Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 6

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Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

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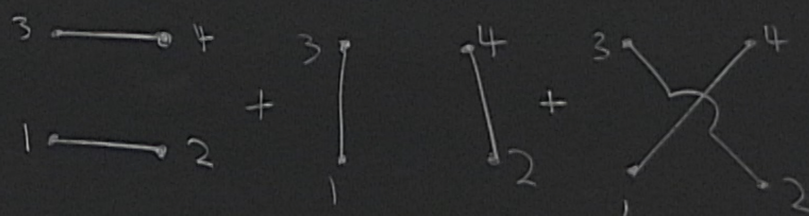
Feynman Diagrams

$$\sigma, \Gamma \longrightarrow i\mathcal{M} \xrightarrow{\text{LSZ}} \langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle \xrightarrow[\text{pict}]{\text{int}} \langle 0 | T \varphi_1 \cdots \varphi_m | 0 \rangle \xrightarrow{\text{Wick}} \Delta_F$$

Feynman diagrams

$$\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12} \Delta_{34} + \Delta_{13} \Delta_{24} + \Delta_{14} \Delta_{23}$$

$$\Delta_{ij} = \Delta_F (x_i - x_j)$$



$$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle = \frac{\langle 0 | T \varphi_1 \varphi_2 \exp\left[i \int d^4y \, \mathcal{L}_{int} \right] | 0 \rangle}{\langle 0 | T \exp\left[i \int d^4y \, \mathcal{L}_{int} \right] | 0 \rangle}$$

$$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi^4(y)$$

numerator: $\langle 0 | T \varphi_1 \varphi_2 | 0 \rangle + \langle 0 | T \varphi_1 \varphi_2 \, i \int d^4y \, \frac{-\lambda}{4!} \varphi_y^4 | 0 \rangle + \cdots$

$$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2\exp[i\int d^4y\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4y\,\mathcal{L}_{int}]|0\rangle}$$

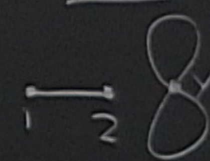
$$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4(y)$$

numerator: $\langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2\, i\int d^4y\,\frac{-\lambda}{4!}\varphi_y^4|0\rangle + \cdots$

1 2

$$\varphi_y\varphi_y\varphi_y\varphi_y$$

$$=3\left(-\frac{i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$$



$$+12\left(-\frac{i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$$

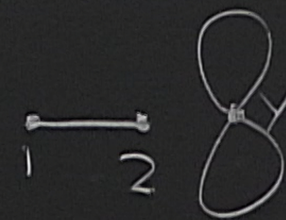
$$\langle 0|T\exp\left[i\int d^4y\,\mathcal{L}_{int}\right]|0\rangle$$

$$\mathcal{L}_{int}=-\frac{\lambda}{4!}\varphi^4(y)$$

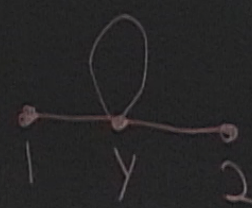
$$+\langle 0|T\,\varphi_1\varphi_2\, i\int d^4y\,\frac{-\lambda}{4!}\varphi_y^4|0\rangle+\cdots$$

$$\varphi_y\varphi_y\varphi_y\varphi_y$$

$$=3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$$



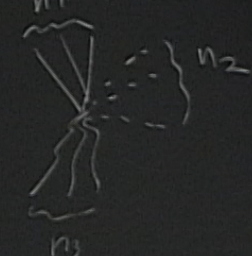
$$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$$



Feynman rules for numerator

$\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4y\, \mathcal{L}_{int}]|0\rangle$ = sum of all diagrams with n external points + m vertices + $\mathcal{O}(\lambda^{m+1})$

1. For each line $\underset{x \quad y}{\bullet\!\!-\!\!-\!\!\bullet} = \Delta_F(x-y)$
2. For each external point $= 1$
3. For each vertex $\underset{z}{\times} = -i\lambda\int d^4z$ $\left(\text{if } \mathcal{L}_{int} \supset -g\dfrac{\varphi^{n_1}\phi^{n_2}\psi^{n_3}}{n_1!\,n_2!\,n_3!} \right.$

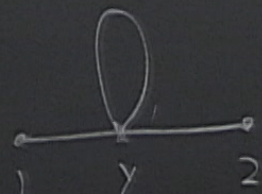


$\left. = -ig\int d^4z \right)$

4. Divide by symmetry factor S

S = # ways a diagram can be mapped to itself with external points held fixed

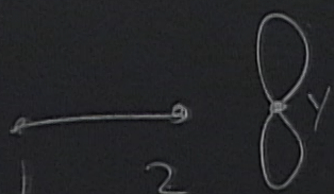
Example:



