

Title: Bosonization and the shear sound of metals

Speakers: Inti Sodemann

Series: Condensed Matter

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URL: <http://pirsa.org/19090113>

Abstract: We will review the bosonization approach to Fermi liquids in dimensions above one. We will use this to study a sharp change in the neutral excitation spectrum of fermi liquids that occurs beyond a critical interaction strength whereby an unconventional collective mode exits the particle-hole continuum. This mode is a collective shear wave that features purely transverse current oscillations, in analogy to the transverse sound of crystals. Because it is hard to "see" due to its transversal nature, the shear sound might be already "hiding" in several metals. We will describe two strategies to "see" the shear sound: the appearance of sharp conductivity dips in ultra clean narrow channels and its coupling to charge-fluctuations under weak magnetic fields.

THE NON-LINEAR HALL EFFECT AND QUANTUM RECTIFICATION SUM RULE

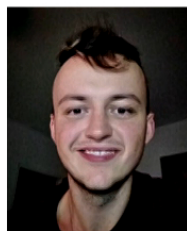
BOSONIZATION AND THE SHEAR SOUND OF METALS

Perimeter Institute for Theoretical Physics

September 24, 2019

Inti Sodemann

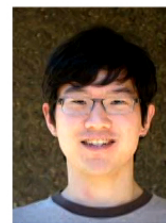
Max Planck Institute for the Physics of Complex Systems - Dresden



Oles Matsyshyn



Junny Khoo

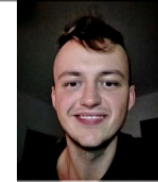


Po-Yao Chang



Falko Pientka

NON-LINEAR HALL ACCELERATION AND QUANTUM RECTIFICATION SUM RULE



Oles Matsyshyn

1) Metals without inversion symmetry have a non-Newtonian and “non-linear Hall acceleration”:

$$\frac{d^2 \mathbf{r}}{dt^2} \sim (\text{Berry dipole}) \mathbf{E}^2$$

2) **Berry dipole** controls a non-linear Hall effect in time reversal invariant metals.

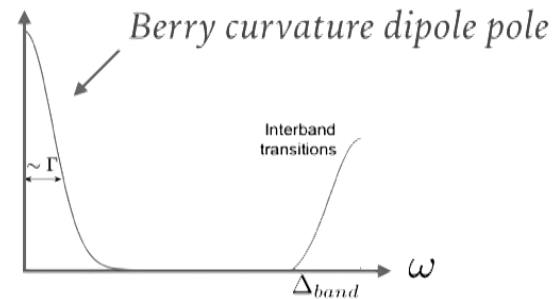
$$\text{Berry dipole} = \left\langle \frac{\partial \Omega}{\partial \mathbf{k}} \right\rangle$$

$\Omega = \text{Berry curvature}$

3) Purely quantum geometric sum rule for rectification conductivity:

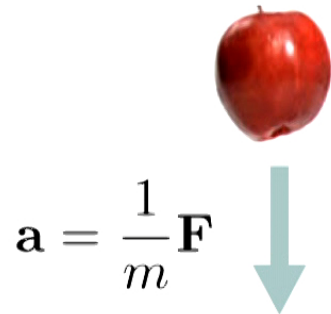
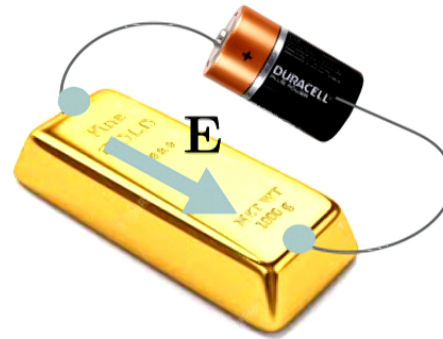
Berry dipole exhausts low frequency weight.

$$\sigma^{(2)}(-\omega, \omega)$$



DRUDE WEIGHT

Measure of the inverse inertia of liquids



$$\mathbf{a} = \frac{1}{m} \mathbf{F}$$

$$\mathbf{v} = \frac{\sigma}{n} \mathbf{E}$$

Ohm

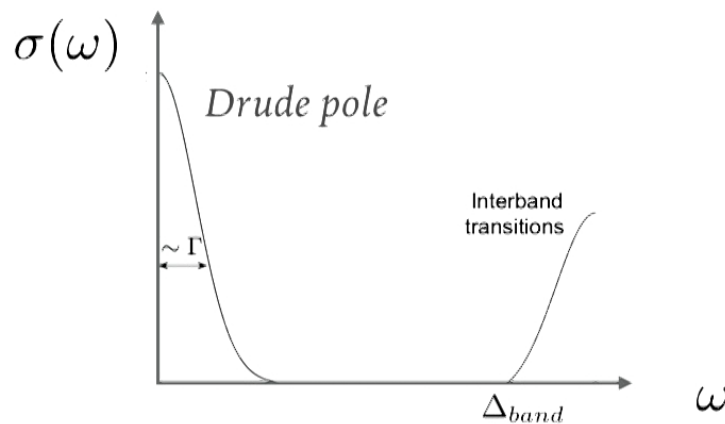
vs.

$$\mathbf{a} = \frac{D}{n} \mathbf{E}$$

Newton

Drude weight

$$D = \frac{n}{m}$$



$$D = \int \frac{d\omega}{\pi} \sigma(\omega)$$

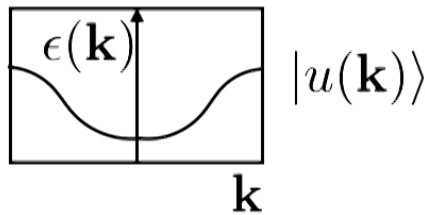
$$\sigma(\omega) = \frac{D}{i\omega + \Gamma}$$

Position operator in crystals

Vacuum

$$\mathbf{r}_\alpha = i \frac{\partial}{\partial \mathbf{p}_\alpha} \quad [\mathbf{r}_\alpha, \mathbf{r}_\beta] = 0 \quad \alpha, \beta \in \{x, y, z\}$$

Bloch band



Berry connection

$$\mathbf{r} = i \frac{\partial}{\partial \mathbf{k}} + \mathbf{A}(\mathbf{k}) \quad \mathbf{A}(\mathbf{k}) = i \langle u(\mathbf{k}) | \partial_{\mathbf{k}} | u(\mathbf{k}) \rangle$$

Berry curvature

$$[\mathbf{r}_\alpha, \mathbf{r}_\beta] = i \epsilon_{\alpha\beta\gamma} \Omega_\gamma$$

$$\Omega(\mathbf{k}) = \partial_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$

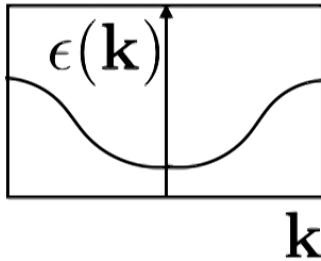


$\mathbf{k} \in$ Brillouin zone

E. Blount, in Solid state physics, Vol. 13 (Elsevier, 1962)

“Anomalous” velocity

Berry connection = momentum-locked dipole



$$H = \epsilon(\mathbf{k}) + e\mathbf{E}(t) \cdot \mathbf{r} = \epsilon(\mathbf{k}) + e\mathbf{E} \cdot \partial_{\mathbf{k}} + e\mathbf{E} \cdot \mathbf{A}(\mathbf{k})$$

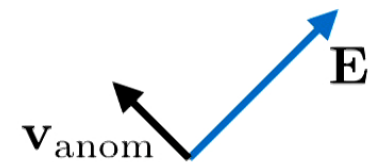
$$\mathbf{r} = i \frac{\partial}{\partial \mathbf{k}} + \mathbf{A}(\mathbf{k})$$

The velocity has an extra piece besides the group velocity:

$$\mathbf{v}_{\alpha} = \frac{d\mathbf{r}_{\alpha}}{dt} = i[H, \mathbf{r}_{\alpha}] = \partial_{\mathbf{k}_{\alpha}} \epsilon - ei[\mathbf{r}_{\alpha}, \mathbf{r}_{\beta}] \mathbf{E}_{\beta}$$

$$[\mathbf{r}_{\alpha}, \mathbf{r}_{\beta}] = i\epsilon_{\alpha\beta\gamma} \Omega_{\gamma}$$

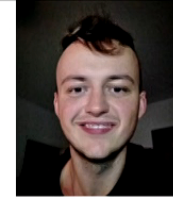
$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{k}} + e\Omega \times \mathbf{E}(t)$$



Karplus and Luttinger, *Phys. Rev.* **95**, 1154 (1954).

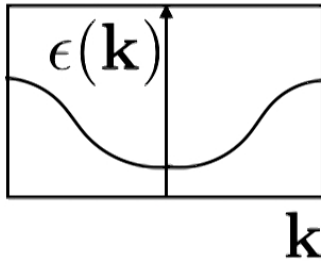
Chang and Niu, *Phys. Rev. Lett.* **75**, 1348, (1995).

