Title: Entanglement entropy, quasiparticle fluctuations, and 1D thermal entropy in topological phases

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Series: Condensed Matter

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Abstract: Entanglement entropy in topologically ordered matter phases has been computed extensively using various methods. In this talk, we study the entanglement entropy of 2D topological phases from the perspective of quasiparticle fluctuations. In this picture, the entanglement spectrum of a topologically ordered system encodes the quasiparticle fluctuations of the system, and the entanglement entropy measures the maximal quasiparticle fluctuations on the entanglement boundary. As a consequence, entanglement entropy corresponds to the thermal entropy of the quasiparticles at infinite temperature on the entanglement boundary. We corroborates our results with explicit computation in the quantum double model with/without boundaries. We then systematically construct the reduced density matrices of the quantum double model on generic 2-surfaces with boundaries.

Entanglement entropy in topological phases: quasiparticle fluctuations, and thermal entropy

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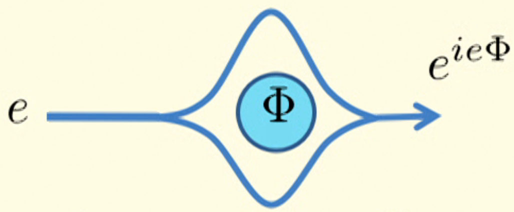
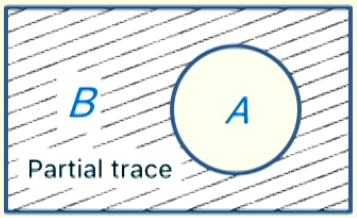
Yuting Hu, Yidun Wan, JHEP 05(2019)110. arXiv:1901.09033

Topological phases

many-body states with topological quantum numbers



Aharanov-Bohm effect
(generally, monodromy of anyons)



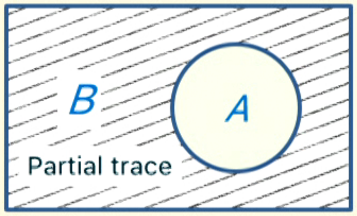
Entanglement entropy
(quantum dimensions, modular S, T matrix)

Effective theories: topological (gauge) field theories

2+1D Discrete model
- (twisted) Quantum double model
- Levin-Wen (string-net) model

Entanglement entropy

$$S_E = -\mathrm{tr}\rho_A \log \rho_A$$
$$\rho_A = \mathrm{tr}_B \left|\Phi\right\rangle \left\langle\Phi\right|$$



Topological phase
- Area law: $S_0 = L \log D$
- topological term $S_1 = -\log D$

Compute

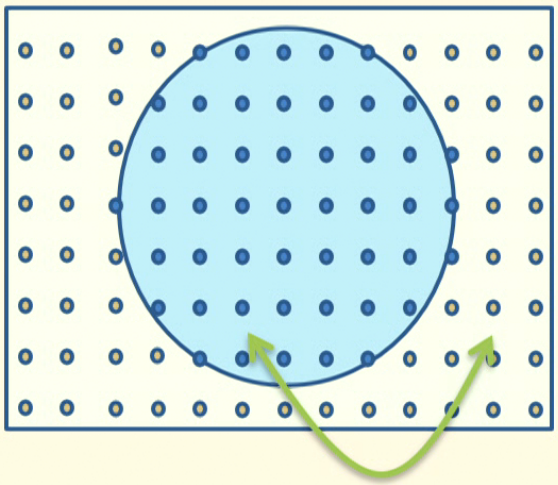
- CFT techniques
- discrete models (QD, Levin-Wen...)

