Title: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Speakers: Dan Wohns, Gang Xu

Collection: PSI 2019/2020 - Quantum Field Theory (Wohns/Xu)

Date: October 07, 2019 - 9:00 AM

URL: http://pirsa.org/19100009

Quantum Field Theory I

Goal: Combine quantum mechanics + special relativity

Why fields?

In single particle QM with $H|\vec{p}\rangle = \sqrt{\vec{p}^{\,2}+m^2}\,|\vec{p}\rangle$

$$\langle \vec{x}|e^{-iHt}|\vec{x}=0\rangle \neq 0 \quad \text{for } |\vec{x}|>t$$

$$\boxed{\hbar = c = 1}$$

Resolution: particle number nonconservation

Why?

$\Delta p \gtrsim \frac{1}{L}$ (QM)

box size

$P \approx E \approx m$ (SR)

$\frac{1}{L} >> m \rightarrow \Delta E > 2m \rightarrow$ pair production

possible routes:

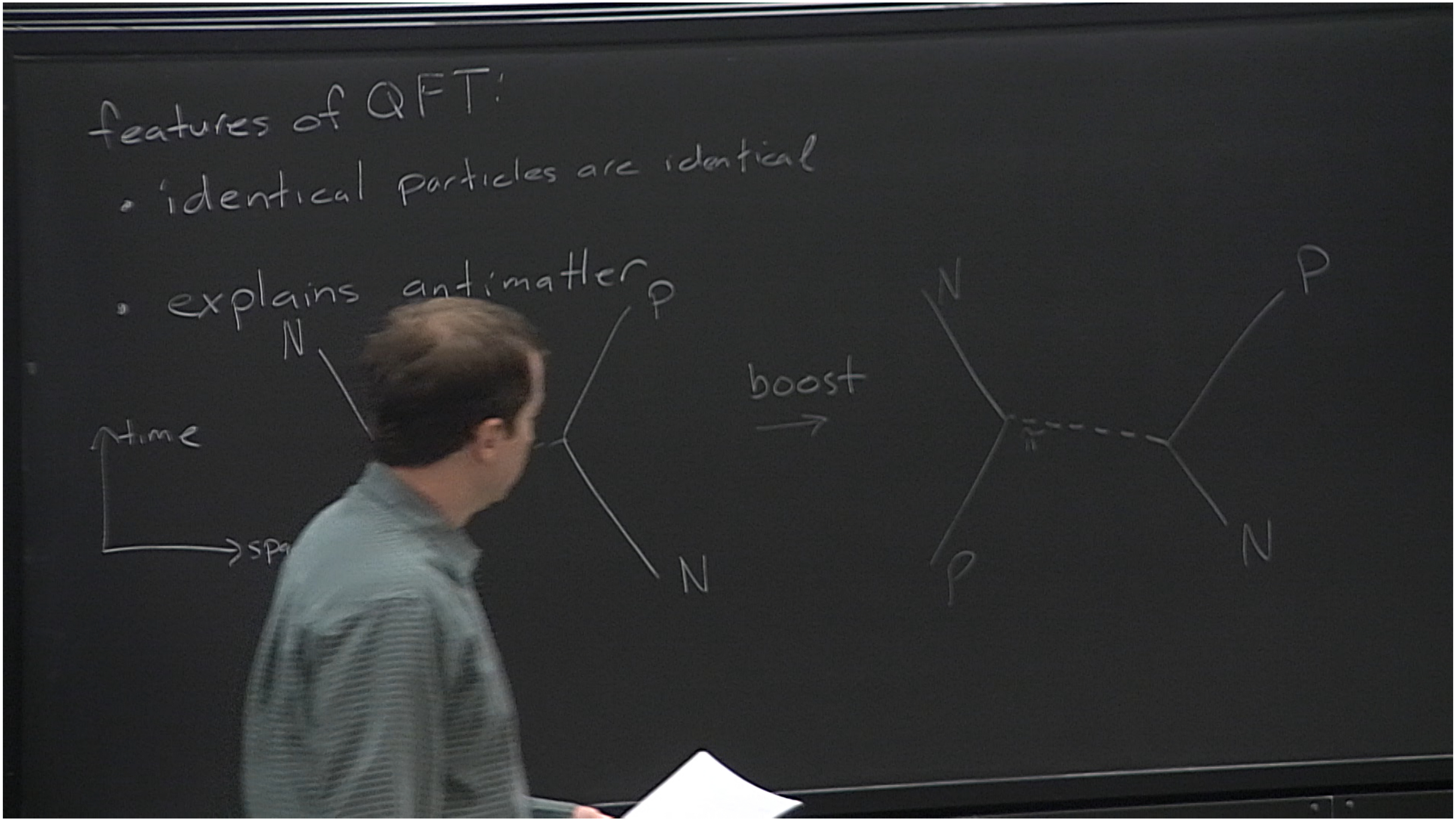
- QM
- SR
- locality/clustering

possible routes:

- QM
- SR
- locality/clustering

$$\underbrace{\text{QM},\ \text{SR},\ \text{locality/clustering}}\ \xrightarrow[]{\text{Weinberg}\ ?}\ \text{QFT}\ \xrightarrow{\text{classical limit}}\ \text{classical field theory}$$

$$\text{QFT}\ \xleftarrow{\text{quantization}}\ \text{classical field theory}$$


features of QFT:
• identical particles are identical
• explains antimatter
N
p
time
N
boost
N
P
P
N

