Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 7

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Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

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Beyond leading order

$$\mathcal{L}_{naive} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3$$

$$\text{LSZ} \longrightarrow \langle\Omega|\varphi(x)|\Omega\rangle = 0 \qquad \langle\vec{k}|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$$

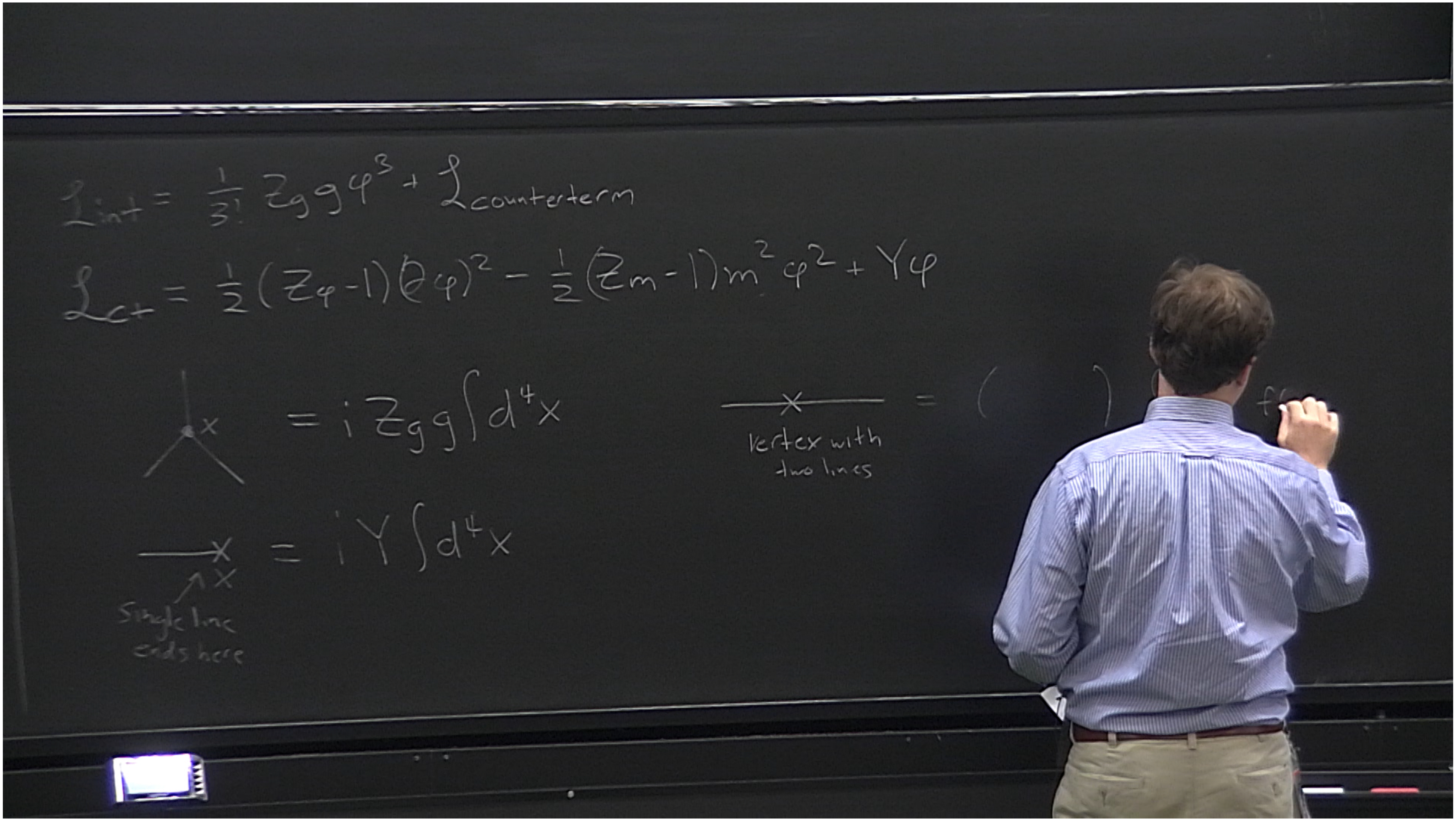
→ shifted
rescaled φ
renamed

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 + \frac{Z_g g}{3!}g\varphi^3 + Y\varphi$$

