Title: PSI 2019/2020 - Statistical Mechanics (Vieira)

Speakers: Pedro Vieira

Collection: PSI 2019/2020 - Statistical Mechanics (Vieira)

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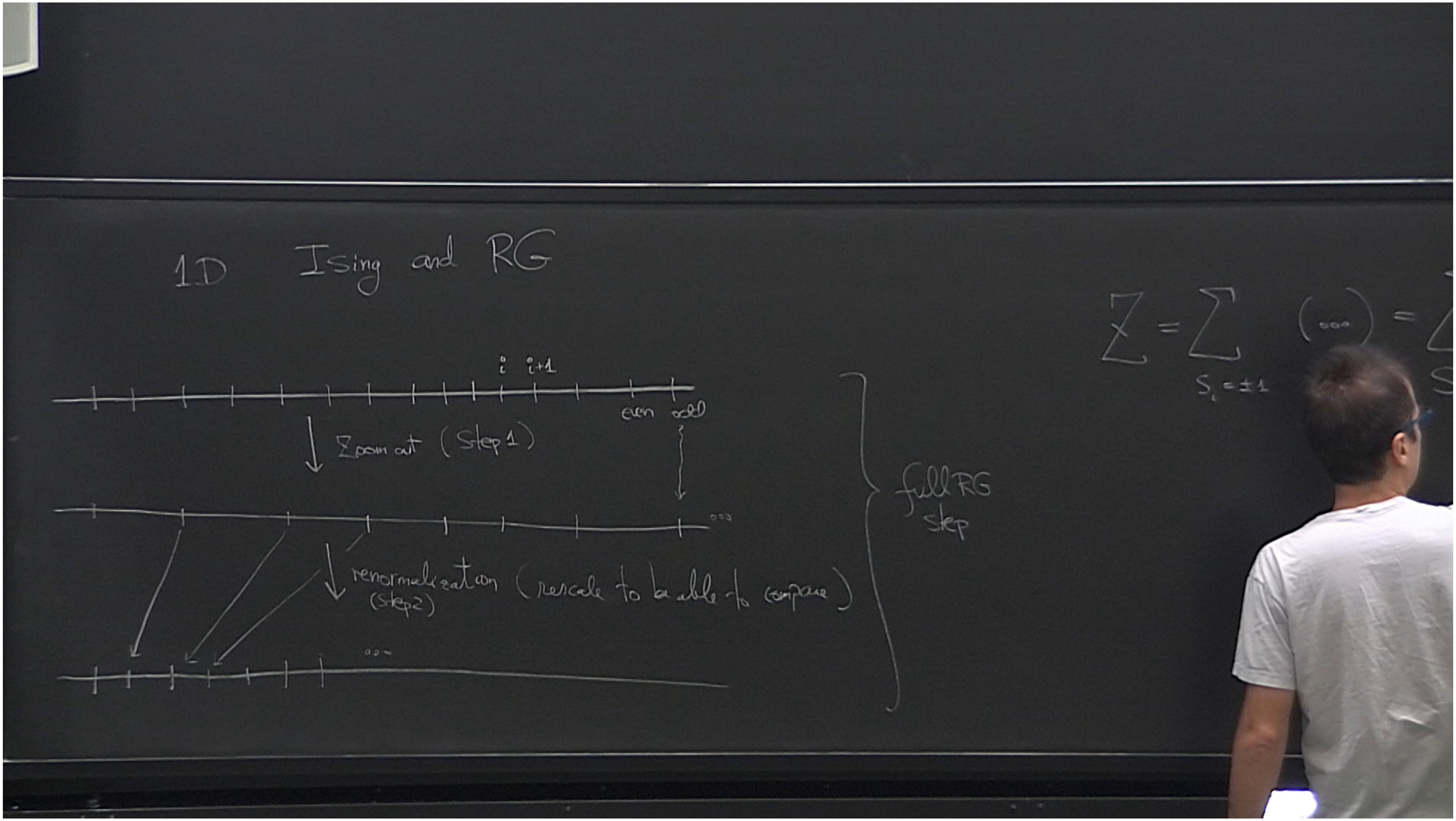
$$Z^{1D}_{\text{Ising}} = \sum_{\{S_i = \pm 1\}} \exp\left(J \sum_i S_i S_{i+1} + h \sum_i S_i \right) =$$

$h = 0$ first

$$\langle S_i \, S_{i+r} \rangle = \frac{1}{Z} \sum \exp(\cdots)\, S_i \, S_{i+r} =$$


1D Ising and RG
i i+1
even odd
Zoom out (Step 1)
renormalization (rescale to be able to compare)
(Step 2)
full RG step
Z = Σ (...) =
S_i = ±1

$$Z = \sum_{S_i=\pm 1} (\dots) = \sum_{S_i \text{ odd}} \Big[\underbrace{\sum_{S_i \text{ even}} \dots}_{\dots} \Big]$$

$h = 0$

$$\sum_{S_2=\pm 1} e^{JS_1S_2 + JS_2S_3} = 2\cosh\big(J(S_1+S_3)\big)$$

$$Z = \sum_{S_i = \pm 1} (\dots) = \sum_{S_i \text{ odd}} \underbrace{\left[\sum_{S_i \text{ even}} \dots \right]}_{\dots}$$

$h = 0$

$$\sum_{S_2 = \pm 1} e^{J S_1 S_2 + J S_2 S_3} = 2 \cosh\left(J(S_1 + S_3)\right) = A e^{B S_1 S_3}$$

boring normalization

effective J after integrating out S_2

$B = J'$

$$Z = \sum_{S_i = \pm 1} (\dots) = \sum_{S_{i\,\mathrm{odd}}} \underbrace{\left[\sum_{S_{i\,\mathrm{even}}} \dots \right]}_{\dots} = \mathcal{N} \sum_{S_{i\,\mathrm{odd}}} e^{J' S_i S_{i+2}}$$

$h = 0$

$$\sum_{S_2 = \pm 1} e^{J S_1 S_2 + J S_2 S_3} = 2 \cosh\left(J(S_1 + S_3)\right) = A e^{B S_1 S_3}$$

boring normalization

effective J after integrating out S_2

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$$Z = \sum_{S_i=\pm 1} (\ldots) = \sum_{S_{i\,\text{odd}}} \Big[\underbrace{\sum_{S_{i\,\text{even}}} \ldots}_{\ldots} \Big] = \mathcal{N} \sum_{S_{i\,\text{odd}}} e^{J' S_i S_{i+2}}$$

$$\begin{cases} 2 & \text{if } S_1 S_3 = -1 \\ 2\cosh 2\beta & \text{if } S_1 S_3 = +1 \end{cases}$$

$h = 0$

$$\sum_{S_2=\pm 1} e^{J S_1 S_2 + J S_2 S_3} = 2\cosh\left(J(S_1+S_3)\right) = A e^{B S_1 S_3}$$

boring normalization

effective J after integrating out S_2

$B = J'$

renormalization (rescale to be able to compare)
(step2)

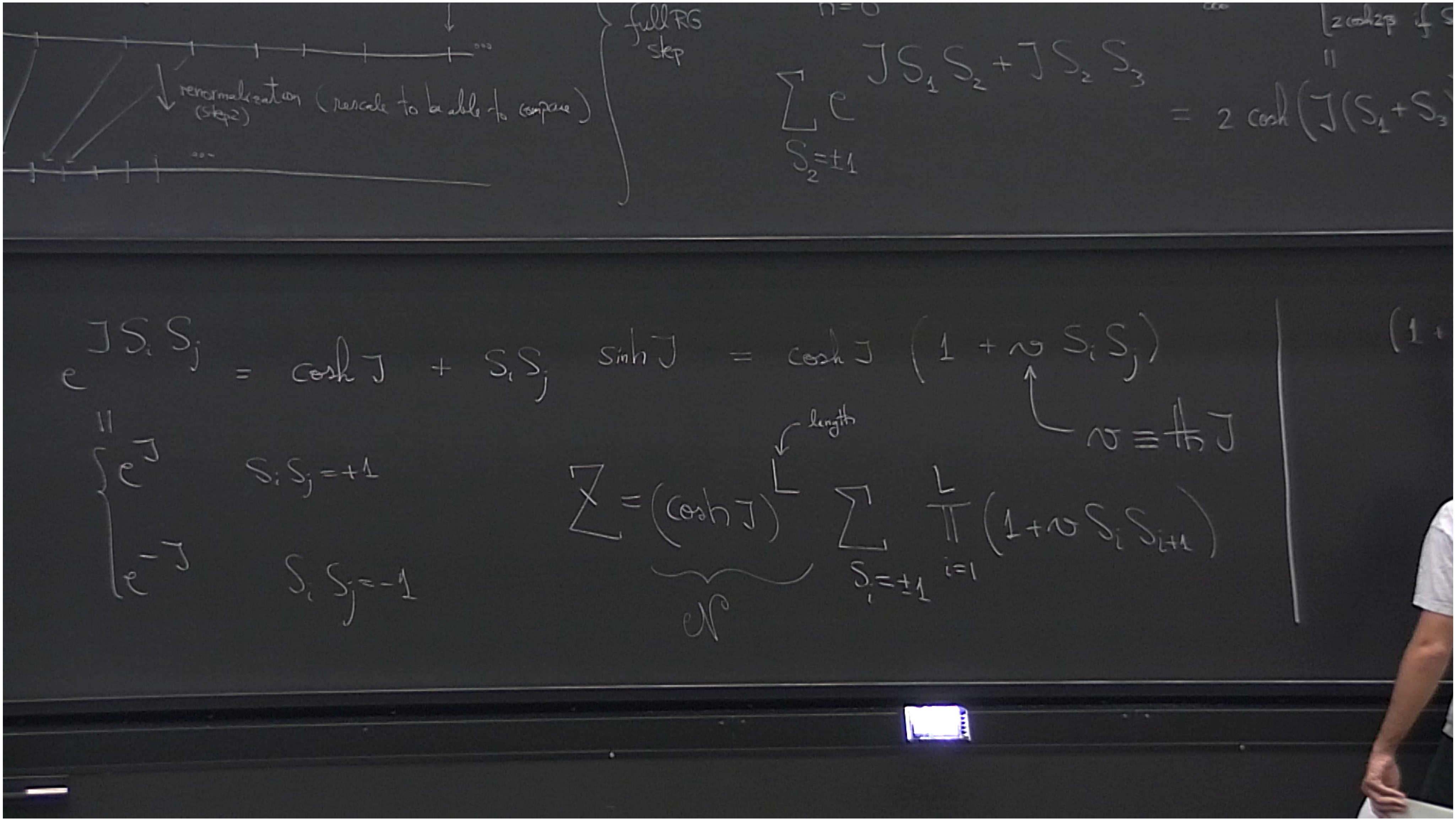
...

