Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 11

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

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REVIEW OF LINEARIZED GRAVITY

$$\boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)}$$

PLUG TO GR MACHINERY

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}\right)$$

$$G_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu}\Box h\right)$$

- SIMPLIFIES

$$\boxed{\overline{h_{\mu\nu}} = h}$$

$$\Updownarrow$$

$$\boxed{h_{\mu\nu} = \overline{h_{\mu\nu}}}$$

REVIEW OF LINEARIZED GRAVITY

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$$\boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)}$$

PLUG TO GR MACHINERY

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}\right)$$

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- SIMPLIFIES

$$\boxed{\bar{h}_{\mu\nu} = h}$$

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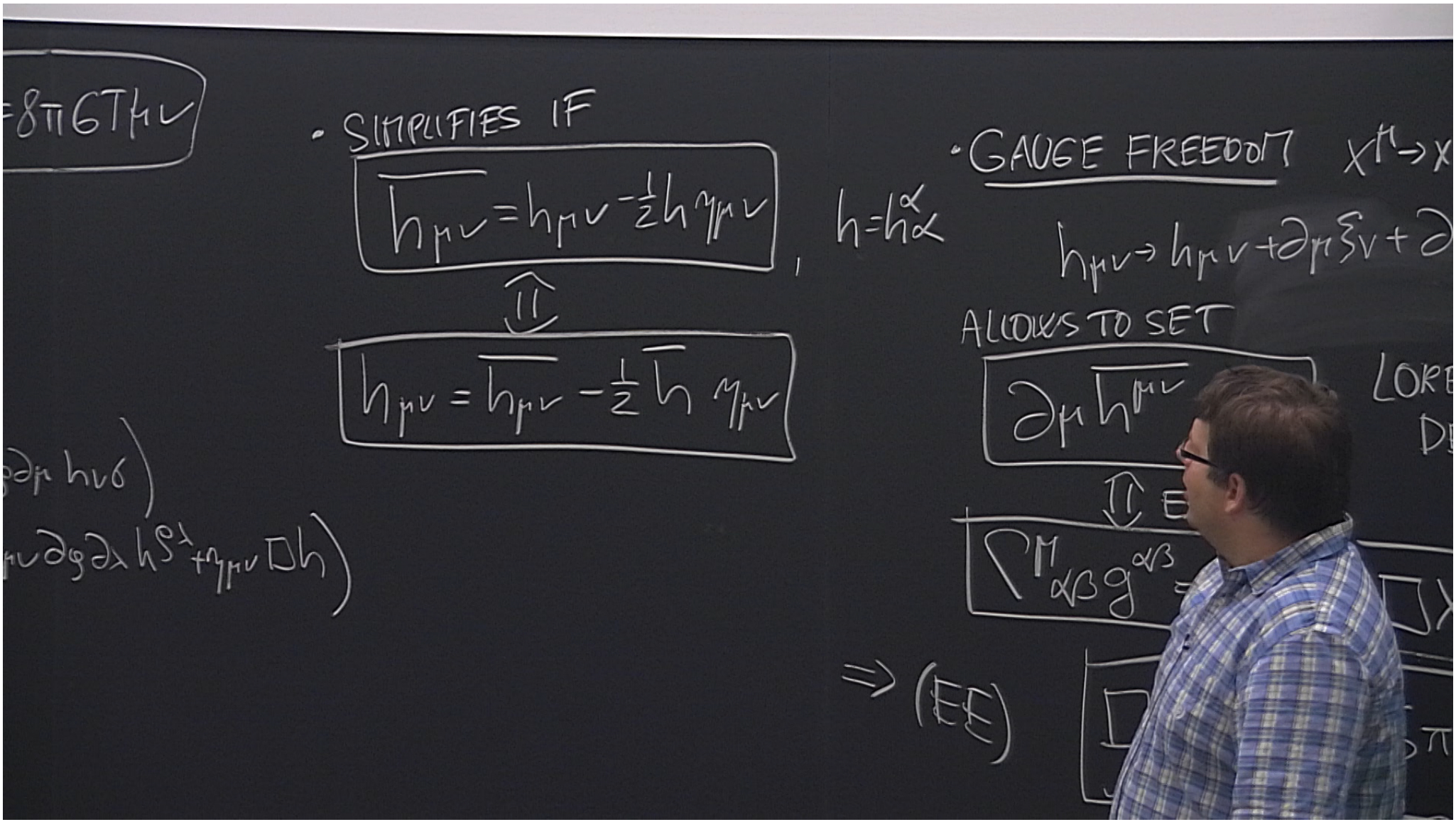

$=8\pi G T_{\mu\nu}$
• SIMPLIFIES IF
$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$
$h = h^{\alpha}_{\alpha}$
$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$
• GAUGE FREEDOM
$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial$
ALLOWS TO SET
$\partial_\mu \bar{h}^{\mu\nu}$
$\Rightarrow$ (EE)

- GAUGE FREEDOM $x^\mu \to x^\mu - \xi^\mu$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

ALLOWS TO SET

$$\boxed{\partial_\mu \bar{h}^{\mu\nu} = 0}$$

LORENZ/HARMONIC/DE DONDER GAUGE

$\Updownarrow$ Ex.

$$\boxed{\Gamma^\mu{}_{\alpha\beta} g^{\alpha\beta} = 0} \Longleftrightarrow \boxed{\Box X^\mu = 0}$$

