Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 4

Speakers: Dan Wohns, Gang Xu

Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Date: October 11, 2019 - 9:00 AM

URL: http://pirsa.org/19100013

Your expectations

particle # not conserved 24/26

$[a_{\vec{k}}(t), a^{+}_{\vec{p}}(t)] = (2\pi)^3 2E_{\vec{k}}\,\delta(\vec{k}-\vec{p})$ 21/26

$\langle \vec{k} | \varphi(x) | \Omega \rangle = e^{ik\cdot x}$ 12/26

$a_{\vec{k}}, a^{+}_{\vec{k}}$ same t dependence 8/26

$\varphi(x)|\Omega\rangle$ is a one-particle state for all t 5/26

LSZ Reduction Formula

$$\Gamma, \sigma \longleftrightarrow \langle f|S|i\rangle \text{ or } iM \overset{LSZ}{\longleftrightarrow} \langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\cdots\varphi(x_m)|$$

Assumptions: $a^\dagger_{\vec p}(t)$ creates a particle with momentum $\vec p$ at time t

$$[a_{\vec p}(t), a^\dagger_{\vec k}(t)] = (2\pi)^3 2E_{\vec k}\,\delta(\vec p - \vec k)$$

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}}\left(a_{\vec p}(t)e^{-ip\cdot x} + a^\dagger_p(t)e^{ip\cdot x}\right)$$

$$a^\dagger_{\vec k}(t) = -i\int d^3x\left(e^{-ik\cdot x}\left(\partial_0\varphi(x) + iE_{\vec k}\varphi(x)\right)\right)$$

...mula

$$\ldots\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\cdots\varphi(x_m)|0\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle$$

...eates a particle with momentum $\vec{p}$ at time t

$$\ldots(t)] = (2\pi)^3 2E_{\vec{k}}\,\delta(\vec{p}-\vec{k})$$

$$\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}}\left(a_{\vec{p}}(t)\,e^{-ip\cdot x} + a_p^\dagger(t)\,e^{ip\cdot x}\right)\Big|_{k^0=E_{\vec{k}}}$$

$$= -i\int d^3x\left(e^{-ik\cdot x}\left(\partial_0\varphi(x) + iE_{\vec{k}}\varphi(x)\right)\right)$$

$2 \to 2$ scattering

vacuum of interacting theory

$$|i\rangle = a^{\dagger}_{\vec{k}_1}(-\infty)\, a^{\dagger}_{\vec{k}_2}(-\infty)\,|\Omega\rangle$$

$$\equiv a^{\dagger}_{1}(-\infty)\, a^{\dagger}_{2}(-\infty)\,|\Omega\rangle$$

$$|f\rangle = a^{\dagger}_{3}(+\infty)\, a^{\dagger}_{4}(+\infty)\,|\Omega\rangle$$

useful identity $\quad a^{\dagger}_{1}(+\infty) - a^{\dagger}_{1}(-\infty) = -i\int d^4x\, e^{-ik\cdot x}(\partial^2+m^2)$

$$\equiv I^{\dagger}_{1}$$

Proof: $a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = \int_{-\infty}^{\infty} dt\, \partial_0 a_1^\dagger(t)$

$$= -i\int d^4x\, \partial_0\left(e^{-ik_1\cdot x}\left(\partial_0\varphi(x) + iE_{\vec{k}_1}\varphi(x)\right)\right)$$

$$= -i\int d^4x\, e^{-ik_1\cdot x}\left(\partial_0^2\varphi(x) + E_{\vec{k}}^2\varphi(x)\right)$$

$$= -i\int d^4x\, e^{-ik_1\cdot x}\left(\partial_0^2 + \underbrace{\vec{k}^2}_{-\vec{\nabla}^2} + m^2\right)\varphi(x)$$

$$= -i\int d^4x\, e^{-ik_1\cdot x}\left(\partial^2 + m^2\right)\varphi(x)$$

$$\equiv a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)\,|\Omega\rangle$$

$$\rangle = a_3^\dagger(+\infty)\, a_4^\dagger(+\infty)\,|\Omega\rangle$$

useful identity $a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = -i\int d^4x\, e^{-ik_1\cdot x}\,(\partial^2+m^2)\,\varphi(x)$

$$\equiv I_1^\dagger$$

$$a_1(+\infty) - a_1(\infty) = I_1$$

$$\langle f|S|i\rangle = \langle\Omega|\, a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)|\Omega\rangle$$

$$= \langle\Omega| T\, a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)|\Omega\rangle$$

$$= \langle\Omega| T\, a_4(-\infty)\, \cancel{a_3(-\infty)\, a_1^\dagger(+\infty)}^{\,0}\, a_2^\dagger(+\infty)|\Omega\rangle +$$

$$+ \cdots\cdots + \langle\Omega| T\, I_4\, I_3\, I_1^\dagger\, I_2^\dagger|\Omega\rangle$$

only term that survives

$T\, \Theta(x)\, \Theta($

$$\boxed{\langle f|S|i\rangle =}$$

$$\theta(y) \equiv \begin{cases} \theta(x)\,\theta(y) & \text{if } x^0 > y^0 \\ \theta(y)\,\theta(x) & \text{if } y^0 > x^0 \end{cases}$$