$S=2$

exchange ends of Δ_{yy}

$$\frac{1}{S} = \frac{1}{2} = \frac{12}{4!} \checkmark$$



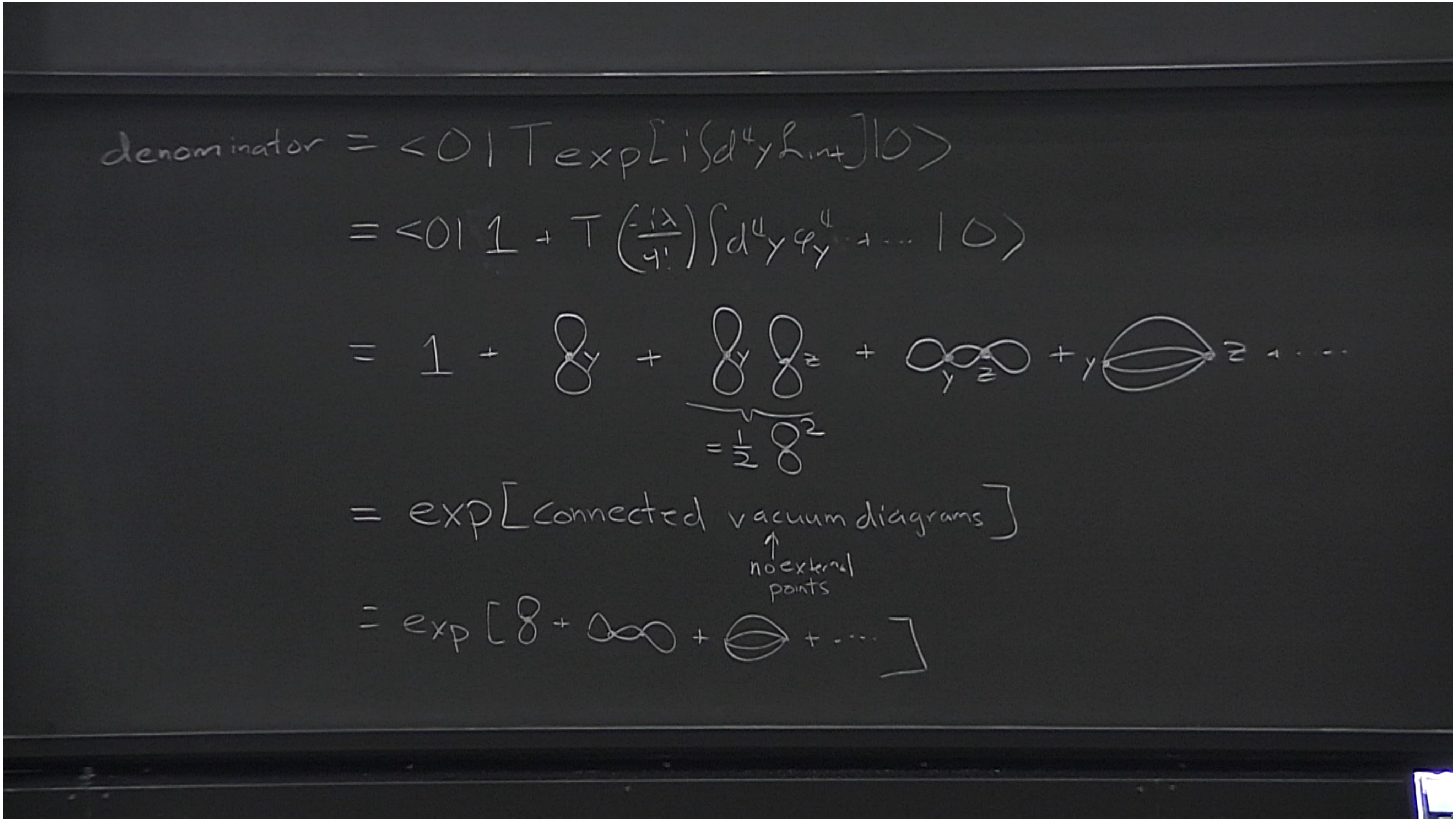
$$S = 2 \cdot 2 \cdot 2 = 8$$

exchanging ends of each line

exchanging lines

$$\frac{1}{8} = \frac{3}{4!} \checkmark$$

$$\frac{1}{S} = \frac{\#\,\text{Wick contractions}}{\text{factorials in } \mathcal{L}}$$


denominator = <0| T exp[i ∫d⁴y ℒint] |0>
= <0| 1 + T (-iλ/4!) ∫d⁴y φy⁴ + ... |0>
= 1 + + + + +
= ½
= exp[connected vacuum diagrams]
no external points
= exp[+ + + ...]