(EE) $$\boxed{\Box \bar{h}^{\mu\nu} = -16\pi G T^{\mu\nu}}$$

- NOTE: GENERAL STRUCTURE

$T_{\mu\nu} =$ (ENERGY DE

- NOTE: GENERAL STRUCTURE OF $T_{\mu\nu}$:

$$T_{\mu\nu} = \left(\begin{array}{c|c} \text{ENERGY DENS.} & \text{MOMENTUM DENS.} \\ \hline & \text{PRESSURE} \end{array}\right)$$

NEWTONIAN LIMIT

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \phi \equiv -\frac{1}{4} h_{00}$$

$$\boxed{\begin{aligned} \nabla^2 \phi &= 4\pi G \rho \\ a &= -\nabla \phi \end{aligned}}$$

"GRAVITOSTATICS"

- NEXT ORDER

$T_{\mu\nu}$

$A^\mu = -\frac{1}{4} \overline{h^{0\mu}} \Rightarrow$

S

- NEXT ORDER

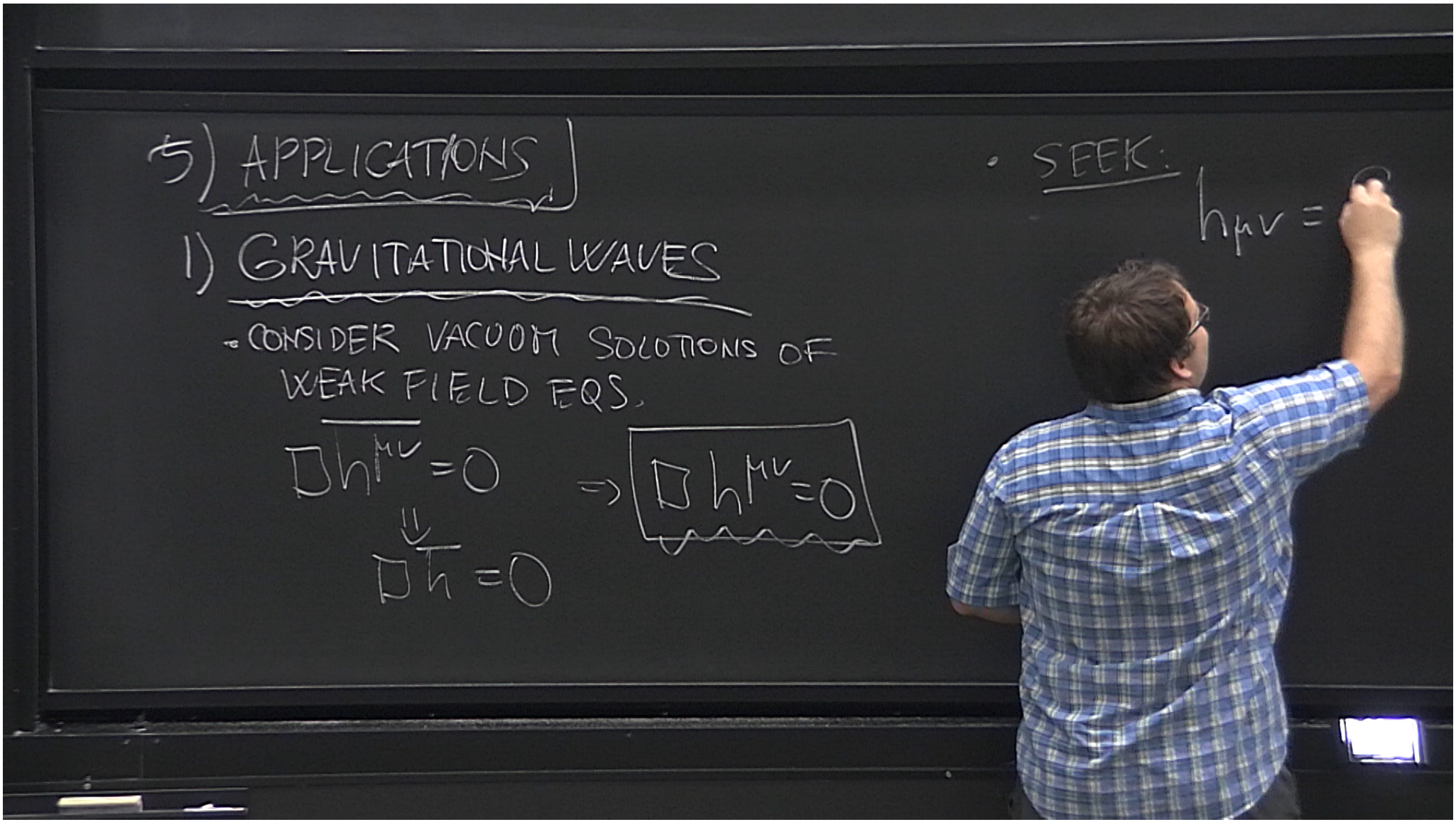
$$T_{\mu\nu} = \begin{pmatrix} \rho & \vec{j} \\ \vec{j} & 0 \end{pmatrix}$$ SLOWLY MOVING SOURCES

$$A^\mu = -\frac{1}{4}\overline{h^{0\mu}} \Rightarrow \boxed{\begin{aligned} \Box A^\mu &= -4\pi j^\mu \\ \partial_\mu A^\mu &= 0 \end{aligned}}$$

"GRAVITOMAGNETISM"

STATIC FIELD $\boxed{a = E - 4\pi \times B}$ $(q = m)$

INDICATION THAT GWS SHOULD EXIST.