Our perspectives:

- quasiparticle fluctuations

(via gauge symmetry breaking)

Compare: free quantum field theories (bosonic/fermionic)



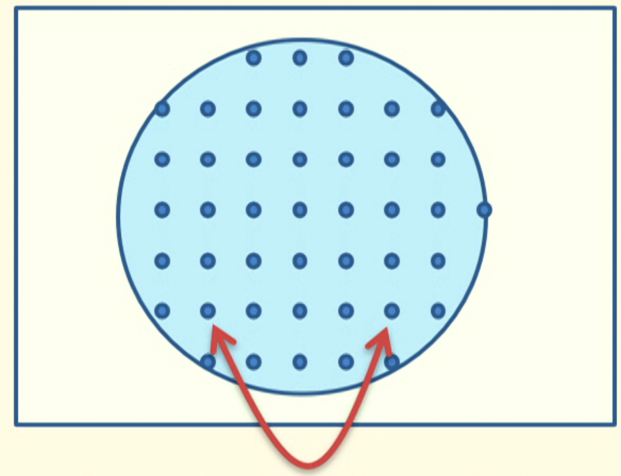
Entanglement

$$\rho_A = \exp\{-\sum_{ij} \beta H_{ji} c_i^\dagger c_j\}$$

(Finite temperature in *A*)

$$\beta H = \ln(\tfrac{1-C}{C})$$

[Peschel 2002]

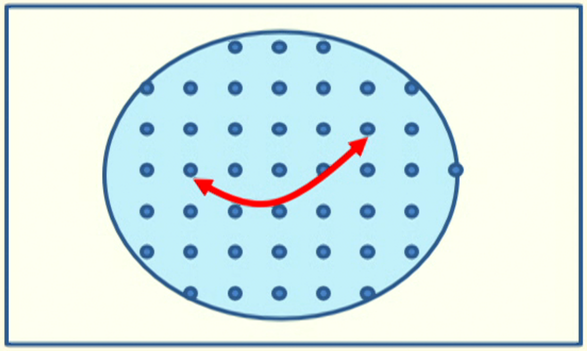


Correlation function

$$C_{ij} = \langle c_i^\dagger c_j \rangle$$

free quantum field theories

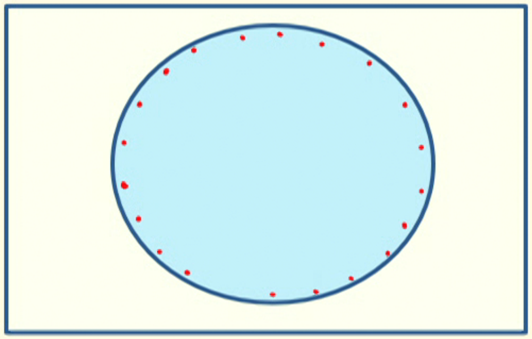
[Peschel 2002]



Hamiltonian → $C_{ij} = \langle c_i^\dagger c_j \rangle$

Finite temperature

Topological phases (topological gauge field theories)?



?

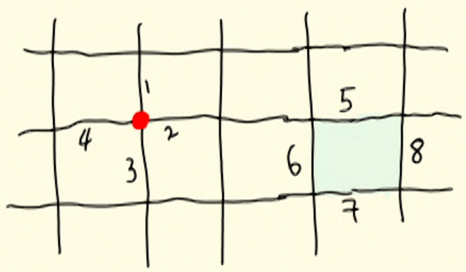
Entanglement:

Gauge Symmetry Breaking

→Quasiparticle fluctuations on the entanglement boundary

Quasiparticles in **Discrete Gauge field theory** $(G = Z_2)$

[Kitaev 2003]
[Levin-Wen 2005]

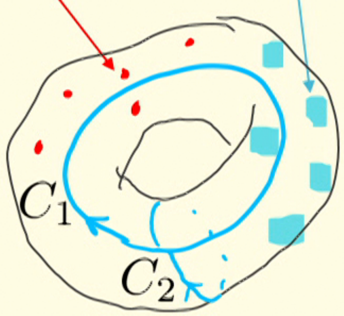


$$H = -\sum_v A_v - \sum_p B_p$$

$$A_v = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$$

$$B_p = \sigma_5^z \sigma_6^z \sigma_7^z \sigma_8^z$$

$A_v = -1$ $B_p = -1$



$$\left\{ \begin{array}{l} A_1, A_2, \ldots = \pm 1 \\ B_1, B_2, \ldots = \pm 1 \\ Z_1 = \prod_{C_1} \sigma^z, Z_2 = \prod_{C_2} \sigma^z \end{array} \right.$$

