Title: Categorical Hikita Duality

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Series: Mathematical Physics

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Abstract:

I will discuss joint work with Roman Bezrukavnikov on a categorical version of Hikita duality, which relates coherent sheaves on a symplectic resolution to constructible sheaves on the loop space of the dual resolution. I will focus on a basic case, where this can be made very explicit, and finish with some wild speculation on further generalisations.

jt w/ R. Bezrukavnikov.

torus $T \hookrightarrow N$

d Gauge theory $\longrightarrow$ higgs branch: $X_h = T^*N /\!/\!/_{\theta}\, T \longrightarrow Y_h = T^*N /\!/ T$

smooth symplectic

singular affine

coulomb branch $X_c \overset{(BFN)}{=} \mathrm{Proj}\, R_\alpha(T,N) \longrightarrow Y_c$

$\longrightarrow Y_h = T^*N /\!/ T$

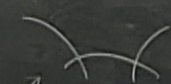
singular affine

$\longrightarrow Y_c$

Ex: $T = \mathbb{C}^* \hookrightarrow \mathbb{C}^3$

$X_h = T^*\mathbb{P}^2 \longrightarrow$ singular cone (contract $\mathbb{P}^2$)

$X_c =$ resolution



$\longrightarrow \{xy = z^3\}$

exceptional fiber

Hil

Hikita's conjecture:

$$H^\bullet(X_{h}, \mathbb{C}) \overset{\text{as rings}}{\cong} \mathcal{O}(Y_c^{\check{T}})$$

Def: $\mathcal{O}(Y_c^{\check{T}}) = \mathcal{O}(Y)/_{y - \tau\cdot y}$ $\quad \tau \in \check{T}$.

Ex: $T = \mathbb{C}^\times \curvearrowright \mathbb{C}^3$

$$H^\bullet(X_h) = \mathbb{C}[u]/u^3$$

Hikita's conjecture:

$$H^\bullet(X_h, \mathbb{C}) \overset{\text{as rings}}{\cong} \mathcal{O}(Y_c^{\check{T}})$$