5) APPLICATIONS
1) GRAVITATIONAL WAVES
• CONSIDER VACUUM SOLUTIONS OF WEAK FIELD EQS.
$\Box \bar{h}^{\mu\nu} = 0$
$\Downarrow$
$\Box \bar{h} = 0$
$\Rightarrow \boxed{\Box h^{\mu\nu} = 0}$
• SEEK:
$h_{\mu\nu} =$

- SEEK:

$$h_{\mu\nu} = \varepsilon_{\mu\nu} e^{i k_\alpha x^\alpha}$$

WAVE VECTOR

POLARIZATION TENSOR

$$D h^{\mu\nu} = 0 \Rightarrow \boxed{k^2 = 0}$$

- SEEK:

$$h_{\mu\nu} = \varepsilon_{\mu\nu} e^{i k_\alpha x^\alpha}$$

WAVE VECTOR

POLARIZATION TENSOR

$$D h^{\mu\nu} = 0 \Rightarrow \boxed{k^2 = 0}$$

GWs PROPAGATE AT THE SPEED OF LIGHT.

E VECTOR

- FIXING THE POLARIZATION TENSOR (UGLY WAY)

- HARMONIC GAUGE

$$\boxed{k^\mu \varepsilon_{\mu\nu} - \frac{1}{2}\varepsilon k_\nu = 0}, \quad \varepsilon = \varepsilon^\alpha{}_\alpha$$

4 CONDITIONS FOR $\varepsilon_{\mu\nu}$.

- THERE IS STILL A RESIDUAL GAUGE FREEDOM

$$x^\mu \to x^\mu - \xi^\mu$$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

SO THAT $\boxed{\Box x^\mu = 0} \Leftrightarrow \boxed{\Box \xi^\mu = 0}$

$$\underbrace{\xi^\mu = \tilde{\xi}^\mu e^{i k_\alpha x^\alpha}}$$

E

$\partial_\nu \xi_\mu$

$\boxed{{}^\mu = 0}$

· SINCE IN VACUUM

$$\boxed{k^2 = 0}$$

SO $\Box \xi^\mu = 0$ IS

AUTOMATICALLY SATISFIED

FOR ANY $\underline{\tilde{\xi}^\mu}$.

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$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + i k_\mu \tilde{\xi}_\nu + i k_\nu \tilde{\xi}_\mu$$

KILL ANOTHER 4 COMPTS OF $\varepsilon_{\mu\nu}$

SPEC: FOR GW PROP IN X-DIRECTION:

$$\varepsilon_{\mu\nu} = \begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & & \\ z & 0 & 0 & & \end{array}$$

ATISFIED

M

$\tilde{\xi}_\mu + ik_\nu \tilde{\xi}_\mu$

COMPTS OF $\varepsilon_{\mu\nu}$

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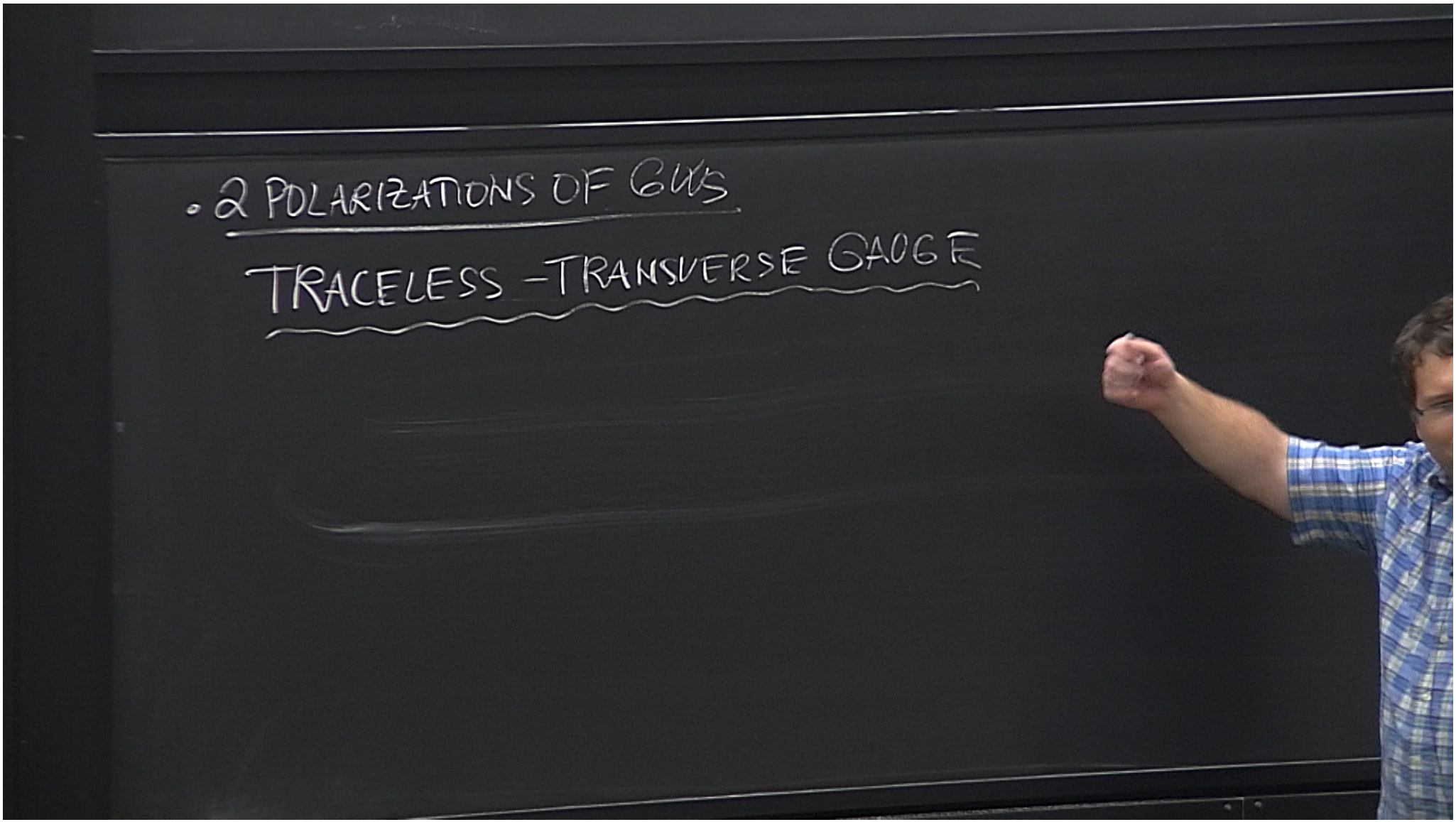
2 COMPTS $\varepsilon_+, \varepsilon_\times$ – 2 DOF

