Title: Observational signatures for extremal black holes

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Abstract: The event horizon and the Cauchy horizon of an extremal black hole admit conserved charges associated with scalar perturbations. We will see that these charges are externally measurable from null infinity. This suggests that these charges have the potential to serve as an observational signature for extremal black holes. The proof of this result is based on obtaining precise late-time asymptotics for the radiation field of outgoing perturbations.

Observational signatures for extremal black holes

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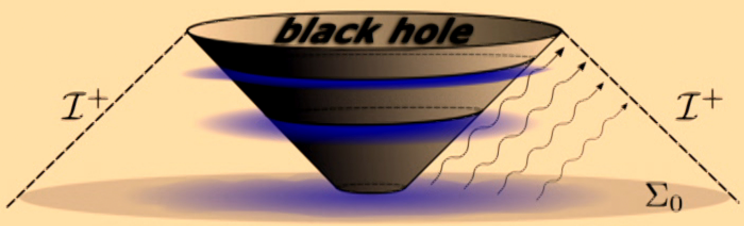
The main question

- Is there an *observational signature* for **extremal** black holes?
- That is, is there a distinguishing feature in the dynamics of perturbations of extremal black holes? If yes, is this *measurable from future null infinity*?



Why extremal black holes?

Theory

- Non-genericity versus reality: $a = M, q = M$, where M mass, q charge, a angular momentum.
- Uniqueness properties (Chruściel, Figueras)
- Mass minimizers (Dain, Kunduri)
- Applications in supersymmetry, quantum gravity, string theory (Strominger, Vafa)
- Electromagnetic and gravitational signatures (Gralla, Lupsasca, Porfyriadis, Khanna)
- Turbulent gravitational behavior (Lehner)

Why extremal black holes?

Practice

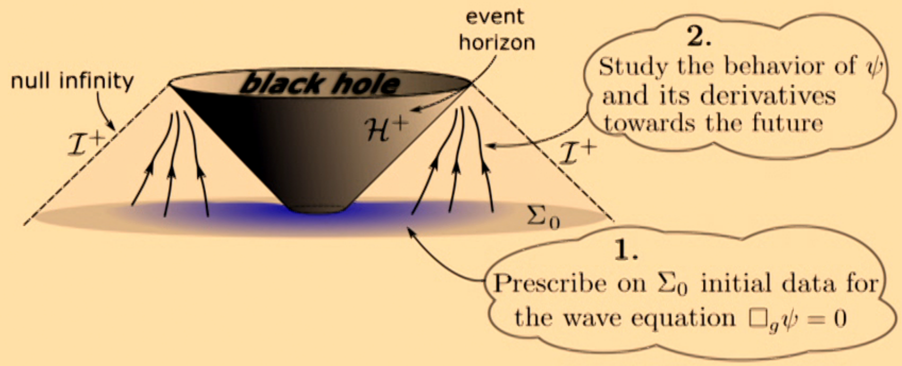
- Vast astronomical evidence for near-extremal black holes.
- Rees et al. (*The distribution and cosmic evolution of massive black hole spins*, Astrophys. J.) report that "the spin distribution is heavily skewed toward fast-rotating Kerr black holes" and that "about 70% of all stellar black holes at all epochs are maximally rotating". Gas accretion dominant effect and spins black holes up.
- First black hole candidate (1971) Cygnus X-1: Gou et al. (*Confirmation via the continuum-fitting method that the spin of the black hole Cygnus X-1 is extreme*, Astrophys. J.).
- Many examples of stellar and supermassive near-extremal black holes.
- We refer to: Brenneman (*Measuring the angular momentum of supermassive black holes*, Springerbrief 2013).
- Highly spinning black holes in Advanced LIGO data (Zackay et al).

Scalar perturbations

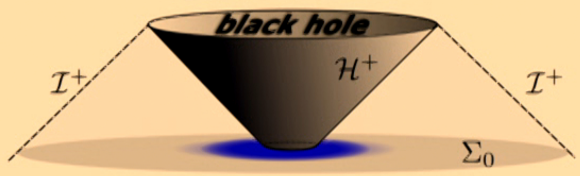
- Investigate the evolution of solutions to the wave equation

$$\Box_g \psi = 0$$

on Reissner–Nordström backgrounds.