“Anomalous” Hall acceleration



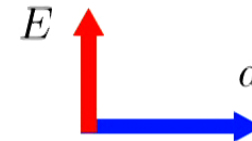
Oles Matsyshyn

Berry connection = momentum-locked dipole



$$H = \epsilon(\mathbf{k}) + e\mathbf{E}(t) \cdot \mathbf{r} = \epsilon(\mathbf{k}) + e\mathbf{E} \cdot \partial_{\mathbf{k}} + e\mathbf{E} \cdot \mathbf{A}(\mathbf{k})$$

$$\mathbf{r} = i \frac{\partial}{\partial \mathbf{k}} + \mathbf{A}(\mathbf{k}) \quad \mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{k}} + e\boldsymbol{\Omega} \times \mathbf{E}(t)$$



The acceleration acquires non-linear Hall correction:

$$\hat{a}_n^k = \underbrace{-e \frac{E^i}{\hbar^2} \frac{\partial^2}{\partial k^i \partial k^k} \epsilon_n}_{\text{Newton's law}} \underbrace{-e \frac{1}{\hbar} \frac{\partial E^i}{\partial t} \hat{\Omega}_n^{ik}}_{\text{Adiabatic change of anomalous Velocity}} \underbrace{+ e^2 \frac{E^i E^j}{\hbar^2} \frac{\partial}{\partial k^i} \hat{\Omega}_n^{jk}}_{\text{Non-linear Hall acceleration}}$$

Newton's law

Adiabatic change of anomalous Velocity

Non-linear Hall acceleration

$$\Omega^{ij} = \partial A^i / \partial k_j - \partial A^j / \partial k_i$$

$$\frac{d\mathbf{k}}{dt} = i[H, \mathbf{k}] = -e\mathbf{E}$$

Matsyshyn & Sodemann, arXiv:1907.02532 (2019).

Non-linear Hall effect

Hall-like effect present time reversal invariant materials

$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{k}} + e\Omega \times \mathbf{E}$$

$$\frac{d\mathbf{k}}{dt} = -e\mathbf{E} \quad \Delta\mathbf{k} = -e\tau\mathbf{E}$$

$$\langle \dots \rangle \equiv \int_{\text{BZ}} \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \dots$$

$$\mathbf{j}_\alpha = -e \int \frac{d^d k}{(2\pi)^d} \mathbf{v}(\mathbf{k}) f_0(\mathbf{k} + e\tau\mathbf{E}) = \text{linear response} +$$

$$- \frac{e^3 \tau^2}{2} \underbrace{\langle \partial_{\mathbf{k}_\alpha \mathbf{k}_\beta \mathbf{k}_\gamma}^3 \epsilon \rangle}_{\text{“Jerk”}} \mathbf{E}_\beta \mathbf{E}_\gamma + e^3 \tau \epsilon_{\alpha\beta\gamma} \underbrace{\langle \partial_{\mathbf{k}_\delta} \Omega_\beta \rangle}_{\text{Berry curvature dipole}} \mathbf{E}_\gamma \mathbf{E}_\delta$$

Time reversal:

$$\partial_{\mathbf{k}_\alpha} \Omega_\beta |_{\mathbf{k}} \rightarrow \partial_{\mathbf{k}_\alpha} \Omega_\beta |_{-\mathbf{k}}$$

$$\langle \partial_{\mathbf{k}_\alpha \mathbf{k}_\beta \mathbf{k}_\gamma}^3 \epsilon \rangle = 0$$

Berry curvature dipole

$$D_{\alpha\beta} = \langle \partial_{\mathbf{k}_\alpha} \Omega_\beta \rangle$$

$$\chi_{\alpha\gamma\delta}^{\text{NLH}} \equiv e^3 \tau \epsilon_{\alpha\beta\gamma} D_{\delta\beta}$$

J. E. Moore and J. Orenstein, *Phys. Rev. Lett.* 105, 026805 (2010)

Inti Sodemann and Liang Fu, *Phys. Rev. Lett.* 115, 216806 (2015)

Non-linear Hall effect

Berry curvature dipole

$$D_{\alpha\beta} = \langle \partial_{\mathbf{k}_\alpha} \Omega_\beta \rangle$$

Insulator :

$$D_{\alpha\beta} = 0$$

Inversion :

$$D_{\alpha\beta} = 0$$

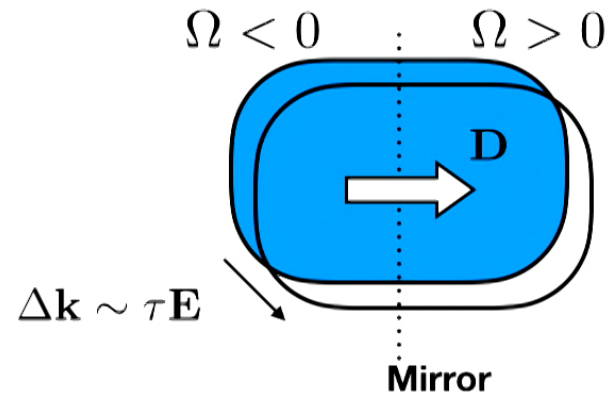
$$\langle \dots \rangle \equiv \int_{\text{BZ}} \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \dots$$

Order parameter for broken inversion symmetry in metals

Example: 2D metal

$$\mathbf{D} \equiv \int \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \frac{\partial \Omega(\mathbf{k})}{\partial \mathbf{k}}$$

$$\mathbf{J} = e^3 \tau (\mathbf{D} \cdot \mathbf{E}) \hat{\mathbf{z}} \times \mathbf{E}$$



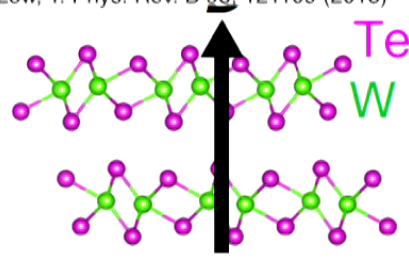
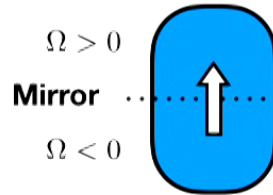
Inti Sodemann and Liang Fu, Phys. Rev. Lett. 115, 216806 (2015)

Experimental Realisation

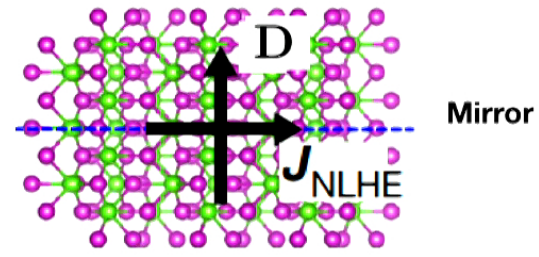
Td-WTe₂ bilayer:

Zhang, Y., Brink, J. v. d., Felser, C. & Yan, B. 2D Mater. 5 (2018) 044001

You, J.-S., Fang, S., Xu, S.-Y., Kaxiras, E. & Low, T. Phys. Rev. B 98, 121109 (2018)

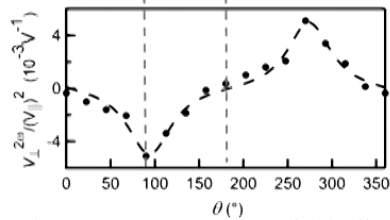
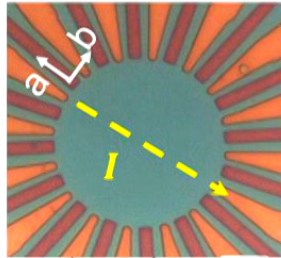


Side view



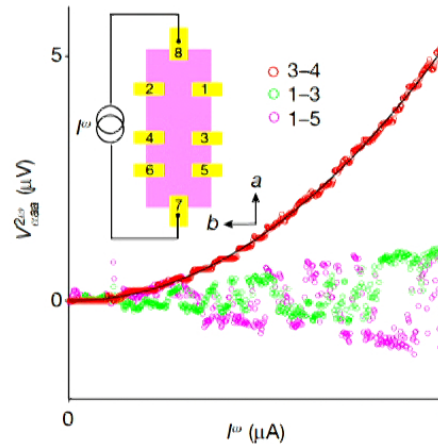
Top view

$$\mathbf{E}_\omega \rightarrow \{\mathbf{j}_0, \mathbf{j}_{2\omega}\}$$



Kaifei Kang, et al. Nat. Mater. 18, 324 (2018)

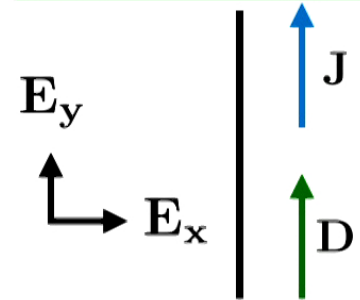
$$\mathbf{J} = e^3 \tau (\mathbf{D} \cdot \mathbf{E}) \hat{\mathbf{z}} \times \mathbf{E}$$