ATISFIED

M

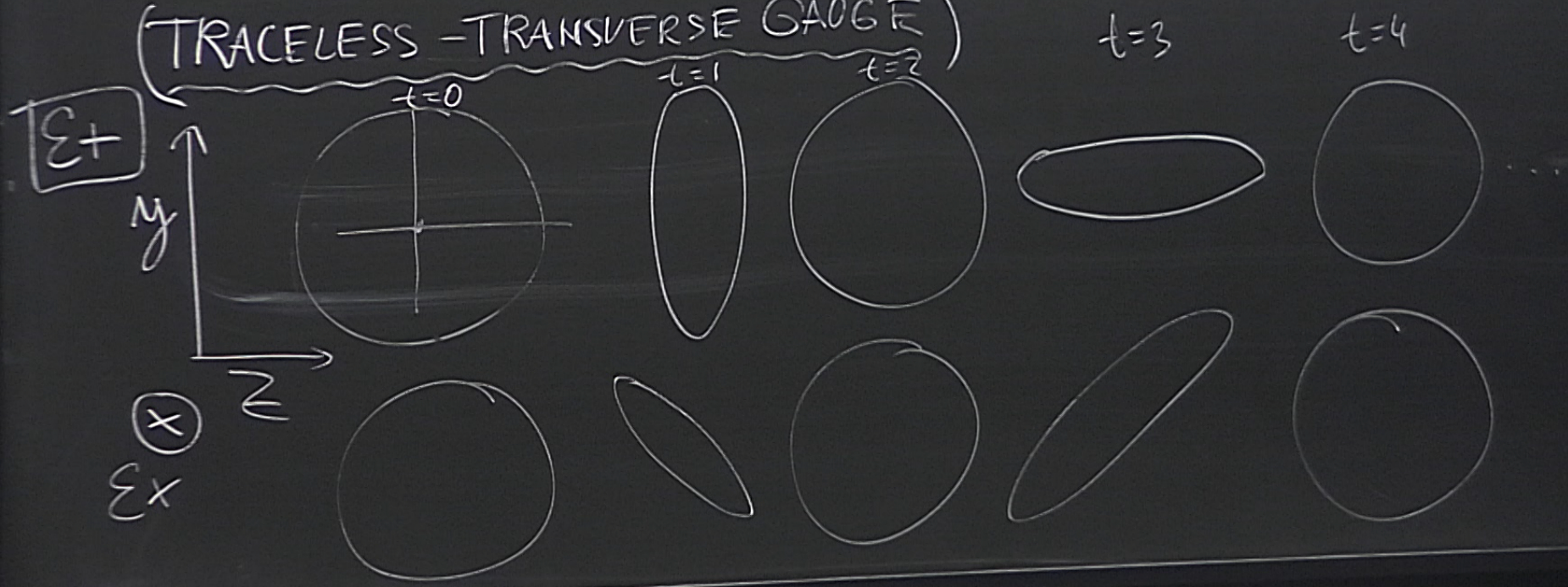
$\tilde{\xi}_\mu + ik_\nu \tilde{\xi}_\mu$