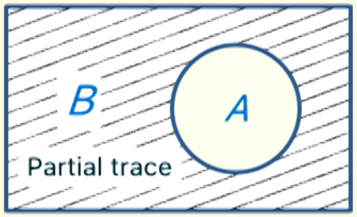
Global Gauss Law

$$\prod_v A_v = 1$$

$$\prod_p B_p = 1$$

Gauge charge + flux + Aharanov-Bohm phase

Entanglement Entropy



Step 1: cut

Schmidt decomposition $|\Phi\rangle = \sum_i \lambda_i \left|\varphi_i^A\right\rangle \otimes \left|\varphi_i^B\right\rangle$

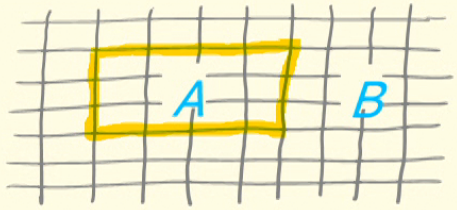
(ground state)

Step 2: partial trace

$$\rho_A = \mathrm{tr}_B \, |\Phi\rangle \langle\Phi| \quad = \sum_i |\lambda_i|^2 \left|\varphi_i^A\right\rangle \left\langle\varphi_i^A\right|$$

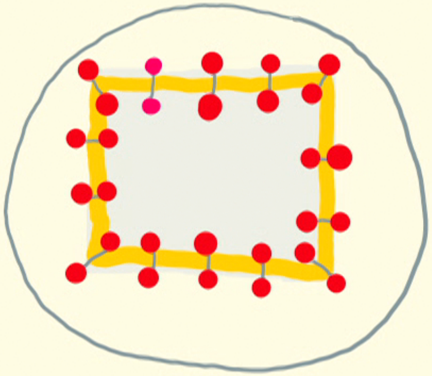
$$S_E = -\mathrm{tr}\rho_A \log \rho_A$$

Step 1: cut



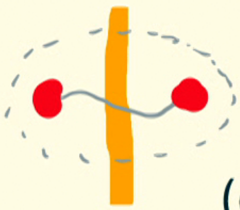
Schmidt decomposition (ground state)

$$|\Phi\rangle = \sum_{\text{all pairs}} (\text{virtual quasiparticle pairs})$$



Local charge pairs

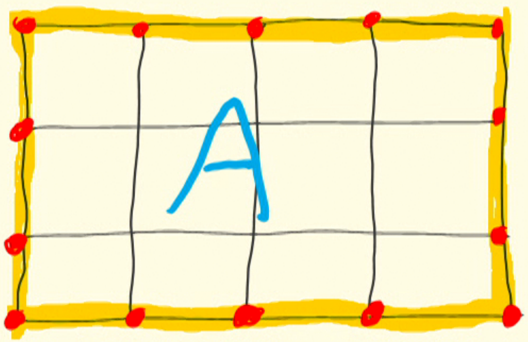
$$\sum_{jm} |j,m\rangle^A \otimes |j^*,m\rangle^B$$



(gauge symmetry)

quasiparticle fluctuation ➔ entangled pairs

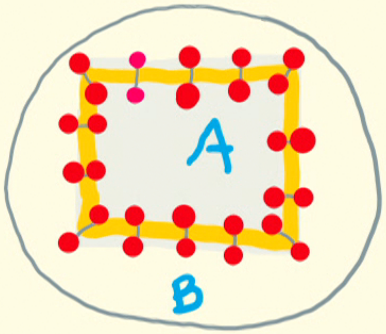
Step 2: partial trace



Quasiparticle states on boundary of subsystem A

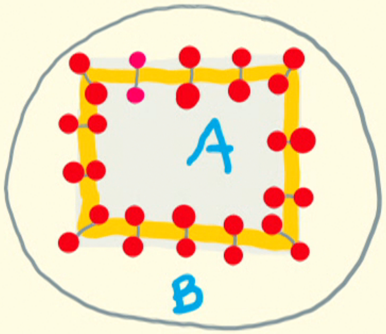
$$\rho_A = ?$$

Step 2: partial trace ➔ Gauge symmetry breaking



Gauge symmetry
in ground state
<entire system>

Step 2: partial trace ➔ Gauge symmetry breaking



Gauge symmetry
in ground state
<entire system>

Partial trace

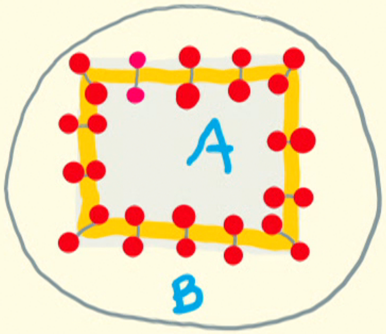
Gauge symmetry breaking
<subsystem boundary>