$$= \mathcal{L}_{free} + \mathcal{L}_{int}$$

$$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$\mathcal{L}_{int} =$

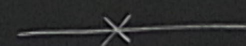


$\mathcal{L}_{int} = \frac{1}{3!} Z_g g \varphi^3 + \mathcal{L}_{counterterm}$
$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi$
x
$= i Z_g g \int d^4x$
vertex with two lines
$= i Y \int d^4x$
x
single line ends here

$$\mathcal{L}_{int} = \frac{1}{3!} Z_g g \varphi^3 + \mathcal{L}_{counterterm}$$

$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi$$

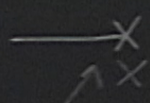
$= i Z_g g \int d^4x$



vertex with two lines

$= \Big(\qquad \Big) \int d^4x \; f(p)$

postpone



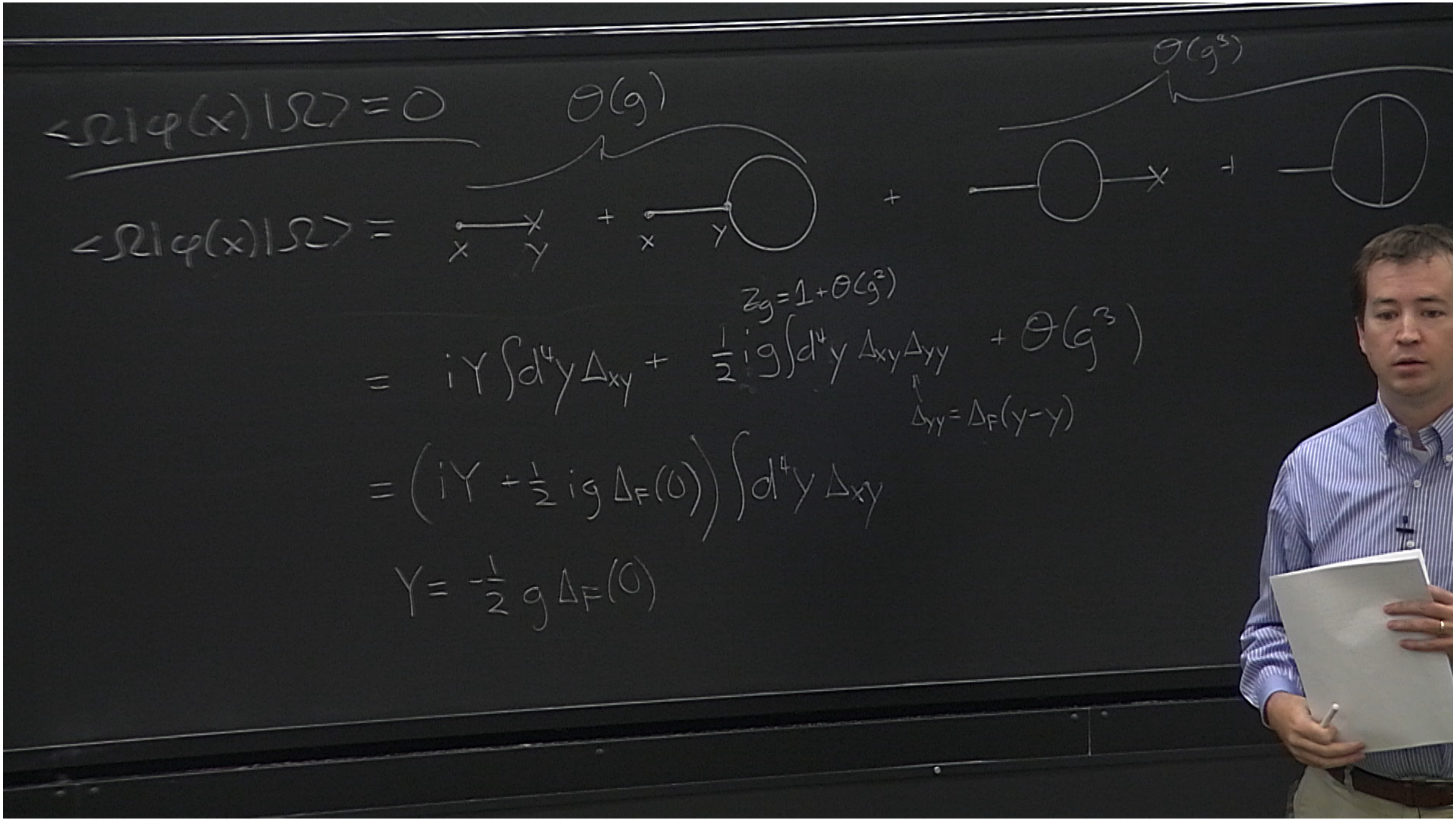
single line ends here

$= iY \int d^4x$

Determine Z_i, Y perturbatively

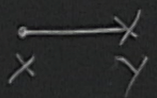
