Title: The cosmological non-constant and Cartan's spiral staircase

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Abstract: We review recent efforts to turn the cosmological constant into a dynamical variable without an ungainly proliferation of free parameters. In a cosmological setting where parity invariance is imposed (along with homogeneity and isotropy) this leads to phenomenological disaster. However, in this theory it is possible to construct parity violating Friedman models due to torsion, a re-enactment of "Cartan's spiral staircase". We examine the Hamiltonian structure of the 2 branches (parity compliant and parity violating) and conclude that they must correspond to different theories, with different numbers of degrees of freedom. Parity violation may save these models phenomenologically, giving observational relevance to the Pontryagin invariant (and possibly the Immirzi parameter) in cosmology.

The cosmological *in*constant and Cartan's spiral staircase

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J. Magueijo and L. Smolin, doi:10.3390/universe5030084 arXiv:1807.01520 [gr-qc].

[2] S. Alexander, J. Magueijo and L. Smolin, arXiv:1807.01381 [gr-qc].

[1] S. Alexander, M. Cortês, A. R. Liddle, J. Magueijo, R. Sims, and L. Smolin "A Zero-Parameter Extension of General Relativity with Varying Cosmological Constant" arXiv:1905.10380 [gr-qc]ed.

Stephon Alexander, Marina Cortês, Andrew R. Liddle, João Magueijo, Robert Sims, and Lee Smolin. The cosmology of minimal varying Lambda arXiv:1905.10380 [gr-qc]

Parity violating Friedmann Universes
João Magueijo, Tom Złośnik arXiv:1908.05184 [gr-qc]

The troubles due to the cosmological constant (Lambda) are endless

- The cosmological constant problem(s):
  - Fine tuning issues of the vacuum energy in HEP
  - Cosmological fine tuning issues
  - Cosmological phase transitions and VEVs
- They were aggravated by evidence for an accelerating Universe.

Lambda's first appearance

- Lambda was first noted because Einstein's equations have this "option":

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa \tau_{\mu\nu}$$

$\kappa = 8\pi G$

- (E's equations are usually derived in the second order formalism, where the connection is a slave to the metric: the Christofell connection. This precludes torsion. It forces the connection to be "compatible with the metric": metricity and torsion-free condition.)

The reason why Lambda is rightly called a "constant":

- Take E's equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa\tau_{\mu\nu}$$

- A combination of Bianchi identities, metricity condition, and energy-momentum conservation imply a constant Lambda:

$$\nabla_\mu G^\mu_{\ \nu} = 0 \qquad \nabla_\alpha g_{\mu\nu} = 0 \qquad \nabla_\mu \tau^\mu_{\ \nu} = 0$$

- (Caveats at this level are irrelevant for this talk.)

The reason why Lambda is called the *cosmological* constant…

"I heard Einstein say to Gamow about the cosmological constant, 'That was my biggest blunder of my life.'"
–John Archibald Wheeler

Let us revisit the argument from the Cartan reformulation point of view:

- Recall that it is possible to formulate GR as a theory where metric and connection start off as independent d.o.f.
- The connection is the gauge field of local Lorentz group:

$$\omega^{ab}$$

- The Riemann tensor is encoded in its field strength:

$$R^{ab}$$

- The metric is represented by the tetrad fields: e^a
- The connection and metric are "compatible" if the torsion-free condition is enforced:

$$T^a \equiv \mathcal{D}e^a = 0$$

From Cartan's viewpoint there is a loophole in the "constancy of Lambda" argument:

- Einstein equation reads:

$$\epsilon_{abcd}\left(e^b \wedge R^{cd} - \frac{1}{3}\Lambda e^b \wedge e^c \wedge e^d\right) = -2\kappa\tau_a$$

$\kappa = 8\pi G$

- In vacuum this is solved by (the self-dual condition):

$$R^{ab} = \frac{\Lambda(x)}{3} e^a \wedge e^b$$

- The Bianchi identity reads: $\mathcal{D}R^{ab} = 0$
- Hence a varying Lambda is consistent with Einstein equations if there is torsion to the tune of:

$T^a \equiv \mathcal{D}e^a$

$$T^a = -\frac{1}{2\Lambda} d\Lambda \wedge e^a$$

To avoid an ungainly proliferation of free parameters let us posit some guiding principles.

- Option 1:

  - **Quasi-topological principle:** *Introduce only new terms in Λ that are topological when Λ is constant. Thus, Λ gets its dynamics from disrupting a topological invariance.*

- Option 2: look for a theory which

1) is at most quadratic in the curvature;
2) leaves the standard Einstein equations (the e equation) unmodified;
3) contains the Palatini action as a term (or reduces to standard torsion-free GR should Λ be constant);
4) is SD, remains invariant under duality $R^{ab} \leftrightarrow \frac{\Lambda(x)}{3} e^a e^b$,

Is this enough?