e

$S_2 = \pm 1$

$$e^{J S_i S_j} = \cosh J + S_i S_j \sinh J$$

$$= \begin{cases} e^{J} & S_i S_j = +1 \\ e^{-J} & S_i S_j = -1 \end{cases}$$



full RG step
renormalization (rescale to be able to compare)
(step2)
$\sum_{S_2=\pm 1} e^{JS_1S_2+JS_2S_3} = 2\cosh(J(S_1+S_3$
$2\cosh 2\beta$ if S
$e^{JS_iS_j} = \cosh J + S_iS_j \sinh J = \cosh J\,(1 + v\,S_iS_j)$
$v \equiv \mathrm{th}\, J$
$= \begin{cases} e^{J} & S_iS_j = +1 \\ e^{-J} & S_iS_j = -1 \end{cases}$
length
$Z = \underbrace{(\cosh J)^{L}}_{\mathcal{N}} \sum_{S_i=\pm 1} \prod_{i=1}^{L} (1 + v\,S_iS_{i+1})$

$B = J$

$S_j)$

$v \equiv \operatorname{th} J$

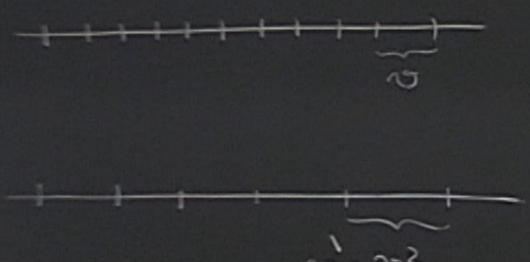
$S_i S_{i+1})$

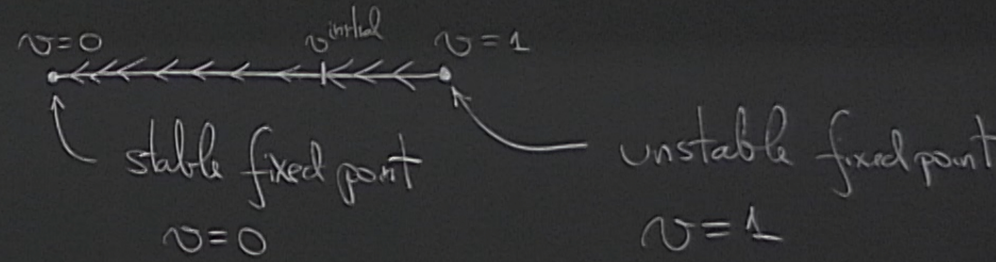
$$\sum_{S_2} (1+v S_1 S_2)(1+v S_2 S_3)$$

$$\| \qquad \sum S_2 = 0, \quad \sum 1 = \sum \underbrace{S_2^2}_{=1} = 2$$

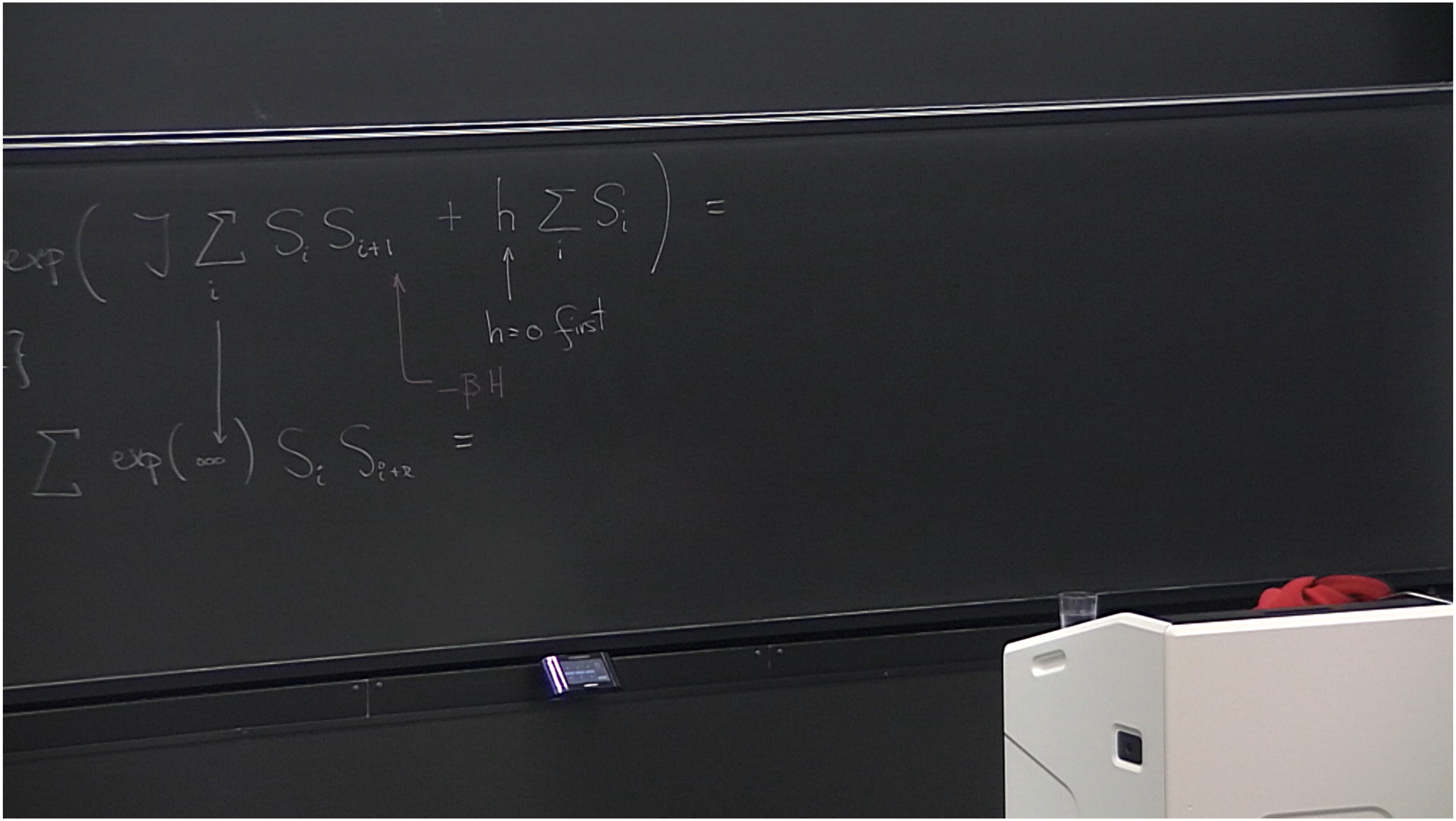
$$2\left(1+v^2 S_1 S_3\right)$$

e^{-J} $\quad S_i S_j = -1$ $\quad \sum \underbrace{(\cosh J)}_{\mathcal{N}} \ldots \quad \sum_{S_i = \pm 1} \prod_{i=1} (1 + v$





$$v = \operatorname{th} J \in [0, 1] \quad , \quad v' < v$$



$\exp\left(J \sum_i S_i S_{i+1} + h \sum_i S_i \right) =$
h=0 first
$-\beta H$
$\sum \exp(\cdots) S_i S_{i+R} =$

$+ v S_i S_{i+1})$

$$2\left(1 + \underbrace{v^2}_{v' = v^2} S_1 S_3\right)$$

$$v = \operatorname{th} \underbrace{\beta J_{\text{spins}}}_{J}$$

pont

Z

$\langle S$

$$\begin{cases} e^{J} & S_i S_j = +1 \\ e^{-J} & S_i S_j = -1 \end{cases}$$

$v \equiv \operatorname{th} J$

$$Z = \underbrace{(\cosh J)^{L}}_{\mathcal{N}} \sum_{S_i = \pm 1} \prod_{i=1}^{L} \left(1 + v\, S_i S_{i+1}\right)$$

v

$v' = v^2$

$v=0$ $\quad v_{\text{initial}}$ $\quad v=1$

stable fixed point
$v = 0$
$(\beta = 0 \text{ or } T = \infty)$
paramagnet

unstable fixed point
$v = 1$
$(\beta = \infty ; T = 0)$
ferromagnetic, phisicaly inaccessible.

$v = \operatorname{th} J \in [0, 1]$, $v' < v$

$$\sum_{S_i=\pm 1} \prod_{i=1} \left(1+v S_i S_{i+1}\right)$$

$$2\left(1+\underbrace{v^2}_{v'=v^2} S_1 S_3\right)$$

Conclusion: no order

$$v = \tanh \underbrace{\beta J_{spins}}_{J}$$

unstable fixed point

$v=1$

$(\beta=\infty, T=0)$

ferromagnetic, phisicaly inaccesible.

Conclusion: no order

1

— unstable fixedpoint

$v = 1$

$(\beta = \infty, T = 0)$

ferromagnetic, phisicaly inassecible.

$$v = \operatorname{th} \underbrace{\beta J_{spins}}_{J}$$

correlation length   typical length scale

$$\boxed{\xi(v) = 2\,\xi(v^2)}$$

Some # of lattice sites

$v = \tanh J \in [0,1]$, $v' < v$ paramagnet ferromagnetic , P

$$\Rightarrow \xi = -\frac{K}{\log v}$$

finite for any $v \in [0,1[$

diverges only for $v \to 1^-$

$$\simeq e^{2J^{\text{spins}}/T}$$

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$$\simeq e^{2J^{\text{spins}}/T}$$

$$\simeq e^{2J}$$

ferromagnetic, phisicaly inassecible.