Charges

Global symmetry on charges
<subsystem boundary>

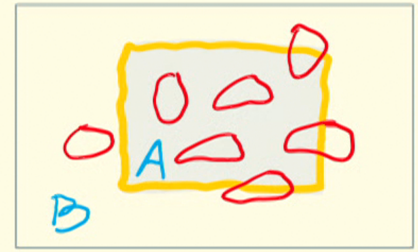
$$P_G = \frac{1}{|G|} \sum_{g \in G} U(g) \otimes U(g) \otimes U(g) \dots$$

Step 2: partial trace ➔ Gauge symmetry breaking



Gauge symmetry
in ground state
<entire system>

Partial trace



Wilson loops

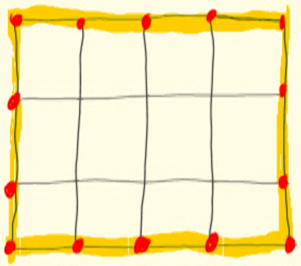
Gauge symmetry breaking
<subsystem boundary>

Charges

Global symmetry on charges
<subsystem boundary>

$$P_G = \frac{1}{|G|} \sum_{g \in G} U(g) \otimes U(g) \otimes U(g) \ldots$$

Step 2: partial trace



Subsystem A

$$\rho_A = P_G P_0 P_G$$

P_0 Projects onto

quasiparticle states on entanglement boundary

$$P_G = \frac{1}{|G|}\sum_{g\in G} U(g)\otimes U(g)\otimes U(g)\ldots$$

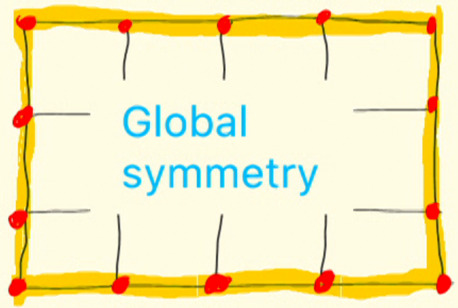
Projects onto states invariant under **global symmetry**

$S_E = \ln W$ (state counting $W = |G|^{L-1}$)

$S_E = S_0 + S_1 = L\ln|G| - \ln|G|$ [Preskill Kitaev 2006] [Levin Wen 2006]

Charges L = 14 (on entanglement boundary)

State counting $S_E = \ln W$

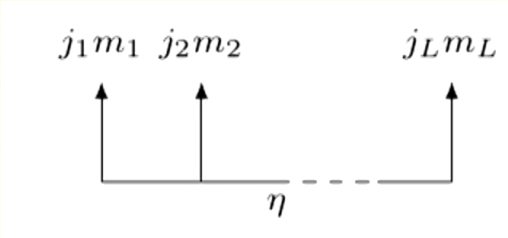


Gauge symmetry broken on boundary

(breaks virtual quasiparticle pairs)