- Yes. Note that if the Einstein equations are unmodified in the new theory, then in vacuum they still reduce to the self-dual condition (with a varying Lambda):

$$R^{ab} = \frac{\Lambda(x)}{3} e^a \wedge e^b$$

- The name self-dual results from considering the duality:

$$R^{ab} \leftrightarrow \frac{\Lambda(x)}{3} e^a e^b$$

- Can guess the correct action just by applying this transformation to the usual Palatini action:

$$S^{\text{grav}} = \int_{\mathcal{M}} \epsilon_{abcd} \left(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d \right)$$

Hence the full theory satisfying these requirements is given by:

- Action – the addition of a modified Euler term without any free parameter:[Indeed with one fewer free parameter than GR!!!!]

$$S^{\text{grav}} = \int_{\mathcal{M}} \epsilon_{abcd} \left(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d - \frac{3}{2\Lambda} R^{ab} \wedge R^{cd} \right)$$

- Field equations – 2 new arrivals join E's equations:

$$S^{ab} \equiv T^{[a} \wedge e^{b]} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{ab}$$

$$\epsilon_{abcd} \left(e^b \wedge R^{cd} - \frac{1}{3} \Lambda e^b \wedge e^c \wedge e^d \right) = -2\kappa \tau_a \, ;$$

$$\epsilon_{abcd} \left(R^{ab} \wedge R^{cd} - \frac{1}{9} \Lambda^2 e^a \wedge e^b \wedge e^c \wedge e^d \right) = 0 \, ,$$

At once we see that in vacuum:

- A possible solution to

$$S^{ab} \equiv T^{[a} \wedge e^{b]} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{ab}$$

$$\epsilon_{abcd}\left(e^b \wedge R^{cd} - \frac{1}{3}\Lambda e^b \wedge e^c \wedge e^d\right) = -2\kappa\tau_a\,;$$

$$\epsilon_{abcd}\left(R^{ab} \wedge R^{cd} - \frac{1}{9}\Lambda^2 e^a \wedge e^b \wedge e^c \wedge e^d\right) = 0\,,$$

  is indeed given by:

$$R^{ab} = \frac{\Lambda(x)}{3} e^a \wedge e^b \qquad T^a = -\frac{1}{2\Lambda} d\Lambda \wedge e^a$$

  as required.

- However, Lambda is not just variable: it is left totally undefined by the field equations:

$$\Lambda^2 = \Lambda^2$$

To give the game away, we can guess what is to come…

- The way we allowed torsion to accommodate an inconstant Lambda is so general that in the absence of matter (and a secret ingredient…) any varying Lambda is possible and is unconstrained by the EOMs.
- But that is the hallmark of a gauge degree of freedom.
- Expect such a varying Lambda to be pure gauge.
- The big questions then would be:
  - What is the underlying symmetry?
  - Can it be broken by pure gravity?
  - What happens if matter is added on?

Lambda is called the *cosmological* constant for a good reason

- Its impact is felt mainly at the level of cosmological models/large scale structure.
- Let us specialize here to homogeneous and isotropic models.
- These usually are also parity invariant.
- Indeed it may seem that that is required.
- (WE WILL FIND IT IS NOT IN THIS CASE!!!!)

Assuming homogeneity, isotropy *and parity* invariance

- We can make the standard ansatz for tetrad and connection (and torsion):

$$e^0 = dt \quad ; \quad e^i = a\,dx^i\,,$$

$$\omega^i{}_0 = g(t)e^i = \left(\frac{\dot a}{a} + T\right) e^i\,; \qquad \omega^i{}_j = 0\,,$$

$$T^0 = 0 \qquad T^i = -T(t)e^0 \wedge e^i\,.$$

- We find that the reduced field equations are:

$$T = \frac{\dot\Lambda}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right);$$

$$g^2 = \left(\frac{\dot a}{a} + \frac{\dot\Lambda}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right)\right)^2 = \frac{\Lambda + \kappa\rho}{3};$$

$$\frac{(ag)^\cdot}{a} = \frac{1}{a}\left(\dot a + \frac{\dot\Lambda}{2\Lambda}a\left(1 + \frac{\kappa\rho}{\Lambda}\right)\right)^\cdot = \frac{\Lambda}{3} - \frac{\kappa}{6}(\rho + 3p);$$

$$(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) = \Lambda^2\,.$$

The status of the conservation (or continuity) cosmological equation:

- It can be proved that these imply:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) \;=\; 0$$

- More generally and adaptation of Noether's theorem can be used to prove that under certain conditions (satisfied, e.g. by a perfect fluid):

$$\tilde{D}\tau_a = 0$$

- Minimally coupled matter has a covariantly conserved energy-momentum with respect to the torsion-free bit of the connection.n