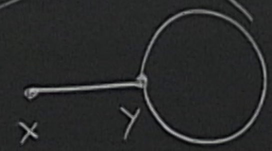
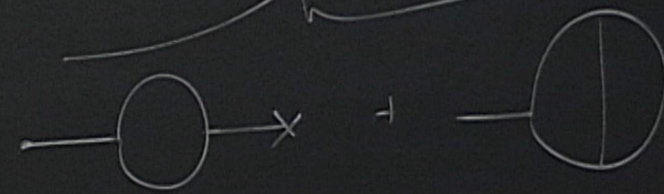
$Z_i = 1 + \mathcal{O}(g)$

$Y = 0 + \mathcal{O}(g)$


$\langle\Omega|\varphi(x)|\Omega\rangle = 0$
$\mathcal{O}(g)$
$\mathcal{O}(g^3)$
$\langle\Omega|\varphi(x)|\Omega\rangle =$
x y + x y +
$Z_g = 1 + \mathcal{O}(g^2)$
$= iY \int d^4y\, \Delta_{xy} + \frac{1}{2} ig \int d^4y\, \Delta_{xy}\Delta_{yy} + \mathcal{O}(g^3)$
$\Delta_{yy} = \Delta_F(y-y)$
$= \left(iY + \frac{1}{2} ig\, \Delta_F(0)\right) \int d^4y\, \Delta_{xy}$
$Y = -\frac{1}{2} g\, \Delta_F(0)$

$\langle\Omega|\varphi(x)|\Omega\rangle = 0$

$\mathcal{O}(g)$ $\qquad\qquad\qquad\qquad$ $\mathcal{O}(g^3)$

$\langle\Omega|\varphi(x)|\Omega\rangle =$  $+$  $+$ 

$Z_g = 1 + \mathcal{O}(g^2)$

$$= iY\int d^4y\,\Delta_{xy} + \frac{1}{2}ig\int d^4y\,\Delta_{xy}\Delta_{yy} + \mathcal{O}(g^3)$$