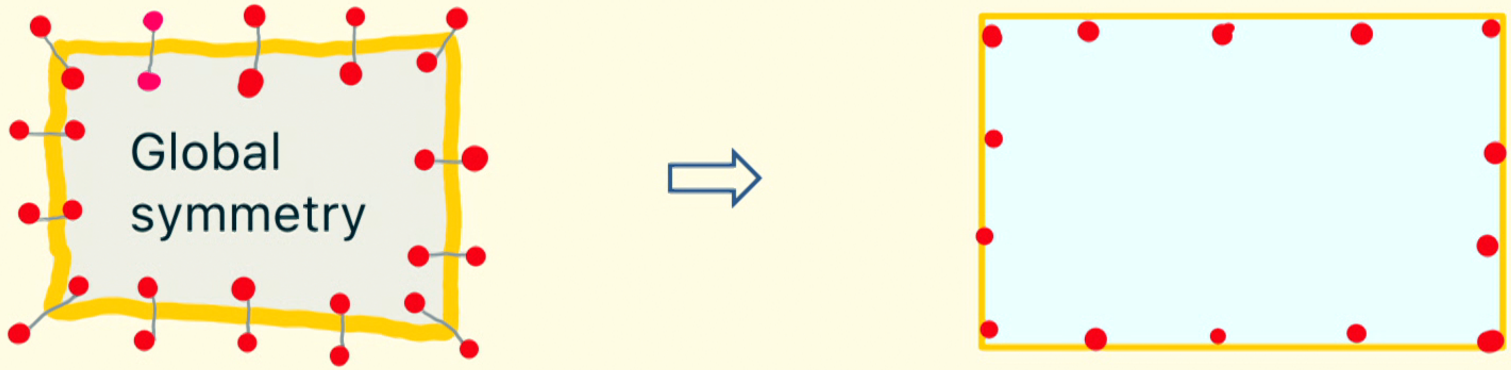
Entanglement entropy

detected quasiparticles (on subsystem boundary)



Eigenvectors of ρ_A

Representation formulation: $|jmn\rangle = \sqrt{\frac{d_j}{|G|}} \sum_{g \in G} \overline{\rho^j_{mn}(g)}\, |g\rangle$



Schmidt decomposition

1D Grand canonical ensemble $(T = \infty)$

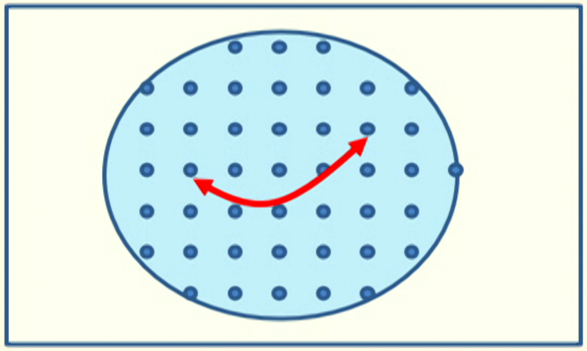
$$\rho_A = e^{-\beta \sum_\alpha n_\alpha (\epsilon_\alpha - \mu_\alpha)}$$

$$\mu_\alpha = 0, \beta \to 0$$

2D Entanglement entropy = 1D Entropy with $T = \infty$

free quantum field theories

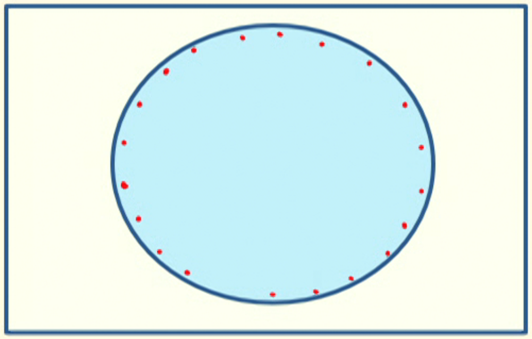
[Peschel 2002]



"Hamiltonian" → $C_{ij} = \langle c_i^\dagger c_j \rangle$

Finite temperature

topological gauge field theories

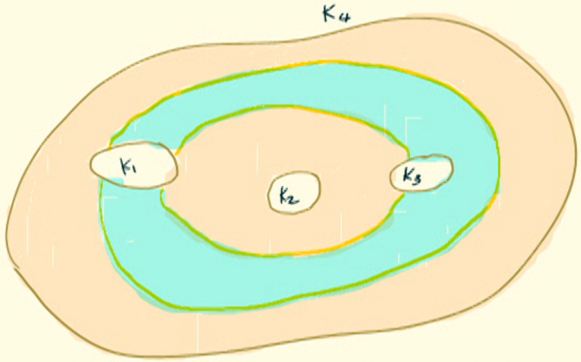


$$\rho_A = P_G P_0 P_G$$

$$T = \infty$$

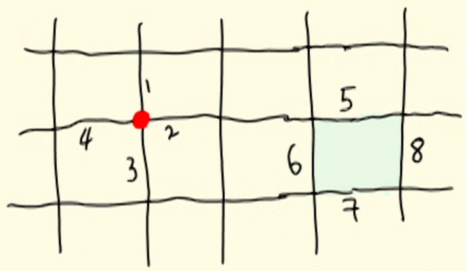
Entanglement = state counting

How generic?