$\left[\xi(v) = 2\,\xi(v^2)\right]$

Some # of lattice sites

no order. Why? $\longleftarrow$ $\exists$ topological excitations $\equiv$ Kinks

$\uparrow\uparrow\uparrow\uparrow \,\vdots\, \downarrow\downarrow\downarrow\downarrow\downarrow \,\vdots\, \uparrow\uparrow\uparrow\uparrow \ldots$

Kink anti-Kink

ferromagnetic, phisicaly inaccesible.

$$\left[\xi(v) = 2\,\xi(v^2)\right]$$

Some # of lattice sites

no order. Why? $\longleftarrow$ $\exists$ topological excitations $\equiv$ Kinks

$\uparrow\uparrow\uparrow\uparrow \;\vdots\; \downarrow\downarrow\downarrow\downarrow\downarrow \;\vdots\; \uparrow\uparrow\uparrow\uparrow \ldots$

Kink   anti-Kink

$E = 2J$

claim: Kinks destroy order.

(periodic) n Kinks, n anti-Kinks

ferromagnetic, phisicaly inaccesible.

$$\left[\xi(v) = 2\,\xi(v^2)\right]$$

Some # of lattice sites

no order. Why? $\longleftarrow$ $\exists$ topological excitations $\equiv$ Kinks

$\uparrow\uparrow\uparrow\uparrow \,\vdots\, \downarrow\downarrow\downarrow\downarrow\downarrow \,\vdots\, \uparrow\uparrow\uparrow\uparrow \ldots$

Kink　　　anti-Kink

claim: Kinks destroy order.

(periodic) n Kinks, n anti-Kinks

$$E = 2J = E(\uparrow \vdots \downarrow) - E(\uparrow \vdots \uparrow)$$

ferromagnetic, phisicaly inassecible.

$$\boxed{\xi(v) = 2\,\xi(v^2)}$$

ξ: Some # of lattice sites

no order. Why? $\longleftarrow$ $\exists$ topological excitations $\equiv$ Kinks

$\uparrow\uparrow\uparrow\uparrow \;\vdots\; \downarrow\downarrow\downarrow\downarrow\downarrow \;\vdots\; \uparrow\uparrow\uparrow\uparrow \dots$

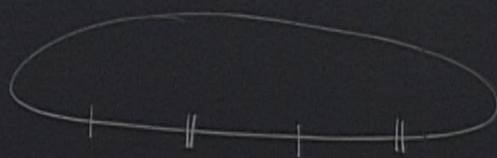
Kink — anti-Kink

claim: Kinks destroy order.

(periodic) n Kinks, n anti-Kinks

$$\boxed{E = 4nJ}$$

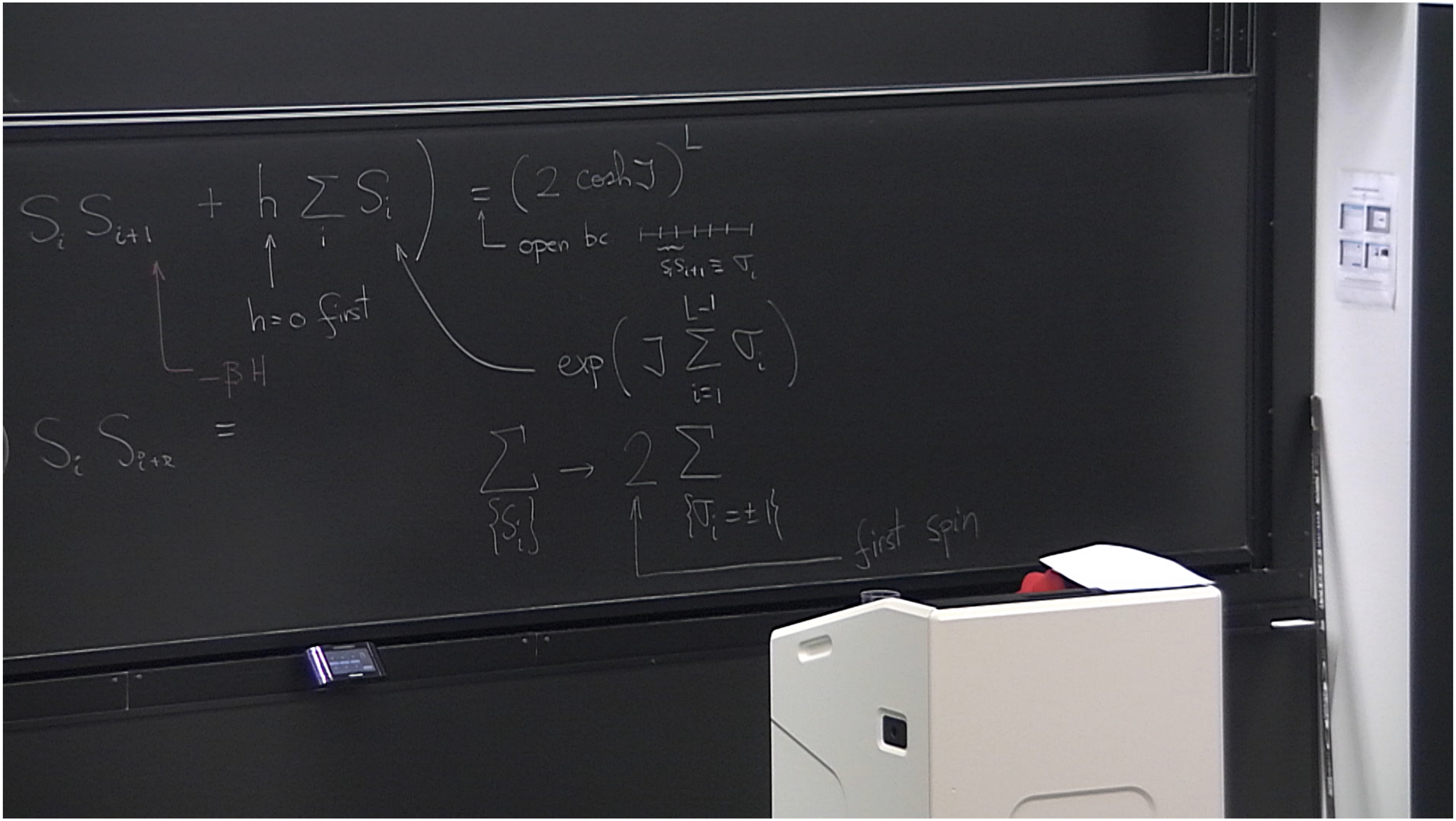
$$E = 2J = E(\uparrow\vdots\downarrow) - E(\uparrow\vdots\uparrow)$$

