Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 11

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 16, 2019 - 10:45 AM

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REVIEW OF LINEARIZED GRAVITY

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

PLUG TO GR MACHINERY

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}\right)$$

$$G_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu}\Box h\right)$$

- SIMPLIFIES

$$\bar{h}_{\mu\nu} = h$$

$$\Downarrow$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu}$$

REVIEW OF LINEARIZED GRAVITY

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- SIMPLIFIES

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

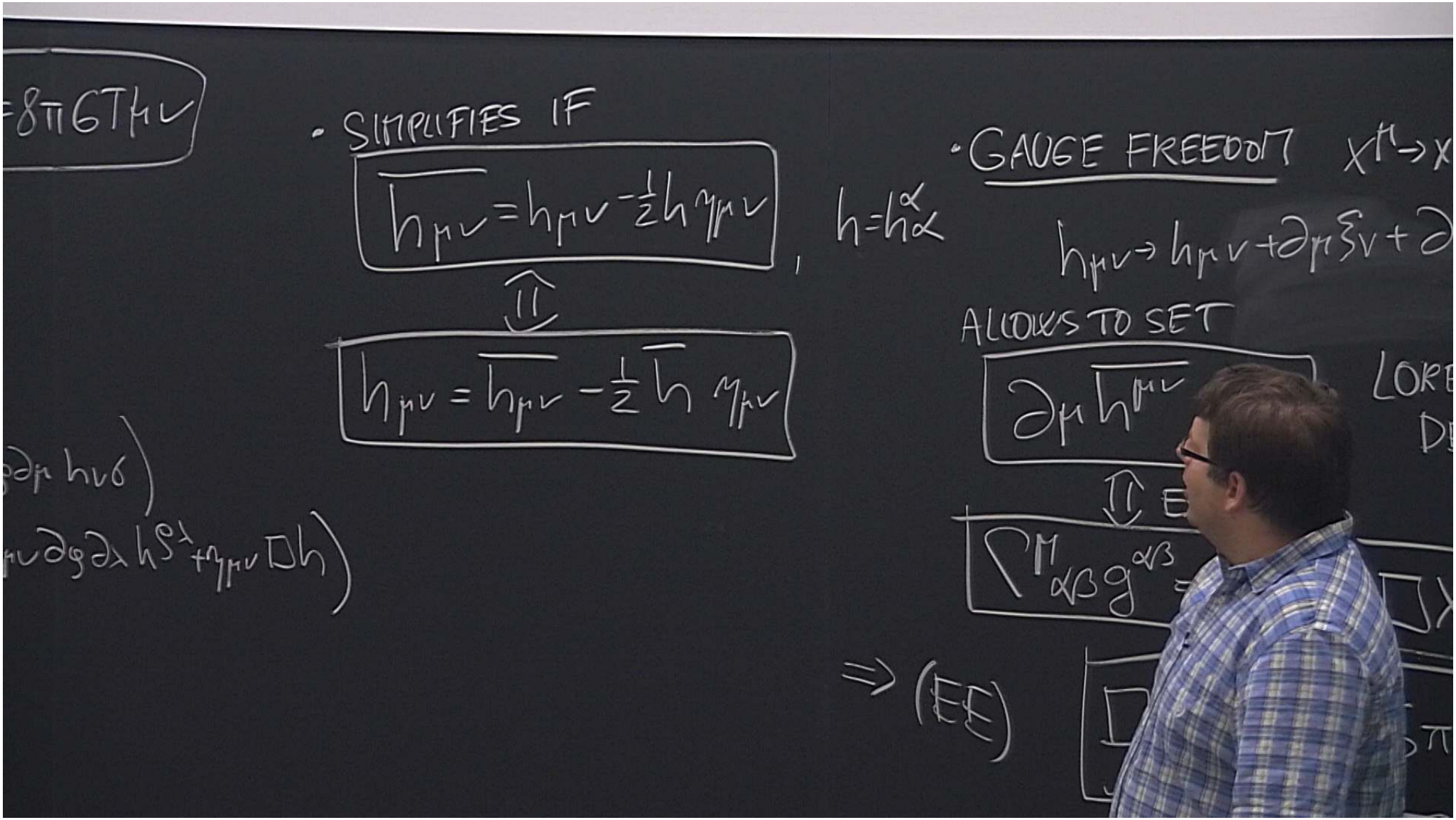
$$\bar{h}_{\mu\nu} = h$$

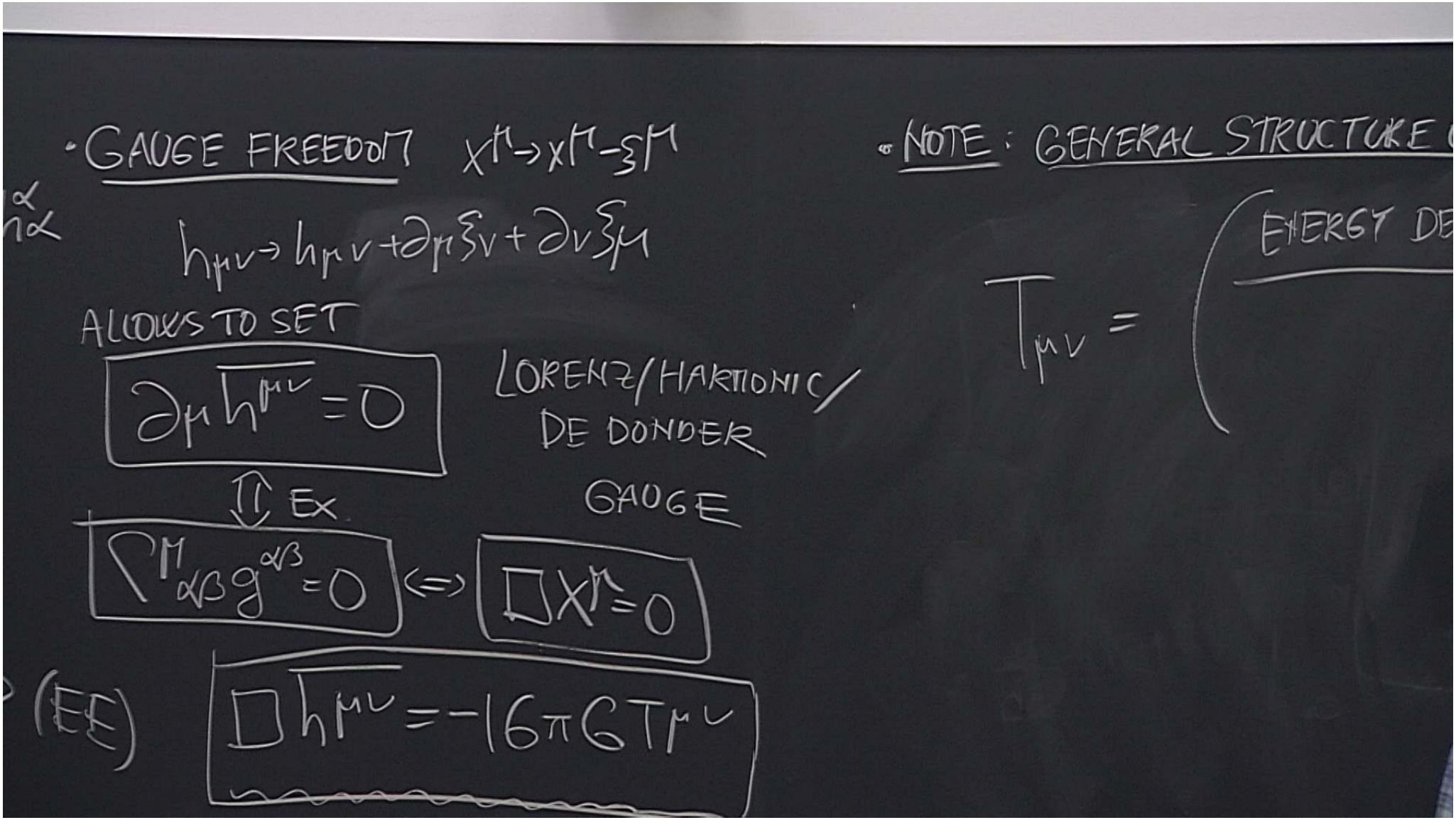
PLUG TO GR MACHINERY

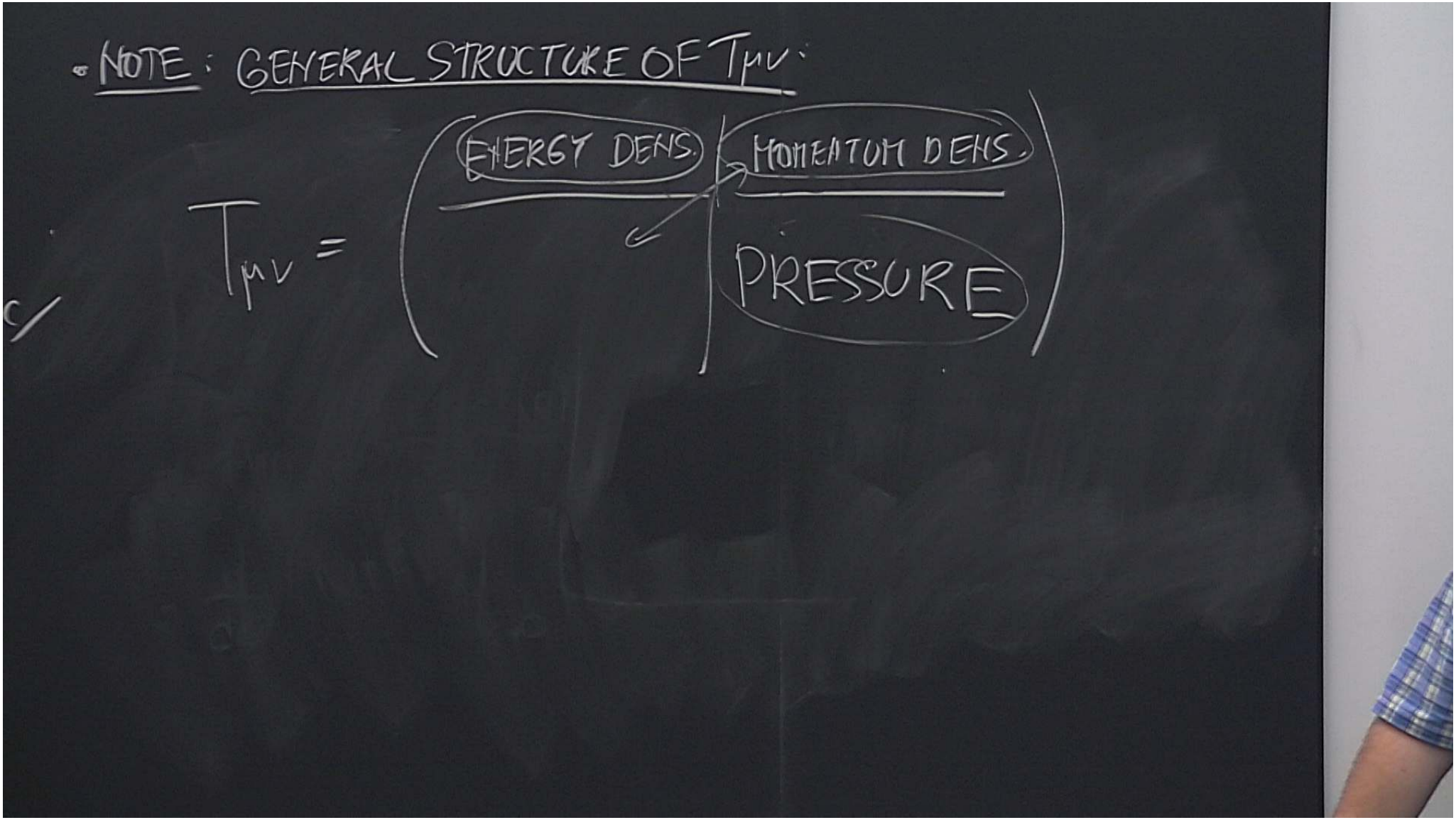
$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}\right)$$