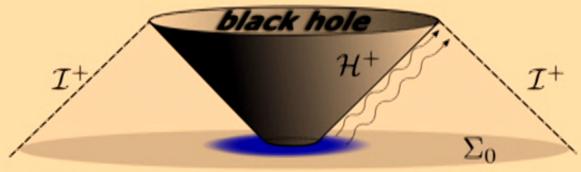


- We assume that the scalar perturbation ψ is initially supported near the event horizon.



Preliminary remarks

For horizon-localized perturbations to register on null infinity we would need measurements along null infinity at very late times.



This suggests that we need to derive the **precise** late-time asymptotics for scalar perturbations and their radiation field. In particular, we need to "see" the *power-law* tails of scalar perturbations (at null infinity). Beyond QNMs.

Define on null infinity:

$$s[\psi](u,\vartheta) := \frac{1}{4M}u^2 \cdot \Big((r\psi)|_{\mathcal{I}^+}(u,\vartheta) \Big) + \frac{1}{8\pi}\int_{\mathcal{I}^+\cap\{u\geq\bar{u}\geq 0\}} r\psi|_{\mathcal{I}^+}(\bar{u},\vartheta)d\bar{u}d\vartheta$$

Here u is a retarded time on $\mathcal{I}^+$. It turns out that for both sub-extremal RN and ERN we have

$$\lim_{\tau\to\infty} s[\psi](u,\vartheta) \to s[\psi] < \infty$$

A signature of extremality at null infinity

The main result is the following:

For all scalar perturbations on sub-extremal RN we have $s[\psi] = 0$

Moreover,

For generic perturbations of ERN we have $s[\psi] \neq 0$

Concluding,

If $s[\psi] \neq 0$ then the black hole is ERN

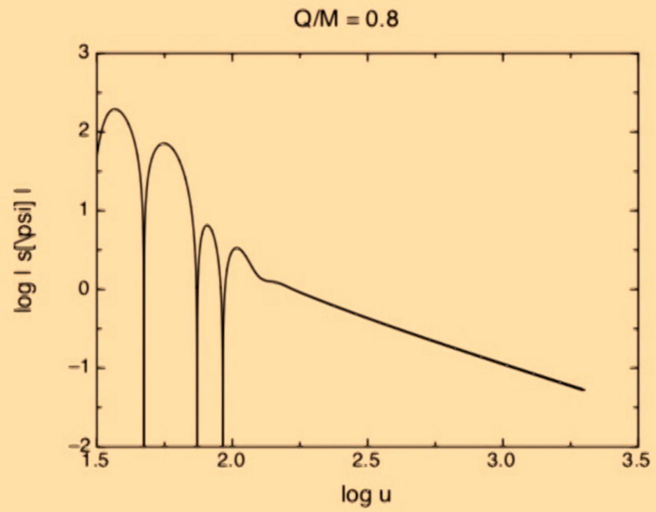
Is it possible to numerically confirm this result?

Yes. Burko, Khanna, Sabharwal can simulate the evolution of the radiation field for long time which enables them to confirm our result and obtain several interesting extensions.

[Figures below are courtesy of Khanna et al who made use of the supercomputing resources of UMass Dartmouth's CSCVR center]