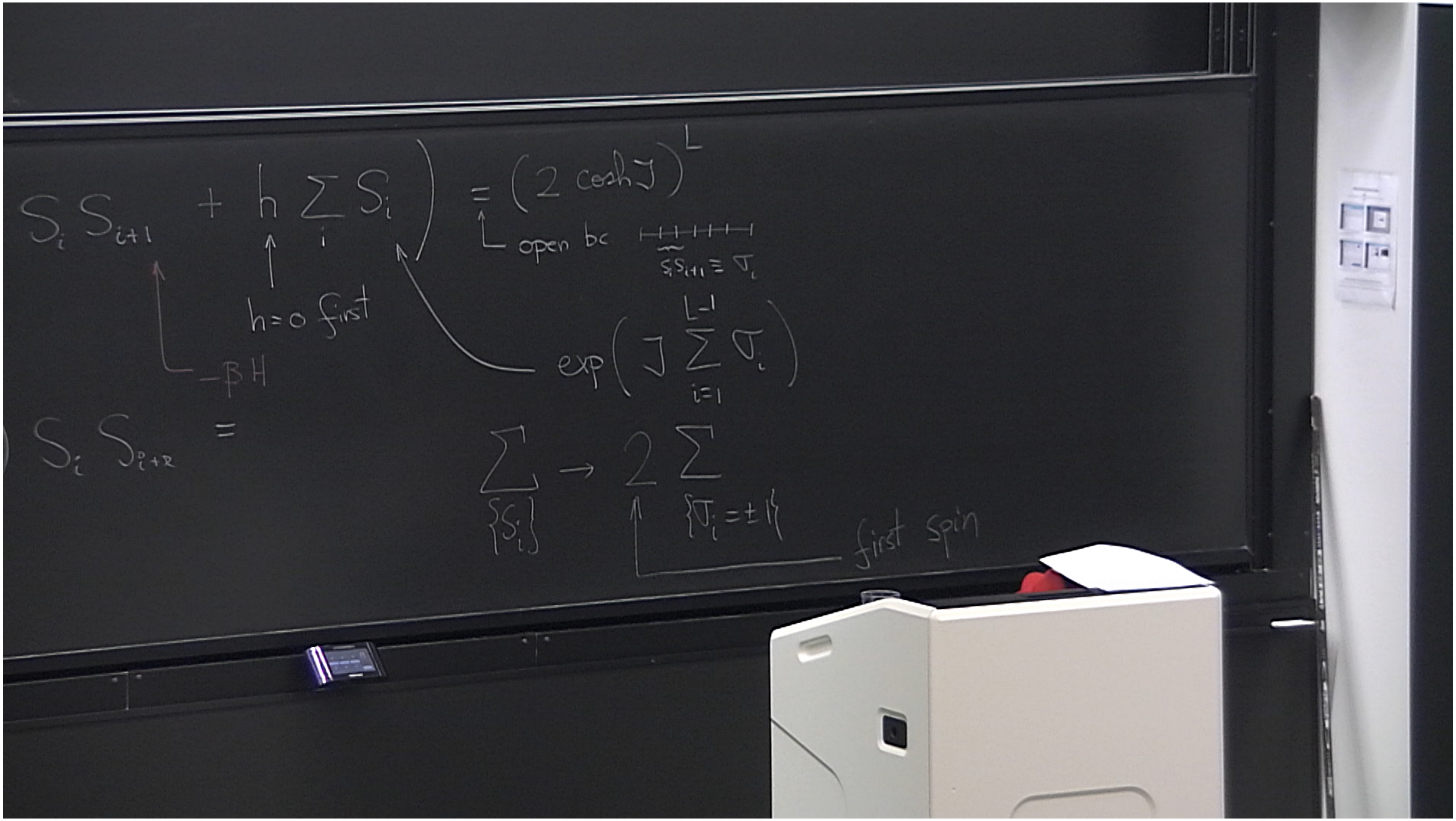


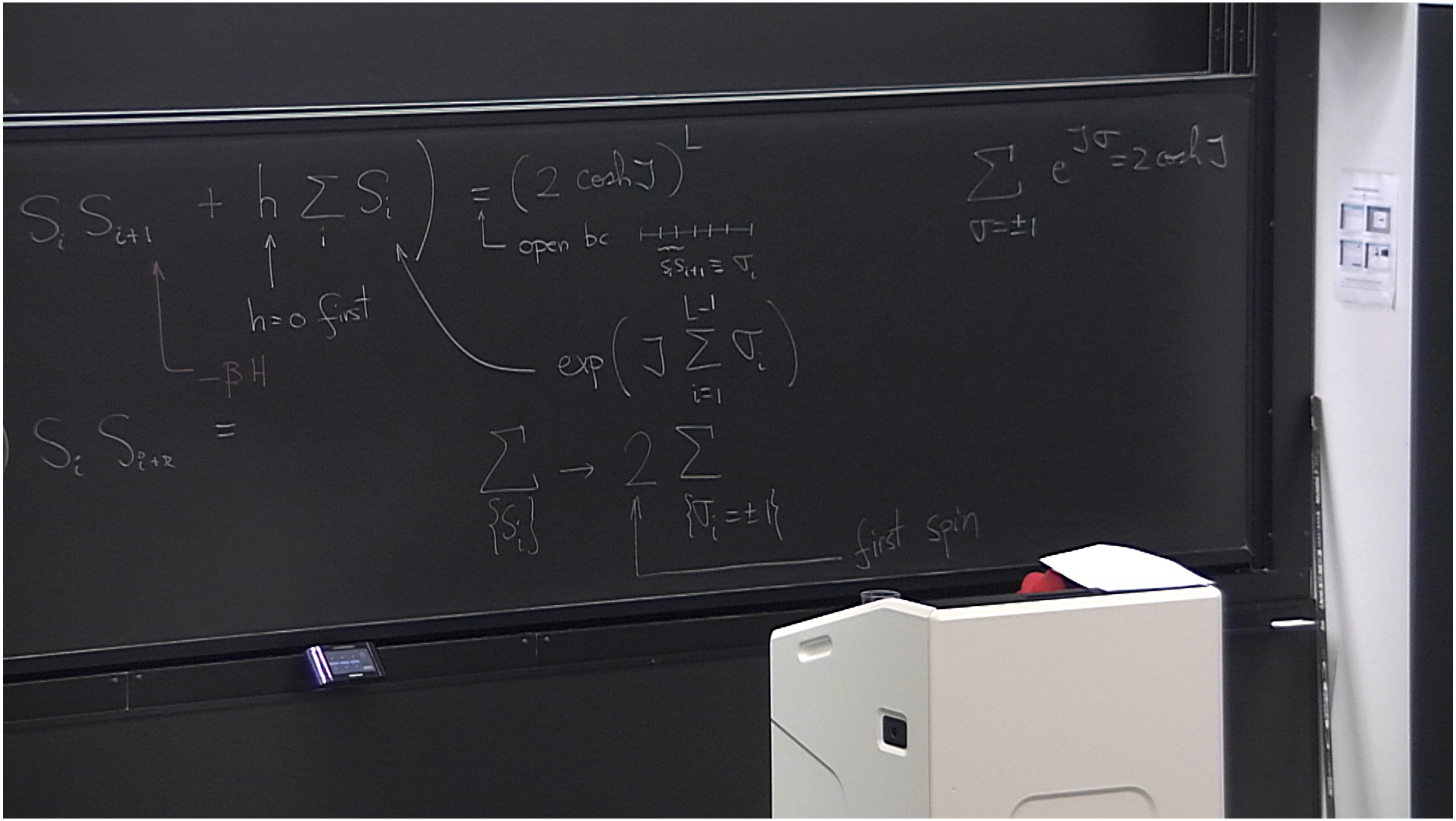
$$\Omega = \binom{L}{2n}$$

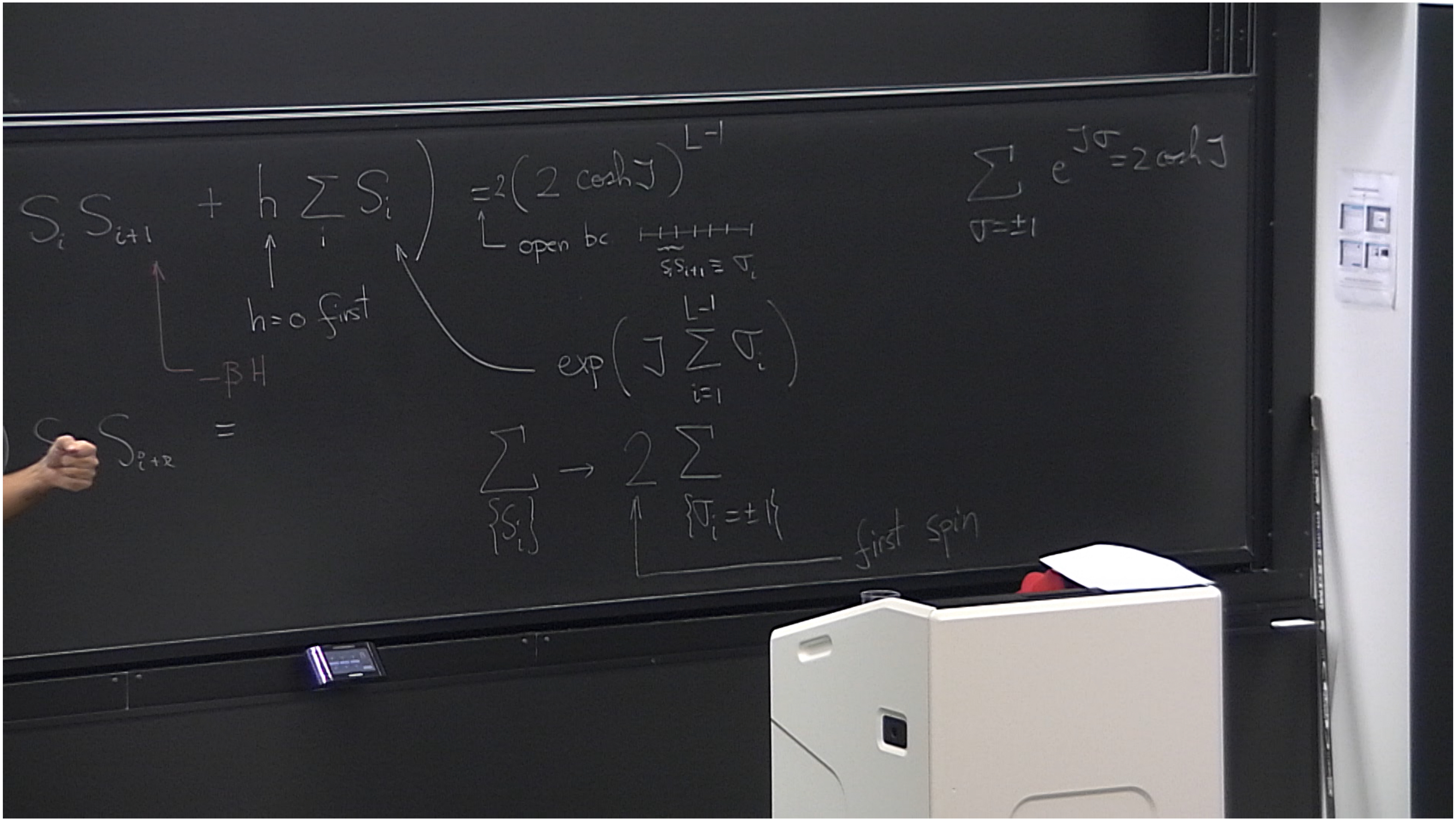
$$\boxed{S = \log \binom{L}{2n} \underset{\substack{\uparrow \\ \text{stirling}}}{\simeq} L \log L - 2n \log 2n - (L-2n) \log (L-2n)}$$