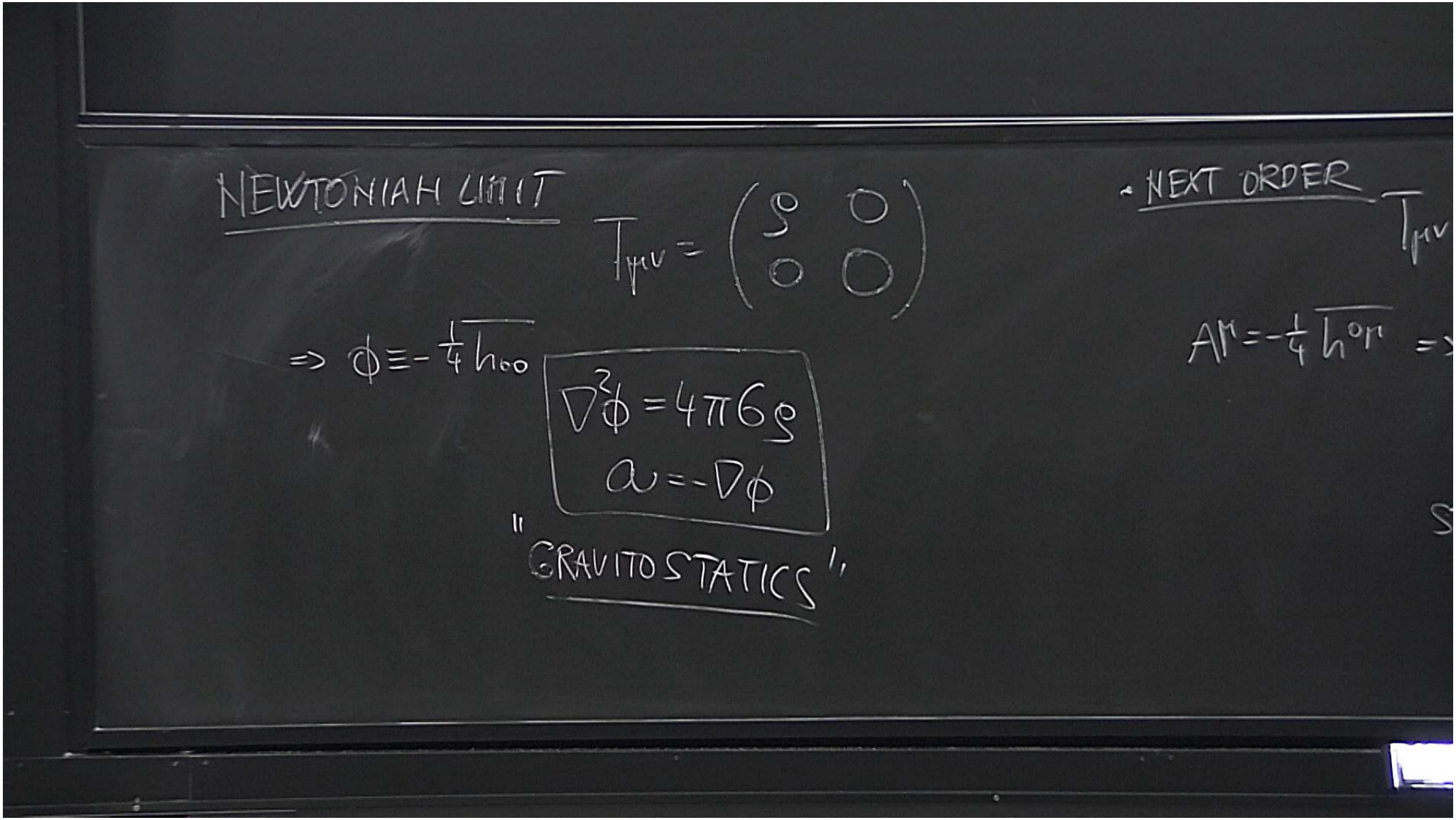
$$G_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu}\Box h\right)$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu}$$