Q. Ma et al. Nature 565, 337 (2019)

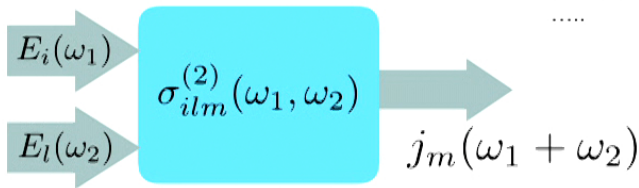
No scattering time:

$$\mathbf{E}_x = - \frac{e \langle \partial_{k_y} \Omega \rangle}{\langle \partial_{k_x}^2 \epsilon \rangle} \mathbf{E}_y^2$$

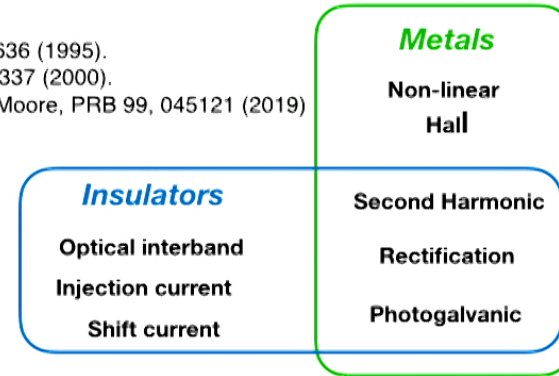


GENERAL THEORY OF SECOND ORDER RESPONSE

Full non-linear conductivity:



Aversa and Sipe, PRB 52, 14636 (1995).
 Sipe and Shkrebtii, PRB 61, 5337 (2000).
 Parker, Morimoto, Orenstein, Moore, PRB 99, 045121 (2019)



$$i\hbar \frac{d}{dt} \hat{\rho}(t) - [\hat{H}_0, \hat{\rho}] = -i\hbar\Gamma(\hat{\rho} - \hat{\rho}^0)$$

$$H_{nm} = \delta_{nm}\epsilon_n(\mathbf{k}) + q\hat{\mathbf{r}}_{nm} \cdot \mathbf{E}(t)$$

$$\hat{\mathbf{r}}_{nm} = i\delta_{nm}\partial_{\mathbf{k}} + \hat{\mathbf{A}}_{nm}$$

↑
Non-Abelian Berry connection

$$\mathbf{A}_{nm}(\mathbf{k}) = i\langle u_{n\mathbf{k}} | \partial_{\mathbf{k}} | u_{m\mathbf{k}} \rangle$$

$$\frac{\sigma_{(2)}^{ji\mu}(-\omega, \omega_1, \omega_2)}{2\pi} = \delta(\omega - (\omega_1 + \omega_2)) \sum_a \left\{ f_a \frac{i\partial^i i\partial^j v_{aa}^\mu}{(\omega + i\Gamma)(\omega_2 + i\Gamma)} + \sum_b f_{ab} \left[\frac{i\partial^j}{\omega_2 + i\Gamma} \left(\frac{A_{ab}^i v_{ba}^\mu}{\omega - \epsilon_{ab} + i\Gamma} \right) + \frac{A_{ab}^j}{\omega_2 - \epsilon_{ab} + i\Gamma} i\partial^i \frac{v_{ba}^\mu}{\omega - \epsilon_{ab} + i\Gamma} \right] + \sum_b f_{ab} A_{ab}^j \sum_c \left[\frac{A_{bc}^i v_{ca}^\mu}{(\omega - \epsilon_{ac} + i\Gamma)(\omega_2 - \epsilon_{ab} + i\Gamma)} - \frac{v_{bc}^\mu A_{ca}^i}{(\omega - \epsilon_{cb} + i\Gamma)(\omega_2 - \epsilon_{ab} + i\Gamma)} \right] \right\}$$

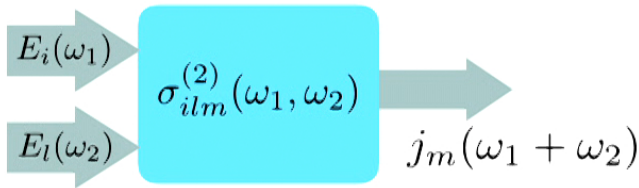
Matsyshyn & Sodemann, arXiv:1907.02532 (2019).

QUANTUM RECTIFICATION SUM RULE

$$i\hbar \frac{d}{dt} \hat{\rho}(t) - [\hat{H}_0, \hat{\rho}] = -i\hbar \Gamma (\hat{\rho} - \hat{\rho}^0)$$

$$H_{nm} = \delta_{nm} \epsilon_n(\mathbf{k}) + q \hat{\mathbf{r}}_{nm} \cdot \mathbf{E}(t)$$

Full non-linear conductivity:



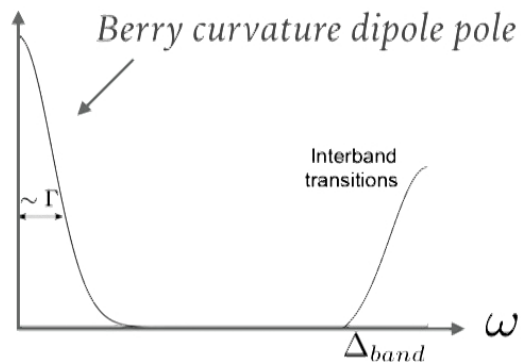
Rectification conductivity:



Quantum geometric sum rule for rectification of linearly polarised light under time reversal invariant conditions:

$$4 \frac{\hbar^2}{q^3} \frac{1}{\pi} \int_0^\infty d\omega \text{Re} [\sigma^{ji\mu}(-\omega, \omega)] = \langle \partial^j \Omega^{i\mu} \rangle + \langle [A^j, i\partial^i A^\mu] \rangle + \langle [A^j, [A^i, \bar{A}^\mu]] \rangle$$

$$\sigma^{(2)}(-\omega, \omega)$$



Select group of quantities

$$P_i = \langle A_i(\mathbf{k}) \rangle \quad \sigma_{ij}^{\text{Hall}} = \langle \Omega_{ij}(\mathbf{k}) \rangle$$

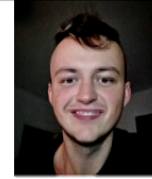
$$\theta = \epsilon_{ijk} \langle A^i \partial^j A^k - (2i/3) A^i A^j A^k \rangle$$

Solves old problem of defining order parameter for inversion breaking in metals

$$\langle \dots \rangle = \sum_n \int \frac{d\mathbf{k}}{(2\pi)^d} f(\epsilon_n(\mathbf{k})) \langle n | \dots | n \rangle$$

Patankar, et al. Phys. Rev. B **98**, 165113 (2018).

NON-LINEAR RESPONSE OF WEYL SEMIMETALS

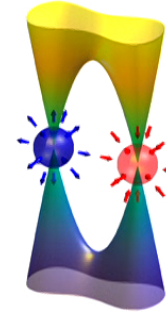


Oles Matsyshyn

Large non-linear optical response:

Liang Wu, et al. Nature Physics 13, 350 (2017)

Quantum rectification sum rule: $j(0)$

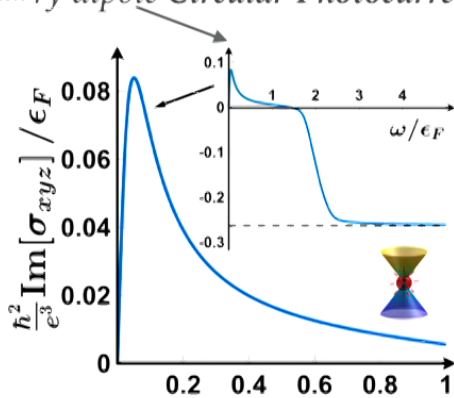


$$4 \frac{\hbar^2}{q^3} \frac{1}{\pi} \int_0^\infty d\omega \text{Re} [\sigma^{j i \mu}(-\omega, \omega)] = \langle \partial^j \Omega^{i \mu} \rangle + \langle [A^j, i \partial^i A^\mu] \rangle + \langle [A^j, [A^i, \bar{A}^\mu]] \rangle$$

RHS dimensionless in 3D $\theta = \epsilon_{ijk} \langle A^i \partial^j A^k - (2i/3) A^i A^j A^k \rangle$

Trace quantisation in Weyl semimetals: $\text{Tr} D_{BCD} = \frac{e^3}{2\pi \hbar^2} Q_W \quad Q_W \in \mathbb{Z}$

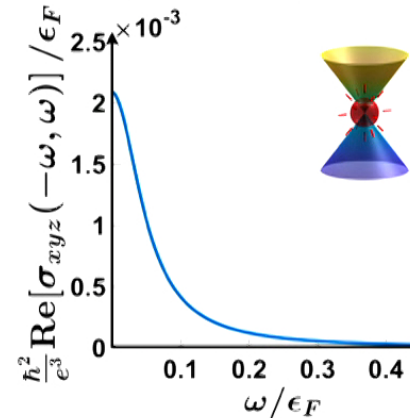
Berry dipole Circular Photocurrent generation