$\Delta_{yy} = \Delta_F(y-y)$

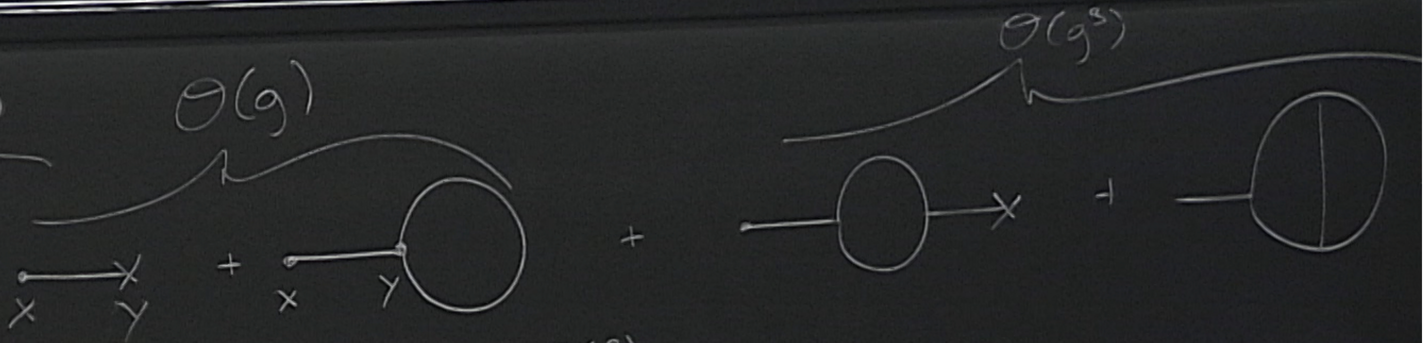
$$= \left(iY + \frac{1}{2}ig\,\Delta_F(0)\right)\int d^4y\,\Delta_{xy}$$

$$Y = -\frac{1}{2}g\,\Delta_F(0) = -\frac{1}{2}g\int\frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\varepsilon}\,e^{\overset{1}{-ip\cdot 0}} \to \infty$$

$$\sim\int\frac{p^3\,dp}{p^2}\to\infty$$

$$\langle\Omega|\varphi(x)|\Omega\rangle = 0$$

$$\langle\Omega|\varphi(x)|\Omega\rangle =$$



$$Z_g = 1 + \theta(g^2)$$

$$= iY\int d^4y\,\Delta_{xy} + \frac{1}{2} i g \int d^4y\,\Delta_{xy}\Delta_{yy} + \theta(g^3)$$

$$\Delta_{yy} = \Delta_F(y-y)$$

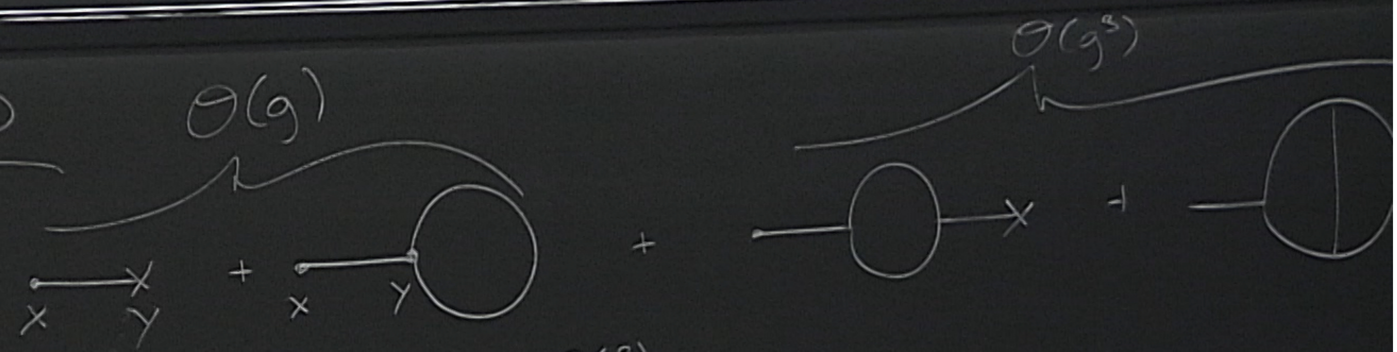
$$= \left(iY + \frac{1}{2} i g\,\Delta_F(0)\right)\int d^4y\,\Delta_{xy}$$

$$Y = -\frac{1}{2} g\,\Delta_F(0) = -\frac{1}{2} g \underbrace{\int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\varepsilon}}_{\sim \int \frac{p^3dp}{p^2} \to \infty} e^{-ip\cdot 0 \to 1} \to \infty$$