$= 8\pi G T_{\mu\nu}$
• SIMPLIFIES IF
$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$
$h = h^{\alpha}_{\alpha}$
$\Leftrightarrow$
$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$
• GAUGE FREEDOM $x^{\mu} \to x$
$h_{\mu\nu} \to h_{\mu\nu} + \partial_{\mu}\xi_{\nu} + \partial$
ALLOWS TO SET
$\partial_{\mu} \bar{h}^{\mu\nu}$
$\Leftrightarrow$ E
$\Gamma^{\mu}_{\alpha\beta} g^{\alpha\beta}$
$\Rightarrow$ (EE)
LORE
D
$\partial_{\mu} h_{\nu\sigma}$)
$\partial_{\sigma}\partial_{\lambda} h^{\sigma\lambda} + \eta_{\mu\nu} \Box h$)


• GAUGE FREEDOM $x^\mu \to x^\mu - \xi^\mu$
$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
ALLOWS TO SET
$\partial_\mu \bar{h}^{\mu\nu} = 0$
LORENZ / HARMONIC / DE DONDER GAUGE
$\Leftrightarrow$ Ex
$\Gamma^\mu_{\alpha\beta} g^{\alpha\beta} = 0 \Leftrightarrow \Box x^\mu = 0$
(EE) $\Box \bar{h}^{\mu\nu} = -16\pi G T^{\mu\nu}$
• NOTE: GENERAL STRUCTURE
$T_{\mu\nu} =$ (
ENERGY DE


NOTE: GENERAL STRUCTURE OF $T_{\mu\nu}$:
$T_{\mu\nu} =$
ENERGY DENS.
MOMENTUM DENS.
PRESSURE


NEWTONIAN LIMIT
$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & 0 \end{pmatrix}$
$\Rightarrow \phi \equiv -\frac{1}{4} h_{00}$
$\nabla^2 \phi = 4\pi G \rho$
$a = -\nabla \phi$
"GRAVITOSTATICS"
- NEXT ORDER
$T_{\mu\nu}$
$A^\mu = -\frac{1}{4} \overline{h^{0\mu}}$

