

Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu) - Lecture 3

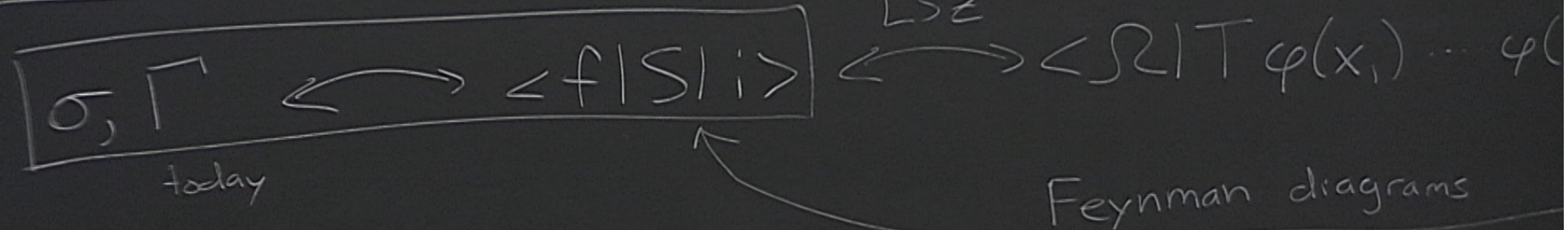
Speakers: Dan Wohns, Gang Xu

Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

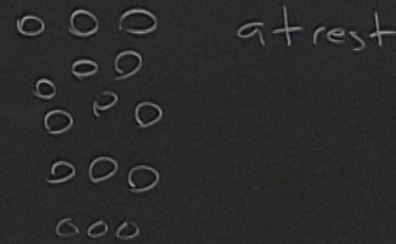
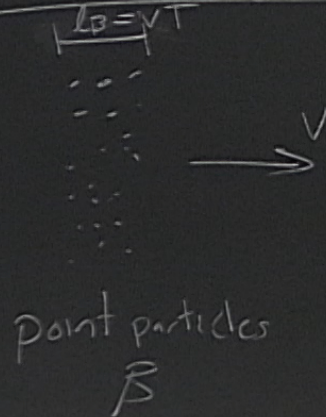
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Cross Sections + Decay Rates



Classical cross section



spherical particles with radius r of type α

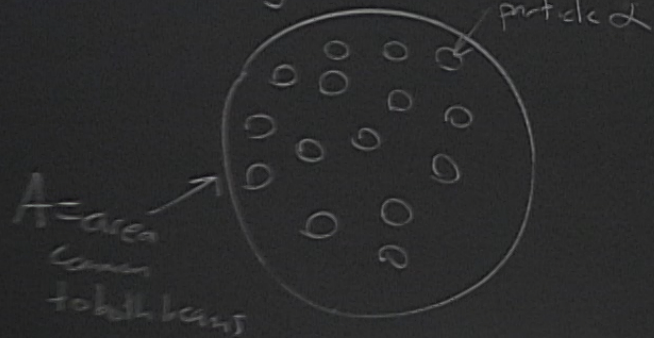
look also

$A = \text{area}$
common
to both beams

$$\varphi(x_1) \dots \varphi(x_n) |\Omega\rangle \xleftrightarrow{\text{int picture}} \langle 0 | T \varphi(x_1) \dots \varphi(x_m) | 0 \rangle \xleftrightarrow{\text{Wick}} \langle 0 | T \varphi(x_1) \dots \varphi(x_m) | 0 \rangle$$

grams

look along direction of motion



$$P = \frac{\sigma}{A} \rightarrow \pi r^2$$

$$P = \frac{\sigma v T}{\text{vol}}$$

int picture $\longleftrightarrow \langle 0 | T \varphi(x_1) \dots \varphi(x_m) | 0 \rangle \xleftrightarrow{\text{Wick}} \langle 0 | T \varphi(x_1) \varphi(x_2) | 0 \rangle$

$$[\varphi(x), \varphi(y)] = 0$$

definition added

$$P = \frac{\sigma}{A} \quad \nearrow \pi r^2$$

$$P = \frac{\sigma v T}{\text{vol}}$$

cross section

$$d\sigma \equiv dP \frac{\text{vol}}{vT}$$

$2 \rightarrow n$ scattering

$$dP = \frac{|\langle f | S | i \rangle|^2}{\langle i | i \rangle \langle f | f \rangle} \prod_{j=1}^n \left(\frac{d^3 p_j}{(2\pi)^3} (\text{vol}) \right) \quad \leftarrow \text{vol of space}$$