- spin-statistics Theorem
  integer spin - Bose-Einstein
  half-integer spin - Fermi-Dirac statistics

Possible approaches

classical field theory

canonical quantization (QFT I)

path or functional integral (QFT II)

QFT

In QM $\hat{x}$ and t treated differently

operator   parameter

options: • $\hat{x}, \hat{t}$

– need time param for evol (Schrödinger eqn)

– use proper time τ or use $\underline{f(\tau)}$

– $\hat{X}^{\mu}(\tau)$ or $\hat{X}^{\mu}(\tau, \sigma)$

- x, t
  - simpler
  - position is <u>label</u> on fields

Goal: Calculate observables at leading order in perturbation theory

Outline: 1. Classical field theory
2. Free quantum field theory
3. Cross Section + Decay Rates $\Leftrightarrow i\mathcal{M}$ — Spin-0
(4+5) How to compute $i\mathcal{M}$ perturbatively
6. Feynman diagrams
7. Beyond leading order

el on fields

observables at leading order in perturbation theory

Classical field theory

Free quantum field theory

Cross Section + Decay Rates $\Leftrightarrow i\mathcal{M}$

How to compute $i\mathcal{M}$ perturbatively

Feynman diagrams

Beyond leading order

Spin-0

Gang - spin $\frac{1}{2}$, 1

Classical Field Theory

continuum limit

finite number of generalized coordinates

$\varphi(x) = \varphi(t, \vec{x})$	$q_a(t)$
$S = \int dt \int d^3\vec{x}\, \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$	$S = \int dt \sum_a L(q_a, \dot{q}_a)$
$\dfrac{\delta S}{\delta \varphi(x)} = 0$	$\dfrac{\delta S}{\delta q_a(t)} = 0$
$\pi(x) = \dfrac{\partial \mathcal{L}}{\partial \dot{\varphi}}$	$p^a = \dfrac{\partial \mathcal{L}}{\partial \dot{q}^a}$

Klein-Gordon Theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$$

$$= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\pi = \dot{\varphi}$$

$$\mathcal{H} = \pi(x)\dot{\varphi}(x) - \mathcal{L}(x)$$

$$= \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\right) = 0$$

$$-m^2\varphi - \partial_\mu\partial^\mu\varphi = 0$$

$$\boxed{(\partial^2 + m^2)\varphi = 0}$$

Noether's Theorem

continuous symmetry $\longrightarrow$ conserved current $\longrightarrow$ conserved charge

$$\delta \mathcal{L} = \partial_\mu F^\mu(\varphi) \qquad \partial_\mu j^\mu = 0 \qquad Q = \int d^3x\, j^0$$

under $\delta\varphi = Y(\varphi)$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} Y(\varphi) - F^\mu(\varphi)$$

Warning

QFT can have anomaly!

Soln of KG equation

$$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \tilde{\varphi}(t,\vec{k})$$

Soln of KG equation

$$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \tilde{\varphi}(t,\vec{k})$$

$$\ddot{\varphi} - \vec{\nabla}^2\varphi + m^2\varphi = 0$$

$$\ddot{\tilde{\varphi}} + (k^2+m^2)\tilde{\varphi} = 0$$

$$\tilde{\varphi}(t,\vec{k}) = A(\vec{k})e^{-iE_{\vec{k}}t} + B(\vec{k})e^{iE_{\vec{k}}t}$$

$$E_{\vec{k}} = \sqrt{\vec{k}^2+m^2}$$

$$\varphi = \varphi^*$$

$$\varphi^*(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}}\, \tilde{\varphi}^*(t, \vec{k})$$

$$\vec{k} \to -\vec{k}$$

$$\varphi^*(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \tilde{\varphi}^*(t, -\vec{k})$$

$$\tilde{\varphi}(t, \vec{k}) = \tilde{\varphi}^*(t, -\vec{k})$$

$$B(\vec{k}) = A^*(-\vec{k})$$

$$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[A(\vec{k})\, \underbrace{e^{-iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}_{e^{-ik\cdot x}} + \underbrace{A^*(-\vec{k})\, e^{iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}_{\vec{k}\to-\vec{k}} \right]$$

$$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[A(\vec{k})\, e^{-ik\cdot x} + A^*(\vec{k})\, e^{ik\cdot x} \right]\Bigg|_{k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2+m^2}}$$

$$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\, \delta(\underbrace{k^2 - m^2}_{E_{\vec{k}}^2 - \vec{k}^2 - m^2})\, \Theta(k^0) = \int \frac{d^3k}{(2\pi)^3 (2E_{\vec{k}})}$$

$$\vec{k} \to -\vec{k}$$

$$k^0 = E_{\vec{k}} = \sqrt{k^2 + m^2}$$

$$\boxed{\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3 (2E_{\vec{k}})} \left[a(\vec{k})\, e^{-ik\cdot x} + a^*(\vec{k})\, e^{ik\cdot x} \right]}$$

$$a(\vec{k}) = A(\vec{k}) \cdot 2E_{\vec{k}}$$