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g^2 = \left(\frac{\dot a}{a} + \frac{\dot\Lambda}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right)\right)^2 &= \frac{\Lambda + \kappa\rho}{3} ; \\
\frac{(ag)^\cdot}{a} = \frac{1}{a}\left(\dot a + \frac{\dot\Lambda}{2\Lambda} a \left(1 + \frac{\kappa\rho}{\Lambda}\right)\right)^\cdot &= \frac{\Lambda}{3} - \frac{\kappa}{6}(\rho + 3p) ; \\
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\end{aligned}$$

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A convenient complete set of cosmological equations:

- Our usual friends can be recognized…

$$\left(\frac{\dot a}{a}+T\right)^2 = \frac{\Lambda+\kappa\rho}{3};$$
$$T = \frac{\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right)$$
$$\dot\rho+3\frac{\dot a}{a}(\rho+p) = 0;$$
$$(\Lambda+\kappa\rho)\left(\Lambda-\frac{\kappa}{2}(\rho+3p)\right) = \Lambda^2.$$

$\kappa = 8\pi G$

- …but there are also some novelties.

Implications - superquintessence:

- In the presence of matter, Lambda is neither an undetermined variable nor a constant, but "scales".

$$\Lambda = \kappa\rho\frac{1+3w}{1-3w}$$

- Not an attractor for a set of ODEs. An algebraic constraint'

$$(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) \;=\; \Lambda^2$$

- Very similar (at this level!) to the cuscaton field of Afshordi and Geshnizjani
- This can be traced to the full EOM using the fact that there is no Weyl curvature in FRW models:

$$\epsilon_{abcd}\left(R^{ab}\wedge R^{cd} - \frac{1}{9}\Lambda^2 e^a \wedge e^b \wedge e^c \wedge e^d\right) = 0$$

Better than quintessence? NO!

- A cataclysm. In the radiation dominated epoch there is no radiation:

$$w = 1/3 \qquad \rho = 0$$

- So in the radiation epoch there is no radiation and in fact Lambda remains undefined by the EOM, as in vacuum.
- No way around this.
- There is a good reason for this, as we shall see…

Even in the matter epoch this is problematic…

- Notice that all terms in the Friedman equation "scale" (go like 1/t^2):

$$\left(\frac{\dot{a}}{a} + T\right)^2 = \frac{\Lambda + \kappa\rho}{3};$$

$$T = \frac{\dot{\Lambda}}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right)$$

- But the proportionality constant between H and rho changes… a varying G theory. A problematic one, phenomenologically.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\bar{\kappa}\rho}{3}$$

$$\bar{\kappa} = \frac{\kappa}{2}\frac{(1+3w)^2}{1-3w}.$$

The Copernican Principle and Parity Symmetry…

Paw choice? Cats show right and left-hand preferences

22 Jan 2018 167

On a roll: blue whales switch 'handedness' when rolling to scoop food

20 Nov 2017 6

It is possible to be homogeneous and isotropic *and break parity*?

- Obviously not in the matter content (1-label 3-forms). τ_a
- Also not in the metric/tetrad (not enough degrees of freedom to do so: 1-label e^a-forms).
- For the connection and torsion (2-label 1-forms and 1-label two forms), however, we can have:

$$\omega^i{}_0 = g(t)e^i = \left(\frac{\dot{a}}{a} + T\right) e^i$$
$$\omega^{ij} = -P\epsilon^{ijk}e^k,$$

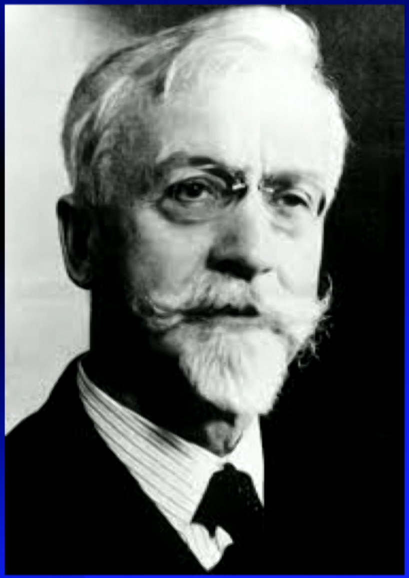
$$T^0 = 0$$
$$T^i = -T(t)e^0 e^i + P(t)\epsilon^i{}_{jk}e^j e^k.$$

- Note that the new piece breaks parity: the extra isotropic contraction is achieved with the Levi-Civita tensor.

We can ask similar questions about everything purely on the grounds of symmetry

- Purely in terms of symmetry, a homogeneous and isotropic parity breaking effect
  - in the CMB temperature: impossible.
  - in the CMB linear polarization: impossible.
  - in the CMB circular polarization: possible.
- A B-mode monopole does not exist.
- A V-mode – circular polarization – monopole does exist and would be consistent with homogeneity and isotropy. It would also break parity invariance.