Quantised circular photo-galvanic effect

Chan, Lindner, Refael, and P. A. Lee, PRB (2017)
de Juan, Grushin, Morimoto, Moore, Nat Comms (2017)

Linear rectification Weyls:



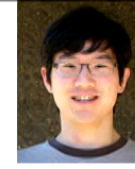
Shvetsov, et al arXiv:1902.02699

THE SHEAR SOUND OF METALS

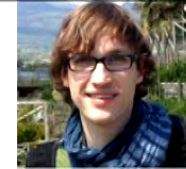
Phys. Rev. B 99, 075434 (2019)



Junny Khoo

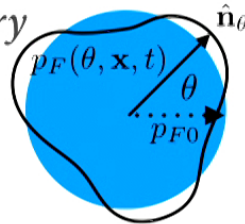


Po-Yao Chang



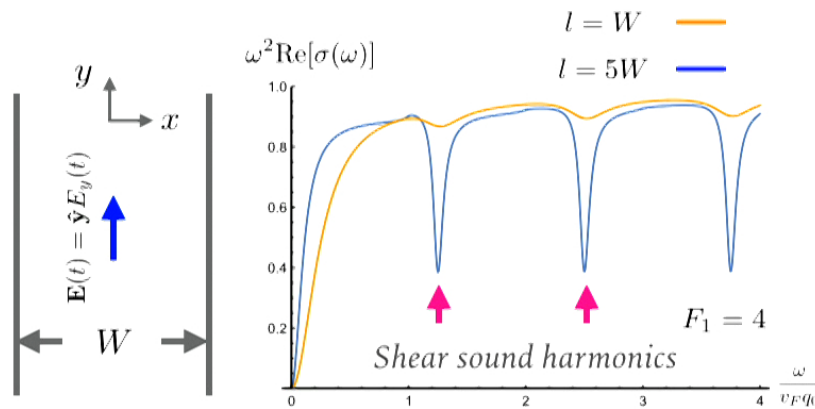
Falko Pientka

- Bosonization as Fermi surface field theory

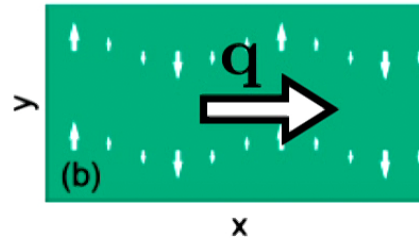


$$[p_F(\mathbf{x}, \theta), p_F(\mathbf{x}', \theta')] = \frac{(2\pi)^2}{ip_{F0}} \delta(\theta - \theta') \hat{\mathbf{n}}_\theta \cdot \partial_{\mathbf{x}} \delta(\mathbf{x} - \mathbf{x}')$$

- Strategy I: conductance dips in clean channels



- Metals can sustain shear sound:

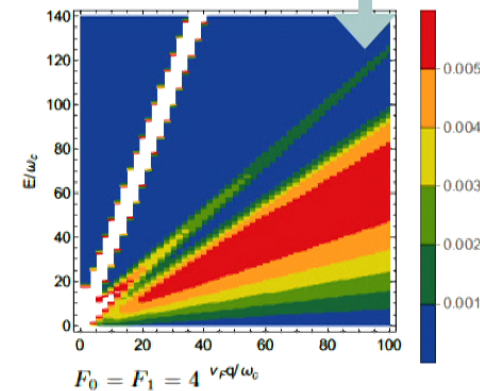


$$m^* > 2m$$

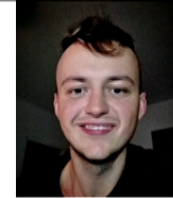
In 2D

- Strategy II: weak magnetic fields

Density spectral weight Shear sound



Summary I: The Quantum rectification sum rule and the non-linear Drude weight



Oles Matsyshyn

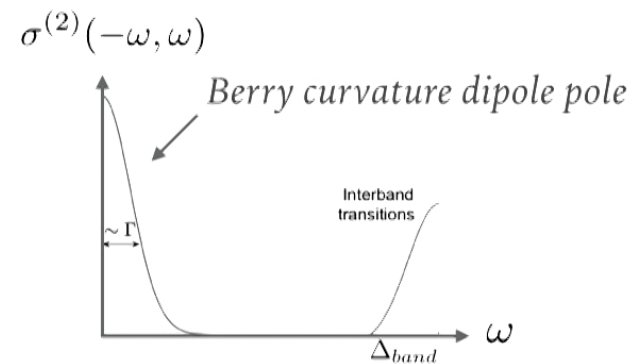
1) Metals without inversion symmetry have a non-Newtonian and non-linear “Hall acceleration”:

$$\frac{d^2 \mathbf{r}}{dt^2} \sim (\text{Berry dipole}) \mathbf{E}^2$$

2) Berry dipole is a non-linear Drude weight that controls a non-linear Hall effect allowed in time reversal invariant conditions.

$$\text{Berry dipole} = \left\langle \frac{\partial \Omega}{\partial \mathbf{k}} \right\rangle$$

3) Purely quantum geometric sum rule for rectification conductivity:
Berry dipole exhaust low frequency weight.

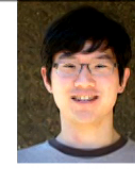


THE SHEAR SOUND OF METALS

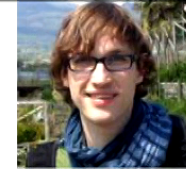
Phys. Rev. B 99, 075434 (2019)



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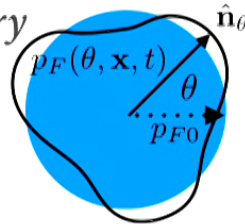


Po-Yao Chang



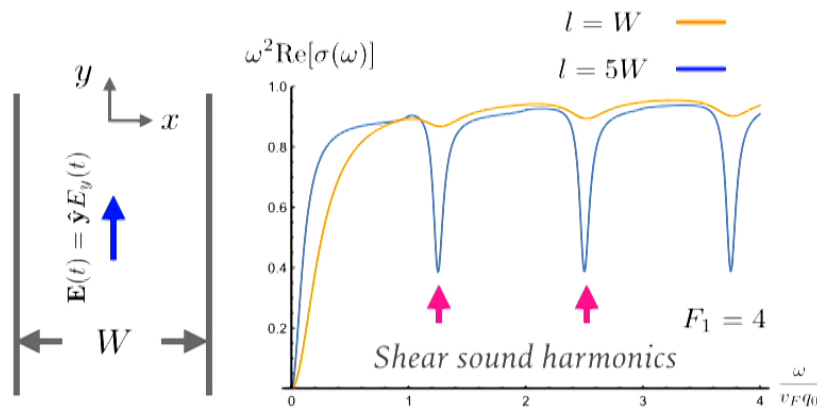
Falko Pientka

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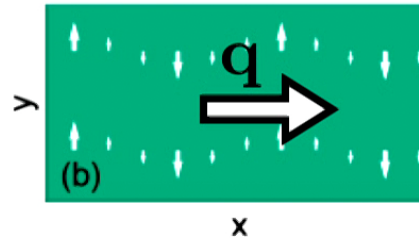


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- Strategy I: conductance dips in clean channels



- Metals can sustain shear sound:

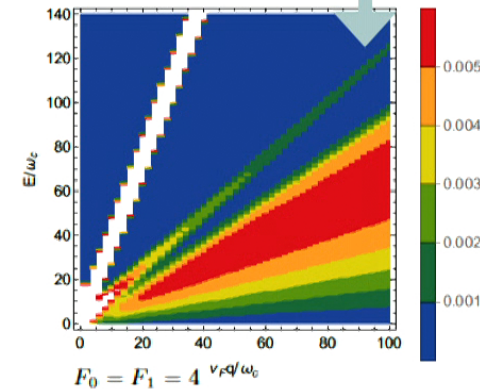


$$m^* > 2m$$

In 2D

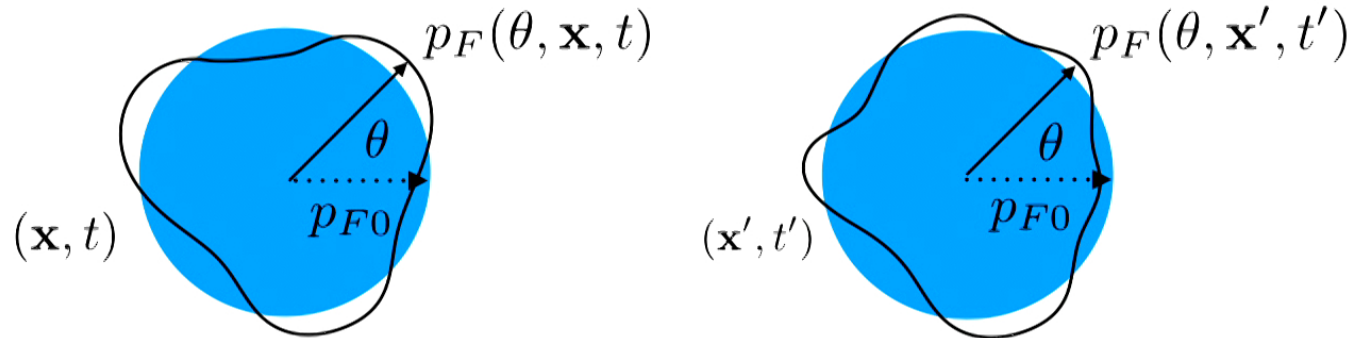
- Strategy II: weak magnetic fields

Density spectral weight Shear sound



Landau Fermi liquid theory

“String theory”



