$$\sigma^\mu = (1, \vec{\sigma}) \qquad \gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}$$

$$\bar{\sigma}^\mu = (1, -\vec{\sigma})$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\psi_a(x) = u_a(\vec{p})\, e^{-ip\cdot x}$$

$$i\partial_\mu \left(e^{-ip\cdot x}\right) = p_\mu e^{-ip\cdot x}$$

$$(\gamma^\mu p_\mu - m)\, u(\vec{p}) = 0$$

$$\begin{pmatrix} -m & \sigma\cdot p \\ \bar{\sigma}\cdot p & -m \end{pmatrix} \begin{pmatrix} u_-(\vec{p}) \\ u_+(\vec{p}) \end{pmatrix} = 0$$

Solve Dirac Equation

Weyl basis

$\sigma^\mu = (1, \vec{\sigma})$ $\quad \gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}$

$\bar{\sigma}^\mu = (1, -\vec{\sigma})$

$(i\gamma^\mu \partial_\mu - m)\psi = 0$

$\psi_a(x) = u_a(\vec{p})\, e^{-ip\cdot x}$

$i\partial_\mu (e^{-ip\cdot x}) = p_\mu e^{-ip\cdot x}$

$(\gamma^\mu \cdot p_\mu - m)\, u(\vec{p}) = 0$

$\begin{pmatrix} -m & \sigma\cdot p \\ \bar{\sigma}\cdot p & -m \end{pmatrix} \begin{pmatrix} u_-(\vec{p}) \\ u_+(\vec{p}) \end{pmatrix} = 0$

$u(\vec{p}) = \begin{pmatrix} u_-(\vec{p}) \\ f(u_-(\vec{p})) \end{pmatrix}$

How to get Weyl

Projection operator

$\Sigma P_i = 1 \quad P_i^2 =$

$\gamma^5 = -i\gamma^0\gamma^1$

$(\gamma^5)^2 = 1$

$$u^s(\vec{p}) = \begin{pmatrix} \sqrt{\sigma \cdot p}\ \xi^s \\ \sqrt{\bar{\sigma} \cdot p}\ \xi^s \end{pmatrix} \rightarrow$$

u: 4-component spinor

2 solutions

$$\xi^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \xi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \xi^{s\dagger}\xi^r = \delta^{sr}$$

$$\psi = v(\vec{p})\, e^{+ip\cdot x}$$

$$v^s(\vec{p}) = \begin{pmatrix} \sqrt{\sigma \cdot p}\ \eta^s \\ -\sqrt{\bar{\sigma} \cdot p}\ \eta^s \end{pmatrix}$$

2 solutions

$$\psi_a(x) = \int \frac{d^3p}{(2\pi)^3(2E_p)} \sum_{s=1}^{2} \Big(u_a^s(\vec{p})\, b^s(\vec{p})\, e^{-ip\cdot x} + v_a^s(\vec{p})\, c^{s*}(\vec{p})\, e^{+ip\cdot x} \Big)$$

$$\psi(x) = \int dV_p \; u^s(\vec{p})\, b^s(\vec{p})\, e^{-ip\cdot x} + v^s(\vec{p})\, c^{s*}(\vec{p})\, e^{+ip\cdot x}$$

The Dirac Lagrangian

first guess

$$\Phi^* \Phi \longrightarrow \psi^+ \psi$$

$\psi \to S\psi$

$\psi^+ \to \psi^+ S^+$

$$\psi^+ \psi \longrightarrow \psi^+ S^+ S \psi$$

$$S^+_{\mu\nu} = -\gamma^0 S_{\mu\nu} \gamma^0$$

$$S^+ = e^{\frac{1}{2}\varepsilon_{\mu\nu}(S^{\mu\nu})^+} = \gamma^0 e^{-\frac{1}{2}\varepsilon_{\mu\nu} S^{\mu\nu}} \gamma^0 = \gamma^0 S^{-1} \gamma^0$$

$$\psi^+ \psi \longrightarrow \psi^+ \gamma^0 \underbrace{S^{-1} \gamma^0 S}_{\gamma^0} \psi$$

$\uparrow \gamma^0$

$$\bar{\psi} \equiv \psi^+ \gamma^0 \quad \text{Dirac conjugate}$$

$$m \bar{\psi} \psi$$

$\bar{\psi}$

$\psi \to \psi^{+}\gamma^{0} \underbrace{S^{-1}\gamma^{0} S}_{\gamma^{0}} \psi$

$\bar{\psi} = \psi^{+}\gamma^{0}$ Dirac conjugate

$\bar{\psi}\psi$

is the $\bar{\psi}\gamma^{\mu}\psi$ a vector?

γ_5

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = 0$$

strongly. e.x

$$\Pi = i\psi^{\dagger}$$

$\varphi \to 1$

complex $\Phi \to 2$

4 component complex $\psi \to 8$ d.o.f

I found 4 solutions

$\bar{\psi} \gamma^\mu \psi$ a vector?