~~ector~~ $\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle$ = sum of all diagrams with no vacuum subdiagrams

Ex

$\mathcal{L}_{int}]|0\rangle$ = sum of all diagrams with n external points + m vertices $+\ \mathcal{O}(\lambda^{m+1})$

$x \quad y \; = \Delta_F(x-y)$

nt $\quad x \; = 1$

$z \; = -i\lambda\int d^4z \qquad \left(\text{if } \mathcal{L}_{int} \supset -g\,\dfrac{\varphi^{n_1}\,\phi^{n_2}\,\psi^{n_3}}{n_1!\,n_2!\,n_3!}\right.$

n_1

$\}\, n_2$

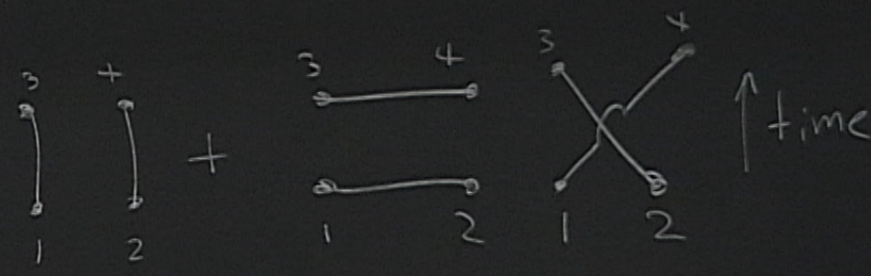
factor S

with external points held fixed

$$\langle f|S|i\rangle = i^4 \int \prod d^4x_i \; e^{i\lambda_i p_i \cdot x_i} \prod (\partial_i^2 + m^2) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$$S = 1 + (2\pi)^4 \delta^4(\Sigma p)\, i\mathcal{M}$$

$$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$$



$\Delta_{13}\, \Delta_{24}$

$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\Delta_{13}$$

$$x_{13} = x_1 + x_3 \qquad p_{13} = \frac{p_1 + p_3}{2}$$

$$\bar{x}_{13} = x_1 - x_3 \qquad \bar{p}_{13} = \frac{p_1 - p_3}{2}$$

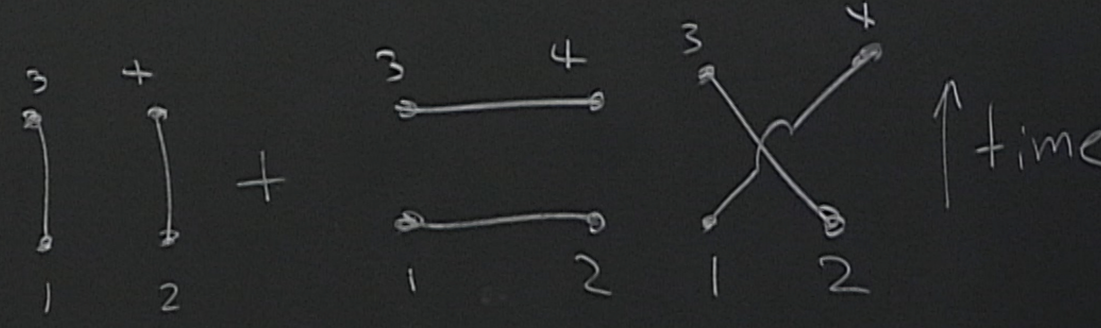
$$\int d^4x_1\, d^4x_3\, e^{i(p_1 \cdot x_1 - p_3 \cdot x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13}\, d^4\bar{x}_{13}\, e^{i(\bar{p}_{13} x_{13} + p_{13}\bar{x}_{13})} \cdot F(\bar{x}_{13})$$

$$= \frac{1}{2}\int d^4x_{13}\, e^{i\bar{p}_{13} x_{13}}\, \tilde{F}(p_{13})$$

$$= (2\pi)^4 \delta^{(4)}(p_1 - p_3)\, \tilde{F}\left(\frac{p_1 + p_3}{2}\right)$$