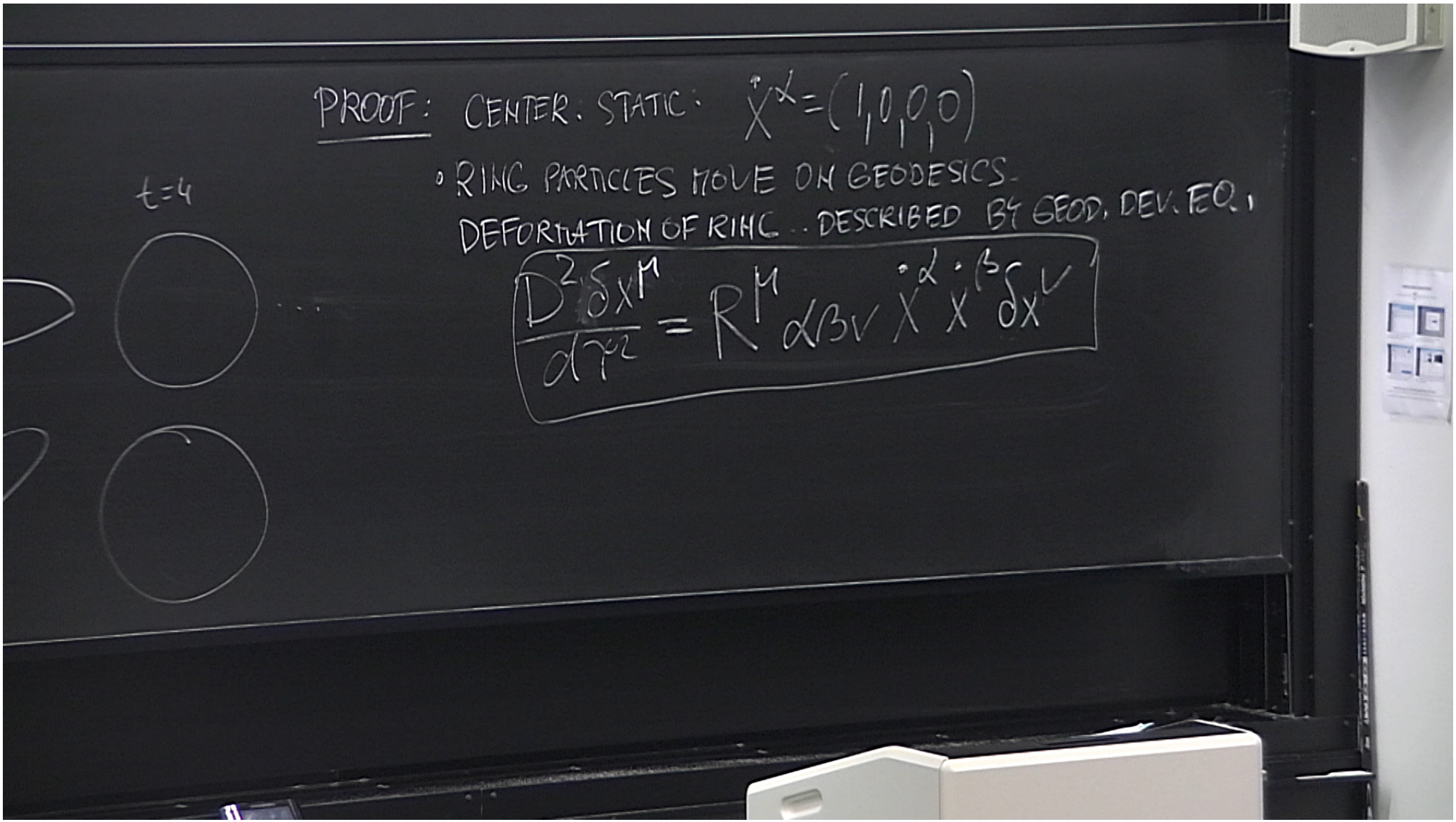
COMPTS OF $\varepsilon_{\mu\nu}$

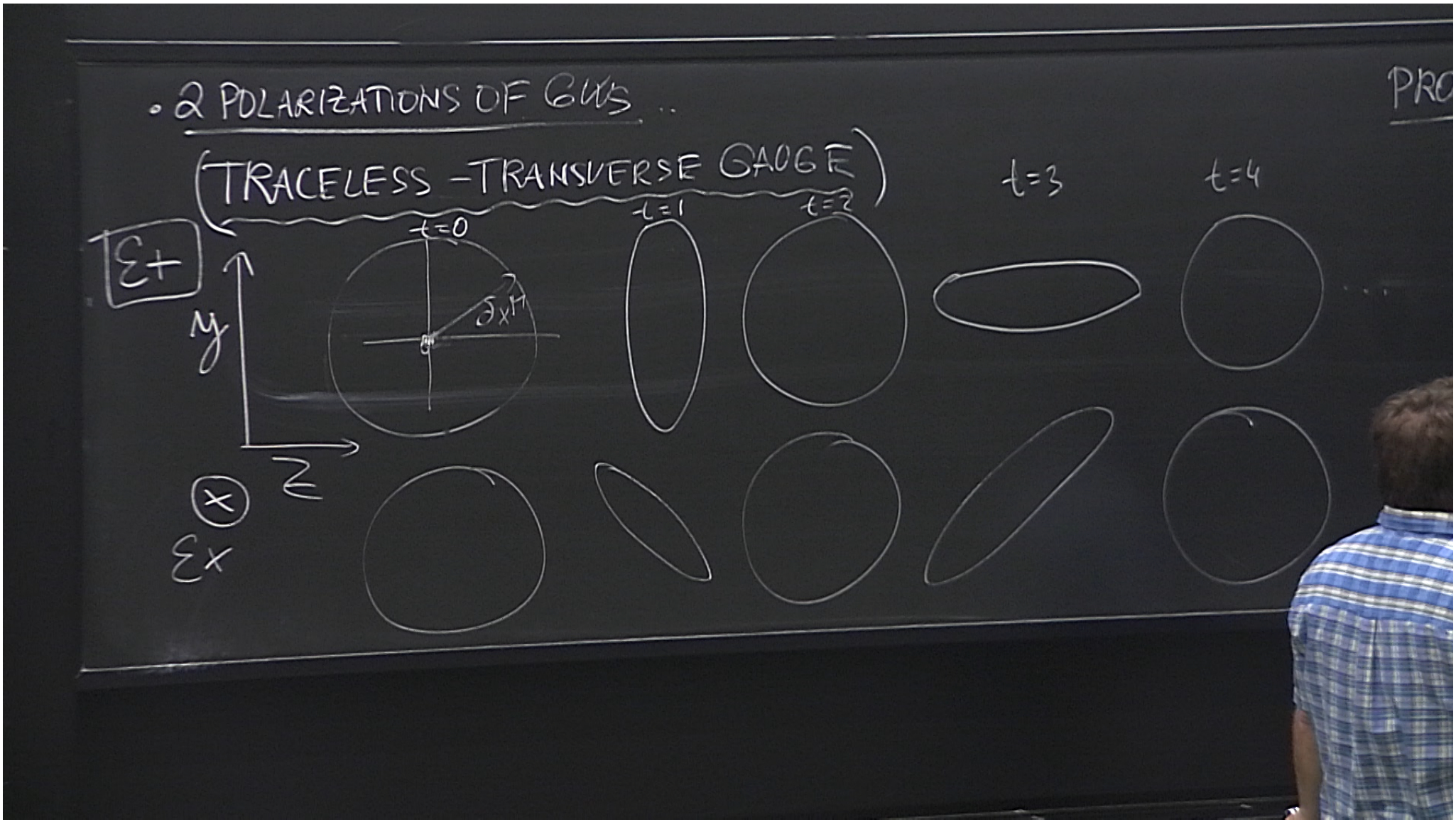

• 2 POLARIZATIONS OF GWS
TRACELESS - TRANSVERSE GAUGE

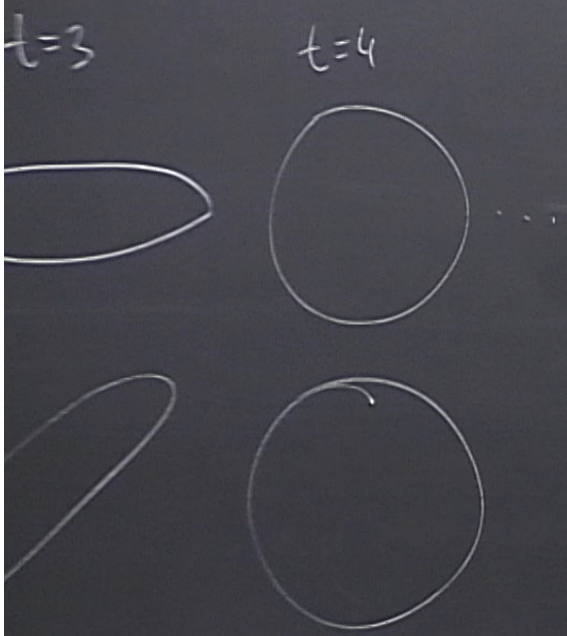
• 2 POLARIZATIONS OF GWS ..

(TRACELESS - TRANSVERSE GAUGE)




PROOF: CENTER. STATIC: $\dot{X}^\alpha = (1,0,0,0)$
• RING PARTICLES MOVE ON GEODESICS.
DEFORMATION OF RING .. DESCRIBED BY GEOD. DEV. EQ.
$\frac{D^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\nu} \dot{X}^\alpha \dot{X}^\beta \delta x^\nu$
t=4


• 2 POLARIZATIONS OF GWS ..
(TRACELESS -TRANSVERSE GAUGE)
PRO
t=0
t=1
t=2
t=3
t=4
ε_+
y
z
$\varepsilon_\times$



PROOF: CENTER. STATIC: $\dot{X}^\alpha = (1,0,0,0)$

- RING PARTICLES MOVE ON GEODESICS.

DEFORMATION OF RING. DESCRIBED BY GEOD. DEV. EQ.:

$$\boxed{\frac{D^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\nu} \dot{X}^\alpha \dot{X}^\beta \delta x^\nu}$$

$$\boxed{\frac{d^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{tt\nu} \delta x^\nu}$$

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$$R_{\mu\sigma} = \frac{1}{2}\left(\partial_\sigma \partial_\mu h_{\mu\sigma} + \partial_\sigma \partial_\mu h_{\nu\sigma} - \partial_\sigma \partial_\sigma h - \partial_\rho \partial_\mu h_{\nu\sigma}\right)$$

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- SIMPLIFIES IF