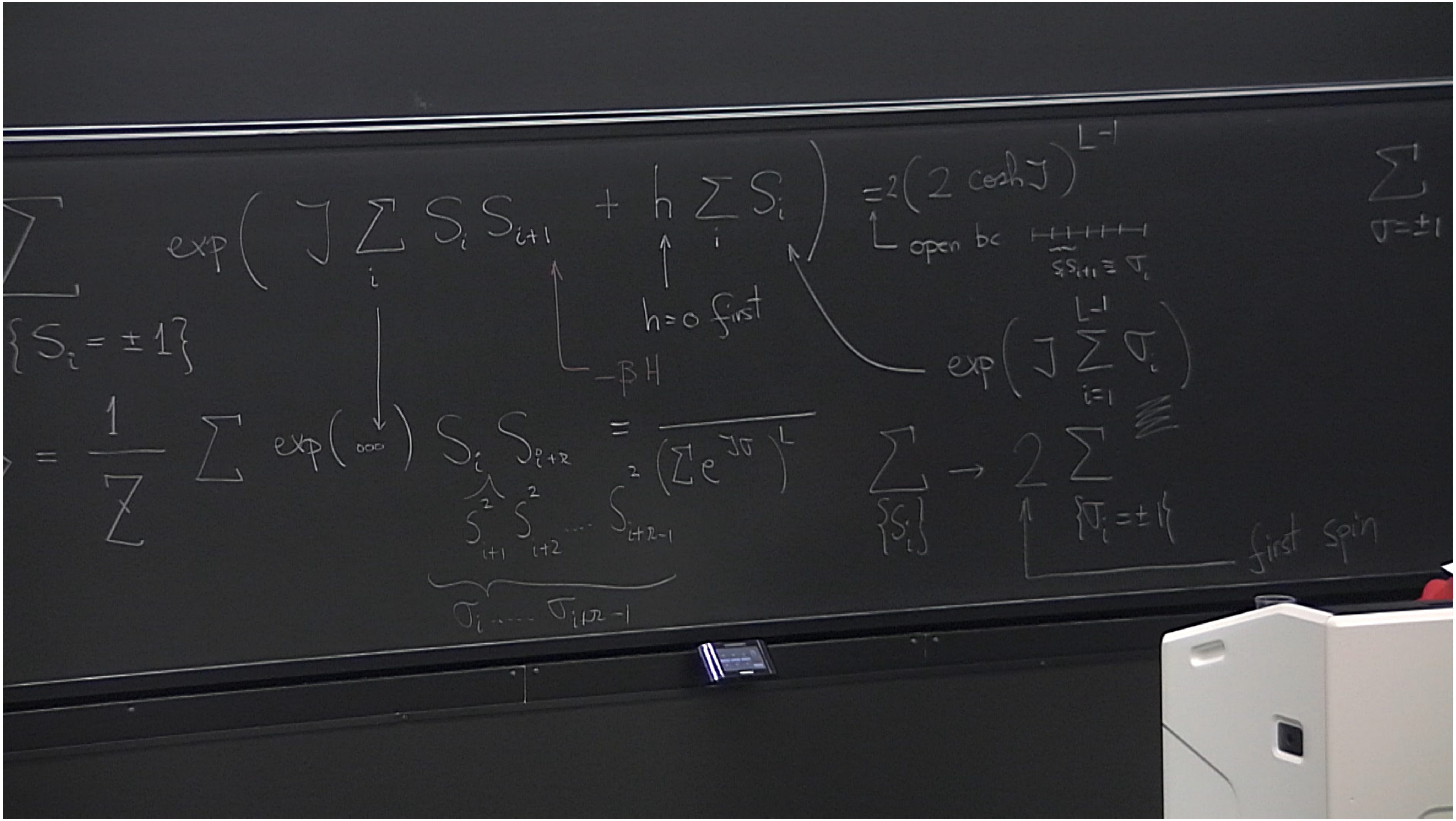

F = E - TS = F(n)
S = log(L 2n) ≃ L log L - 2n log 2n - (L-2n) log (L-2n)
stirling
stable fixed point
unstable fixed point
v = th J ∈ [0,1]
paramagnet
ferromagnetic , physically inaccessible
correlation lengths
typical lengths
same # of lattice sites
finite for any v ∈ [0,1[
no order . Why?
∃ topological excitations ≡ Kinks
Kink
anti-Kink
(periodic)
claim Kinks destroy order
n Kinks , n anti-Kinks
E = 4nJ


$S_i S_{i+1} + h \sum_i S_i \Big) = (2\cosh J)^L$
open bc
$s_i s_{i+1} \equiv \sigma_i$
h = 0 first
$-\beta H$
$\exp\left(J \sum_{i=1}^{L-1} \sigma_i \right)$
$S_i S_{i+2} =$
$\sum_{\{S_i\}} \to 2 \sum_{\{\sigma_i = \pm 1\}}$
first spin

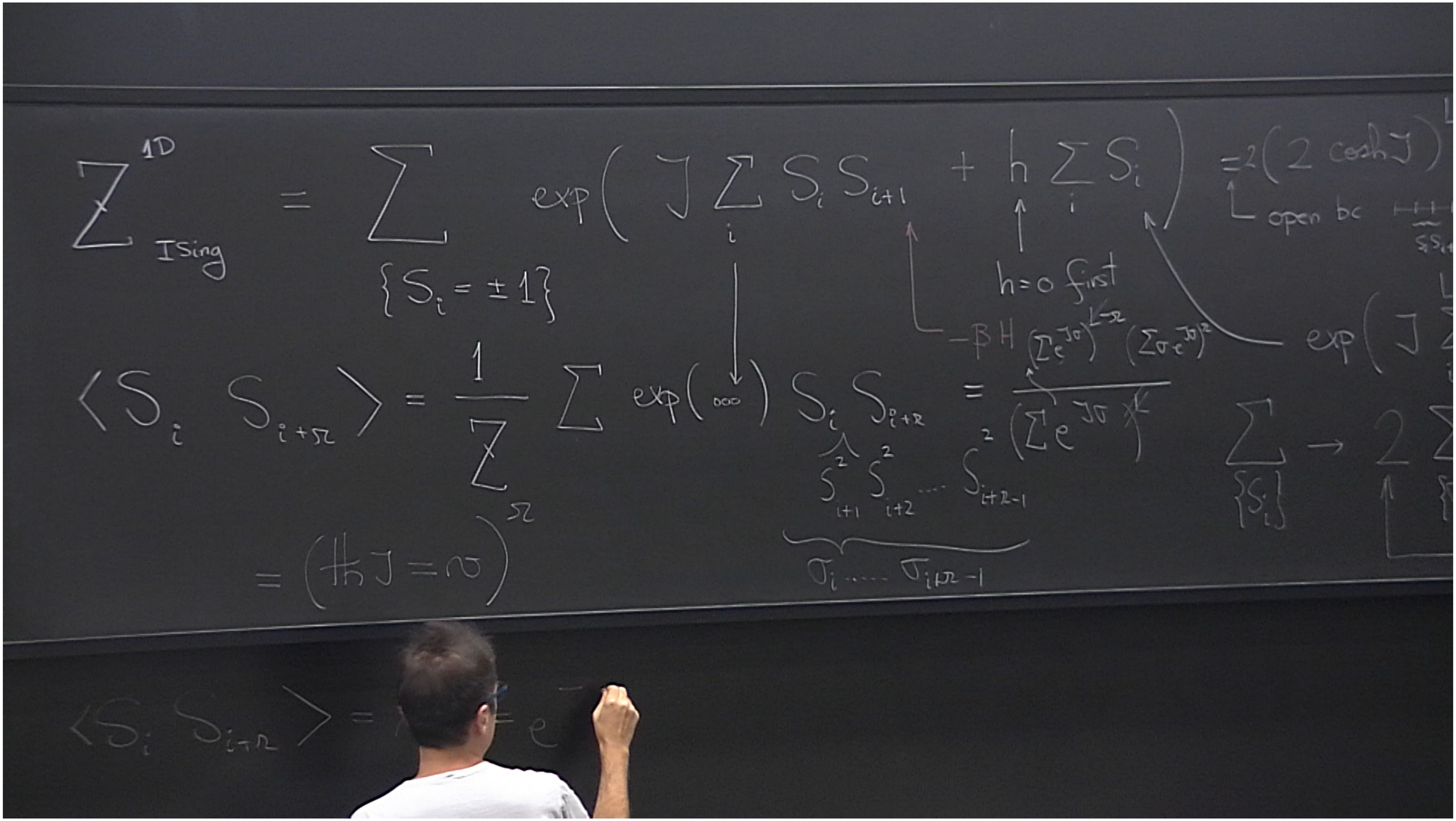

$S_i S_{i+1} + h \sum_i S_i$
$= (2 \cosh J)^L$
open bc
$s_i s_{i+1} \equiv \sigma_i$
h = 0 first
$-\beta H$
$\exp\left(J \sum_{i=1}^{L-1} \sigma_i \right)$
$S_i S_{i+R} =$
$\sum_{\{S_i\}} \to 2 \sum_{\{\sigma_i = \pm 1\}}$
first spin


$S_i S_{i+1} + h \sum_i S_i) = (2 \cosh J)^L$
open bc
$s_i s_{i+1} \equiv \sigma_i$
h=0 first
$-\beta H$
$\exp\left(J \sum_{i=1}^{L-1} \sigma_i \right)$
$\sum_{\sigma=\pm 1} e^{J\sigma} = 2 \cosh J$
$S_i S_{i+R} =$
$\sum_{\{S_i\}} \to 2 \sum_{\{\sigma_i = \pm 1\}}$
first spin


S_i S_{i+1} + h Σ_i S_i) =2(2 cosh J)^{L-1}
open bc
S_i S_{i+1} ≡ σ_i
h=0 first
−βH
exp(J Σ_{i=1}^{L-1} σ_i)
Σ_{σ=±1} e^{Jσ} = 2 cosh J
S_i S_{i+R} =
Σ_{\{S_i\}} → 2 Σ_{\{σ_i=±1\}}
first spin



$\sum_{\{S_i=\pm1\}} \exp\left(J \sum_i S_i S_{i+1} + h \sum_i S_i \right) = 2(2\cosh J)^{L-1}$
open bc
$S_i S_{i+1} \equiv \sigma_i$
$-\beta H$
h=0 first
$\exp\left(J \sum_{i=1}^{L-1} \sigma_i \right)$
$\sum_{\sigma=\pm1}$
$= \frac{1}{Z} \sum \exp(\cdots) S_i S_{i+r} = \frac{}{(\sum e^{J\sigma})^{L}}$
$S_{i+1}^2 S_{i+2}^2 \cdots S_{i+r-1}^2$
$\sigma_i \cdots \sigma_{i+r-1}$
$\sum_{\{S_i\}} \rightarrow 2 \sum_{\{\sigma_i=\pm1\}}$
first spin