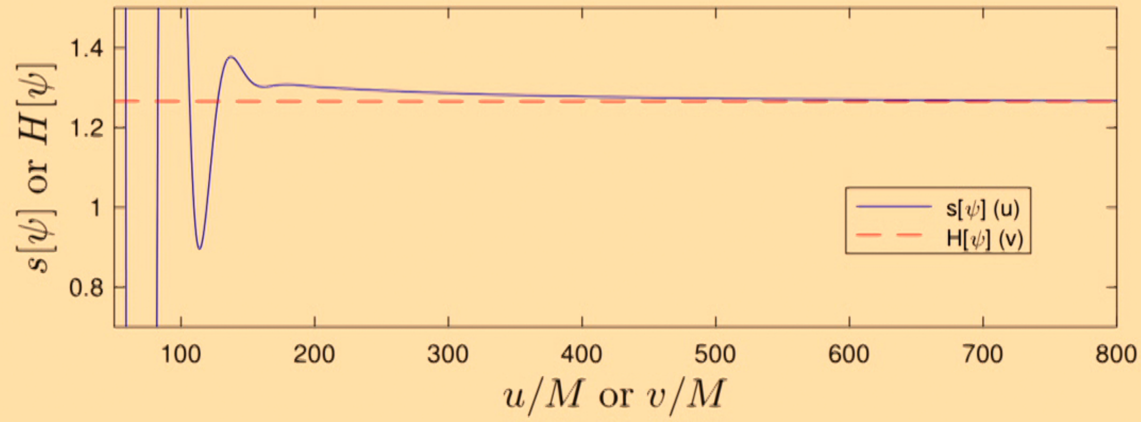
Numerical confirmation

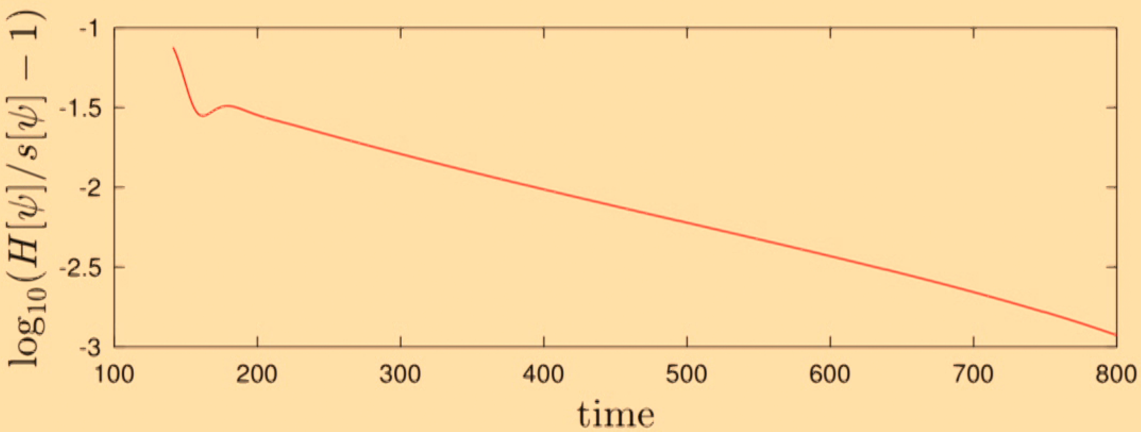
The plot below confirms that $s[\psi](u,\vartheta) \to 0$ as $u \to \infty$ for sub-extremal RN.