Gapped boundaries

[Beigi-Shor-Whalen 2011]
[Bullivant Hu Wan 2017]



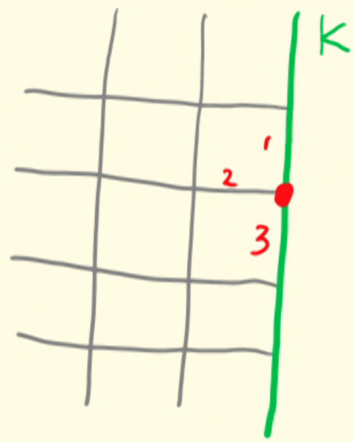
$$H_{bulk} = -\sum_v A_v - \sum_p B_p$$

$$A_v = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$$

$$B_p = \sigma_5^z \sigma_6^z \sigma_7^z \sigma_8^z$$

$$H_{\mathrm{bdry}} = -\sum_v \bar{A}_v$$

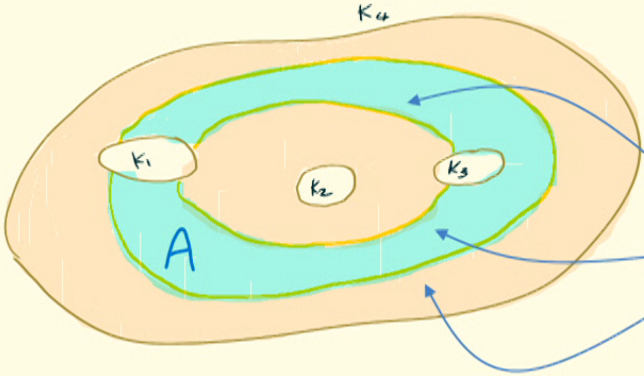
$$\bar{A}_v = \sigma_1^x \sigma_2^x \sigma_3^x$$



$$\bar{A}_v = \frac{1}{|K|} \sum_{k \in K} \bar{A}_v^k$$

$$K \subseteq G$$

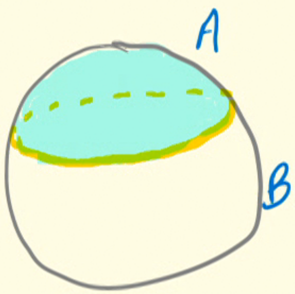
$$\bar{A}_v^k \left| \; g_2 \; k_1 \; k_3 \right\rangle = \left| k g_2 \; k k_1 \; k k_3 \right\rangle$$



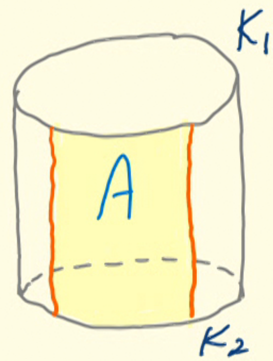
$$\rho_A = P_A P_B P_0 P_B P_A$$

P_0 : projection

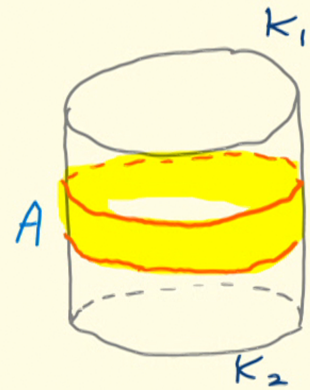
quasiparticles on entanglement boundary



P_A: G-symmetry



P_A: G-symmetry

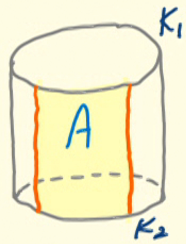


P_A: G-symmetry

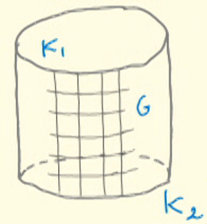
P_B: K_1 symmetry on top
K_2 symmetry on bottom