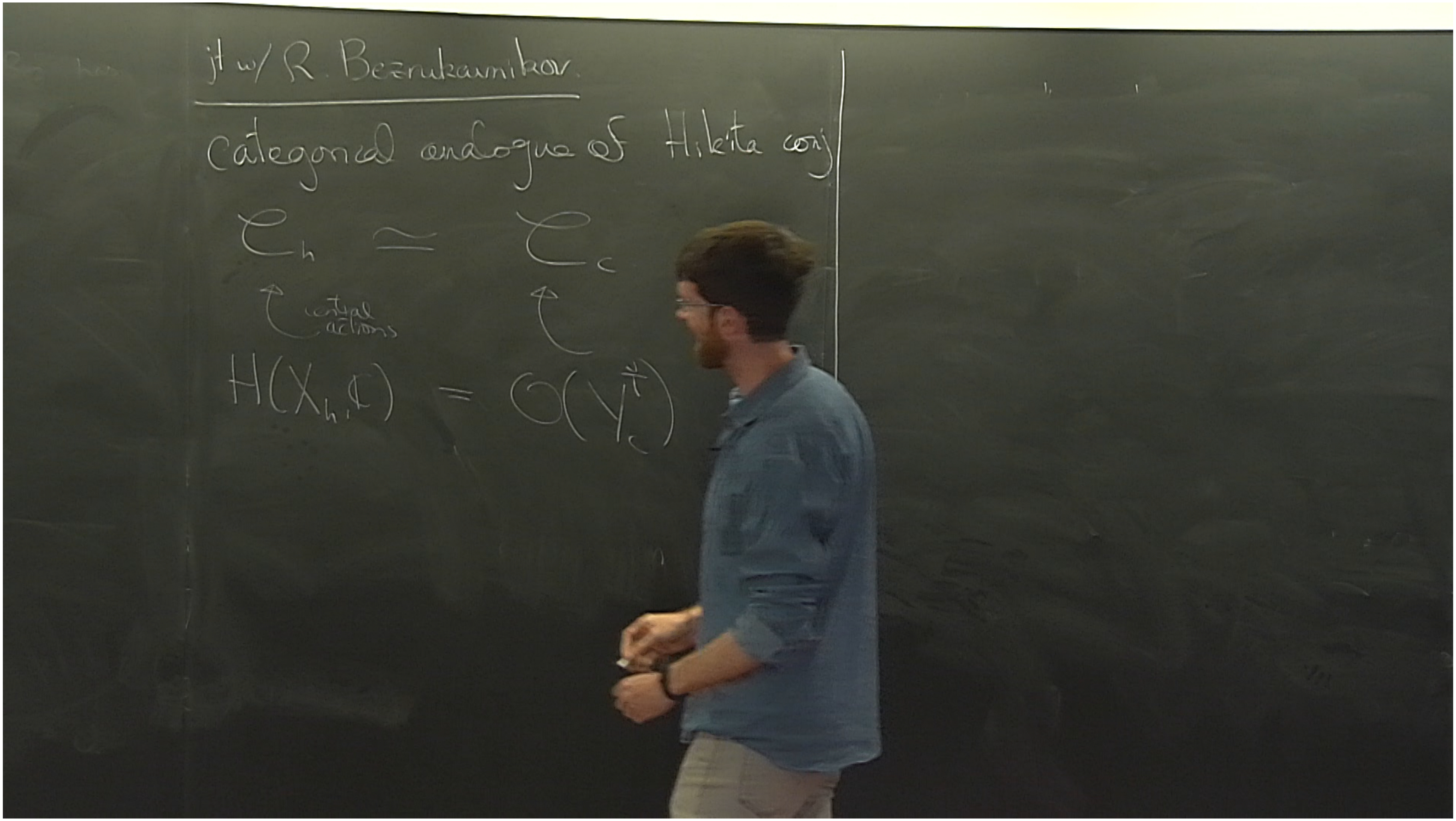
Def. $\mathcal{O}(Y_c^{\check{T}}) = \mathcal{O}(Y) / y - \tau \cdot y \qquad \tau \in \check{T}.$

Ex: $T = \mathbb{C}^* \hookrightarrow \mathbb{C}^3$

$$H^\bullet(X_h) = \mathbb{C}[u]/u^3.$$

$$\mathcal{O}(Y_c^{\check{T}}) = \mathbb{C}[x,y,z]/xy = z^3,\ x=0\ \ y=0$$

$$= \mathbb{C}[z]/z^3.$$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categorical analogue of Hikita conj
$\mathcal{C}_h \simeq \mathcal{C}_c$
central actions
$H(X_h, \mathbb{C}) = \mathcal{O}(Y_c^{\mathbb{T}})$

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categorical analogue of Hikita conj