ERN simulation

For perturbations of ERN, Khanna et al obtain



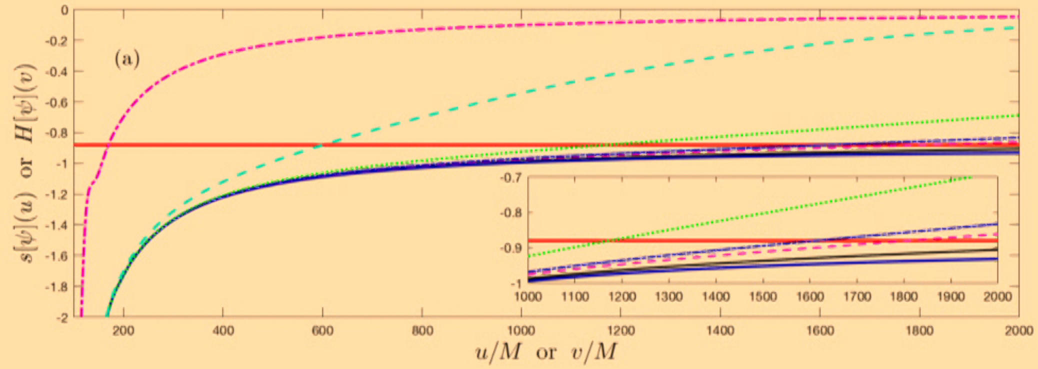


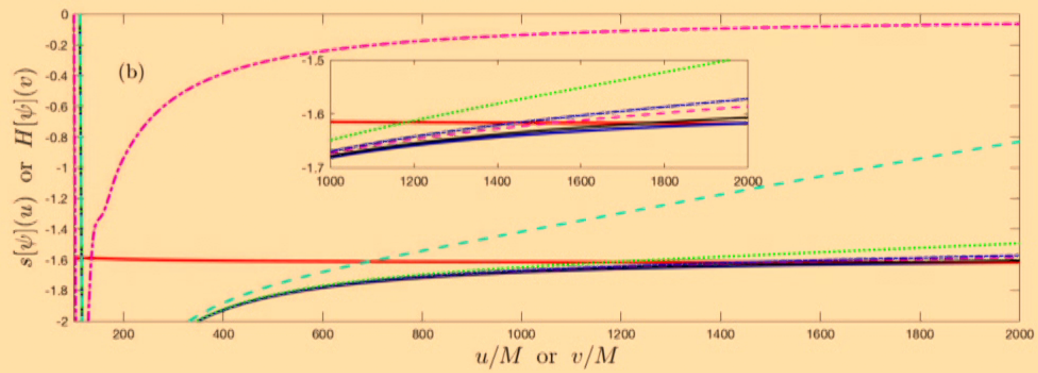
Hence, $s[\psi](u, \vartheta) \to c$ where $c \neq 0$ as $u \to \infty$ on ERN.

Transient behavior of the signature $s[\psi](u)$

The plot of $s[\psi](u)$ for various values of a/M (plot (a)) and q/M (plot (b)). From bottom to top the values of either $1 - \frac{a}{M}$ or $1 - \frac{q}{M}$ are:

$0,\ 4.5 \cdot 10^{-8},\ 1.25 \cdot 10^{-7},\ 1.8 \cdot 10^{-7},\ 5 \cdot 10^{-6},\ 4.5 \cdot 10^{-6},\ 5 \cdot 10^{-5}.$



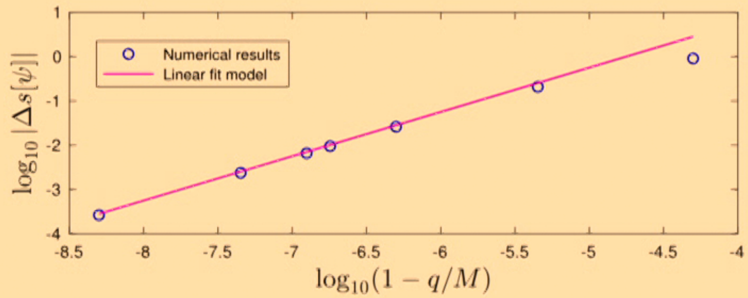


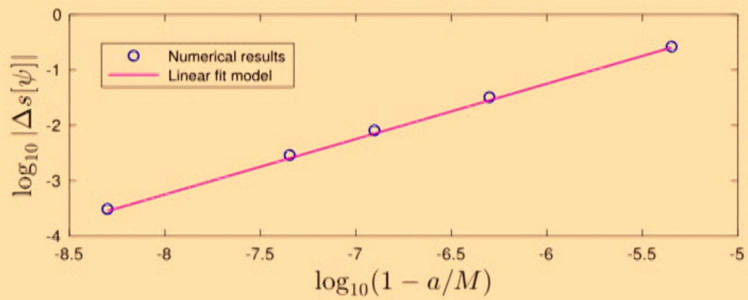
To capture the transient signature more appropriately we define