$\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$

$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = 0$

$i\psi^\dagger$

$\varphi \rightarrow 1 \quad \dot{\varphi}$

complex $\Phi \rightarrow 2$

4 component complex $\psi \rightarrow 8$ d.o.f

I found 4 solutions

phase space

particle x, y, z $\quad \frac{3+3}{2} = 3$

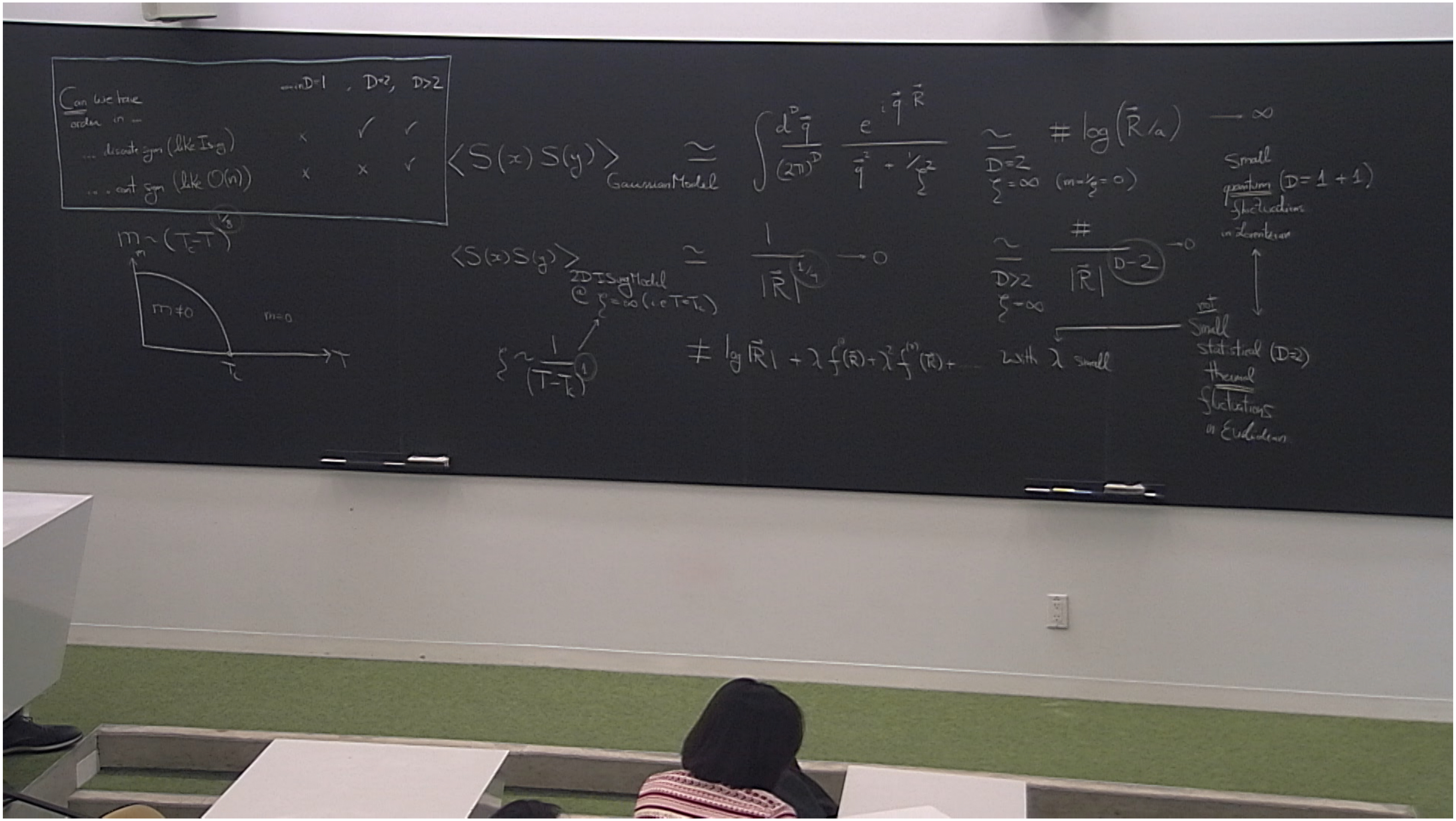
p_x, p_y, p_z

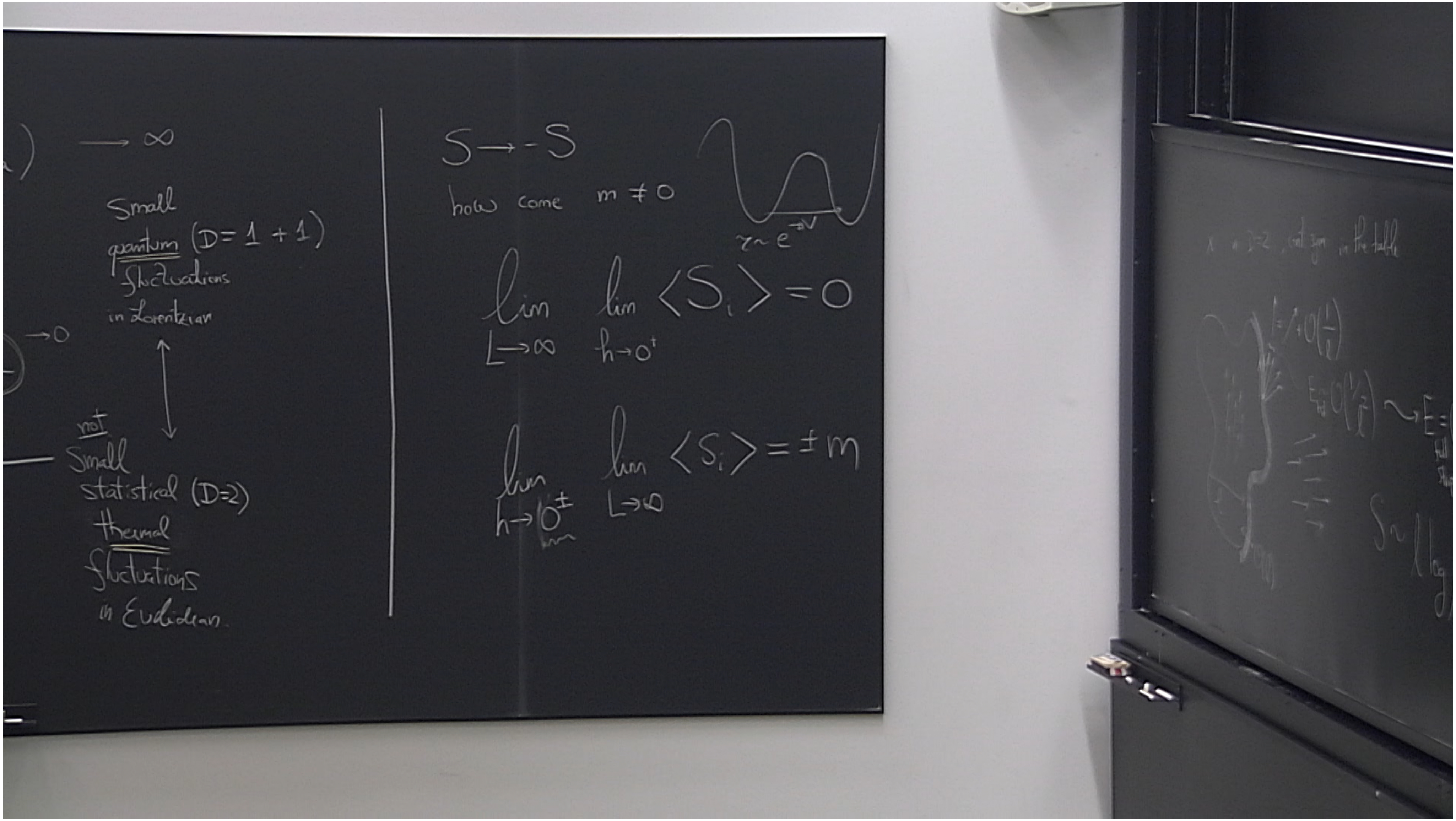
d.o.f = $\frac{\text{dim(phase space)}}{2}$