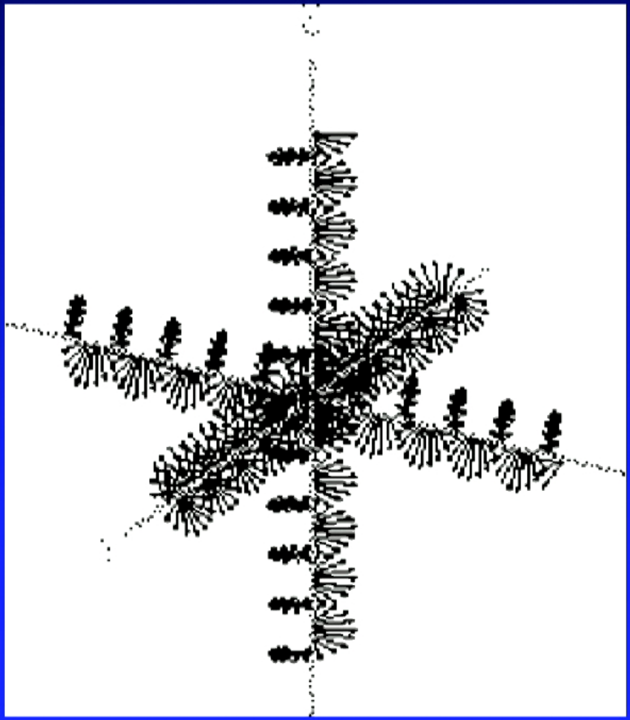
The new term in the torsion has a simple geometrical interpretation: the Cartan spiral staircase (1922)



M. Lazar and F. W. Hehl, Cartan's spiral staircase in physics and, in particular, in the gauge theory of dislocations, Found. Phys. 40 (2010) 1298 [arXiv:0911.2121].

The Cartan spiral staircase

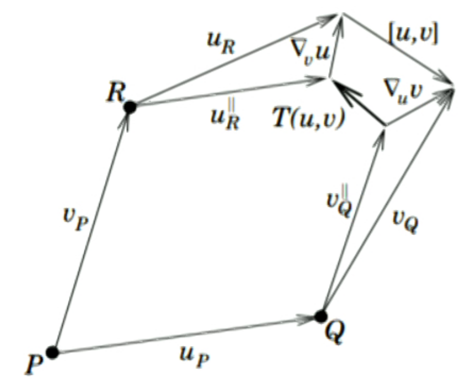
Most general flat 3D space: a connection with the following parallel transport pattern:



Used to describe dislocations in crystals

Clearly, a connection with torsion:

$$T(X, Y) := \nabla_X Y - \nabla_Y X - [X, Y]$$



$$\omega^i{}_0 = g(t) e^i = \left(\frac{\dot{a}}{a} + T\right) e^i$$

$$\omega^{ij} = -P \epsilon^{ijk} e^k,$$

It turns out this changes completely the nature of the EOMs for FRW:

- EOMs if parity is imposed (left), and otherwise (right):

$$g^2 = \left(\frac{\dot a}{a} + \frac{\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right)\right) = \frac{\Lambda+\kappa\rho}{3};$$

$$\frac{(ag)^{\cdot}}{a} = \frac{1}{a}\left(\dot a + \frac{\dot\Lambda}{2\Lambda}a\left(1+\frac{\kappa\rho}{\Lambda}\right)\right)^{\cdot} = \frac{\Lambda}{3} - \frac{\kappa}{6}(\rho+3p);$$

$$T = \frac{\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right)$$

$$(\Lambda+\kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho+3p)\right) = \Lambda^2 .$$

$$g^2 - P^2 = \frac{\Lambda+\kappa\rho}{3}$$

$$\frac{(ag)^{\cdot}}{a} = \frac{\Lambda}{3} - \frac{\kappa}{6}(\rho+3p)$$

$$T = \frac{\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right)$$

$$P = \frac{3\dot\Lambda}{\Lambda^2}gP$$

$$18gP\frac{(aP)^{\cdot}}{a} = (\Lambda+\kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho+3p)\right) - \Lambda^2.$$

- Culprit is easy to find:

$$P = \frac{3\dot\Lambda}{\Lambda^2}gP$$

$P = 0$ $\quad$ $0 = 0$

$P \neq 0$ $\quad$ $1 = \frac{3\dot\Lambda}{\Lambda^2}g.$

Notice that this only happens because…

- At the same time parity violation is allowed by torsion into FRW Universes…
- …the doors open up to a non-vanishing Weyl tensor:

$$R^{ab} = \mathcal{R}^{ab} + \mathcal{W}^{ab}$$

$$\mathcal{R}^{0i} = \frac{(ag)^{\cdot}}{a} e^0 e^i$$
$$\mathcal{R}^{ij} = (g^2 - P^2) e^i e^j$$

$$\mathcal{W}^{0i} = gP\epsilon^i{}_{jk} e^j e^k$$
$$\mathcal{W}^{ij} = -\frac{(aP)^{\cdot}}{a} \epsilon^{ij}{}_k e^0 e^k$$

- No longer can we eliminate the Euler invariant in terms of matter – there is a pure gravity degree of freedom in the Lambda EOM:

$$\epsilon_{abcd}\left(R^{ab} \wedge R^{cd} - \frac{1}{9}\Lambda^2 e^a \wedge e^b \wedge e^c \wedge e^d\right) = 0$$

What does this mean?