LSZ Reduction Formula

$$\boxed{= i^{2+2}\left(\prod_{j=1}^{2+2}\int d^4x_j\, e^{-i\lambda_j k_j \cdot x_j}\left(\partial_j^2 + m^2\right)\right)\langle\Omega|T\,\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle}$$

$$\lambda_j = \begin{cases} +1 & \text{initial} \\ -1 & \text{final} \end{cases} \qquad \partial_j^2 = \eta^{\mu\nu}\frac{\partial}{\partial x_j^\mu}\frac{\partial}{\partial x_j^\nu} \qquad \varphi_i \equiv \varphi(x_i)$$

Checking Assumptions

Assumed that $\varphi(x)|\Omega\rangle$ is a one particle state

$$\langle \vec{k} | \varphi(x) | \Omega \rangle = \underline{e^{+ik\cdot x}}$$

need to check: • $\langle \Omega | \varphi(x) | \Omega \rangle = 0$

• $\langle \vec{k} | \varphi(x) | \Omega \rangle = e^{ik\cdot x}$

• can distinguish one + multiparticle states

$\langle\Omega|\varphi(x)|\Omega\rangle$

$$\varphi(x) = e^{i\hat{P}\cdot x}\,\varphi(0)\,e^{-i\hat{P}\cdot x} \qquad \hat{P}^{\mu} = (H, \vec{P})$$

momentum oper

$$\langle\Omega|\varphi(x)|\Omega\rangle = \langle\Omega|\,e^{i\hat{P}\cdot x}\,\varphi(0)\,e^{-i\hat{P}\cdot x}\,|\Omega\rangle$$