$Z^{1D}_{Ising} = \sum_{\{S_i = \pm 1\}} \exp\left(J \sum_i S_i S_{i+1} + h \sum_i S_i \right) = 2(2\cosh J)^{L}$
open bc
h=0 first
$-\beta H$
$\langle S_i \, S_{i+r} \rangle = \frac{1}{Z} \sum \exp(\ldots) S_i S_{i+r} =$
$= (\tanh J = \infty)^{r}$
$\sigma_i \ldots \sigma_{i+r-1}$
$\langle S_i \, S_{i+r} \rangle = e^{-}$

$$= (\operatorname{th} J = v)^{r}$$

$$\underbrace{S_{i+1}^2 \, S_{i+2}^2 \cdots S_{i+r-1}^2}_{\sigma_i \cdots\cdots \sigma_{i+r-1}}$$

$$\sum_{\{S_i\}} \to 2 \sum_{\{\sigma_i =}$$

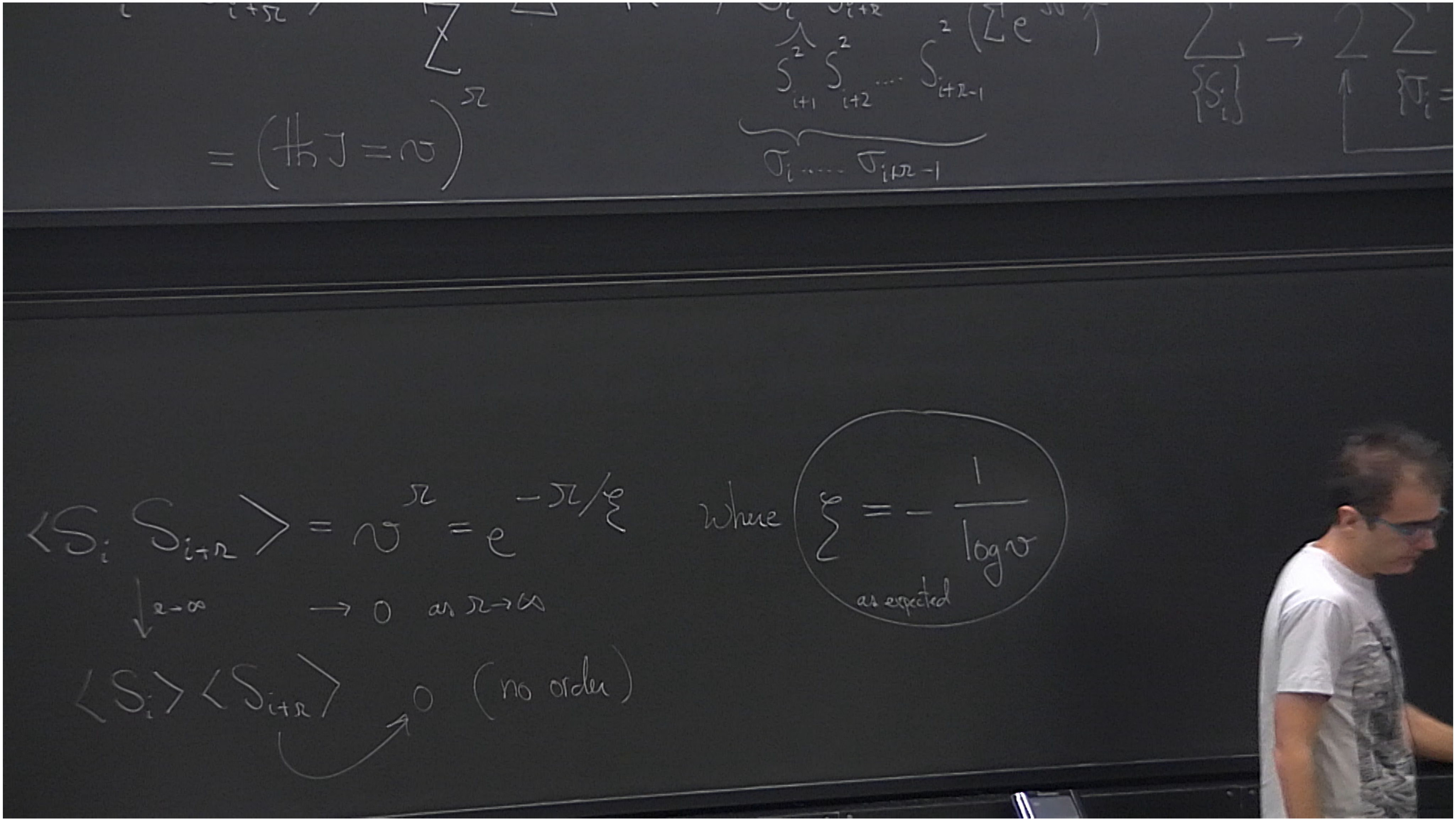
$$\langle S_i \, S_{i+r} \rangle = v^{r} = e^{-r/\xi} \quad \text{where} \quad \xi = -\frac{1}{\log v}$$

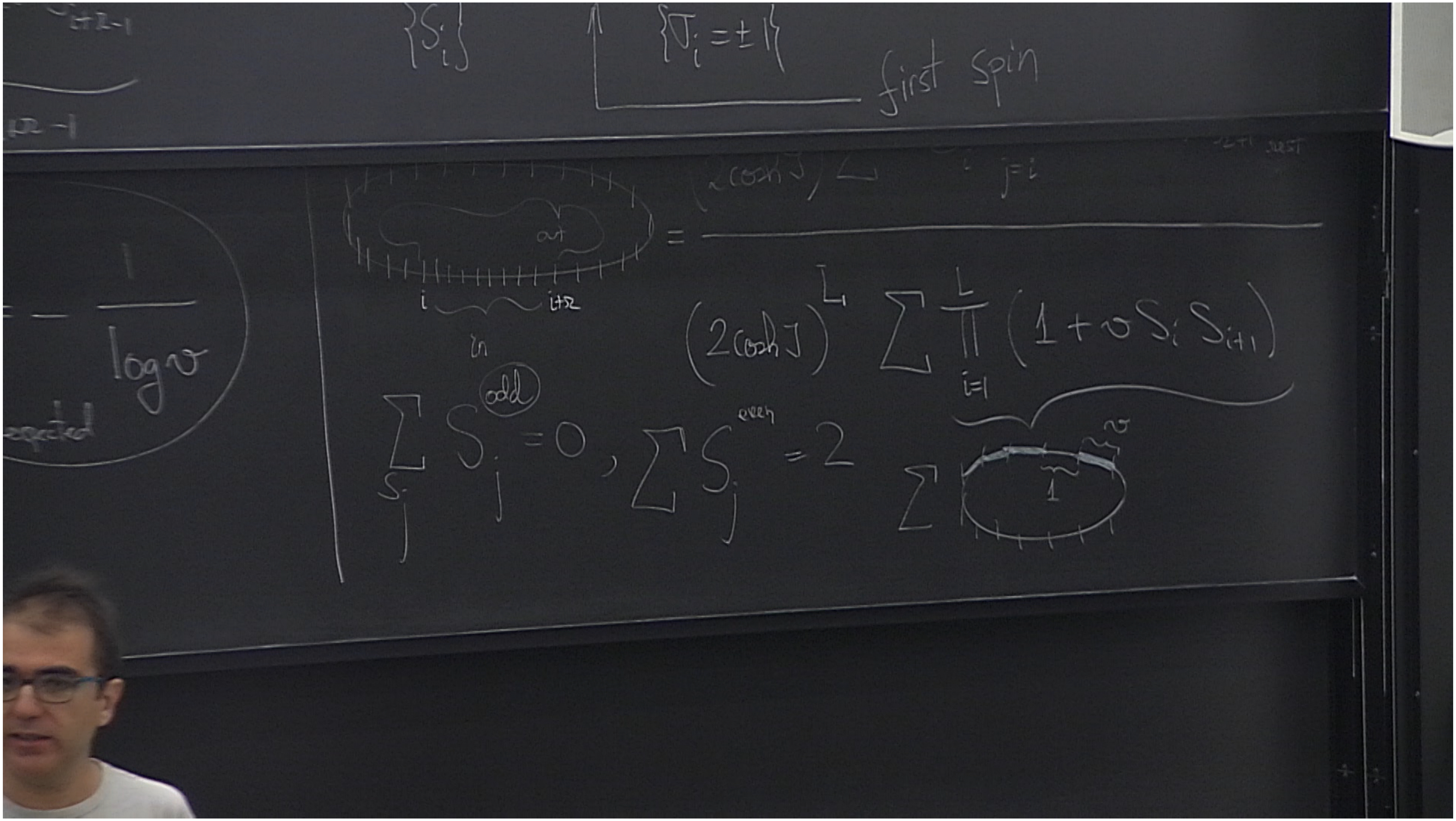
$$= \left(\mathrm{tr}\, T = v\right)^{r}$$