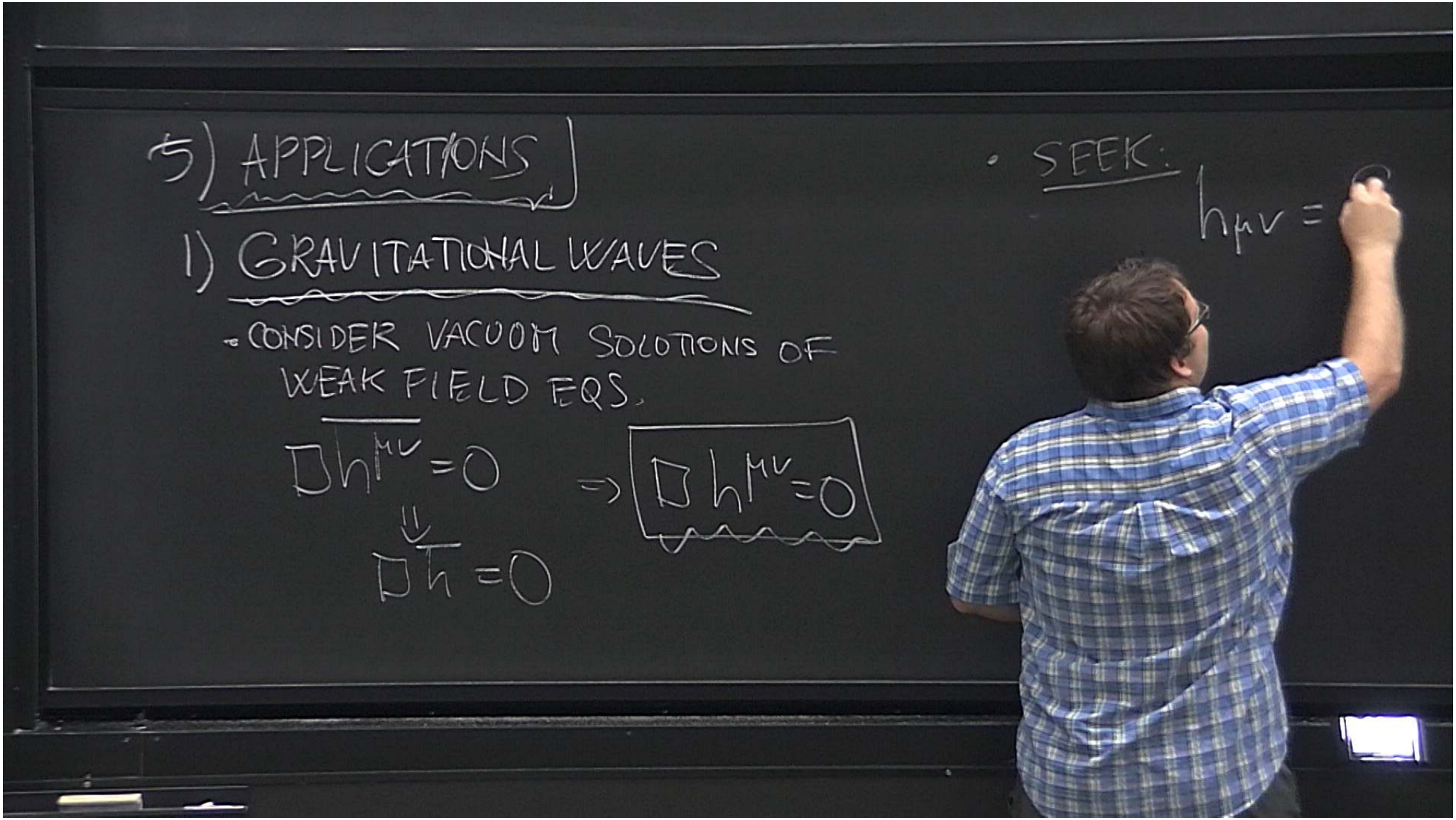
- NEXT ORDER $T_{\mu\nu} = \begin{pmatrix} \rho & \vec{g} \\ \vec{g} & 0 \end{pmatrix}$ — SLOWLY MOVING SOURCES