in periodic space with $\text{vol} = L^3$

$$\vec{p}_i = \frac{2\pi}{L} \vec{n}_i$$

$\uparrow$
vector of integers

$$\sum_{n_j} \rightarrow \int \frac{d^3 p_j}{(2\pi)^3} \text{vol}$$

$$S \equiv \underset{\substack{\uparrow \\ \text{free} \\ \text{theory}}}{1} + i \underset{\substack{\uparrow \\ \text{transfer} \\ \text{matrix}}}{\tilde{\mathcal{L}}}$$

$$\tilde{\mathcal{L}} = (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^n p_j - k_1 - k_2\right) \mathcal{M}$$

$$|\langle f | S | i \rangle|^2 = (2\pi)^8 \left(\delta^{(4)}(\sum p) \right)^2 \underbrace{|\langle f | \mathcal{M} | i \rangle|^2}$$

$$\delta^{(4)}(0) = \int d^4x e^{i0 \cdot x} = \text{vol} \cdot T \quad i\mathcal{M} \text{ if process is understood}$$

$\frac{d^3 p_j}{(2\pi)^3}$

ering

$$\frac{|1\rangle^2}{\langle f|f\rangle} \prod_{j=1}^n \left(\frac{d^3 p_j}{(2\pi)^3} \text{vol} \right)$$

← vol of space

periodic space with $\text{vol} = L^3$

$$\vec{p}_i = \frac{2\pi}{L} \vec{n}_i$$

← vector of integers

$$\sum_{n_j} \rightarrow \int \frac{d^3 p_j}{(2\pi)^3} \text{vol}$$

$$|i\rangle = |\vec{k}_1, \vec{k}_2\rangle$$

$$|f\rangle = |\vec{p}_1, \dots, \vec{p}_n\rangle$$

$$\langle \vec{k} | \vec{k} \rangle = (2\pi)^3 2E_{\vec{k}} \delta^3(\vec{k} - \vec{k}) = 2E_{\vec{k}} \cdot \text{vol}$$

$$\langle i | i \rangle = 2E_{\vec{k}_i} 2E_{\vec{k}_i} (\text{vol})^2$$

$$\langle f | f \rangle = \prod_{j=1}^n (2E_{\vec{k}_j} \cdot \text{vol})$$

$$dP = d\Phi \frac{|iM|^2 T}{2E_{\vec{k}_1} 2E_{\vec{k}_2} \text{vol}}$$

phase space $\rightarrow$

$$d\Phi = (2\pi)^4 \delta^{(4)}(\Sigma p) \prod_{j=1}^n \frac{d^3 p_j}{(2\pi)^3 2E_{p_j}}$$

$$|\vec{k}_1, \vec{k}_2\rangle$$

$$|\vec{p}_1, \dots, \vec{p}_n\rangle$$

$$S \equiv \underbrace{1}_{\text{free theory}} + i \underbrace{\tilde{\mathcal{L}}}_{\text{transfer matrix}}$$

$$\tilde{\mathcal{L}} = (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^n p_j - k_1 - k_2\right) \mathcal{M}$$

$$|\langle f | S | i \rangle|^2 = (2\pi)^8 \left(\delta^{(4)}(\sum p) \right)^2 \underbrace{|\langle f | \mathcal{M} | i \rangle|^2}_{\text{if process is understood}}$$

$$\delta^{(4)}(0) = \int d^4x e^{i0 \cdot x} = \text{vol} \cdot T \underset{\substack{\uparrow \\ \text{time periodic}}}{i\mathcal{M}}$$

$$\langle \vec{k} | \vec{k} \rangle = (2\pi)^3 2E_{\vec{k}} \delta^{(3)}(\vec{k} - \vec{k}) = 2E_{\vec{k}} \cdot \text{vol}$$

$$\langle 1 | 1 \rangle = 2E_{\vec{k}_1} 2E_{\vec{k}_2} (\text{vol})^2$$

$$\langle f | f \rangle = \prod_{j=1}^n (2E_{\vec{k}_j} \cdot \text{vol})$$

$$\Sigma_P = \sum_{j=1}^n p_j - k_1 - k_2$$

$$dP = d\Phi \frac{|M|^2 T}{2E_{\vec{k}_1} 2E_{\vec{k}_2} \text{vol}}$$

phase
space $\rightarrow$

$$d\Phi = (2\pi)^4 \delta^{(4)}(\Sigma_P) \prod_{j=1}^n \frac{d^3 p_j}{(2\pi)^3 2E_{p_j}}$$

Decay Rates

$$\Gamma_{\text{total}} = \frac{1}{\tau} = \sum_{\text{decay channels}} \Gamma_{\text{partial}}$$

$$d\Gamma_{\text{partial}} = \frac{dP}{T} = d\Phi \frac{|M|^2}{2E_{\vec{k}_1}}$$

if identical particles restrict $d\Phi$
or divide by symmetry factor

Phase space

If final state has two massless particles

$$d\Phi_2 = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \frac{d^3 p_1}{(2\pi)^3 2E_{\vec{p}_1}} \frac{d^3 p_2}{(2\pi)^3 2E_{\vec{p}_2}}$$

$$\text{COM: } \vec{k}_1 + \vec{k}_2 = 0 \quad k_1^0 + k_2^0 = E_{\text{CM}}$$

$$\int d\Phi_2 \frac{1}{(2\pi)^3 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{\text{CM}}) d^3 p_1 \Big|_{\vec{p}_1 = -\vec{p}_2}$$