ψ: 8

$\psi^\dagger$ π: 8, but it's same 8

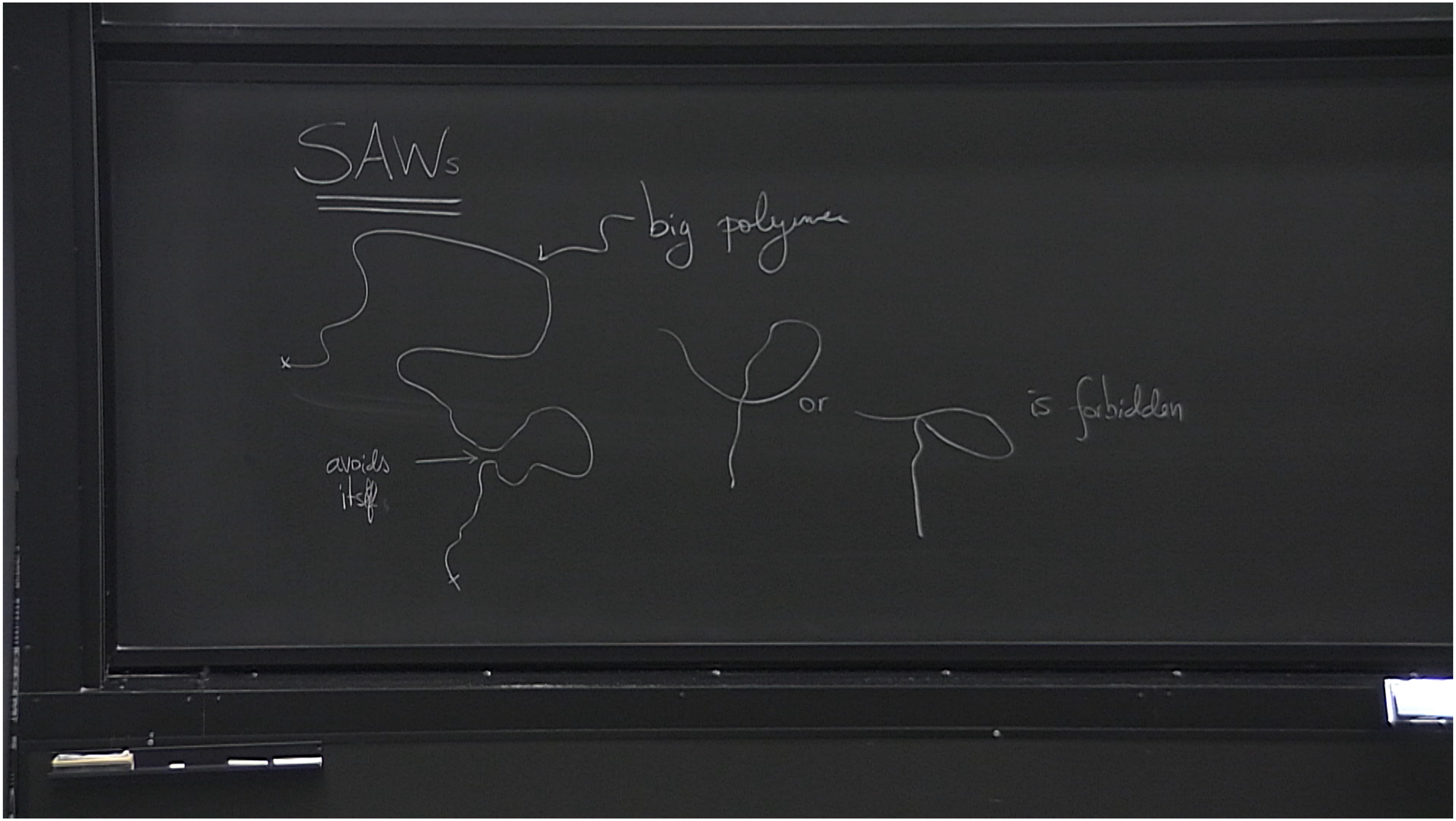
dof: $\frac{8}{2} = 4$

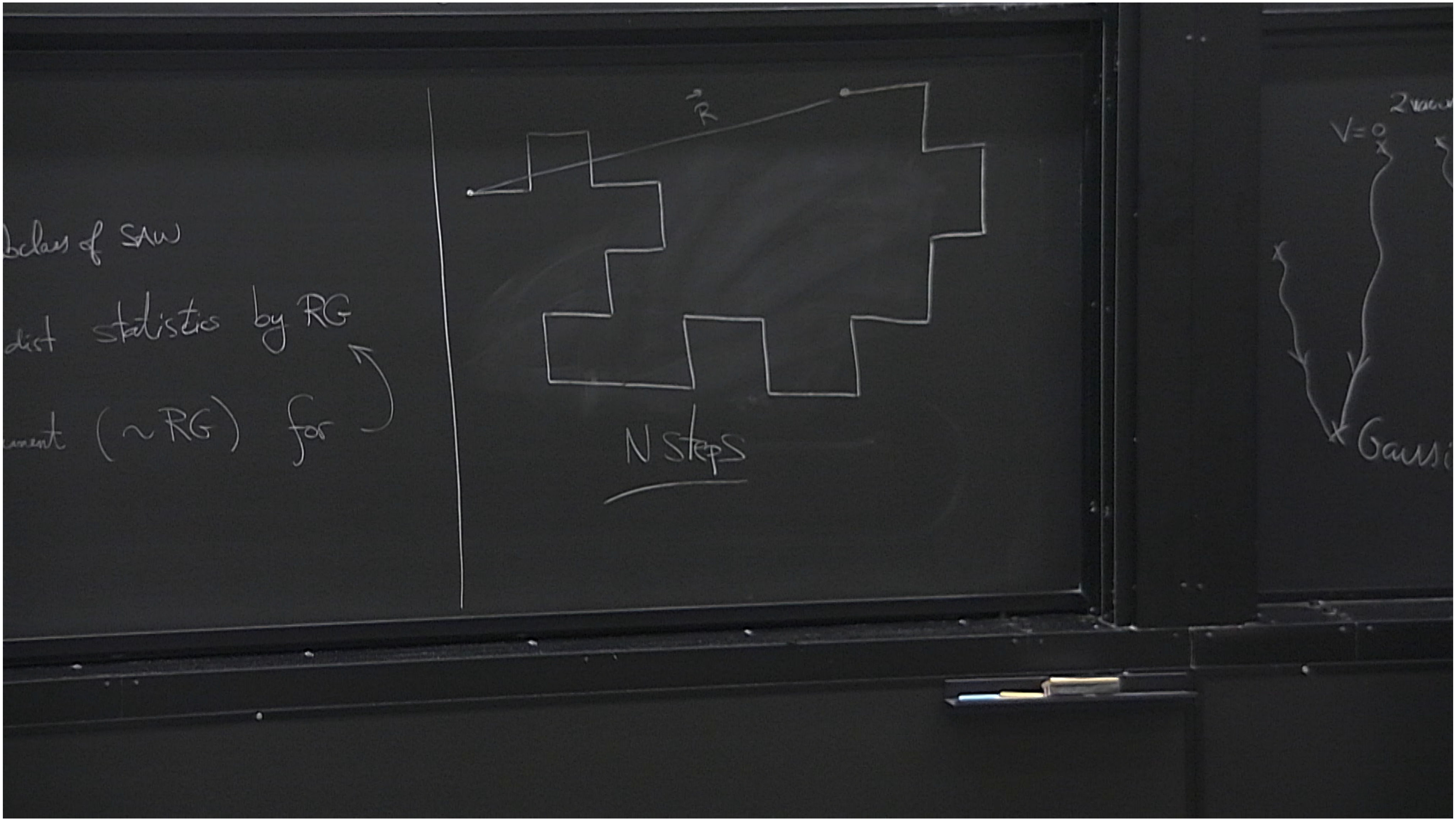

Can we have order in ...
... in D=1 , D=2, D>2
... discrete sym (like Ising)
... cont sym (like O(n))
⟨S(x)S(y)⟩ Gaussian Model
D=2 ξ=∞
# log(R/a) → ∞
Small quantum (D = 1 + 1) fluctuations in Lorentzian
⟨S(x)S(y)⟩ 2D Ising Model @ ξ=∞ (i.e. T=Tc)
D>2 ξ=∞
m≠0
m=0
with λ small
not small statistical (D=2) thermal fluctuations in Euclidean