$$\mathcal{C}_h \simeq \mathcal{C}_c$$

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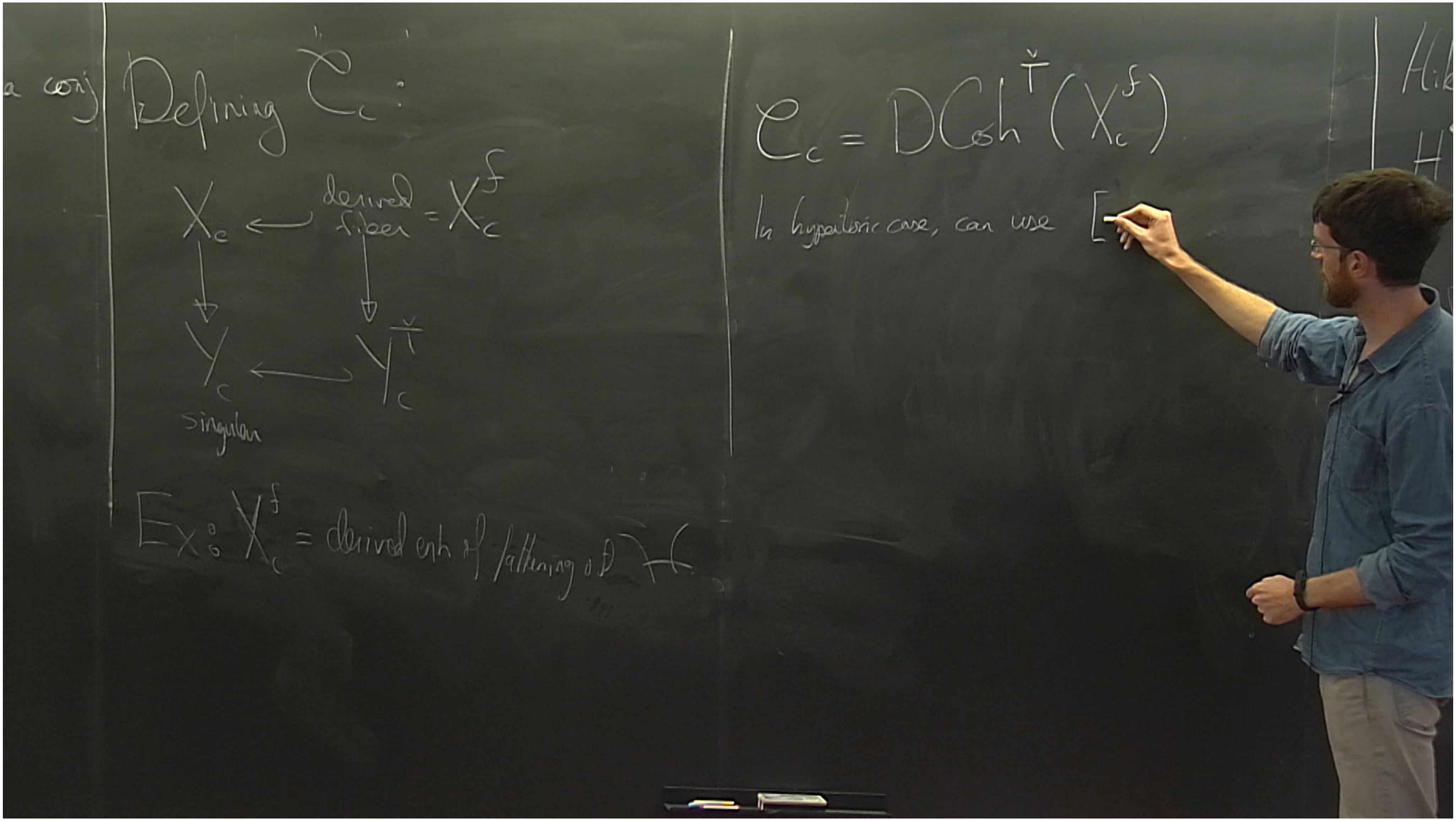
$$H(X_h, \mathbb{C}) = \mathcal{O}(Y_c^T)$$

Defining $\mathcal{C}_c$:

$$\begin{array}{ccc} X_c & & \\ \downarrow & & \\ Y_c & \longleftarrow & Y_c^T \end{array}$$

singular


a conj
Defining $\check{\mathcal{C}}_c$:
$X_c \leftarrow$ derived fiber $= X_c^f$
$Y_c \leftarrow Y_c^{\check{T}}$
singular
Ex: X_c^f = derived enh of flattening of $\mathcal{H}$
$\mathcal{C}_c = DCoh^{\check{T}}(X_c^f)$
In hypertoric case, can use [

a conj

Defining $\check{\mathcal{C}}_c$:

$$\begin{tikzcd} X_c \arrow[d] & \underset{\text{fiber}}{\text{derived}} = X_c^f \arrow[l] \arrow[d] \\ Y_c & Y_c^{\check{T}} \arrow[l] \end{tikzcd}$$