$$\int \frac{d^3p}{(2\pi)}$$

$$\langle\Omega|\varphi(x)|\Omega\rangle = 0$$

$$\langle\Omega|\varphi(x)|\Omega\rangle =$$



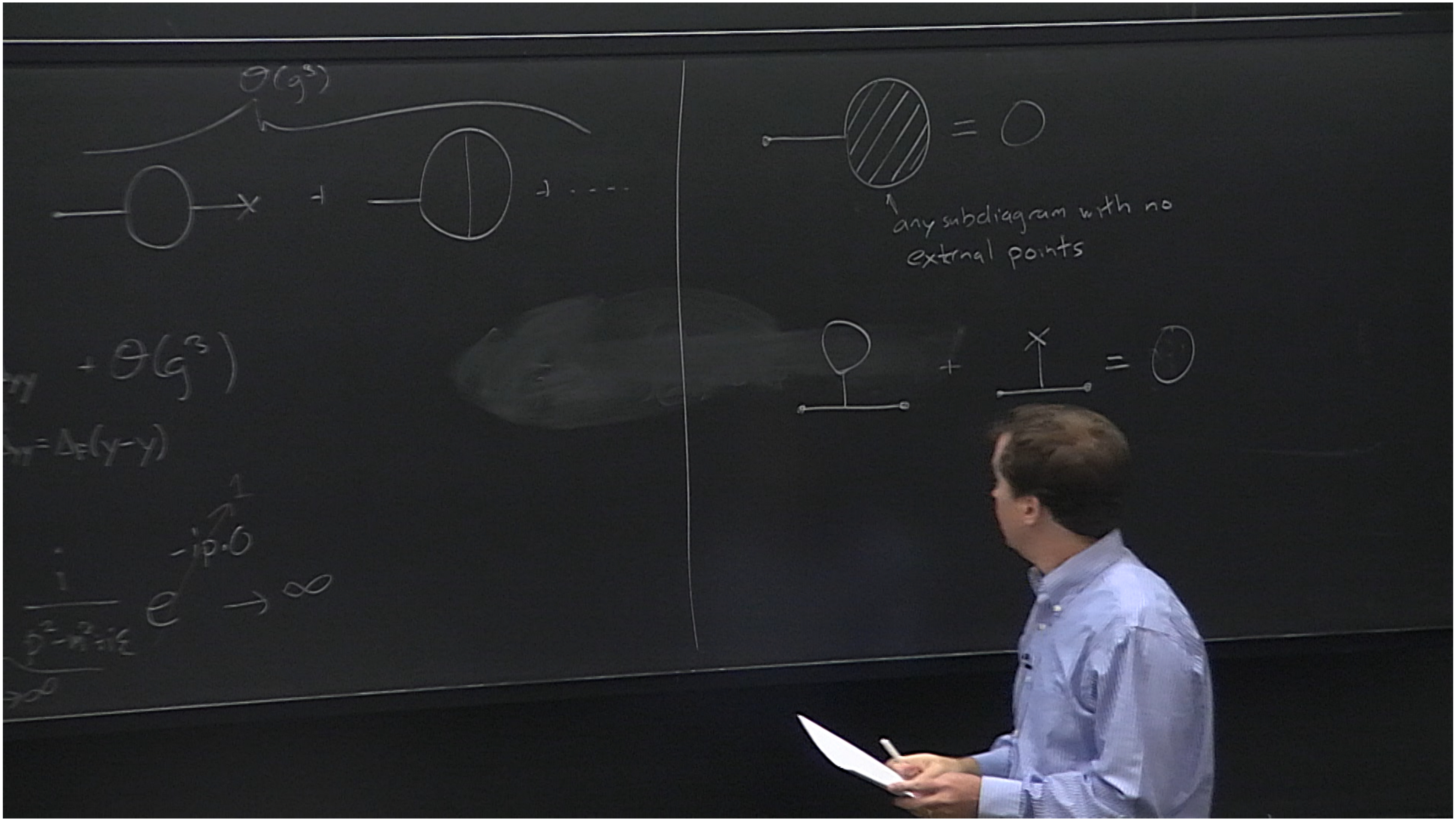
$$Z_g = 1 + O(g^2)$$

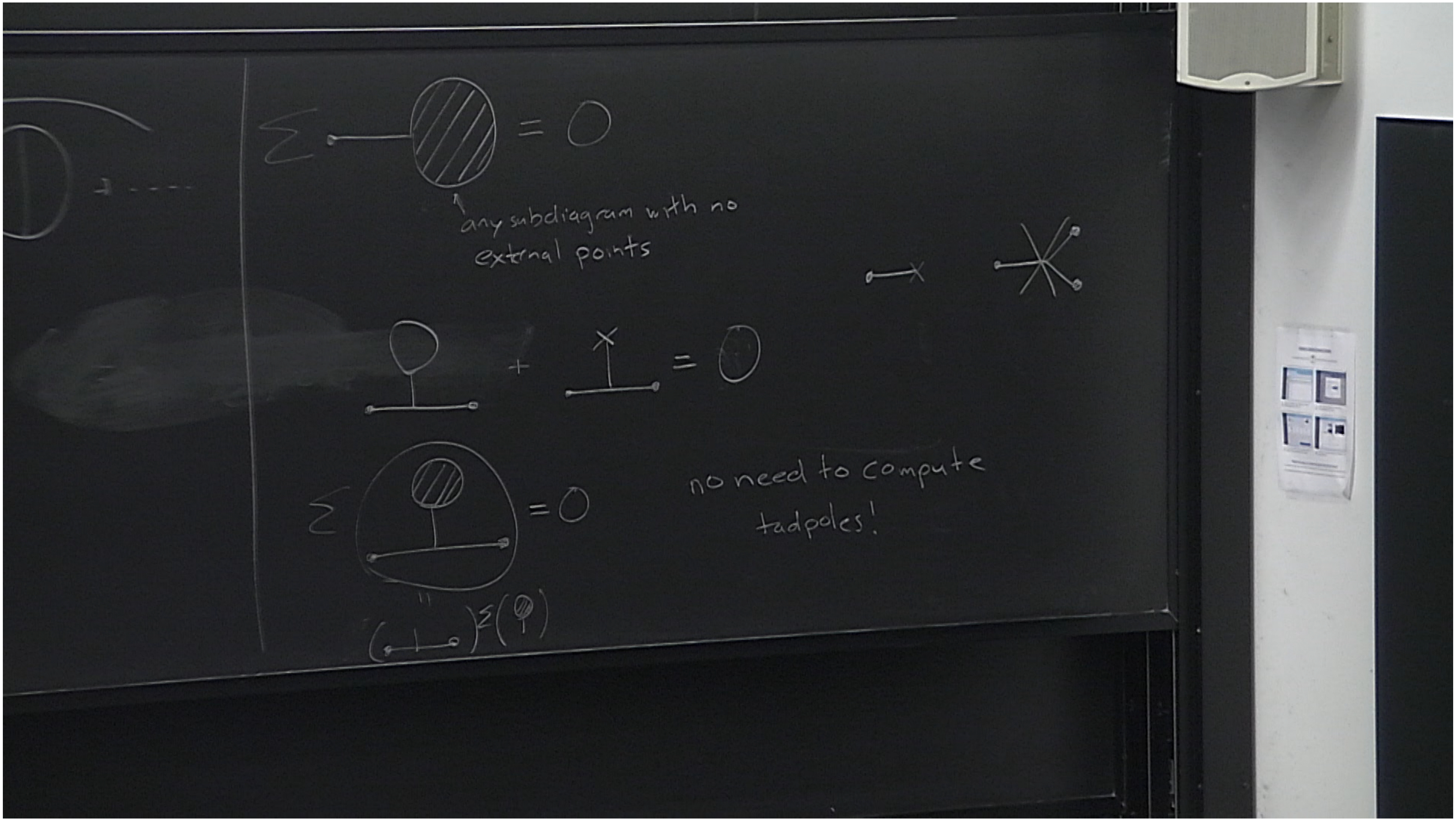
$$= iY\int d^4y\,\Delta_{xy} + \frac{1}{2}ig\int d^4y\,\Delta_{xy}\Delta_{yy} + O(g^3)$$

$$\Delta_{yy} = \Delta_F(y-y)$$

$$= \left(iY + \frac{1}{2}ig\,\Delta_F(0)\right)\int d^4y\,\Delta_{xy}$$

$$Y = -\frac{1}{2}g\,\Delta_F(0) = -\frac{1}{2}g\underbrace{\int\frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\varepsilon}}_{\sim\int\frac{p^3dp}{p^2}\to\infty}\underbrace{e^{-ip\cdot 0}}_{1} \to \infty$$