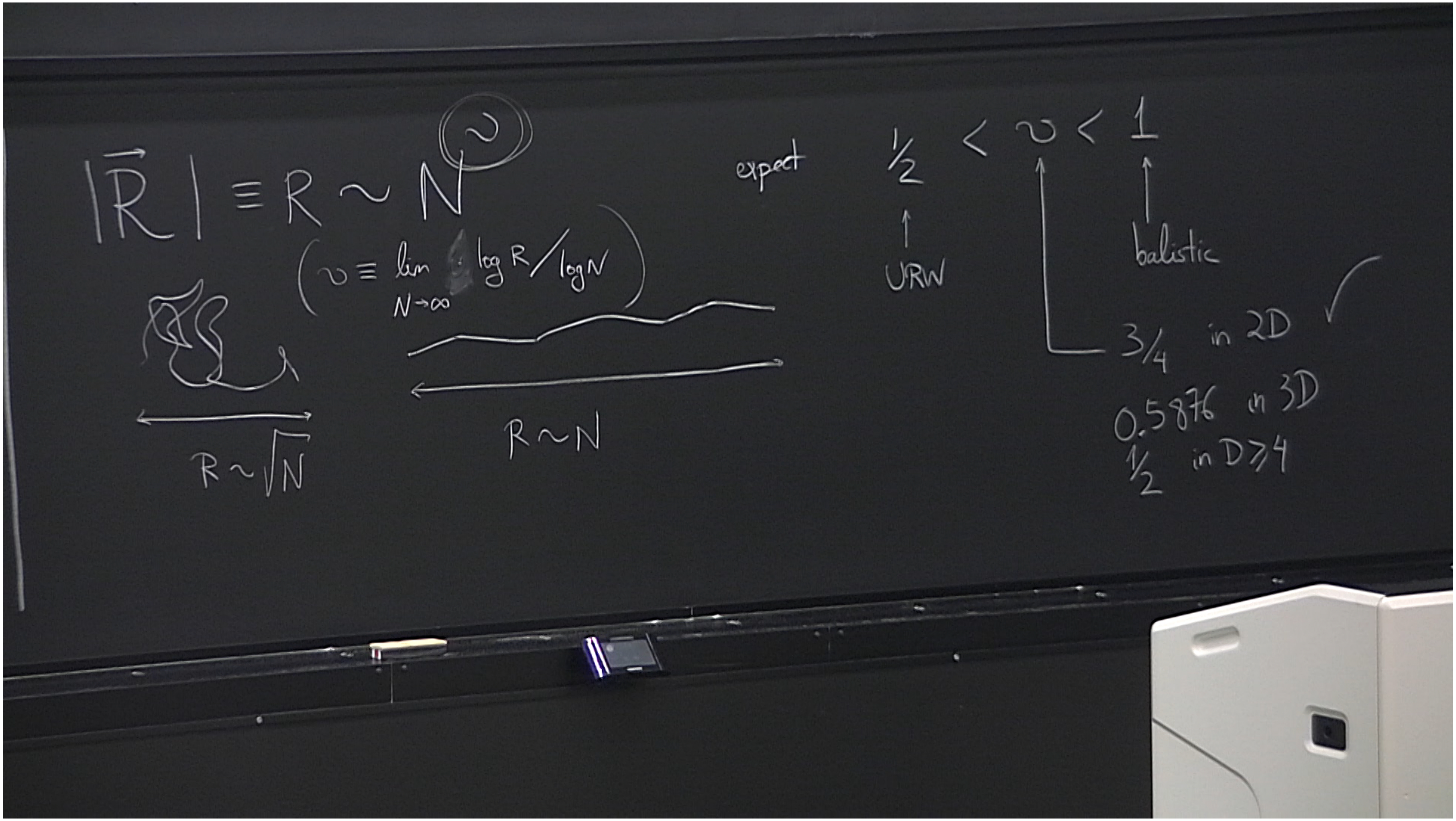

$\to \infty$
Small
quantum (D = 1 + 1)
fluctuations
in Lorentzian
$\to 0$
not
Small
statistical (D=2)
thermal
fluctuations
in Euclidean
$S \to -S$
how come $m \neq 0$
$\tau \sim e^{\pm V}$
$\lim_{L\to\infty} \lim_{h\to 0^+} \langle S_i \rangle = 0$
$\lim_{h\to 0^\pm} \lim_{L\to\infty} \langle S_i \rangle = \pm m$