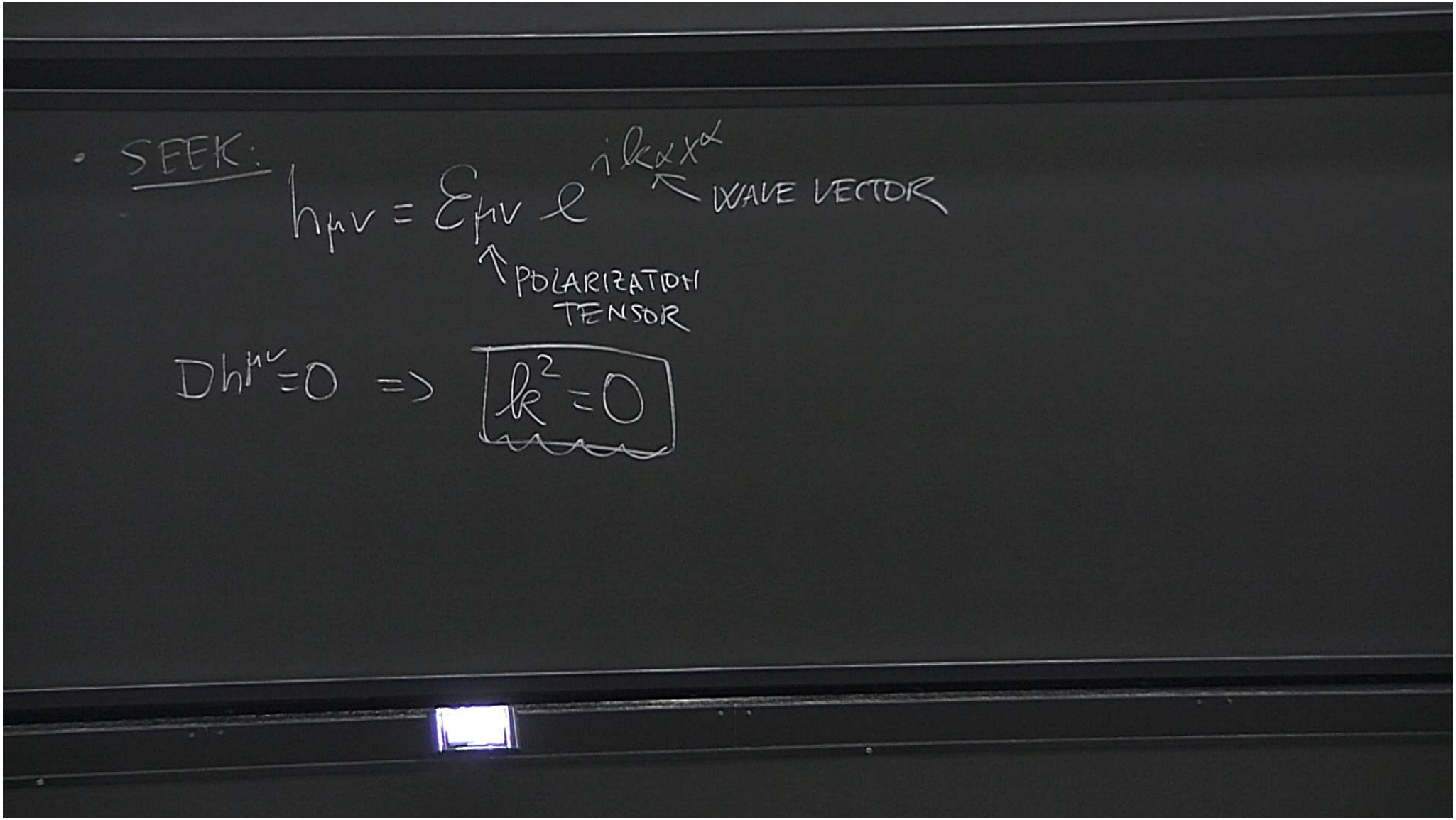
$A^\mu = -\frac{1}{4}\bar{h}^{0\mu}$ $\Rightarrow$ $\boxed{\begin{array}{c} \Box A^\mu = -4\pi g^\mu \\ \partial_\mu A^\mu = 0 \end{array}}$

"GRAVITOMAGNETISM"

STATIC FIELD $\boxed{\omega = E - 4\pi \times B}$ $(q = m)$

INDICATION THAT GWs SHOULD EXIST.


5) APPLICATIONS
1) GRAVITATIONAL WAVES
- CONSIDER VACUUM SOLUTIONS OF WEAK FIELD EQS.
$\Box h^{\mu\nu} = 0$
$\Rightarrow \boxed{\Box h^{\mu\nu} = 0}$
$\Downarrow$
$\Box h = 0$
• SEEK:
$h_{\mu\nu} =$


• SEEK:
$h_{\mu\nu} = \varepsilon_{\mu\nu} e^{i k_\alpha x^\alpha}$
WAVE VECTOR
POLARIZATION TENSOR
$Dh^{\mu\nu} = 0 \Rightarrow \boxed{k^2 = 0}$