$$\Delta s[\psi](u; a/M) = s[\psi](u; a/M) - s[\psi](u; a/M = 1)$$

Signature for near-extremal black holes I: fixed intermediate u

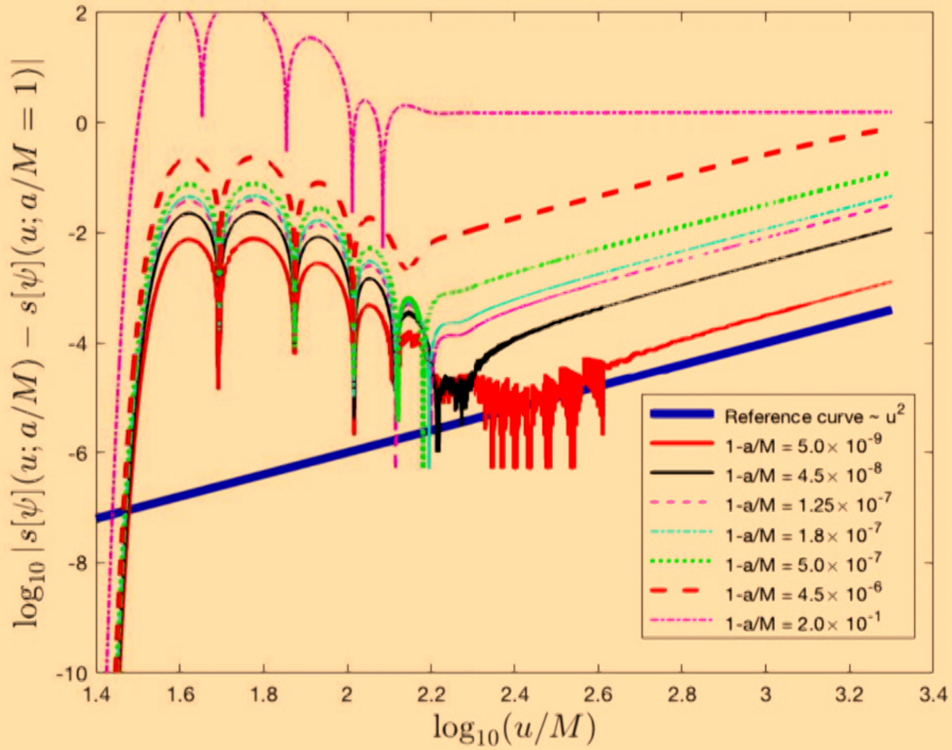
Plot of $\Delta s[\psi](u; a/M)$ for a fixed intermediate time u. Slope of lines $=1$.





Signature for near-extremal black holes II: fixed $1 - a/M$

Plot of $\Delta s[\psi](u; a/M)$ for all intermediate times u.



Transient signature formula

Combining the previous results yield for intermediate retarded times (soon after the QNM phase at u=100M and until around u=2000M) the following

$$s[\psi](u,a/M) = s[\psi](u,a/M=1) + 0.065u^2 \cdot \left(1 - \frac{a}{M}\right)$$

for NERN and

$$s[\psi](u,q/M) = s[\psi](u,q/M=1) + 0.15u^2 \cdot \left(1 - \frac{q}{M}\right)$$

for NEK. The "transient hair" grows quadratically in the intermediate regime until its length becomes short and eventually vanishes. For values less or equal to $1 - \frac{q}{M} = 10^{-6}$ this transient behavior is present.

Next: Physical meaning of $s[\psi]$. Background and some ideas

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Late-time asymptotics

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a sub-extremal Reissner–Nordström space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$8I^{(1)}[\psi]\cdot\tau^{-3}$	$8I^{(1)}[\psi]\cdot\tau^{-3}$	$-2I^{(1)}[\psi]\cdot\tau^{-2} - 8MI^{(1)}[\psi]\log\tau\cdot\tau^{-3}$

where

$$I^{(1)}[\psi] = \frac{M}{4\pi}\int_{\{t=0\}\cap S_{\mathrm{BF}}} \psi\, d\Omega + \frac{M}{4\pi}\int_{\{t=0\}} \frac{1}{1-\frac{2M}{r}} \partial_t\psi\, r^2 dr d\Omega.$$

Comments:

- Generically $I^{(1)}[\psi] \neq 0$
- We further obtain $(2\ell+3)$-asymptotics (Price's law).