$T_c \longleftarrow$ not universal

critical exponents $\longleftarrow$ universal


SAWs
big polymer
or
is forbidden
avoids itself


bclass of SAW
dist statistics by RG
ment (~RG) for
R
N steps
V=0
Gauss

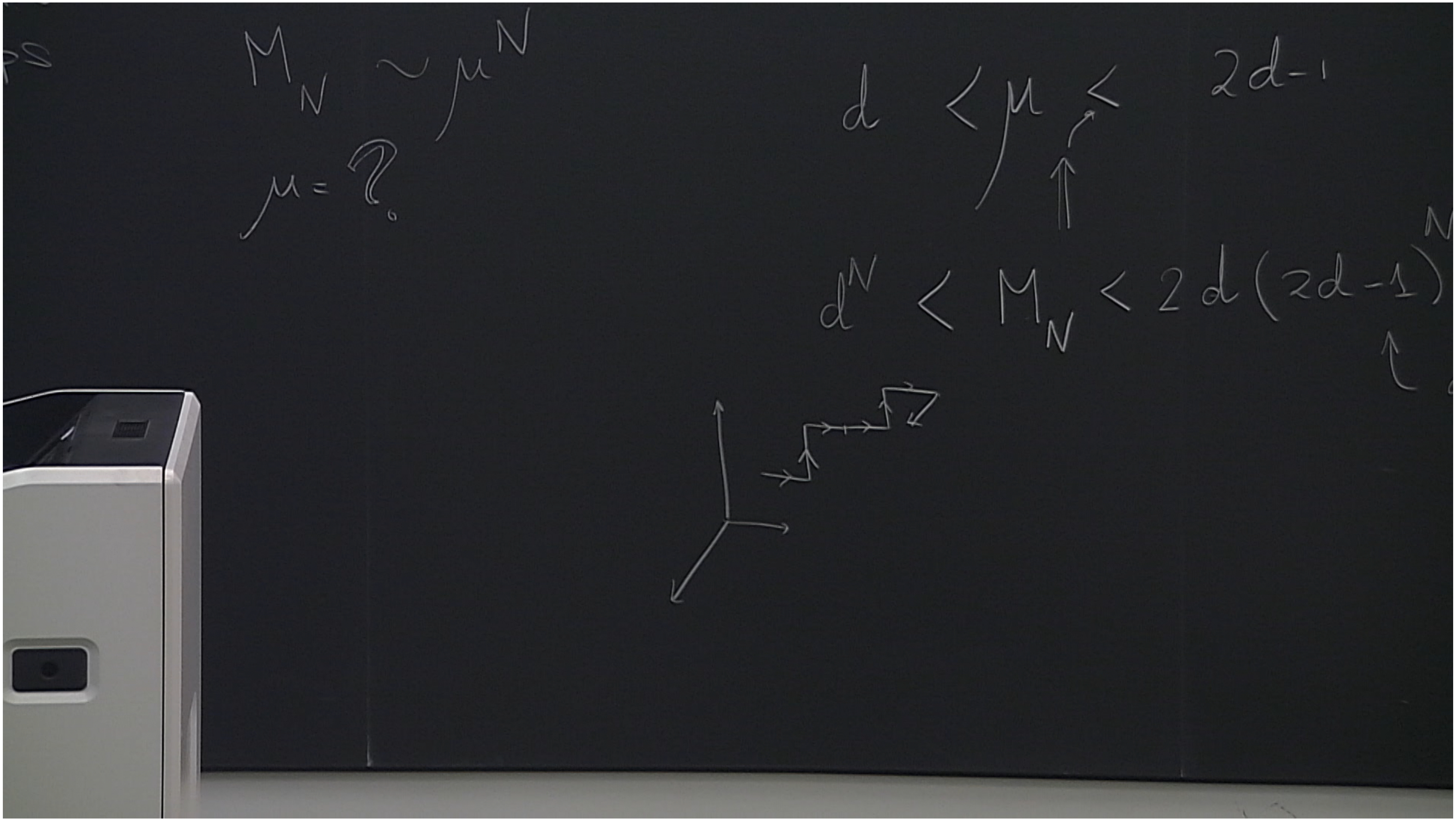

$|\vec{R}| \equiv R \sim N^{\nu}$
$\left(\nu \equiv \lim_{N\to\infty} \log R / \log N\right)$
$R \sim \sqrt{N}$
$R \sim N$
expect $\frac{1}{2} < \nu < 1$
URW
balistic
$3/4$ in 2D
0.5876 in 3D
$1/2$ in $D \geq 4$