- Two separate branches?
- Two separate theories?
- Two phases of the same theory?

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$$T = \frac{\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right)$$

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- No longer can we eliminate the Euler invariant in terms of matter – there is a pure gravity degree of freedom in the Lambda EOM:

$$\epsilon_{abcd}\left(R^{ab} \wedge R^{cd} - \frac{1}{9}\Lambda^2 e^a \wedge e^b \wedge e^c \wedge e^d\right) = 0$$

It turns out that the Hamiltonian formulation is revealing…

$$c = Pa$$
$$b = ga.$$

$$g = \frac{\dot{a}}{a} + T.$$

$$S^g = \int dt \left(2a^2\dot{b} + \frac{6}{\Lambda}\frac{d(bc^2 - \frac{1}{3}b^3 - bk)}{dt} + 2Na\left(k - c^2 + b^2 - \frac{\Lambda}{3}a^2\right)\right),$$

- The newly promoted-to-a-variable Lambda has the Chern-Simons time as conjugate momentum. This forms a primary constraint.

$$S^{g'} = \int dt \left(2a^2\dot{b} + \Pi\frac{d\Lambda^{-1}}{dt} + 2Na\left(k - c^2 + b^2 - \frac{\Lambda}{3}a^2\right) + V\left(\frac{\Pi}{6} + bc^2 - bk - \frac{1}{3}b^3\right)\right). \quad (70)$$

$$\tau_{CS} = 6b(c^2 - \frac{1}{3}b^2 - k)$$

The parity violating degree of freedom does not have a conjugate momentum:

- It has an algebraic equation of motion:

$$2Nac - bVc = 0$$

- This is clearly behind the 2 branches of the theory:
  - Either $c = 0$ and the Hamiltonian constraint and the new constraint are independent.
  - Or we must have: $V = N\frac{2a}{b}$
  - …and then, we only have one independent constraint:

$$S^{g'} = \int dt \left(2a^2\dot{b} + \Pi \frac{d\Lambda^{-1}}{dt} + 2Na\left(\frac{2}{3}b^2 - \frac{\Lambda}{3}a^2 + \frac{\Pi}{6b} \right) \right).$$

Hence we have two theories for the price of one… 2 FOR 1 Sale!

$$S = \int dt \left(p\frac{da^2}{dt} + \Pi\frac{d\Lambda^{-1}}{dt} - H(a^2, p; \Lambda^{-1}, \Pi) \right)$$

$$\{a^2, p\} = 1$$
$$\{\Lambda^{-1}, \Pi\} = 1$$

- If parity-violation is allowed ($c \neq 0$) we have theory 1:

$$H_1 = -2N\sqrt{a^2}\left(\frac{2}{3}\left(\frac{p}{2}\right)^2 - \frac{2\Pi}{6p} - \frac{\Lambda}{3}a^2\right)$$

- If parity-violation is not allowed ($c = 0$) we have theory 2:

$$H_2 = -2N\sqrt{a^2}\mathcal{H}_2 - V\mathcal{V}_2$$

$$\mathcal{H}_2 = k + \left(\frac{p}{2}\right)^2 - \frac{\Lambda}{3}a^2$$
$$\mathcal{V}_2 = \frac{\Pi}{6} + \frac{p}{2}k + \frac{1}{3}\left(\frac{p}{2}\right)^3$$

$$\{\mathcal{V}_2, \mathcal{H}_2\} = \frac{\Lambda}{6}\mathcal{H}_2$$

Clearly according to Dirac they have different numbers of d.o.f.:

- Recall that according to the great man:

$$N_{dof} = \frac{1}{2}\Big(Dim_{ph} - 2F - S\Big)$$

- Hence, only if we do allow for parity violation in FRW (theory 1) do we have something truly different from GR:

	Dim_{ph}	F	S	N_{dof}
GR (no matter)	2	1	0	0
Theory 1 (no matter)	4	1	0	1
Theory 2 (no matter)	4	2	0	0

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Theory 1 (no matter)	4	1	0	1
Theory 2 (no matter)	4	2	0	0

The new constraint represents "conformal" symmetry

- Can be seen from the action of the constraint on the FRW reduced variables:

$$\delta_V a^2 = \{a^2, \mathcal{V}_2\} = \frac{k}{2} + \frac{1}{2}\left(\frac{p}{2}\right)^2 \approx \frac{\Lambda}{6}a^2$$
$$\delta_V p = \{p, \mathcal{V}_2\} = 0$$
$$\delta_V \Lambda^{-1} = \{\Lambda^{-1}, \mathcal{V}_2\} = \frac{1}{6}$$
$$\delta_V \Pi = \{\Pi, \mathcal{V}_2\} = 0.$$