O(g³)
= 0
any subdiagram with no external points
+ O(g³)
→ ∞


any subdiagram with no
external points
no need to compute
tadpoles!

$$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\underbrace{\langle \vec{k}|}_{\text{one-particle state}}\ \underbrace{\varphi(x)|\Omega\rangle}_{\text{one-particle state}} = e^{ik\cdot x}$$

$$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle$$

$$\mathbb{1} = \underbrace{|\Omega\rangle\langle\Omega|}_{\text{zero particle}} + \underbrace{\int\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}|\vec{k}\rangle\langle\vec{k}|}_{\text{one-particle}} + \underbrace{\sum_\sigma\int\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}|\vec{k},\sigma\rangle\langle\vec{k},\sigma|}_{\text{multiparticle states}}$$

total momentum

all other parameters

zero particle ↑ gives zero

$$\langle\Omega|\varphi(x)|\Omega\rangle = 0$$

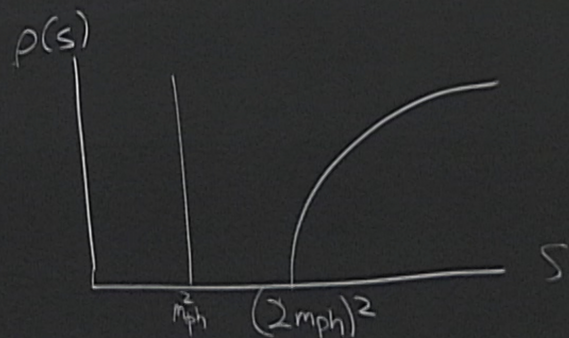
$$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4} 2\pi\,\delta(k^2 - m^2)\,\Theta(k^0)$$

$$\rho(s) = 2\pi\,\delta(s - m_{ph}^2) + \sum_\sigma (2\pi)\,\delta(s - M_\sigma^2)\,|\langle \vec{k},\sigma|\varphi(0)|\vec{k},\sigma\rangle|^2$$

spectral density

invariant mass of a multiparticle state

no bound states $\Rightarrow M_\sigma \geq 2m_{ph}$



ther
eters
l momentum
$\sigma\rangle\langle\vec{k},\sigma|$
tes

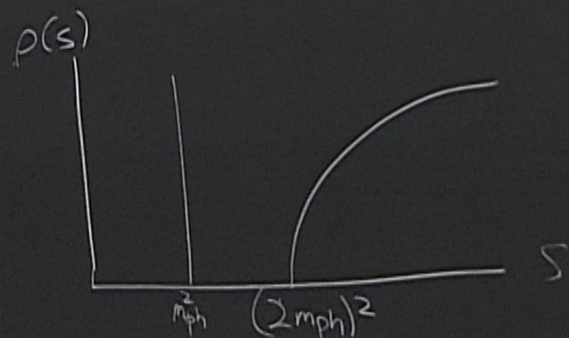
$$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4} 2\pi\, \delta(k^2 - m^2)\, \Theta(k^0)$$

$$\rho(s) = 2\pi\, \delta(s - m_{ph}^2) + \sum_\sigma (2\pi)\, \delta(s - M_\sigma^2)\, |\langle \vec{k}, \sigma | \varphi(0) | \Omega \rangle|^2$$

$\rho(s)$: spectral density

M_σ^2: invariant mass of a multiparticle state

no bound states $\Rightarrow M_\sigma \geq 2 m_{ph}$



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...l momentum

$\ldots, \sigma \rangle \langle \vec{k}, \sigma |$

...tes