$$\boxed{\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}}$$

$$\Updownarrow$$

$$\boxed{h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}}$$

$$R_{\mu tt\nu} = \frac{1}{2}\partial_t^2 h_{\mu\nu}$$

$$\boxed{\frac{\partial^2 \delta x^\mu}{\partial t^2} = \frac{1}{2}\delta x^\nu \frac{\partial^2}{\partial t^2} h_{\mu\nu}}$$

$$R_{\mu tt\nu} = \frac{1}{2}\partial_t^2 h_{\mu\nu}$$

$\boxed{\varepsilon_t}$ $\underline{\delta y}$: $\dfrac{\partial^2 \delta y}{\partial t^2} = \dfrac{1}{2}\delta y \dfrac{\partial^2}{\partial t^2}\varepsilon_t e^{ik\cdot x}$

$$\boxed{\frac{\partial^2 \delta x^\mu}{\partial t^2} = \frac{1}{2}\delta x^\nu \frac{\partial^2}{\partial t^2} h_{\mu\nu}}$$

$$\delta x^\mu = (\ \ , 0, \delta y, \delta z)$$

$\boxed{\varepsilon_+}$

$\underline{\delta y}$: $$\frac{\partial^2 \delta y}{\partial t^2} = \frac{1}{2}\delta y \frac{\partial^2}{\partial t^2}\varepsilon_+ e^{i k\cdot x}$$

TO LEADING ORDER

$$\delta y = \delta y(0) + \frac{1}{2}\delta y(0)\,\varepsilon_+ e^{i k\cdot x}$$

$\underline{\delta z}$: $$\frac{\partial^2 \delta z}{\partial t^2} = -\frac{1}{2}\delta z \frac{\partial^2}{\partial t^2}\varepsilon_+ e^{i k\cdot x}$$

$$\delta z = \delta z(0) - \frac{1}{2}\delta z(0)\,\varepsilon_+ e^{i k\cdot x}$$

- IF FIELD QUANTIZED - SPIN-2 PARTICLE (GRAVITON)