singular

Ex: X_c^f = derived enh of flattening of $\mathcal{H}$

$\mathcal{C}_c = DCoh^{\check{T}}(X_c^f)$

In hypertoric case, can use $[-W]$ to describe

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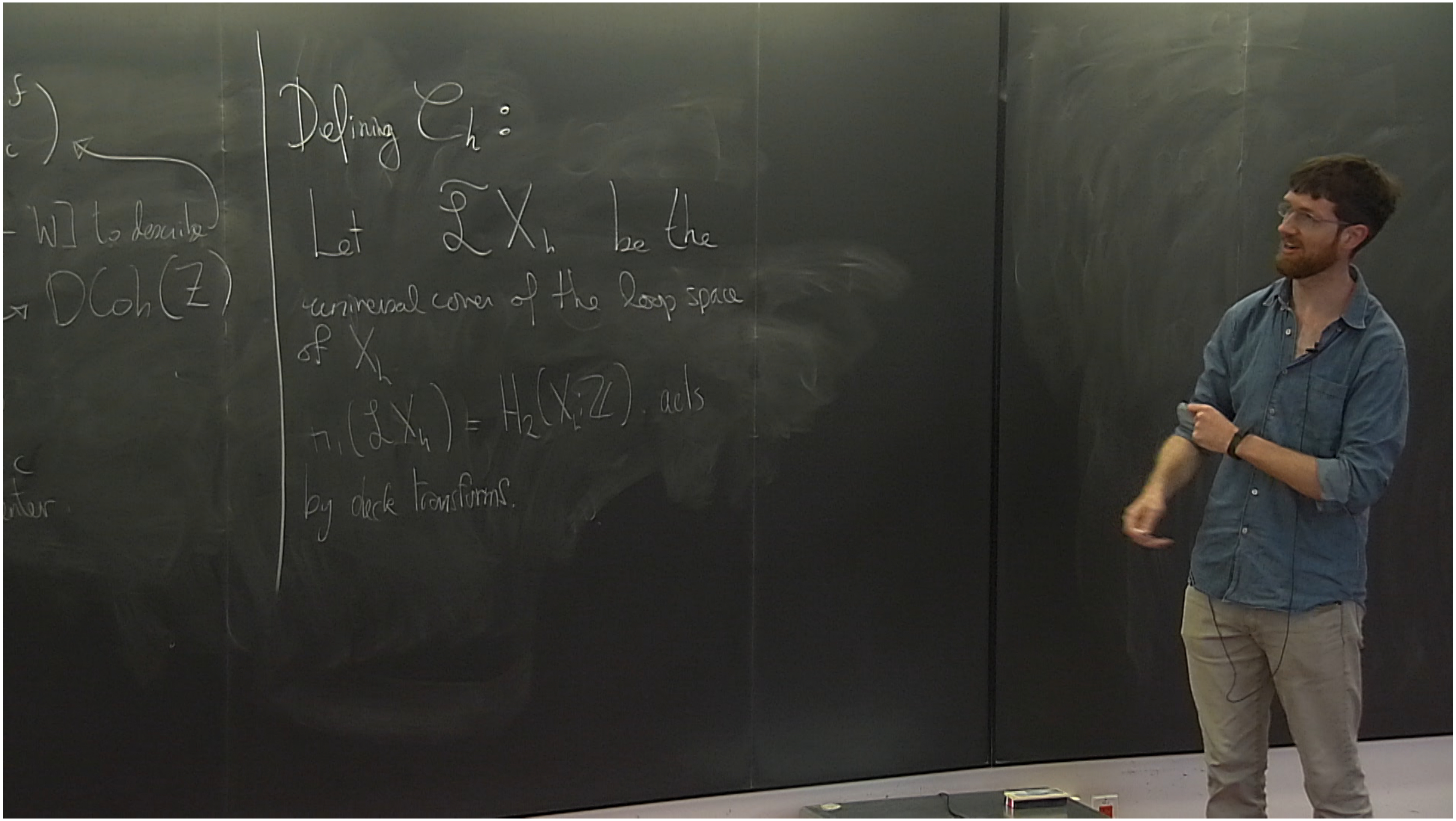
Defining $\mathcal{C}_h$:
Let $\widetilde{\mathcal{L}}X_h$ be the
universal cover of the loop space
of X_h
$\pi_1(\mathcal{L}X_h) = H_2(X_h;\mathbb{Z})$ acts
by deck transforms.
W] to describe
DCoh(Z)

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Defining $\mathcal{C}_h$:

Let $\widetilde{\mathcal{L}X_h}$ be the universal cover of the loop space of X_h

$\pi_1(\mathcal{L}X_h) = H_2(X_h;\mathbb{Z})$ acts by deck transforms.