$$E = \int d^2 \mathbf{x} \frac{D_F v_F^2}{2} \left(\int \frac{d\theta}{2\pi} \delta p_F^2(\theta) + \underbrace{\int \frac{d\theta}{2\pi} \frac{d\theta'}{2\pi} F(\theta - \theta') \delta p_F(\theta) \delta p_F(\theta')}_{\text{Landau parameters}} \right) \quad D_F = \frac{p_{F0}}{2\pi v_F}$$

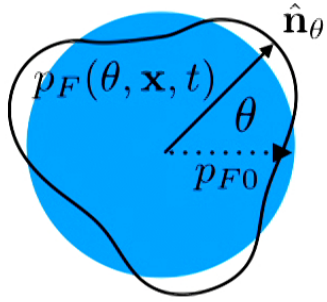
Kinetic Equation:

$$\frac{\partial p_F}{\partial t} = v_F \hat{n}_\theta \cdot \partial_{\mathbf{x}} p_F(\mathbf{x}, \theta) + v_F \hat{n}_\theta \cdot \partial_{\mathbf{x}} \int \frac{d\theta'}{2\pi} F(\theta - \theta') p_F(\mathbf{x}, \theta')$$

$$n(\mathbf{x}) \approx \int \frac{p_{F0} d\theta}{(2\pi)^2} \delta \mathbf{p}_F(\theta)$$

Pines and Nozieres, Vol I, Benjamin, 1966

“Quantized” Landau Fermi liquid theory: $\{A, B\} \rightarrow -i[A, B]$



Fermi-surface algebra

$$[p_F(\mathbf{x}, \theta), p_F(\mathbf{x}', \theta')] = \frac{(2\pi)^2}{ip_{F0}} \delta(\theta - \theta') \hat{\mathbf{n}}_\theta \cdot \partial_{\mathbf{x}} \delta(\mathbf{x} - \mathbf{x}')$$

$$\frac{\partial p_F(\mathbf{x}, \theta)}{\partial t} = i[H, p_F(\mathbf{x}, \theta)]$$

1D “Quantized” Landau Fermi liquid



$$H = \frac{v_F}{2} \sum_{IJ} \int \frac{dx}{2\pi} (\delta_{IJ} + f_{IJ}) p_{FI}(x) p_{FJ}(x)$$

$$H = \frac{v_F}{2} \sum_{IJ} \int dx (\delta_{IJ} + f_{IJ}) \partial_x \phi_I(x) \partial_x \phi_J(x)$$

$$\frac{p_{FI}}{\sqrt{2\pi}} = \partial_x \phi_I$$

$$g = \sqrt{\frac{1 + f_{RR} - f_{RL}}{1 + f_{RR} + f_{RL}}}$$

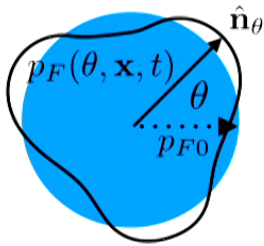
$$D_{CDW}(x) \propto \frac{\cos(2k_F x)}{|x|^{2g}}$$

F. D. M. Haldane, [eprint arXiv:cond-mat/0505529](https://arxiv.org/abs/cond-mat/0505529)

1D “Quantized” Landau Fermi liquid = Luttinger Liquid \neq Landau Fermi liquid



2D “Quantized” Landau Fermi liquid \neq Landau Fermi liquid ?

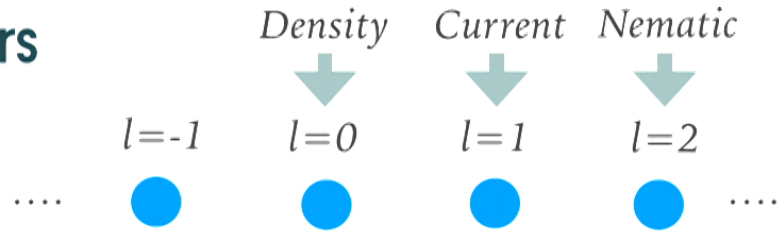


$$[p_F(\mathbf{x}, \theta), p_F(\mathbf{x}', \theta')] = \frac{(2\pi)^2}{ip_{F0}} \delta(\theta - \theta') \hat{\mathbf{n}}_\theta \cdot \partial_{\mathbf{x}} \delta(\mathbf{x} - \mathbf{x}')$$

$$\frac{\partial p_F(\mathbf{x}, \theta)}{\partial t} = i[H, p_F(\mathbf{x}, \theta)]$$

Infinite chain of bosonic operators

$$p_F(\mathbf{q}, \theta) = \sum_l p_F(\mathbf{q}, l) e^{il\theta}$$

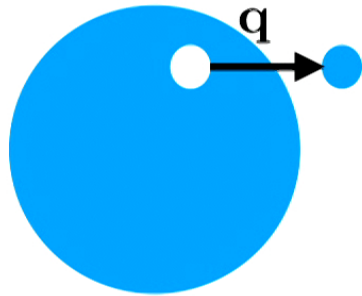


F. D. M. Haldane, [eprint arXiv:cond-mat/0505529](https://arxiv.org/abs/cond-mat/0505529)
 A. H. Castro Neto and E. Fradkin, *Phys. Rev. Lett.* **72**, 1393 (1994).
 A. Houghton and J. B. Marston, *Phys. Rev. B* **48**, 7790 (1993).

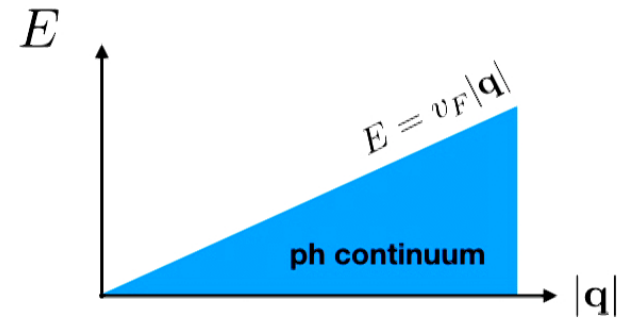
D. F. Mross and T. Senthil, *Phys. Rev. B* **84**, 165126 (2011).
 S. Golkar, D. X. Nguyen, M. M. Roberts, and D. T. Son, *Phys. Rev. Lett.* **117**, 216403 (2016).
Khoo and Sodemann, *Phys. Rev. B* **99, 075434 (2019)**

Neutral excitations in Fermi liquids

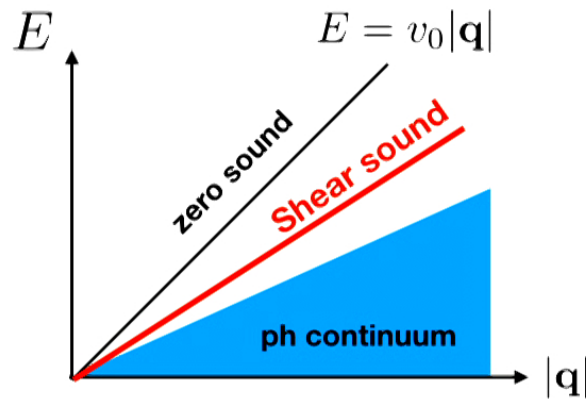
Particle-hole excitations:



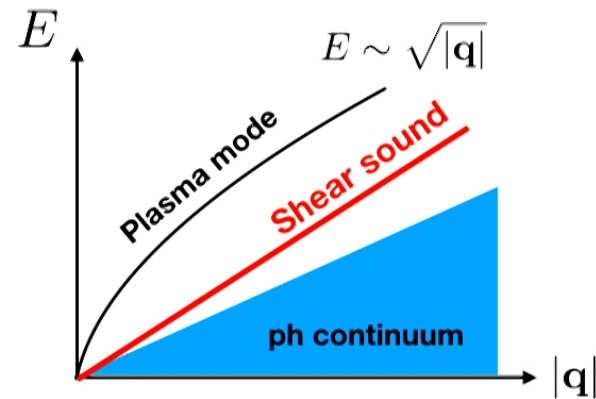
Non-interacting:



Short range repulsive:



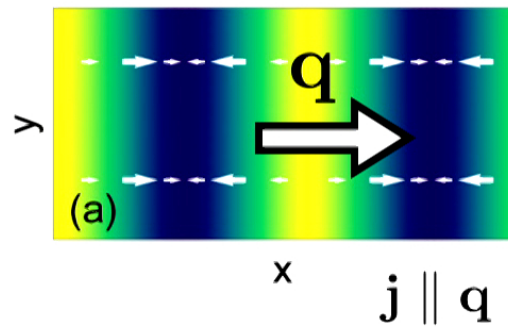
Coulomb:



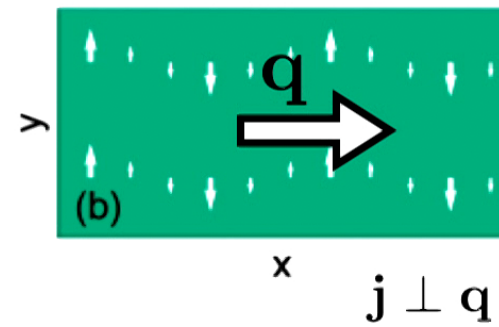
Sound in classical solids vs classical liquids

Classical solids: have shear modulus

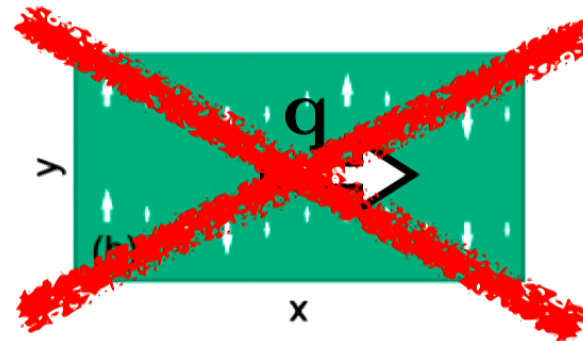
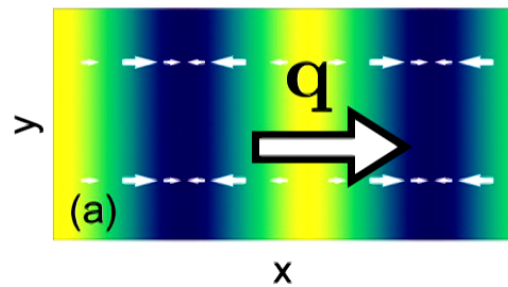
Longitudinal
compressional sound



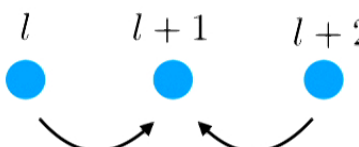
Transverse
shear sound



Classical liquids: zero shear modulus



Shear Sound

$$E_{\lambda}^{\sigma} \psi_{\lambda, l+1}^{\sigma} = t_l \psi_{\lambda, l}^{\sigma} + t_{l+2} \psi_{\lambda, l+2}^{\sigma}$$


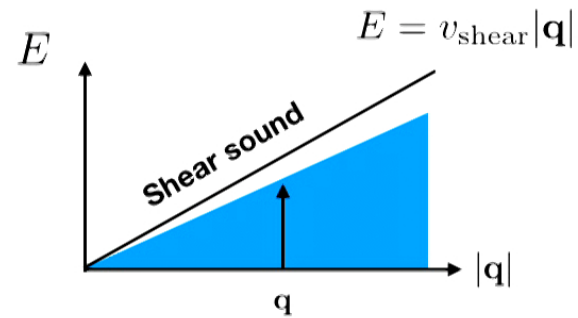
$$t_l = v_F q (1 + F_l) / 2$$

$$v_{\text{shear}} = \frac{1 + F_1}{2\sqrt{F_1}}$$

Sharp peak on the transverse current correlator

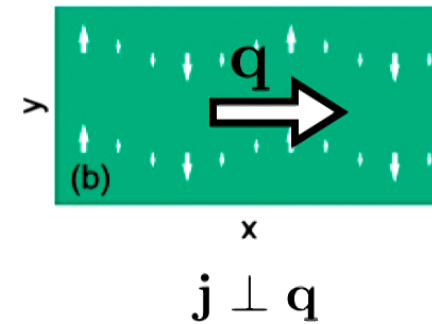
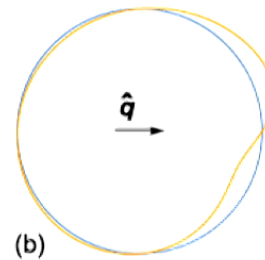
$$w_{j_{\perp} j_{\perp}} = \frac{p_F^3 v_F q^2}{32 m^2 E_1} \left(1 - \frac{1}{F_1^2} \right)$$

Bound state in the odd sector

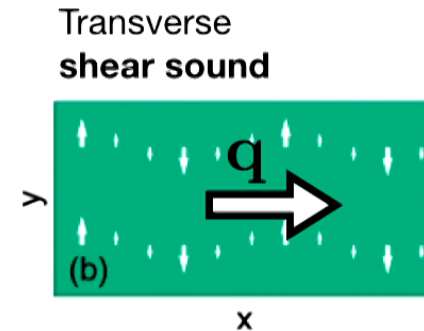
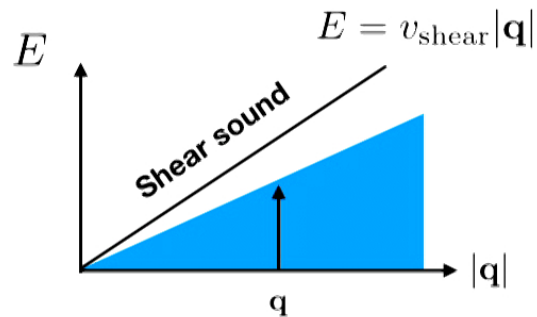


$$v_{\text{shear}} > v_F \quad \rightarrow \quad F_1 > 1$$

Transverse shear sound



Shear Sound



Shear sound should be present in quasiparticles become twice as heavy

Quasiparticle mass $\frac{m^*}{m} = 1 + F_1$

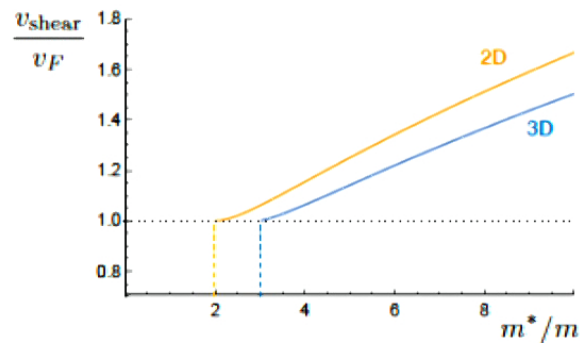
Transport mass

$v_{\text{shear}} > v_F$

$\rightarrow F_1 > 1$

$m^* > 2m$

2D vs 3D:



Shear sound was attempted to be measured in 3D Helium in 70's but results remained controversial (too close to ph continuum)

P. R. Roach and J. B. Ketterson, *Phys. Rev. Lett.* **36**, 736 (1976).

E. G. Flowers, R. W. Richardson, and S. J. Williamson, *Phys. Rev. Lett.* **37**, 309 (1976).

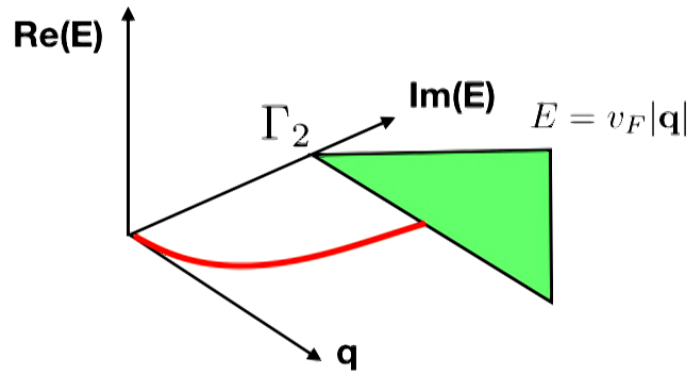
Visco-elastic properties of electron liquid:

S. Conti and G. Vignale, *Phys. Rev. B* **60**, 7966 (1999).

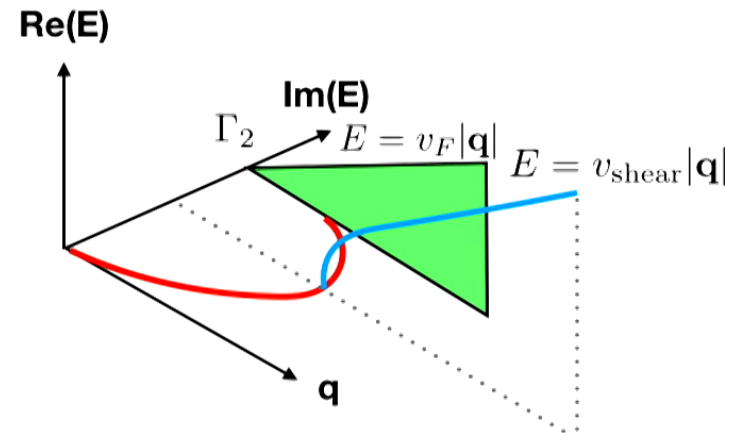
X. Gao, J. Tao, G. Vignale, and I. V. Tokatly, *Phys. Rev. B* **81**, 195106 (2010).