SPIN $\underline{S}$ IS RELATED TO THE ANGLE θ UNDER WHICH THE POLARIZATION MODE IS INVARIANT

$$\boxed{S = \frac{360^\circ}{\theta}}$$

- IF FIELD QUANTIZED · SPIN-2 PARTICLE (GRAVITON)

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THE POLARIZATION MODE IS INVARIANT

$$\boxed{S = \frac{360^\circ}{\theta}}$$

GW: $\theta = 180^\circ$ $\quad S = 2$

- IF FIELD QUANTIZED · SPIN-2 PARTICLE (GRAVITON)

SPIN $\underline{S}$ IS RELATED TO THE ANGLE θ UNDER WHICH THE POLARIZATION MODE IS INVARIANT

$$\boxed{S = \frac{360^\circ}{\theta}}$$

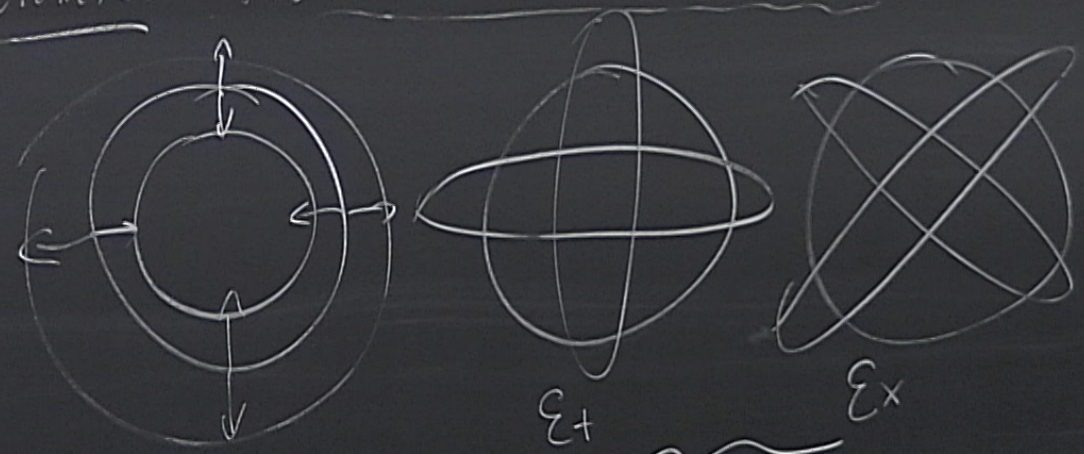
GW: $\theta = 180^\circ$ $\quad S = 2$

EM: $\theta = 360^\circ$ $\quad S = 1$

GRAVITON)

R WHICH

REMARK: VIBRATIONS OF CLOSED STRING



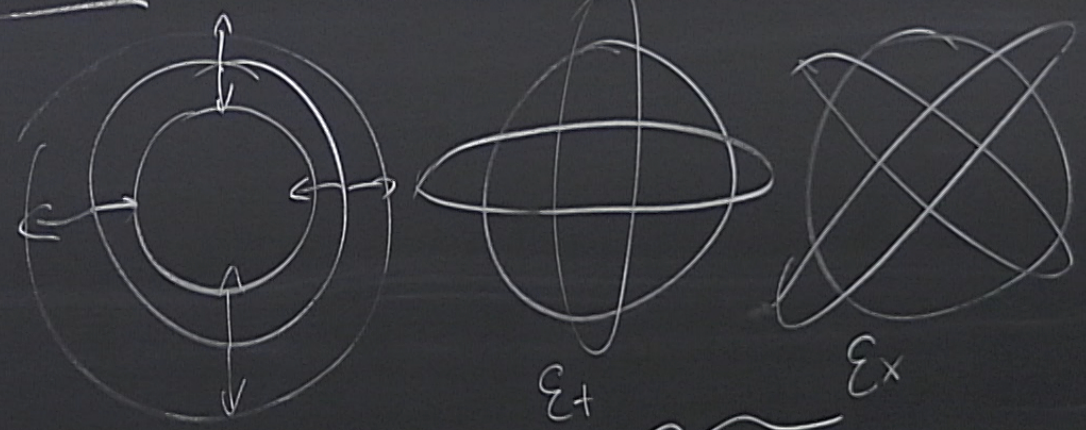
"BREATHING MODE"

SPIN 2 - GRAVITON

GRAVITON)

R WHICH

REMARK: VIBRATIONS OF CLOSED STRING



"BREATHING MODE"

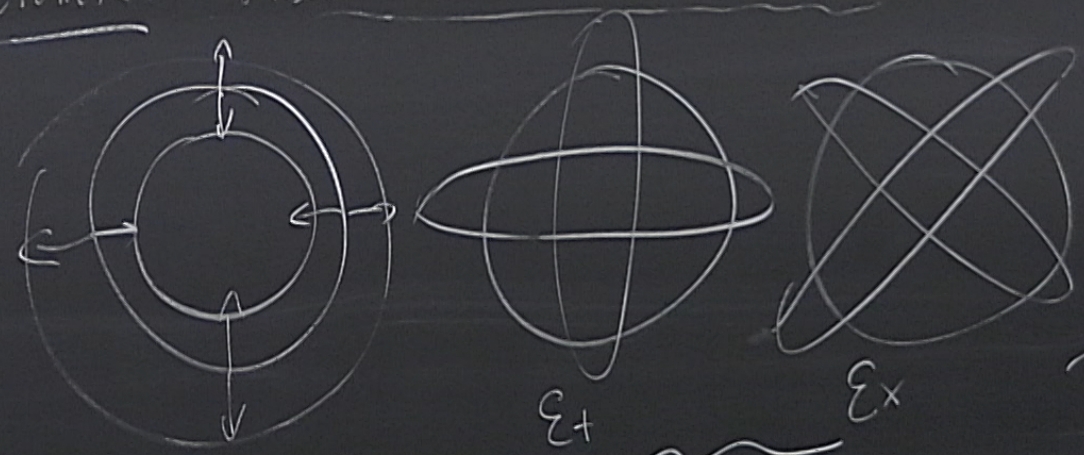
SPIN-0 .. DILATON

SPIN 2 - GRAVITON

GRAVITON)