$\widetilde{\mathcal{L}X_h}$ should be symplectic.

Would like:

$\mathcal{C}_h \subset \mu loc(\widetilde{\mathcal{L}X_h})$

subcategory defined rel to stab cond.

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$\mathcal{C}_h$ should contain quantizations of lagrangians L_σ for $\sigma \in H_2^F(X_h,\mathbb{Z})_\alpha \subset H_2^F(X_h,\mathbb{Z})$

extension of cocharacter lattice of $F \curvearrowright X_h$ by $H_2(X_h,\mathbb{Z}) \underset{\text{deck trans.}}{\curvearrowright} \widetilde{\mathcal{L}X_h}$

Defining $\check{\mathcal{C}}_c$:

$$\begin{tikzcd}
X_c \arrow[d] & \text{derived fiber} = X_c^f \arrow[l] \arrow[d] \\
Y_c & Y_c^{\vee} \arrow[l]
\end{tikzcd}$$

singular

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Defining $\check{\mathcal{C}}_c$:

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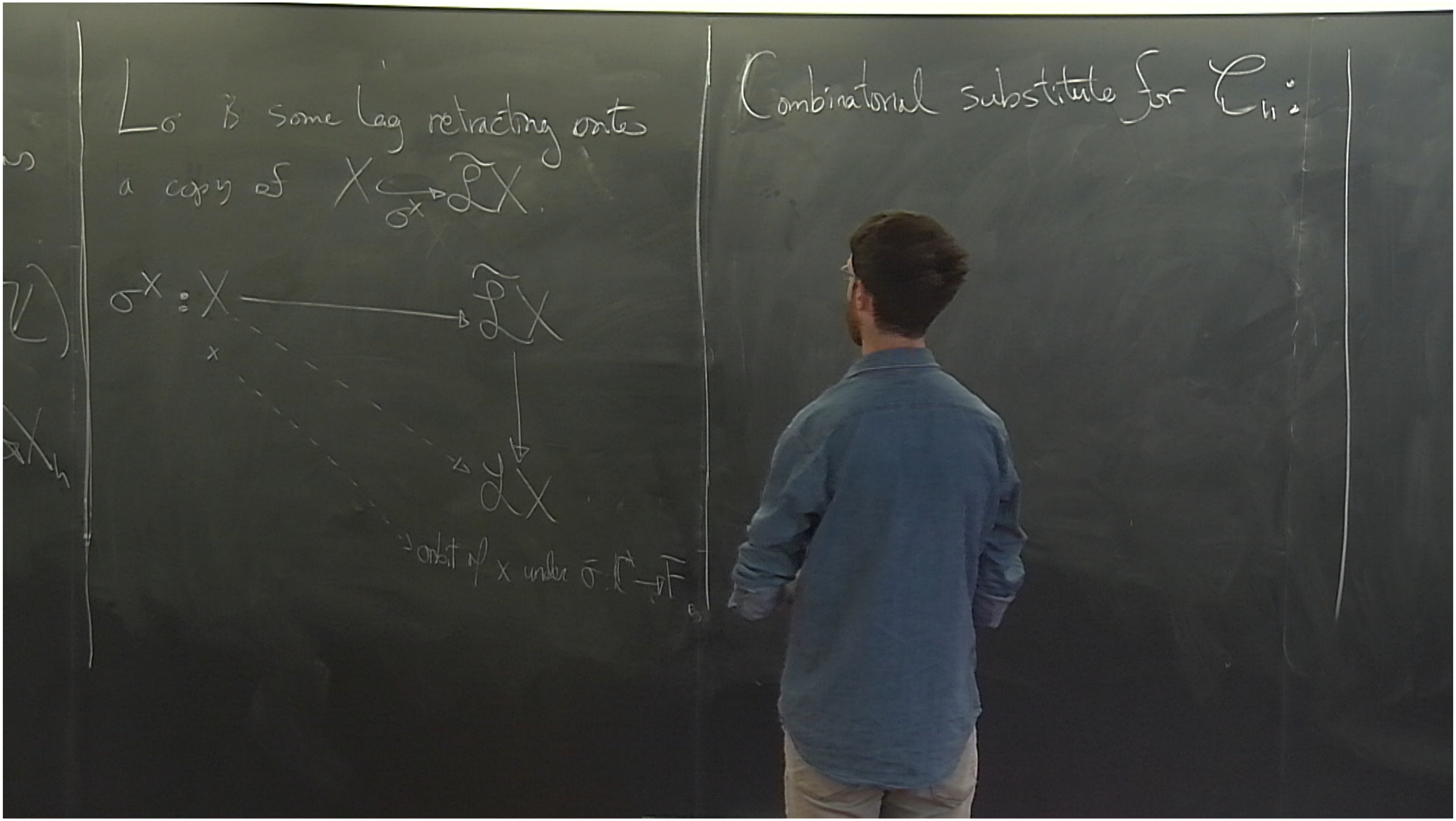
singular