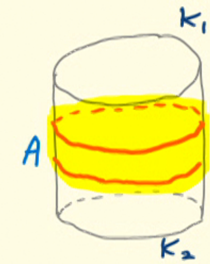
$$\rho_A = P_A P_B P_0 P_B P_A$$

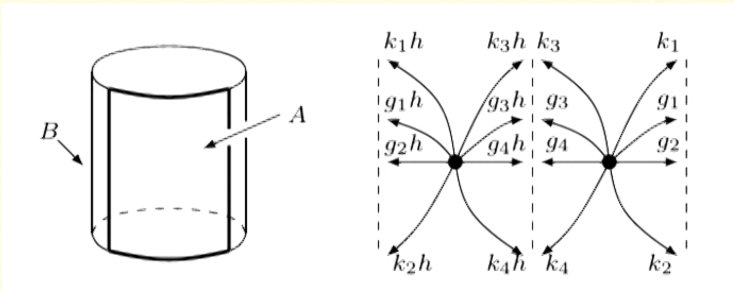
$$S_E = S_0 + S_1$$



P_A: G-symmetry

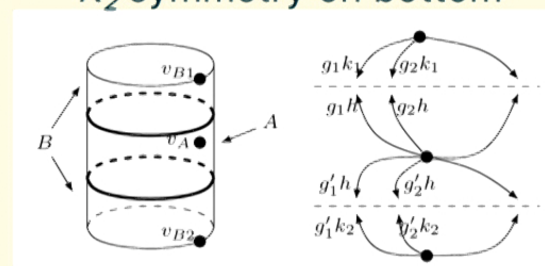






P_A: G-symmetry

P_B: K_1 symmetry on top
K_2 symmetry on bottom



$$S_0 = L\log|G| + 2\log|K_1| + 2\log|K_2|$$

$$S_1 = -\log|K_1K_2|$$

$$S_0 = L\log|G| \qquad N_\eta = \frac{1}{|K_1||K_2|}\frac{1}{\dim_\eta}\sum_{k_1k_2}\chi_\eta(k_1k_2)$$

$$S_1 = -\log|G| - \log|K_1K_2| - \sum_{\eta\in\mathrm{Rep}_G}\frac{d_\eta^2|K_1K_2|}{|G|}N_\eta\log N_\eta$$

Levin-Wen model

finite groups → quantum groups

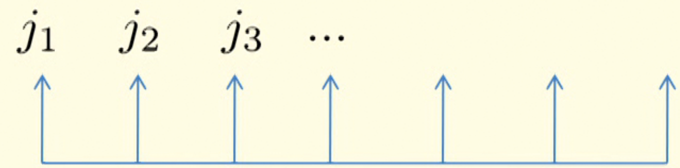
d.o.f.: representations

Gauge symmetry

→ invariant tensor in Tensor network state

(6j symbols)

Weighted state counting



$$\sum d_{j_1} d_{j_2} d_{j_3} \dots$$

Mathematically

ρ_A is a morphism in $\mathrm{Hom}(Q^{\otimes L}, Q^{\otimes L}), Q = \oplus_j j$

2D Entanglement entropy = 1D Entropy with $T = \infty$

$$\rho_A = e^{-\beta \sum_\alpha n_\alpha (\epsilon_\alpha - \mu_\alpha)}$$

fugacity $e^{\beta \mu_\alpha} = d_\alpha, \beta \to 0$

[Zhu-Xi Luo, Brendan G. Pankovich, Yuting Hu, Yong-Shi Wu, Phys. Rev. B 99, 205137 (2019). arXiv: 1806.07794]

Quasiparticle approach to Entanglement

Entanglement
Spectrum/Entropy

Gauge symmetry breaking
(Anyon condensation)

Boundary theory

1D entropy $T = \infty$

Quasiparticle
fluctuations

Yuting Hu, Yidun Wan, JHEP 05(2019)110. arXiv:1901.09033