COLLISIONS AND CLASSICAL LIMIT

Fermi liquids w Shear diffusion $F_1 < 1$



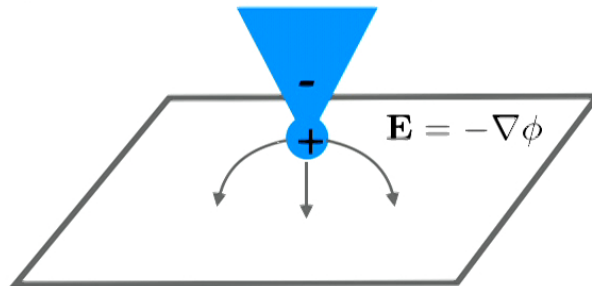
Fermi liquids w shear sound $F_1 > 1$



HOW TO EXCITE & MEASURE SHEAR SOUND?

@ $q=0$: no distinction between longitudinal and transverse.

Near field electrostatic electric fields are longitudinal:



Translational invariance needs to be broken

$$\nabla \times \mathbf{E} = 0$$

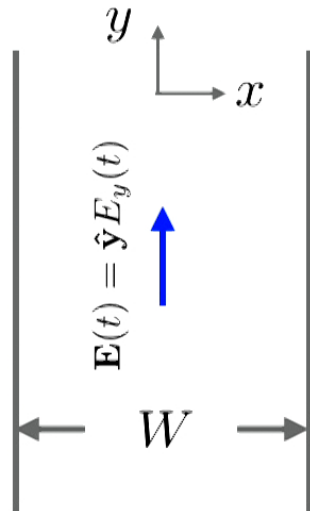
$$\mathbf{j}_{\parallel}(\mathbf{q}, \omega) = \sigma_{\parallel}(\mathbf{q}, \omega) \mathbf{E}_{\parallel}(\mathbf{q}, \omega)$$

$$\mathbf{j}_{\perp}(\mathbf{q}, \omega) = \sigma_{\perp}(\mathbf{q}, \omega) \mathbf{E}_{\perp}(\mathbf{q}, \omega)$$

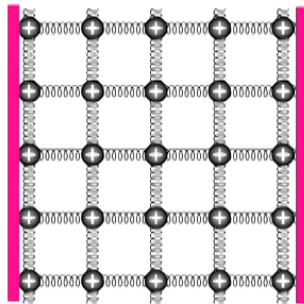
Within linear response:

$$\mathbf{j}_{\perp}(\mathbf{q}, \omega) = 0$$

STRATEGY I - STRIP GEOMETRY



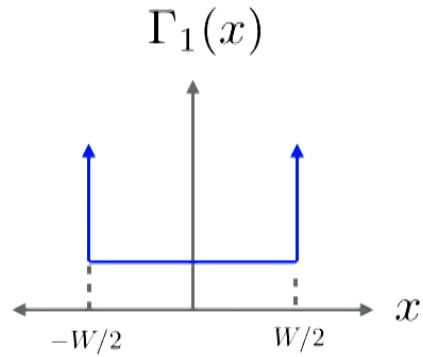
Pinned boundaries



Resonant condition

$$\omega = n v_{\perp} q_0 = v_{\perp} \frac{2\pi n}{W}$$

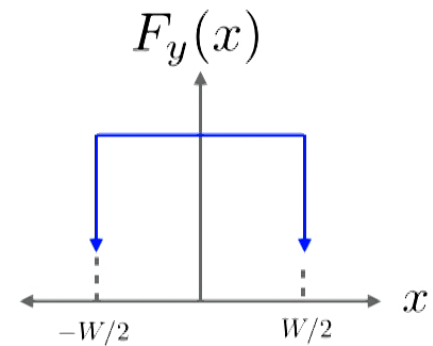
Momentum relaxing collisions:



$$\Gamma = \frac{v_F}{l}$$

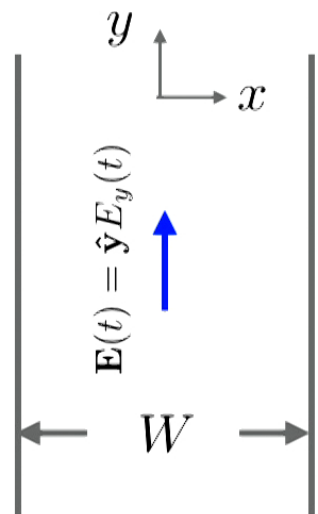
Net self-consistent force:

Transverse

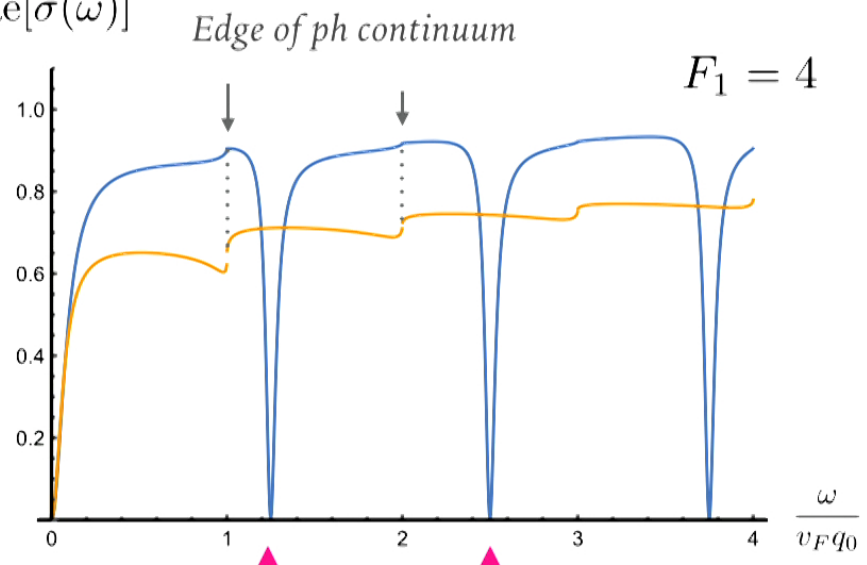


Guo, Ekin Ilseven, Gregory Falkovich, and Leonid S. Levitov, PNAS (2017)

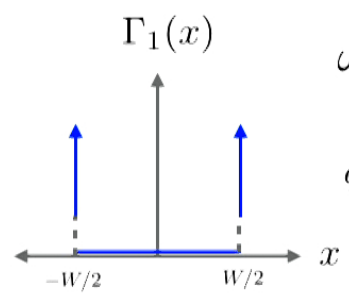
SHEAR SOUND RESONANCES



$$\omega^2 \text{Re}[\sigma(\omega)]$$



Collisions:



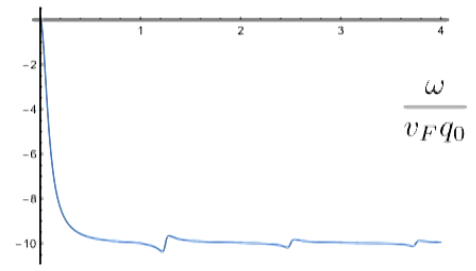
Resonant condition

$$\omega = n v_{\perp} q_0 = v_{\perp} \frac{2\pi n}{W}$$

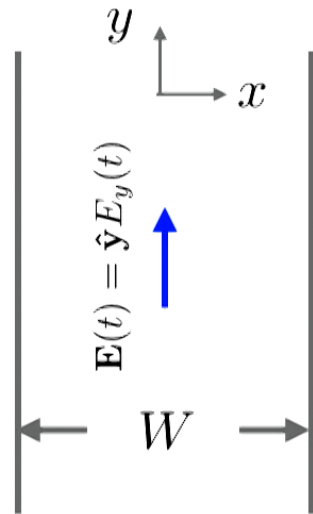
$$q_0 = \frac{2\pi}{W}$$

Shear sound harmonics

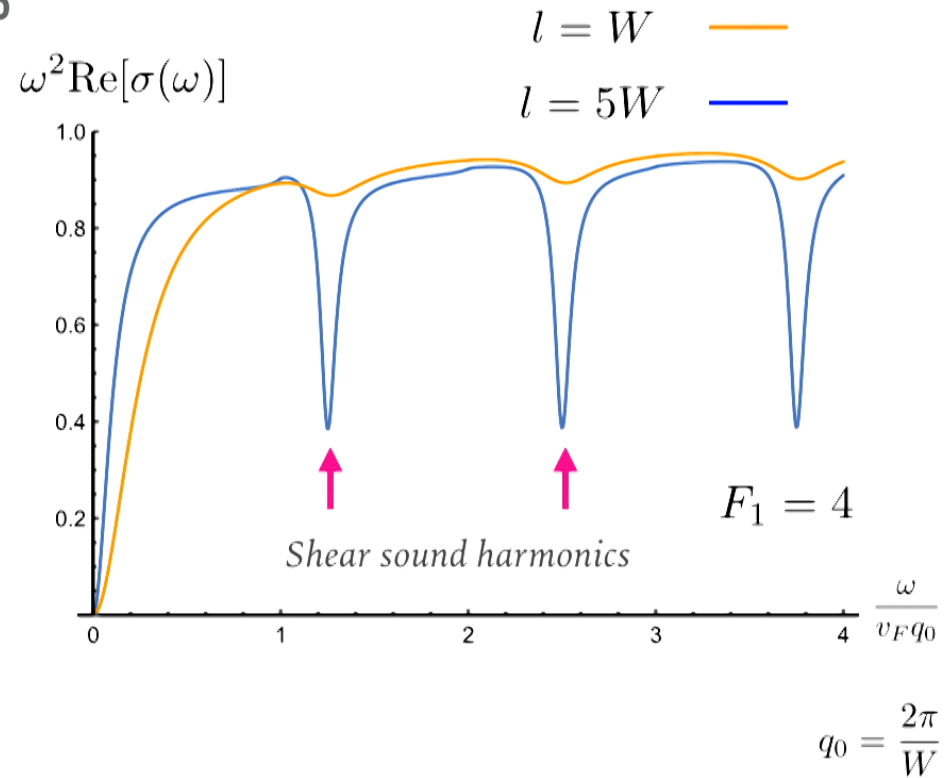
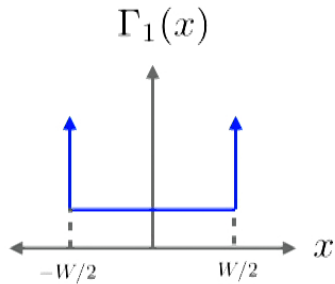
$$\omega \text{Im}[\sigma(\omega)]$$



SHEAR SOUND RESONANCES

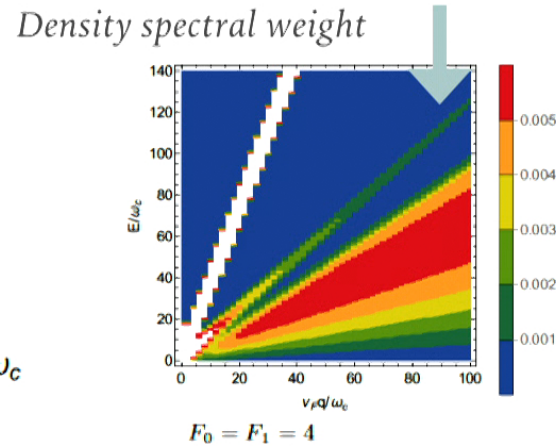
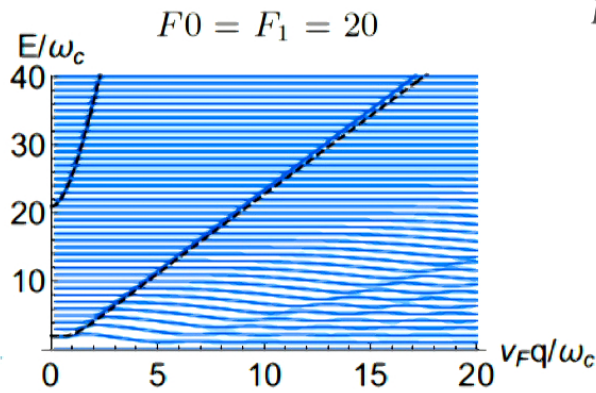
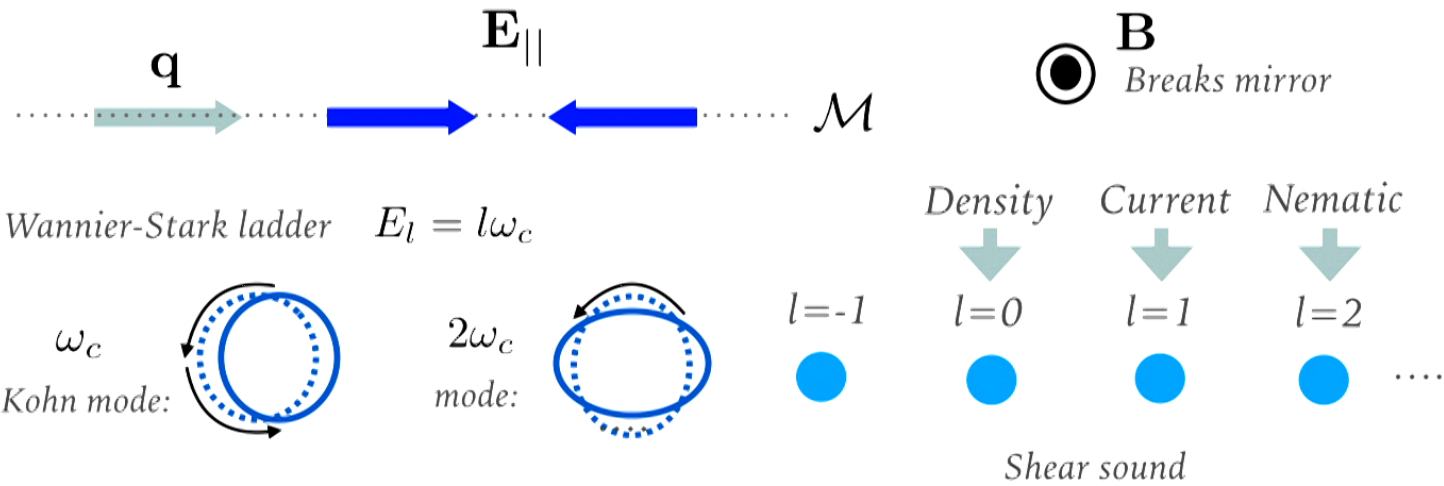


Collisions:



Width needs to be smaller than mean free path

STRATEGY II - WEAK MAGNETIC FIELDS

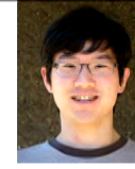


THE SHEAR SOUND OF METALS

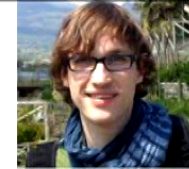
Phys. Rev. B 99, 075434 (2019)



Junny Khoo

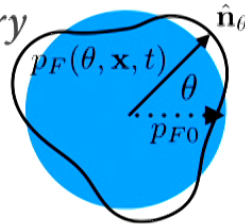


Po-Yao Chang



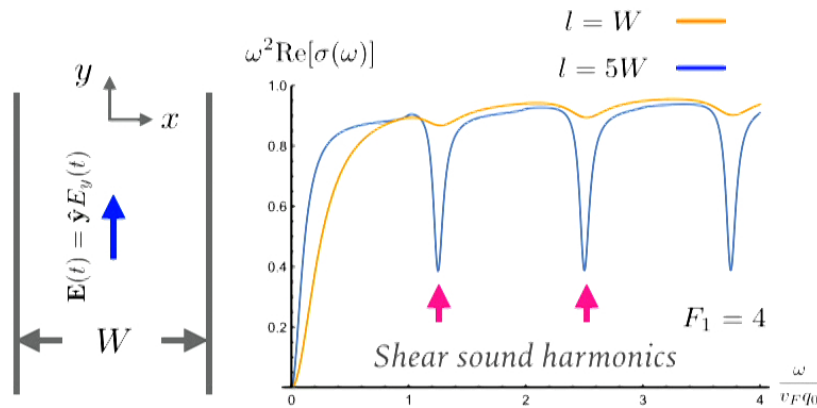
Falko Pientka

- Bosonization as Fermi surface field theory

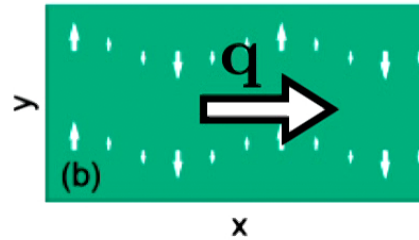


$$[p_F(\mathbf{x}, \theta), p_F(\mathbf{x}', \theta')] = \frac{(2\pi)^2}{ip_{F0}} \delta(\theta - \theta') \hat{\mathbf{n}}_\theta \cdot \partial_{\mathbf{x}} \delta(\mathbf{x} - \mathbf{x}')$$

- Strategy I: conductance dips in clean channels



- Metals can sustain shear sound:



$$m^* > 2m$$

In 2D

- Strategy II: weak magnetic fields

Density spectral weight Shear sound