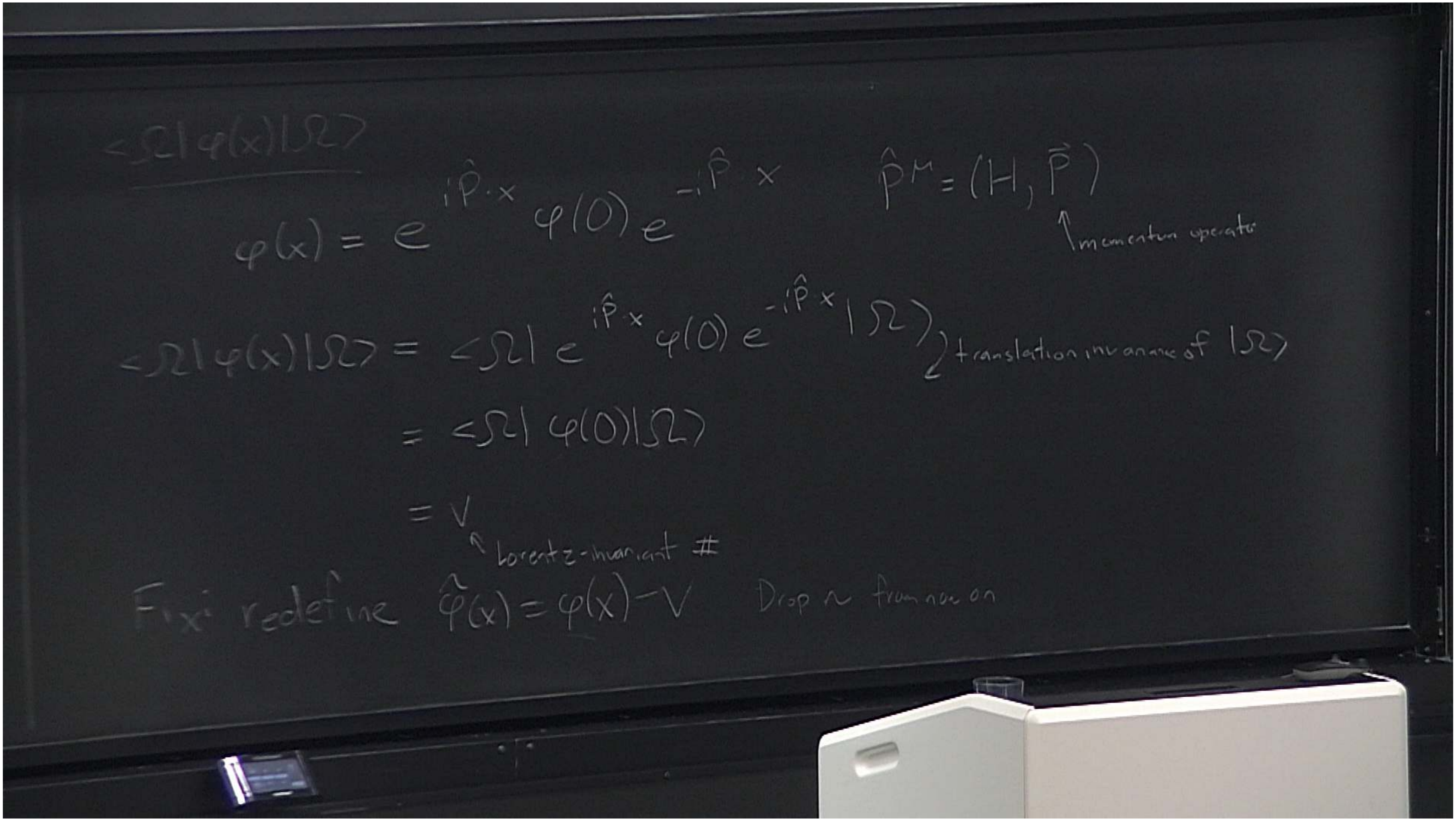
translation invariance of

$$= \langle\Omega|\varphi(0)|\Omega\rangle$$

$$= v$$

Lorentz-invariant #

Fix: redefine $\tilde{\varphi}(x) = \varphi(x) - v$ Drop $\sim$ from now on


$\langle\Omega|\varphi(x)|\Omega\rangle$
$\varphi(x) = e^{i\hat{P}\cdot x}\varphi(0)e^{-i\hat{P}\cdot x}$
$\hat{P}^\mu = (H, \vec{P})$
momentum operator
$\langle\Omega|\varphi(x)|\Omega\rangle = \langle\Omega|e^{i\hat{P}\cdot x}\varphi(0)e^{-i\hat{P}\cdot x}|\Omega\rangle$
translation invariance of $|\Omega\rangle$
$= \langle\Omega|\varphi(0)|\Omega\rangle$
$= V$
Lorentz-invariant #
Fix: redefine $\tilde{\varphi}(x) = \varphi(x) - V$
Drop ~ from now on

$$\underline{\langle \vec{k} | \varphi(x) | \Omega \rangle = e^{ik\cdot x}}$$

$$\langle k | \varphi(x) | \Omega \rangle = \langle \vec{k} | e^{i\hat{P}\cdot x} \varphi(0) e^{-i\hat{P}\cdot x} | \Omega \rangle$$

$$= e^{ik\cdot x} \langle \vec{k} | \varphi(0) | \Omega \rangle$$

U is a unitary operator that boosts from $\vec{k}$ to 0

$$= e^{ik x} \underbrace{\langle \vec{k} | U^{-1}}_{\langle \vec{p}=0|} \underbrace{U \varphi(0) U^{-1}}_{\varphi(0)} \underbrace{U | \Omega \rangle}_{|\Omega\rangle}$$

$$= e^{ik x} \underbrace{\langle \vec{p}=0 | \varphi(0) | \Omega \rangle}_{\neq 1 \text{, function of } m^2}$$

unitary operator

boosts from $\vec{k}$ to 0

Fix: $\tilde{\varphi}(x) = N\varphi(x)$

Multiparticle states vs one particle states

$|\vec{p}, \sigma\rangle$

total 3-momentum

all other parameters

$$\langle \vec{p}, \sigma | \varphi(x) | \Omega \rangle = \langle \vec{p}, \sigma | e^{i\hat{P}\cdot x} \varphi(0) e^{-i\hat{P}\cdot x} | \Omega \rangle$$

$$= e^{ip\cdot x} \langle \vec{p}, \sigma | \varphi(0) | \Omega \rangle$$

$$\neq 0$$

Good enough to be able to distinguish

one + multiparticle states asymptotically $t \to \pm\infty$

Can show: one + multiparticle wavepackets

have zero overlap as $t \to \pm\infty$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3$$

have to shift + rescale

$$\mathcal{L} = \frac{1}{2}Z_\varphi\,\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}Z_m m^2\varphi^2 - \frac{Z_g}{3!}g\varphi^3 + Y\varphi$$

new normalization

new term

$$\langle\Omega|\varphi(x)|\Omega\rangle = 0$$
$$\langle\vec{k}|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$$

At leading order $Z_i = 1$, $Y = 0$