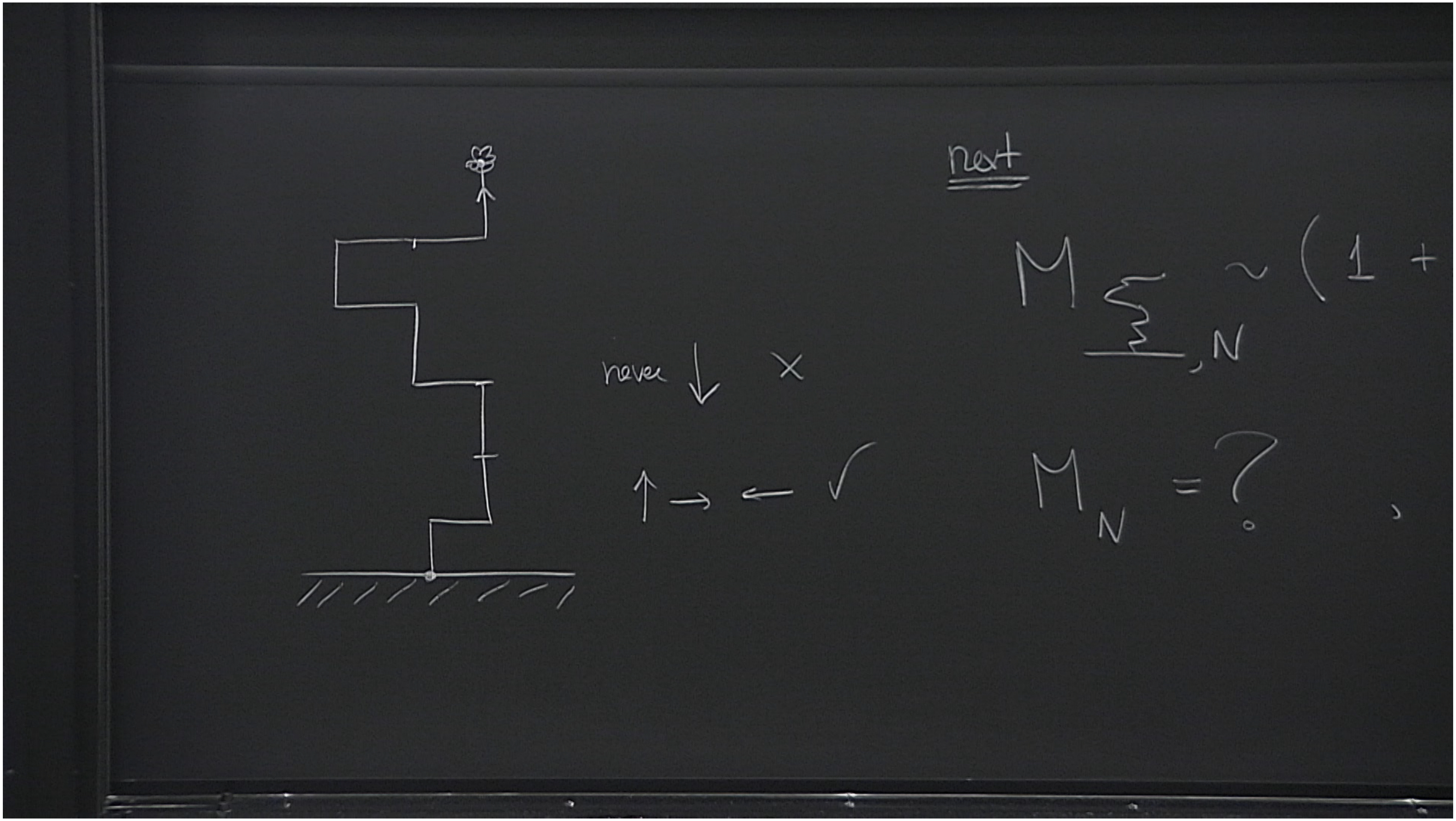
$< \mu <$ $2d-1$

$\Uparrow$

$< M_N < 2d(2d-1)^N$

all paths that do not do this:


$M_N \sim \mu^N$
$\mu = ?$
$d < \mu < 2d-1$
$d^N < M_N < 2d(2d-1)^N$


next
$M_{\xi, N} \sim (1 +$
never ↓ ×
↑ → ← ✓
$M_N = ?$

next

$$M_{\,,N} \sim \left(1+\sqrt{2}\right)^N$$

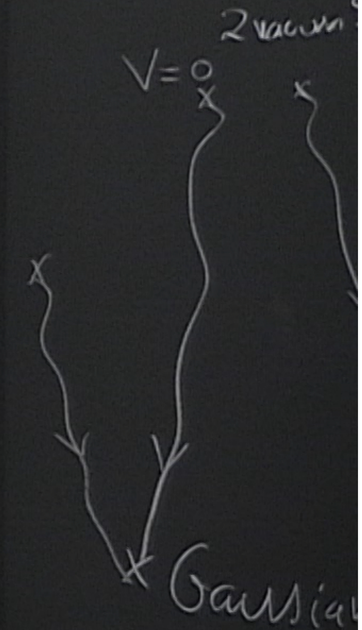
paths ending at $\rightarrow$ or $\leftarrow$

paths ending w/ $\uparrow$

$$M_N = ? \quad , \quad M_N = H_N + V_N$$

$$H_{N+1} = 2\,V_N + 1\,H_N$$

$$V_{N+1} = 1\,V_N + 1\,H_N$$



ths ending at $\rightarrow$ or $\leftarrow$

paths ending w/ $\uparrow$

V_N

$$H_{N+1} = 2\,V_N + 1\,H_N$$

$$V_{N+1} = 1\,V_N + 1\,H_N$$

$$\begin{pmatrix} V \\ H \end{pmatrix}_{N+1} = \mathbb{T} \begin{pmatrix} V \\ H \end{pmatrix}_N , \quad \mathbb{T} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

next

$$M_{\;,N} \sim \left(1+\sqrt{2}\right)^N$$

×

← ✓

$$M_N = ?\qquad , \qquad M_N = H_N + V_N$$

paths ending at → or —

paths ending

$$\begin{pmatrix}V\\H\end{pmatrix}_{N+1} = T^N \begin{pmatrix}V\\H\end{pmatrix}_1$$

$$M_{\Xi_{,N}} \sim (1+\sqrt{2})^N$$

paths ending at $\rightarrow$ or $\leftarrow$

paths ending w/ $\uparrow$

$$M_N = ?\,, \quad M_N = H_N + V_N$$

$$\begin{pmatrix} V \\ H \end{pmatrix}_{N+1} = \mathbb{T}^N \begin{pmatrix} V \\ H \end{pmatrix}_1, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \boxed{(1\ 1)\cdot\mathbb{T}^{N-1}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} = M_N}$$

$$M_N = \underbrace{(1,1)}_{\text{could be anything}} \cdot T^{N-1} \underbrace{\left[c_+ |+\rangle + c_- |-\rangle \right]}_{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$$T |\pm\rangle = \lambda_\pm |\pm\rangle$$

do not do this

$|\lambda_+| > |\lambda_-|$ #

μ

$\begin{pmatrix}1\\0\end{pmatrix} \equiv |\text{start}\rangle$

$N \gg 1$

$T|\pm\rangle = \lambda_\pm |\pm\rangle$

$\lambda_\pm = 1 \pm \sqrt{2}$, $\lambda_+ = \sqrt{2} + 1$

$\det\begin{pmatrix}1-\lambda & 1\\ 2 & 1-\lambda\end{pmatrix} = (1-\lambda)^2 - 2 \Rightarrow$

$$(\lambda_+)^{N-1}_{1+x+1} + (-) \qquad \langle + | \text{start} \rangle$$

$$M_N = \overbrace{(1,1)}^{\langle \text{end}|}_{\text{could be anything}} \cdot \mathbb{T}^{N-1} \underbrace{\left[c_+ |+\rangle + c_- |-\rangle \right]}_{\binom{1}{0} \equiv |\text{start}\rangle} = \quad |\lambda$$