R WHICH

REMARK: VIBRATIONS OF CLOSED STRING



"BREATHING MODE"

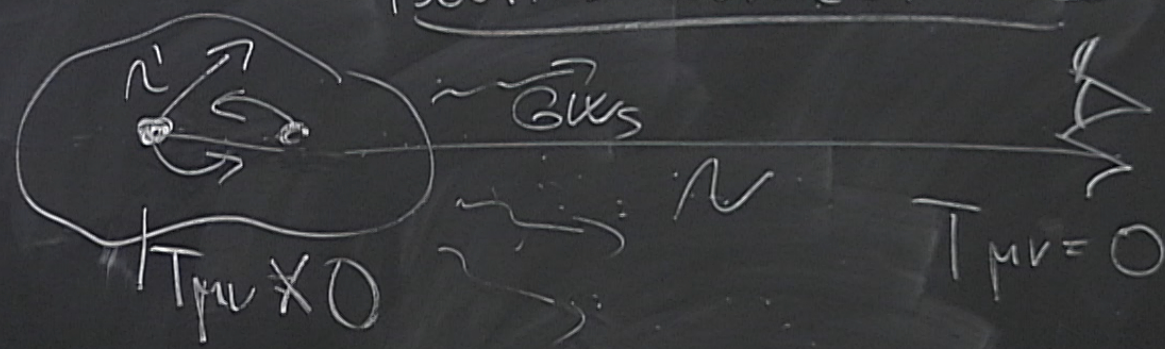
SPIN-0 .. DILATON

SPIN 2 - GRAVITON

A THEORY OF QG.

GRAVITATIONAL RADIATION FROM AN ISOLATED SYSTEM

ISOLATED SOURCE OF GWS



$$|\vec{r}'| \ll |\vec{r}|$$

$$\omega \ll \frac{1}{|\vec{r}'|}$$ SLOW MOTION

HAVE TO SOLVE $\Box \bar{h}^{\mu\nu} = -16\pi G T^{\mu\nu}$

SPECIAL SOL. .. GIVEN BY RETARDED POTENTIAL.

$$\boxed{\bar{h}^{\mu\nu}(t,\vec{r}) = 4G \int d^3r' \, \frac{T^{\mu\nu}(t_R,\vec{r}')}{|\vec{r}-\vec{r}'|} \qquad t_R = t - |\vec{r}-\vec{r}'|}$$

$$\omega \ll \frac{1}{|\vec{r}'|}$$ SLOW MOTION

$$\tilde{h}_{\mu\nu}(\omega, \vec{r}) = \frac{1}{\sqrt{2\pi}} \int dt\, e^{i\omega t}\, h_{\mu\nu} = \frac{4G}{\sqrt{2\pi}} \int dt\, d^3r'\, e^{i\omega t}\, \frac{T_{\mu\nu}(t_R, r')}{|r - r'|}$$

$|\vec{r}'| \ll |\vec{r}|$

$\omega \ll \frac{1}{|\vec{r}'|}$ SLOW MOTION

$t_R = t - |\vec{r} - \vec{r}'|$

$$\tilde{h}_{\mu\nu}(\omega, \vec{r}) = \frac{1}{\sqrt{2\pi}} \int dt\, e^{i\omega t}\, h_{\mu\nu} = \frac{4G}{\sqrt{2\pi}} \int dt\, d^3r'\, e^{i\omega t} \frac{T_{\mu\nu}(t_R, r')}{|r - r'|}$$

. CHANGE $t \to$

$$= \frac{4G}{\sqrt{2\pi}} \int dt_R\, d^3r'\, e^{i\omega(t_R + |r - r'|)} \frac{T_{\mu\nu}(t_R, r')}{|r - r'|} = 4G \int d^3r'\, e^{i\omega|r - r'|} \frac{\tilde{T}_{\mu\nu}(\omega, r')}{|r - r'|}$$

$$|\vec{r}'| \ll |\vec{r}|$$

$$\omega \ll \frac{1}{|\vec{r}'|} \quad \text{SLOW MOTION}$$

$$t_R = t - |\vec{r} - \vec{r}'|$$

$$\tilde{h}_{\mu\nu}(\omega, \vec{r}) = \frac{1}{\sqrt{2\pi}} \int dt\, e^{i\omega t}\, h_{\mu\nu} = \frac{4G}{\sqrt{2\pi}} \int dt\, d^3r'\, e^{i\omega t} \frac{T_{\mu\nu}(t_R, r')}{|r - r'|}$$

CHANGE $t \to$

$$= \frac{4G}{\sqrt{2\pi}} \int dt_R\, d^3r'\, e^{i\omega(t_R + |r - r'|)} \frac{T_{\mu\nu}(t_R, r')}{|r - r'|} = 4G \int d^3r'\, e^{i\omega|r - r'|} \frac{\tilde{T}_{\mu\nu}(\omega, r')}{|r - r'|}$$

$$\boxed{\tilde{h}_{\mu\nu} \simeq 4G \frac{e^{i\omega r}}{r} \int d^3r'\, T_{\mu\nu}(\omega, r')}$$