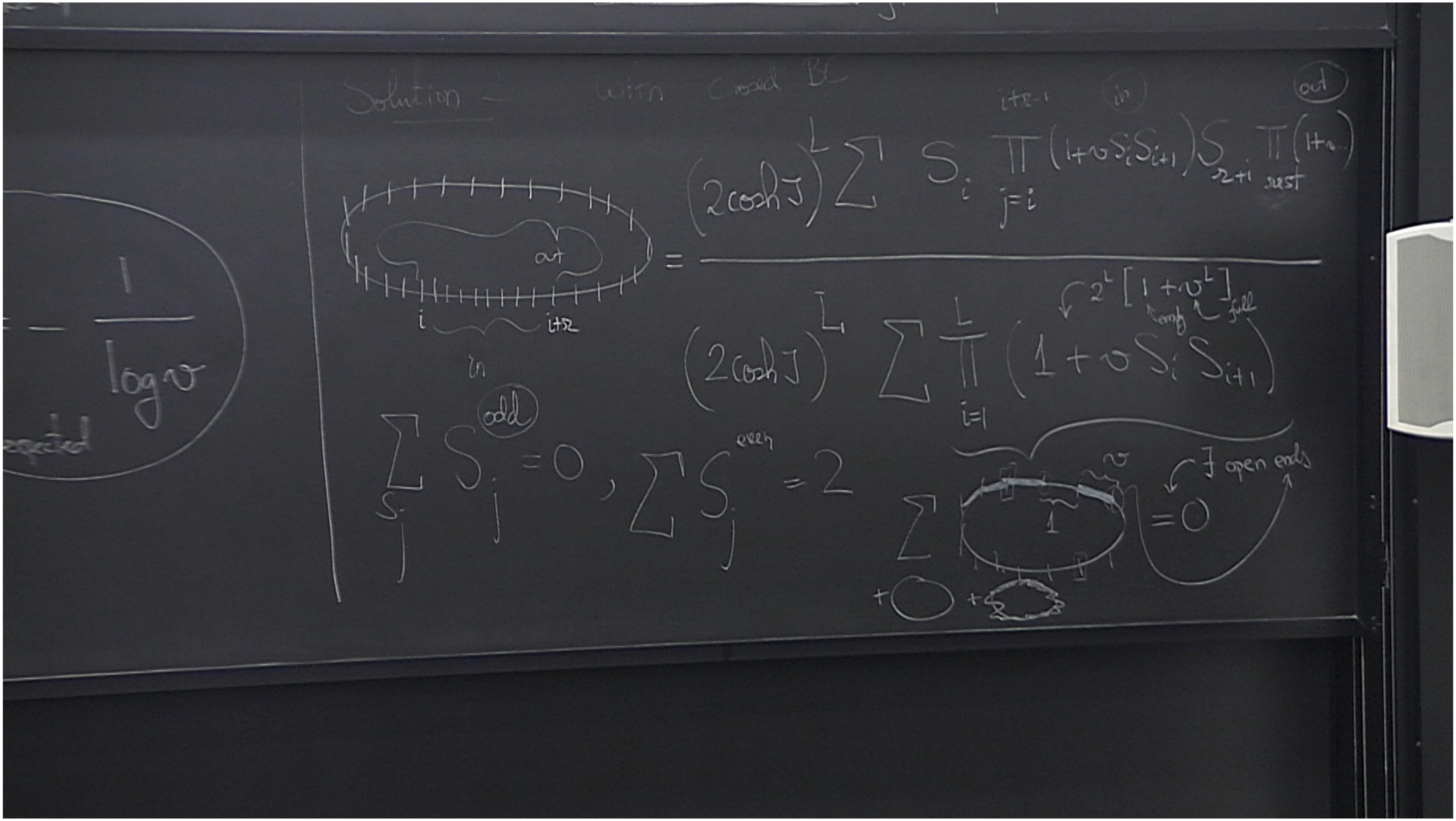
$$\underbrace{S^2_{i+1}\, S^2_{i+2} \cdots S^2_{i+r-1}}_{\sigma_i \cdots\cdots \sigma_{i+r-1}}$$

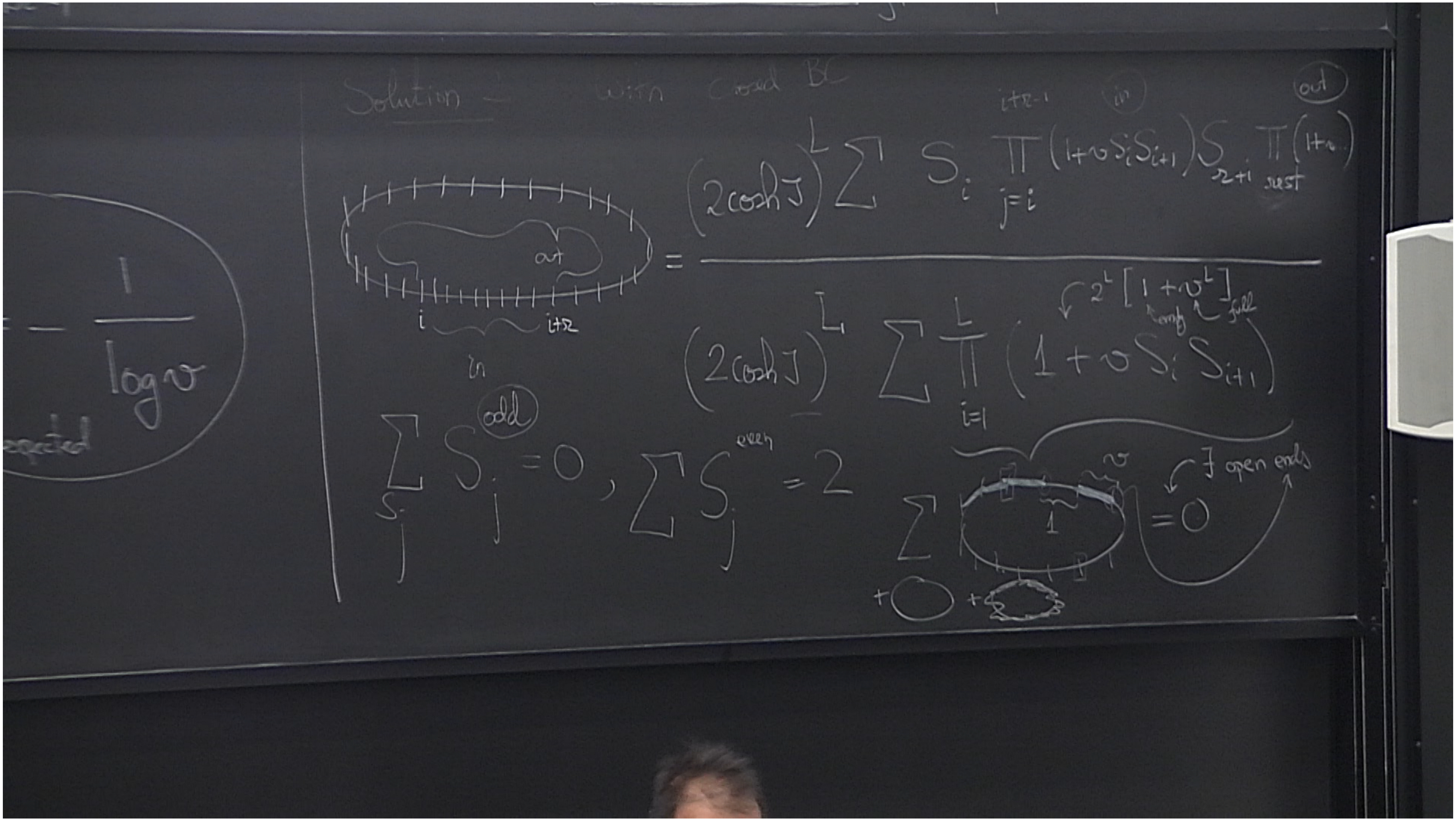
$$\langle S_i\, S_{i+r} \rangle = v^{r} = e^{-r/\xi} \quad \text{where} \quad \xi = -\frac{1}{\log v}$$

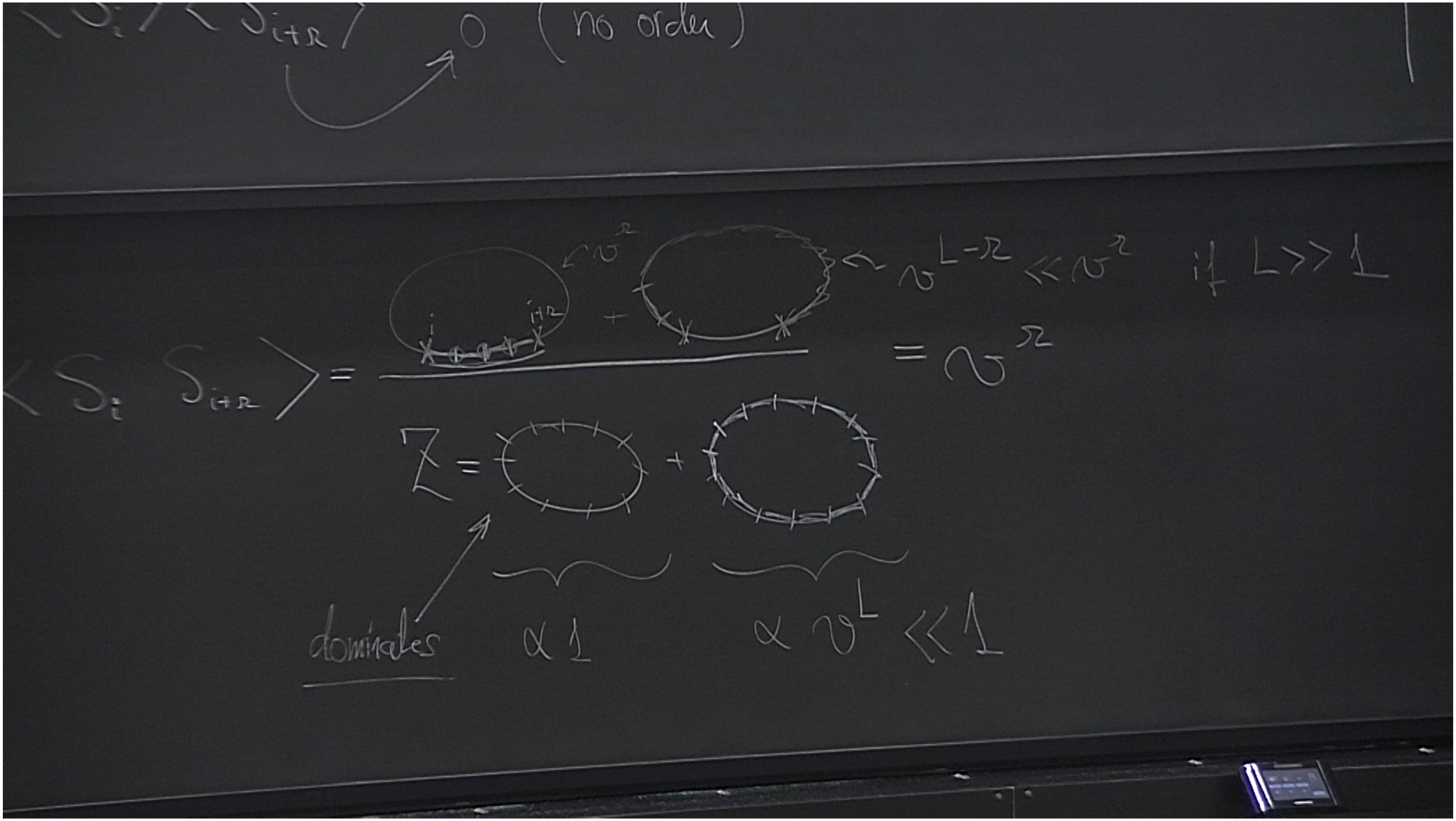
as expected


$= (\operatorname{th} J = v)^r$
$S^2_{i+1} S^2_{i+2} \cdots S^2_{i+r-1}$
$\sigma_i \cdots \sigma_{i+r-1}$
$\sum_{\{S_i\}} \to 2 \sum_{\{\sigma_i\}}$
$\langle S_i S_{i+r} \rangle = v^r = e^{-r/\xi}$ where $\xi = -\frac{1}{\log v}$
as expected
$r \to \infty$
$\to 0$ as $r \to \infty$
$\langle S_i \rangle \langle S_{i+r} \rangle$ 0 (no order)