Ex: X_c^f = derived enh of flattening of ...


L_σ is some Lag retracting onto
a copy of X ↪ L̃X
σ^X
σ^X : X → L̃X
x
LX
orbit of x under σ: Γ → F
Combinatorial substitute for C_n:

L_σ is some Lag retracting onto a copy of $X \overset{\sigma^x}{\hookrightarrow} \widetilde{\mathcal{L}}X$.

$$\begin{tikzcd}
\sigma^x : \underset{x}{X} \arrow[r] \arrow[dr, dashed] & \widetilde{\mathcal{L}}X \arrow[d] \\
 & \mathcal{L}X
\end{tikzcd}$$

$\to$ orbit of x under $\sigma : \mathbb{C}^* \to F_\sigma$

Combinatorial substitute for $\mathcal{C}_h$:

uses GKM theory:

Let $\underset{\text{torus}}{F} \hookrightarrow X$

$$H^\bullet_F(X) \xrightarrow{\text{res}} H^\bullet_F(X^F)$$

isomorphism after localization (BV)

For spaces satisfying the GKM cond:

$$H_F^\bullet(X) \xrightarrow{\text{restriction}} H_F^\bullet(X^{GKM})$$

is an isom

where X^{GKM} = fxd pts
+ 1D orbits
between fxd points.

Ex: $\mathbb{P}^2 \rightsquigarrow$  $\Gamma_{\mathbb{P}^2}$

Can also compute Ext's of certain eq. constructible sheaves this way.

Given $\mathcal{G} \in D\,Con_F(X)$

$\rightsquigarrow \overline{\mathcal{G}} \in Sh(\Gamma_X)$

graph of X^{GKM}

$$Ext(\mathcal{G},\mathcal{G}') = Hom(\overline{\mathcal{G}},\overline{\mathcal{G}'})$$