- This is a FRW expression of the (almost) conformal invariance of the full theory:

$$\tilde{e}^a = \phi^{1/2} e^a$$
$$\tilde{R}^{ab} = R^{ab}$$
$$\tilde{\Lambda} = \frac{\Lambda}{\phi},$$

$$S^{\mathrm{grav}} = \int_{\mathcal{M}} \epsilon_{abcd}\left(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d - \frac{3}{2\Lambda} R^{ab} \wedge R^{cd}\right)$$

$$S = \int L$$

$$\tilde{L} = \phi L$$

So, in Theory 2 (parity invariant) conformal invariance is preserved

- Lambda can be seen as a gauge mode associated with conformal invariance.
- We can use the conformal invariance to find a "gauge" where Lambda is a constant:

$$\phi = \frac{\Lambda}{\Lambda_0}$$

$$\begin{aligned} \tilde{e}^a &= \phi^{1/2} e^a \\ \tilde{R}^{ab} &= R^{ab} \\ \tilde{\Lambda} &= \frac{\Lambda}{\phi}, \end{aligned}$$

- (Note that this would change the gravitational coupling, as observed in our calcs…) $\tilde{L} = \phi L$

In theory 1, the spiral staircase not only breaks parity but breaks the conformal invariance of the theory.

- Note that the action is not invariant but a conformal density with weight 1.
- Hence, it affects the field equations when integrations by parts are involved.
- This is indeed what happens for the connection/torsion equation. However, the non-invariant correction only affects the parity violating component:

$$T = \frac{\dot{\Lambda}}{2\Lambda},$$
$$P = \frac{3\dot{\Lambda}}{\Lambda^2} g P$$

$$\tilde{T} = \frac{1}{\sqrt{\phi}}\left(T - \frac{\dot{\phi}}{2\phi}\right)$$
$$\tilde{P} = \frac{P}{\sqrt{\phi}}.$$

Addition of matter complicates calculations but reinforces this:

- Dirac's count now reads:

	Dim_{ph}	F	S	N_{dof}
GR (no matter)	2	1	0	0
GR (matter)	4	1	0	1
Theory 1 (no matter)	4	1	0	1
Theory 1 (w. matter)	6	1	0	2
Theory 2 (no matter)	4	2	0	0
Theory 2 (w. dust)	6	1	2	1
Theory 2 (w. radiation)	6	2	2	0

- Only by breaking parity (and conformal invariance) do we have a new degree of freedom.
- Otherwise Lambda is a gauge degree of freedom in vacuum or with radiation (conformal matter)
- With non-conformal matter it is a slave degree of freedom.

They would have allowed for extra terms, with a single new parameter…

- A possible parameterization of a theory with self-duality:

$$S^g = -\int \frac{3}{2\Lambda}\left(\epsilon_{abcd} + \frac{2}{\gamma}\eta_{ac}\eta_{bd}\right)\left(R^{ab} - \frac{\Lambda}{3}e^a e^b\right)\left(R^{cd} - \frac{\Lambda}{3}e^c e^d\right) - \frac{2}{\gamma}\int T^a T_a.$$

$$S_{Pal} = \int \epsilon_{abcd}\left(e^a e^b R^{cd} - \frac{\Lambda}{6}e^a e^b e^c e^d\right),$$

$$S_{Eul} = -\frac{3}{2\Lambda}\int \epsilon_{abcd}R^{ab}R^{cd},$$

$$S_{NY} = \frac{2}{\gamma}\int e^a e^b R_{ab} - T^a T_a,$$

$$S_{Pont} = -\frac{3}{\gamma\Lambda}\int R^{ab}R_{ab}.$$

The field equations for FRW feel the Pontryagin term if parity violation is allowed

- General equations of motion:

$$\epsilon_{abcd}\left(e^b R^{cd} - \frac{1}{3}\Lambda e^b e^c e^d\right) = -2\kappa\tau_a$$

$$T^{[a}e^{b]} = -\frac{3}{2\Lambda^2}d\Lambda R^{ab} + \frac{3}{4\gamma\Lambda^2}\epsilon^{abcd}d\Lambda R_{cd}$$

$$\epsilon_{abcd}\left(R^{ab}R^{cd} - \frac{1}{9}\Lambda^2 e^a e^b e^c e^d\right) + \frac{2}{\gamma}R^{ab}R_{ab} = 0$$