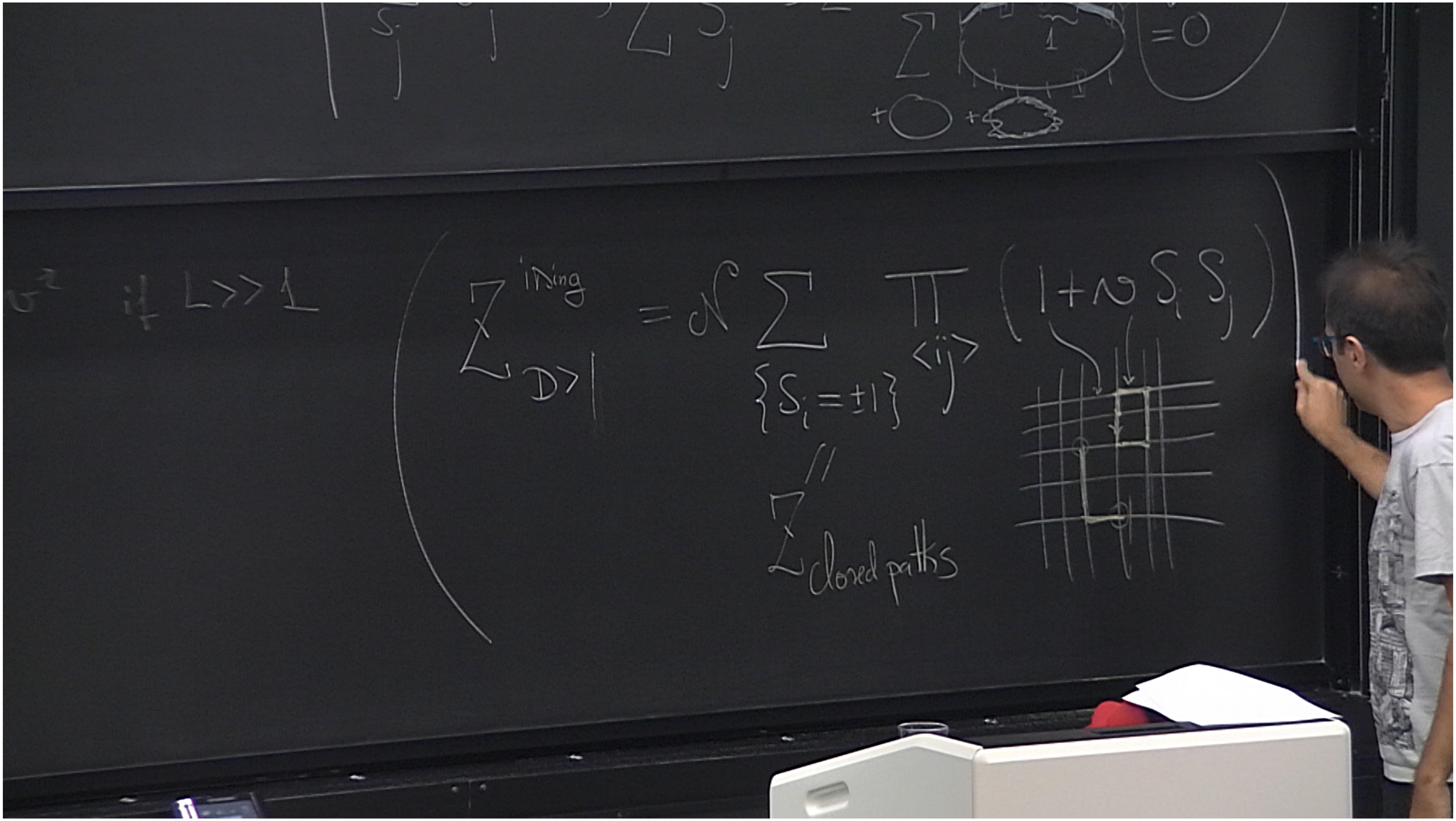

{S_i}
{σ_i = ±1}
first spin
out
i
i+R
in
log v
expected
(2cosh J)^L Σ Π_{i=1}^{L} (1 + v S_i S_{i+1})
odd
Σ_{S_j} S_j = 0 , Σ S_j^{even} = 2

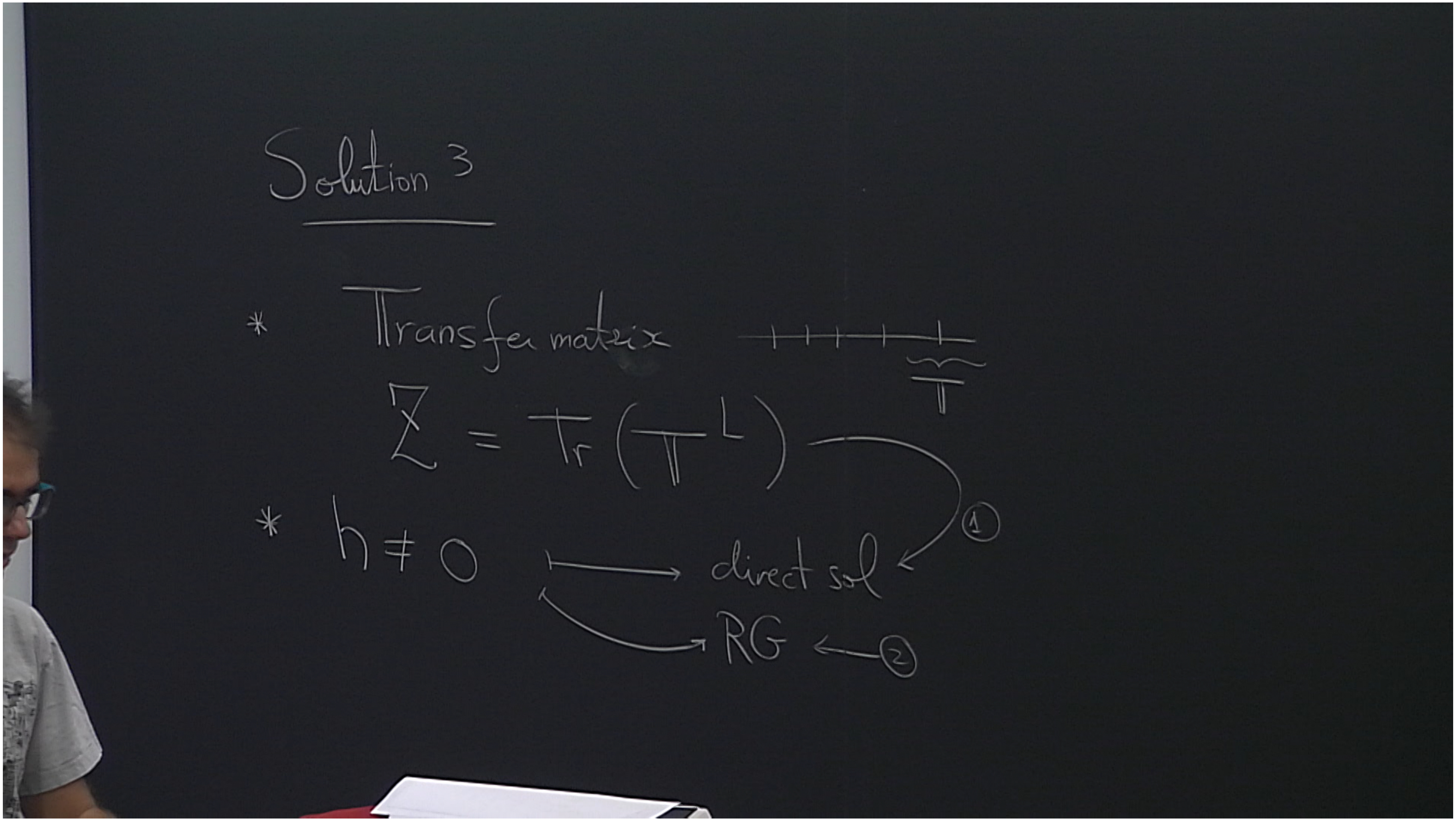

Solution – with closed BC
out
in
i
i+r
in
odd
even
∃ open ends
=0
open ends
log v


Solution with closed BC
in
out
odd
even
full
open ends
expected
= 0
= 2



$\langle S_i \rangle \langle S_{i+r} \rangle$ 0 (no order)
$\sim v^{r}$
$\sim v^{L-r} \ll v^{r}$ if $L \gg 1$
i $i+r$
$\langle S_i \, S_{i+r} \rangle =$
$= \sim v^{r}$
$Z =$ +
dominates
$\propto 1$
$\propto v^{L} \ll 1$


if L >> 1
Z Ising D>1 = N Σ {Si = ±1} Π <ij> (1 + v Si Sj)
Z closed paths


Solution 3
* Transfer matrix
$Z = \mathrm{Tr}(T^L)$
* $h \neq 0$ ⟶ direct sol
RG ⟵ 2
1
T