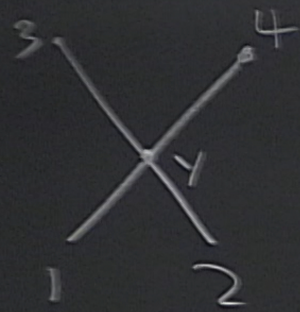
$$S = \mathbb{1} + (2\pi)^4 \delta^4\left(\sum p\right) i\mathcal{M}$$

$$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$$



$$\Delta_{13}\, \Delta_{24}$$

$$\langle f | S | i \rangle_{\mathbb{1}} \propto \delta^4(p_1 - p_3)\, \delta^4(p_2 - p_4)$$



$$= -i\lambda \int d^4y \; \Delta_{1y}\Delta_{2y}\Delta_{3y}\Delta_{4y}$$

$$(\partial_1^2 + m_{ph}^2)\,\Delta_{1y} = -i\,\delta^{(4)}(x_1 - y)$$

$$\langle f|S|i\rangle_X = -i\lambda \int d^4y \; e^{i(p_1+p_2-p_3-p_4)\cdot y}$$

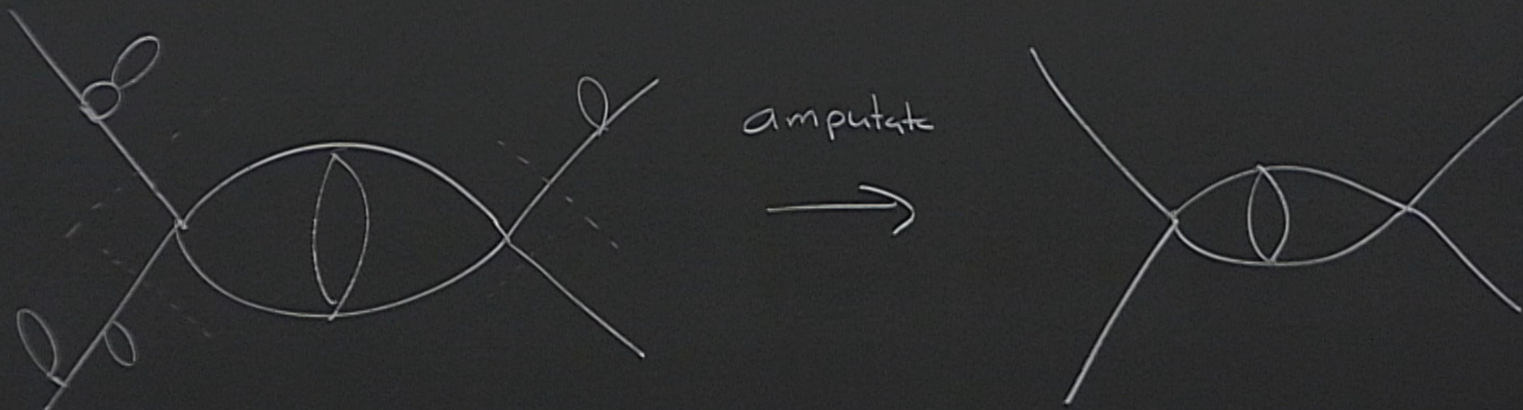
$$= -i\lambda\,(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$(iM)_X = -i\lambda$$

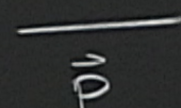
Amputation

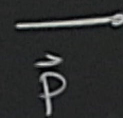
m^2 in LSZ is physical mass

m^2 in Δ_F is not necessarily $= m_{ph}^2$



$i\mathcal{M}$ = sum of completely connected, amputated diagrams

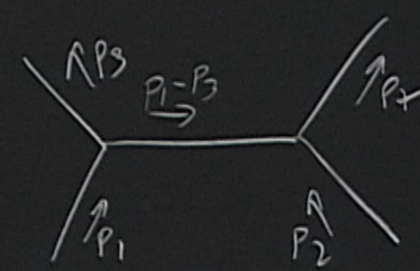
1. internal line  $= \dfrac{i}{p^2 - m^2 + i\epsilon}$

2. external line  $= 1$

3. vertex  $= -i\lambda$

4. Impose momentum conservation at each vertex

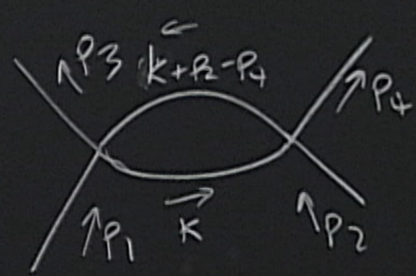
e.g.



5. Divide by symmetry factor

6. Integrate over undetermined momenta

e.g.



$$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\varepsilon} \frac{i}{(k+p_2-p_4)^2-m^2+i\varepsilon}$$