Phase space

If final state has two massless particles

$$d\Phi_2 = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \frac{d^3 p_1}{(2\pi)^3 2E_{\vec{p}_1}} \frac{d^3 p_2}{(2\pi)^3 2E_{\vec{p}_2}}$$

$$\text{COM: } \vec{k}_1 + \vec{k}_2 = 0 \quad k_1^0 + k_2^0 = E_{\text{cm}}$$

$$\int d\Phi_2 \frac{1}{(2\pi)^3 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{\text{cm}}) d^3 p_1 \Big|_{\vec{p}_1 = -\vec{p}_2}$$

$$E_{\vec{p}_1} = |\vec{p}_1| = |\vec{p}_2| = E_{\vec{p}_2} \quad \text{massless}$$

$$d\Phi_2 = \int \frac{1}{16\pi^2 |\vec{p}_1|^2} |\vec{p}_1|^2 d|\vec{p}_1| d\Omega \delta(2|\vec{p}_1| - E_{cm})$$

$$\delta(f(x)) = \sum_{x_i} \frac{\delta(x-x_i)}{|f'(x_i)|}$$

$$= \int \frac{d\Omega}{32\pi^2}$$

$$\boxed{d\Phi_2 = \frac{d\Omega}{32\pi^2}}$$

$$\rightarrow \int d\Phi_2 f(p_1, p_2) = \int \frac{d\Omega}{32\pi^2} f(p_1, p_2) \Big|_{\Sigma p = \Sigma k}$$

Example:

$$\mathcal{L} = \underbrace{\partial_\mu \rho^* \partial^\mu \rho}_{\text{Scalar "nucleon"}} + \frac{1}{2} \underbrace{\partial_\mu \varphi \partial^\mu \varphi}_{\text{meson}} - g \rho^* \rho \varphi$$

$$\rho\rho \rightarrow \rho\rho$$

$$iM(\rho(k_1) + \rho(k_2) \rightarrow \rho(p_1) + \rho(p_2)) = (-ig)^2 \left(\frac{i}{(k_1 - p_1)^2 + i\varepsilon} + \frac{i}{(k_1 - p_2)^2 + i\varepsilon} \right)$$

$$d\sigma = \frac{1}{2} d\Phi_2 \frac{|M|^2}{2E_{k_1} 2E_{k_2} v_{\text{rel}}} \quad v=2$$

In COM:

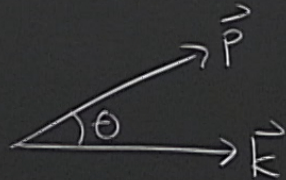
$$k_1 = (E, \vec{k})$$

$$k_2 = (E, -\vec{k})$$

$$p_1 = (E, \vec{p})$$

$$p_2 = (E, -\vec{p})$$

$$k_1^2 = p_1^2$$

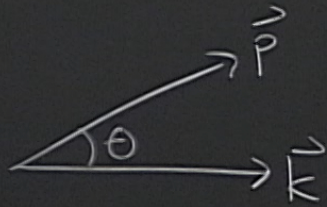


$$k_1 \cdot p_1 = E^2 - \vec{k} \cdot \vec{p} = E^2(1 - \cos\theta)$$

$$k_1 \cdot p_2 = E^2(1 + \cos\theta)$$

$$\frac{1}{2} M^2$$

$$2E_{k_1} 2E_{k_2}^v \quad v=2$$



$$k_1 \cdot p_1 = E^2 - \vec{k} \cdot \vec{p} = E^2(1 - \cos \theta)$$

$$k_1 \cdot p_2 = E^2(1 + \cos \theta)$$

$$\begin{aligned}
 |M|^2 &= g^4 \left(\frac{1}{(k_1 - p_1)^2} + \frac{1}{(k_1 - p_2)^2} \right)^2 \\
 &= g^4 \left(\frac{1}{-2k_1 \cdot p_1} + \frac{1}{-2k_1 \cdot p_2} \right)^2 \\
 &= \frac{g^4}{E^4} \left(\frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta} \right)^2 \\
 &= \frac{g^4}{4E^4} \left(\frac{2}{1 - \cos^2 \theta} \right)^2
 \end{aligned}$$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{512\pi^2} \left(\frac{1}{1 - \cos^2 \theta} \right)^2$$