- FRW reduction allowing for parity violating torsion P:

$$g^2 - P^2 = \frac{\Lambda + \kappa\rho}{3}$$

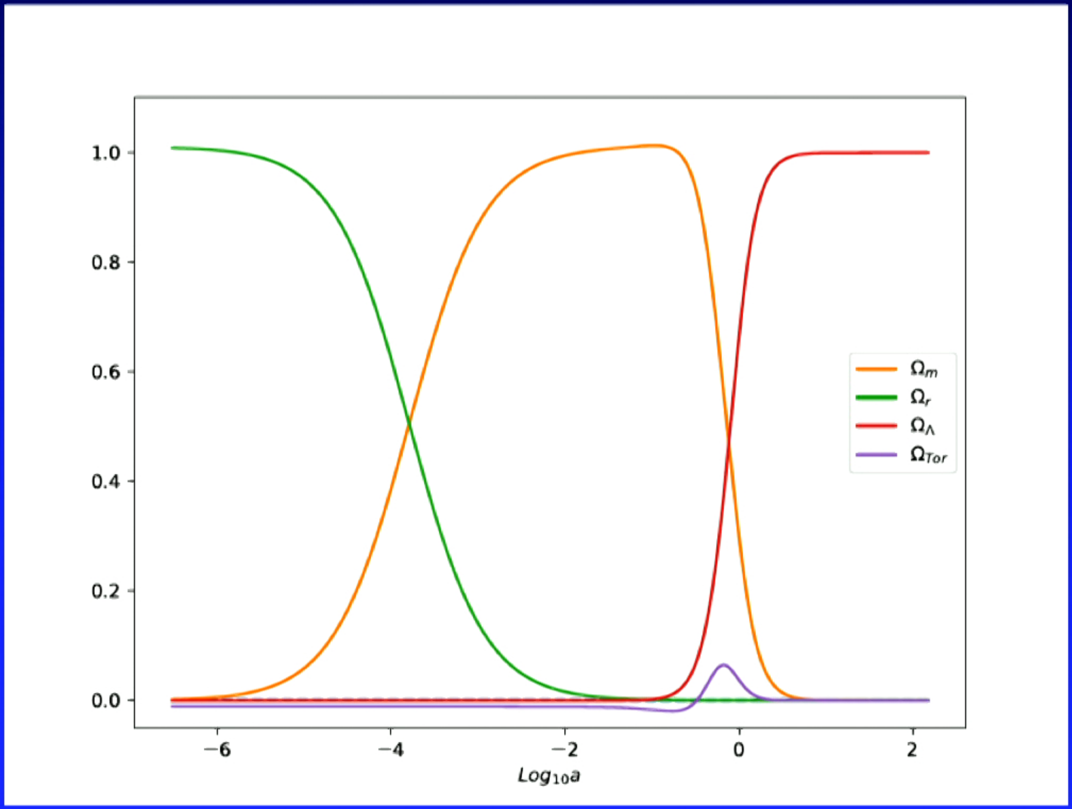
$$\frac{(ag)^{\cdot}}{a} = \frac{\Lambda}{3} - \frac{\kappa}{6}(\rho + 3p)$$

$$T = \frac{\dot{\Lambda}}{2\Lambda^2}\left(\Lambda + \kappa\rho - \frac{6}{\gamma}gP\right)$$

$$P = \frac{3\dot{\Lambda}}{\Lambda^2}\left(gP + \frac{\Lambda + \kappa\rho}{6\gamma}\right)$$

$$(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) - \Lambda^2 = 18gP\frac{(aP)^{\cdot}}{a} + \frac{9}{\gamma}\left(\frac{\Lambda + \kappa\rho}{3}\frac{(aP)^{\cdot}}{a} + \frac{2}{3}\left(\Lambda - \kappa\frac{\rho + 3p}{2}\right)gP\right)$$

An example of a successful cosmology:



International Journal of Modern Physics D | Vol. 24, No. 12, 1543001 (2015)

| Award-Winning Essays

Local conformal symmetry: The missing symmetry component for space and time

Gerard 't Hooft

Abstract

Local conformal symmetry is usually considered to be an approximate symmetry of nature, which is explicitly and badly broken. Arguments are brought forward here why it has to be turned into an exact symmetry that is *spontaneously* broken. As in the BEH mechanism in Yang–Mills theories, we then will have a formalism for disclosing the small-distance structure of the gravitational force. The symmetry could be as fundamental as Lorentz invariance, and guide us towards a complete understanding of physics at the Planck scale.

This essay is awarded first prize in the 2015 Essay Competition of the Gravity Research Foundation.

International Journal of Modern Physics D | Vol. 24, No. 12, 1543002 (2015)