N

paths that do not do this

$$\mathbb{T} |\pm\rangle = \lambda_\pm |\pm\rangle$$

$$\det \begin{pmatrix} 1-\lambda & 1 \\ & \end{pmatrix} = (1-\lambda)^2 -$$

$$= \underbrace{\frac{1}{\lambda_+}\langle \mathrm{end}|+\rangle\langle +|\mathrm{start}\rangle}_{\#}\,\lambda_+^N + \underbrace{\left[\begin{smallmatrix}+\\ \downarrow \\ -\end{smallmatrix}\right]}_{\ll(\dots)}$$

μ

$|\lambda_+| > |\lambda_-|$

$N \gg 1$

$$\Rightarrow \quad \lambda_\pm = 1 \pm \sqrt{2}, \quad \lambda_+ = \sqrt{2} + 1$$

$(\ldots)^2 - 2 \Rightarrow$

μ^N

$N \doteq 4$

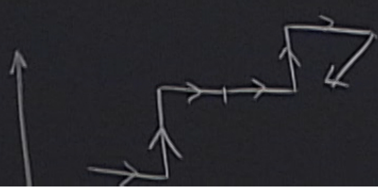
$1+\sqrt{2} \simeq 2.41$

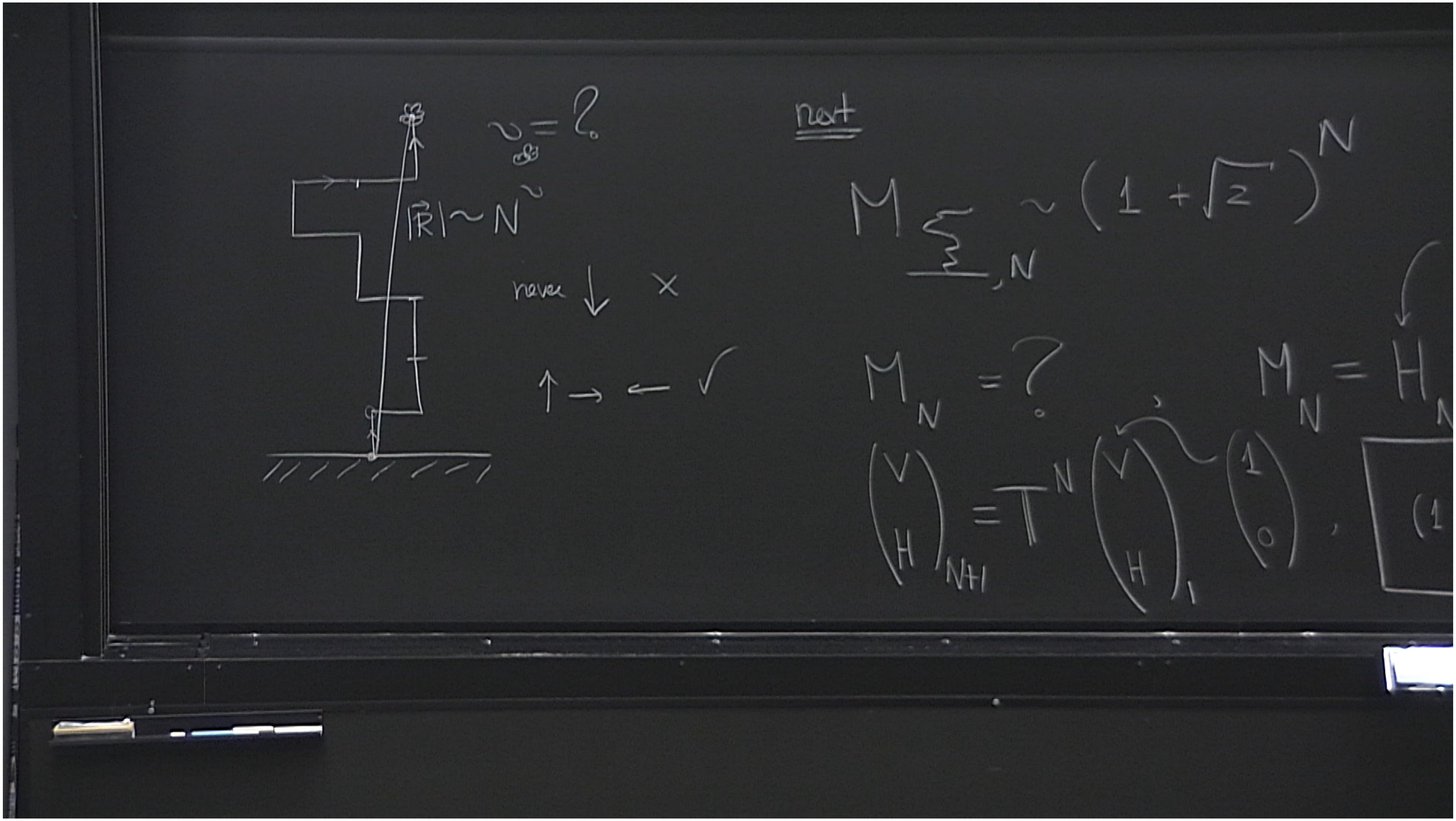
$2 < \mu < 3$ in $d=2$

$d < \mu < 2d-1$

$d^N < M_N < 2d(2d-1)^N$

all paths that




$v = ?$
$|\vec{R}| \sim N^{\nu}$
never ↓ ×
↑ → ← ✓
next
$M_{N} \sim (1+\sqrt{2})^N$
$M_N = ?$
$M_N = H_N$
$\begin{pmatrix} V \\ H \end{pmatrix}_{N+1} = T^N \begin{pmatrix} V \\ H \end{pmatrix}_1$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$