$I^{(1)}[\psi]$ in terms of the radiation field on $\mathcal{I}^+$

$$I^{(1)}[\psi] = \frac{M}{4\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi$$

Weak-field dynamics

- In view of the expression of $I^{(1)}[\psi]$, in terms of the radiation field, we obtain that the late time tails are dictated by the weak-field dynamics, namely by dynamics at very large r.

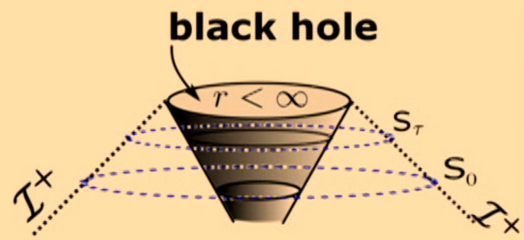
$$\lim_{\tau\to\infty}\left(\tau^2\cdot(r\psi)|_{\mathcal{I}}\right) = -\frac{M}{2\pi}\int_{\mathcal{I}^+\cap\{\tau\geq 0\}} r\psi|_{\mathcal{I}},$$
$$\lim_{\tau\to\infty}\left(\tau^3\cdot\psi|_{r=R}\right) = \frac{2M}{\pi}\int_{\mathcal{I}^+\cap\{\tau\geq 0\}} r\psi|_{\mathcal{I}},$$
$$\lim_{\tau\to\infty}\left(\tau^3\cdot\psi|_{\mathcal{H}}\right) = \frac{2M}{\pi}\int_{\mathcal{I}^+\cap\{\tau\geq 0\}} r\psi|_{\mathcal{I}},$$

- The first identity above implies that for any sub-extremal RN we have

$$\boxed{s[\psi]=0}$$

The coefficient $I^{(1)}$ and the Newman–Penrose constant

- The Newman–Penrose constant gives rise to a conservation law along null infinity.



The constant is equal to

$$NP[\psi] = \int_{S_\tau} \lim_{r\to\infty} r^2 \cdot \partial_v(r\psi)$$

- $NP[\psi] = 0$ if ψ has compactly supported initial data.
- Remark: $NP[\partial_t\psi] = 0$. So, $NP[\psi]$ is an obstruction to inverting $\partial_t\psi$.
- If $NP[\psi] = 0$ then we can canonically define $\partial_t^{-1}\psi$ (as long as $\partial_t \neq 0$) and, generically, obtain

$$NP[\partial_t^{-1}\psi] \neq 0.$$

- In fact: $I^{(1)}[\psi] = NP[\partial_t^{-1}\psi]$ and can be determined in terms of the initial data of ψ or the radiation field of ψ.
- $I^{(1)}$ is an obstruction to inverting ∂_t^2.

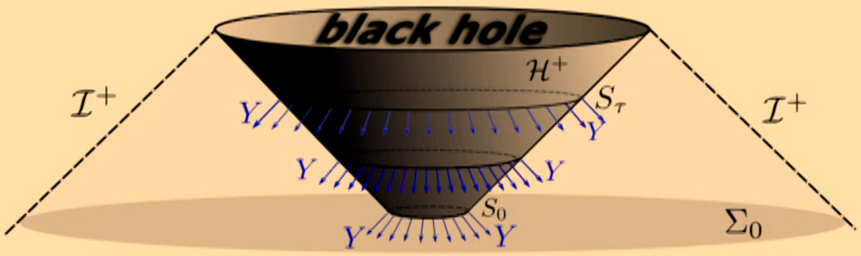
Firstly, we have the following

Proposition (A.)

If ψ satisfies the wave equation on extremal Reissner–Nordström then the integral

$$H[\psi] = -\int_{S_\tau} \left(Y\psi + \frac{1}{2M}\psi \right) dvol$$

is independent of τ. Here Y is transversal to the horizon.