| Award-Winning Essays

Gravity as the breakdown of conformal invariance

Giovanni Amelino-Camelia, Michele Arzano, Giulia Gubitosi and João Magueijo

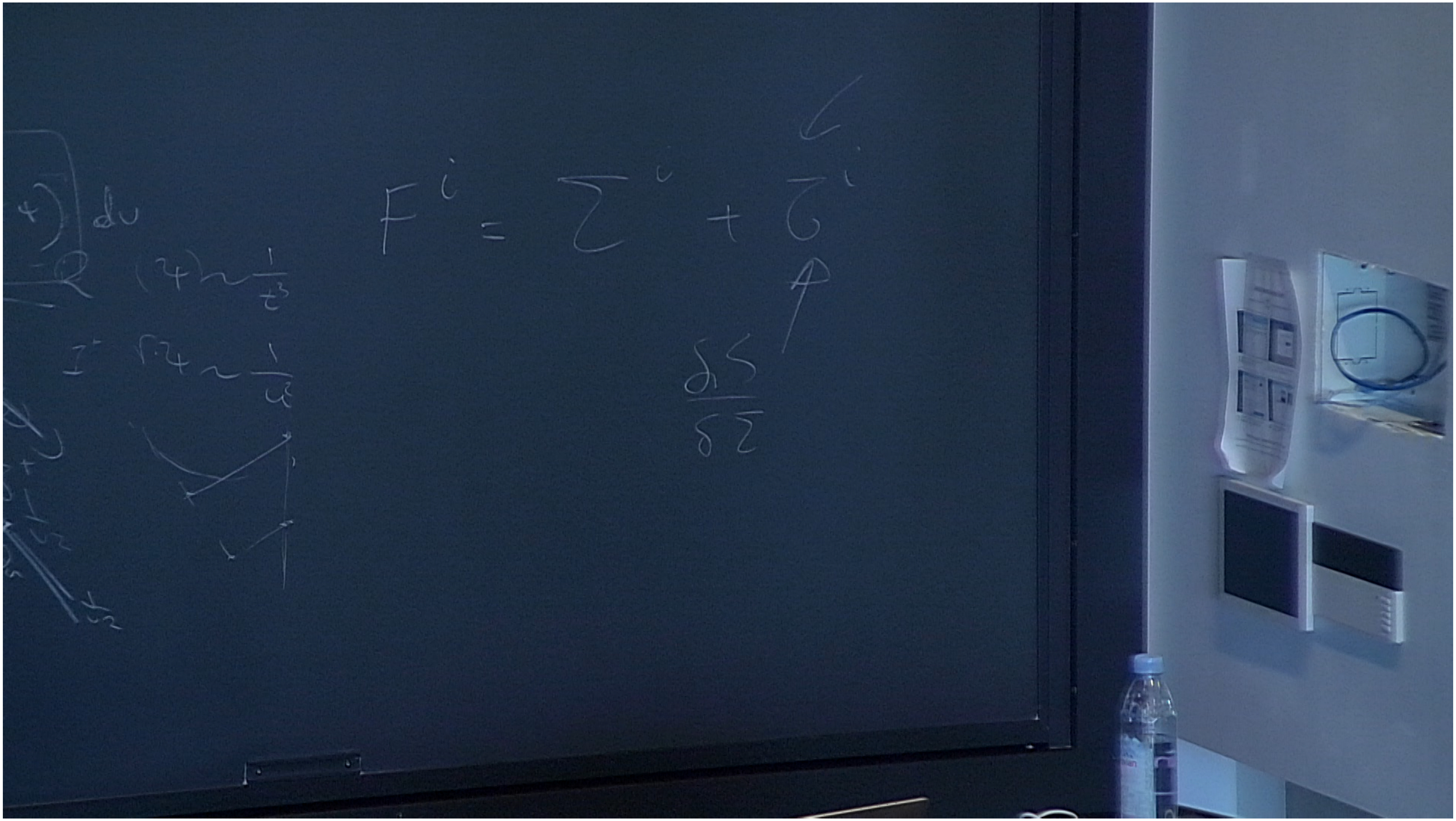
Abstract

In this paper, we propose that at the beginning of the universe gravity existed in a limbo either because it was switched off or because it was only conformally coupled to all particles. This picture can be reverse-engineered from the requirement that the cosmological perturbations be (nearly) scale-invariant without the need for inflation. It also finds support in recent results in quantum gravity suggesting that spacetime becomes two-dimensional at super-Planckian energies. We advocate a novel top-down approach to cosmology based on the idea that gravity and the Big Bang Universe are relics from the mechanism responsible for breaking the fundamental conformal invariance. Such a mechanism should leave clear signatures in departures from scale-invariance in the primordial power spectrum and the level of gravity waves generated.

This essay is awarded second prize in the 2015 Essay Competition of the Gravity Research Foundation.

Another piece to the puzzle…

- It now appears that conformal invariance may be essential to understand the cosmological constant promoted to a dynamical field.
- The cosmological constant may be the gauge field associated with conformal invariance.
- This can be broken spontaneously in pure gravity, in conjunction with spontaneous parity breaking.


F^i = Σ^i + σ̄^i
δS/δΣ

Transposed to modern cosmology this creates a headache: a “timelike tower of turtles”

As a side remark note that without matter in theory 1 solutions are not SD

- Set rho to zero, and let Lambda vary.
- In theory 2 (parity invariant):

$$R^{ab} = \frac{\Lambda(x)}{3} e^a \wedge e^b$$

- In theory 1 this is not true. Solutions can be found in Magueijo and Zlosnik.