Can prove: $\exists$ a tilting bundle

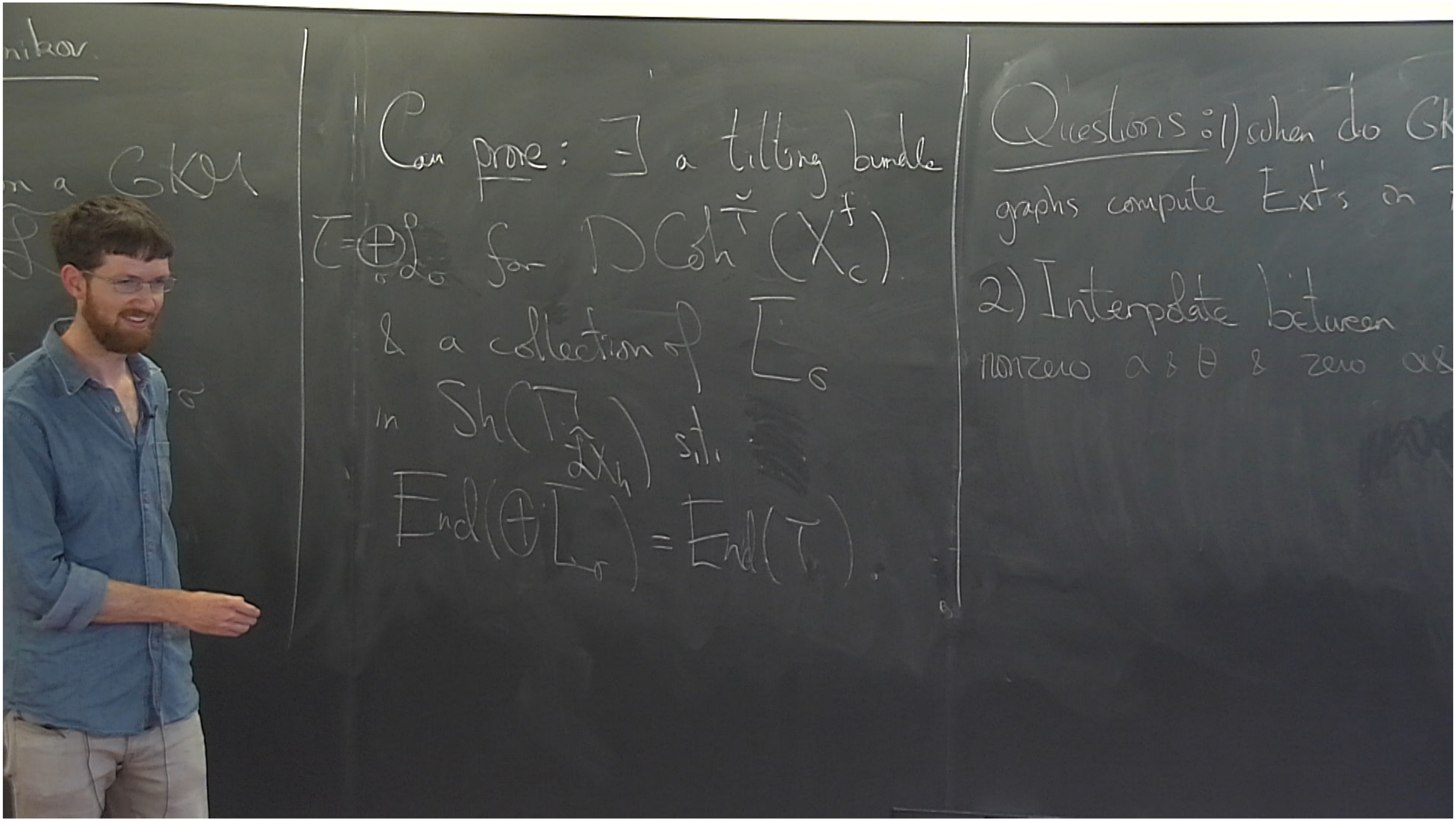
$\mathcal{T} = \bigoplus_{\sigma} \mathcal{L}_{\sigma}$ for $DCoh^{\check{T}}(X_c^f)$

& a collection of L_{σ}

in $Sh(T_{\hat{\Sigma}X_h})$ s.t.

$End(\bigoplus L_{\sigma}) = End(\mathcal{T})$.

Questions: when do GKM graphs compute Ext's in FS?


nikov.
n a GKM
Can prove: ∃ a tilting bundle
T = ⊕ L_σ for DCoh^T(X_c^f).
& a collection of L_σ
in Sh(T_{LX_h}) s.t.
End(⊕ L_σ) = End(T).
Questions: 1) when do GK
graphs compute Ext's in
2) Interpolate between
nonzero α & θ & zero α&