“Outgoing radiation”

Solutions ψ with $H[\psi] \neq 0$ and compactly supported initial data

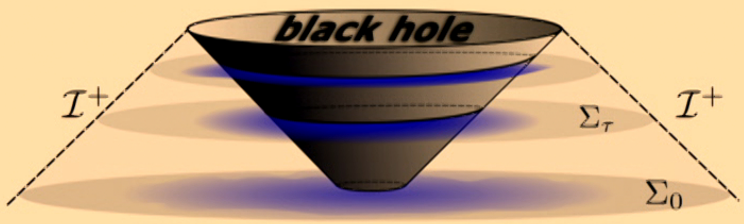
$H[\psi]$ as a "horizon hair"

- Outgoing perturbations and perturbations ($H[\psi] \neq 0$) satisfy along the event horizon:

  1) **Slow decay**: $\psi(\tau, \vartheta) \sim 2H[\psi] \cdot \frac{1}{\tau}$
  2) **Non-decay**: $Y\psi(\tau, \vartheta) \sim -\frac{1}{M} H[\psi]$
  3) **Blow-up**: $YY\psi(\tau, \vartheta) \sim \frac{1}{M^3} H[\psi] \cdot \tau$

- $H[\psi]$: "horizon" "hair" since

  1) Energy density measured by incoming observers: $\boldsymbol{T}_{rr}[\psi] \sim H[\psi]$ where $\boldsymbol{T}$ is the E-M tensor,
  2) $|Y^k\psi|, |\boldsymbol{T}_{rr}[\psi]| \leq 0$ away from the horizon.



- Later extensions/applications by: Reall, Murata, Casals, Zimmerman, Gralla, Tanahashi, Bizon, Lucietti, Angelopoulos, Gajic, Ori, Sela, Tsukamoto, Kimura, Harada, Hadar, Dain, Dotti, Godazgar, Burko, Khanna, Bhattacharjee, Cvetic, Pope, Chow, Berti et al, Cardoso et al,...

Measurements at $\mathcal{I}^+$

Let's consider outgoing radiation.

- Along $\mathcal{I}^+$: The horizon hair registers in the asymptotics

$$r\psi|_{\mathcal{I}} \sim \left(4MH - 2I^{(1)}\right) \cdot \tau^{-2}.$$

Recalling that $I^{(1)} = \frac{M}{4\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi \, d\Omega d\tau$ yields precisely that

$$\frac{1}{4M} \lim_{\tau \to \infty} \tau^2 \cdot (r\psi)|_{\mathcal{I}^+} + \frac{1}{8\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}^+} = H[\psi]$$

or

$$\boxed{s[\psi] = H[\psi]}$$

Observational signature $s[\psi]$ on $\mathcal{I}^+$

Recall that

$$s[\psi] = \frac{1}{4M} \lim_{\tau \to \infty} \tau^2 \cdot (r\psi)|_{\mathcal{I}^+} + \frac{1}{8\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}^+}$$

and

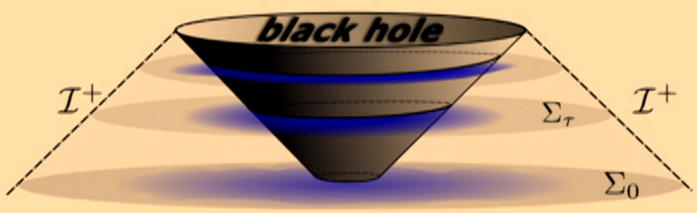
$$s[\psi] = H[\psi]$$

We conclude that

- **Information "leaks" from the event horizon of extremal black holes to null infinity.**
- **Extremal black holes admit classical externally measurable hair.**
- **The horizon hair $H[\psi]$ could potentially serve as an observational signature.**

Physics Literature

- Work by Reall, Murata and Tanahashi suggests that perturbations of initial data of extremal R–N in the context of the Cauchy problem for the Einstein–Maxwell-scalar field equations exhibit a version of the horizon instability.
- Work by Casals–Gralla–Zimmerman and subsequently by Hadar–Reall obtained that the decay rate for non-zero azimuthal frequencies along the event horizon on extremal Kerr is $\frac{1}{\sqrt{\tau}}$ and for the first-order transversal derivative is $\sqrt{\tau}$ (amplified instability).
- Recent work by Gralla–Zimmerman provided a systematic approach to decay (or growth) rates of modes of fields in terms of their scaling properties on extremal backgrounds.



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Thank you!