- SEEK:

$$h_{\mu\nu} = \varepsilon_{\mu\nu} e^{i k_\alpha x^\alpha}$$

(k_α: WAVE VECTOR; $\varepsilon_{\mu\nu}$: POLARIZATION TENSOR)

$$\Box h^{\mu\nu} = 0 \Rightarrow \boxed{k^2 = 0}$$

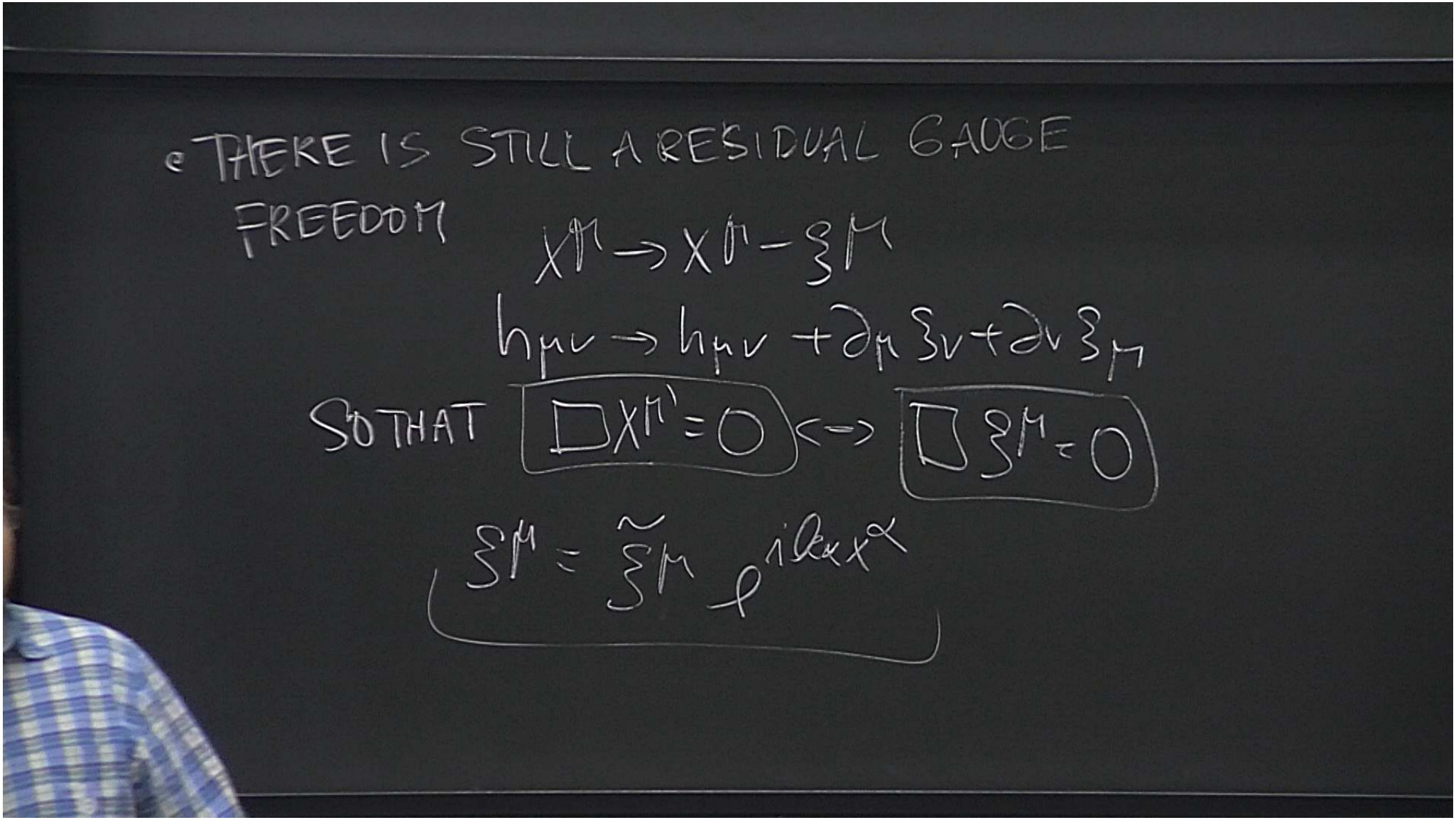
GWs PROPAGATE AT THE SPEED OF LIGHT.

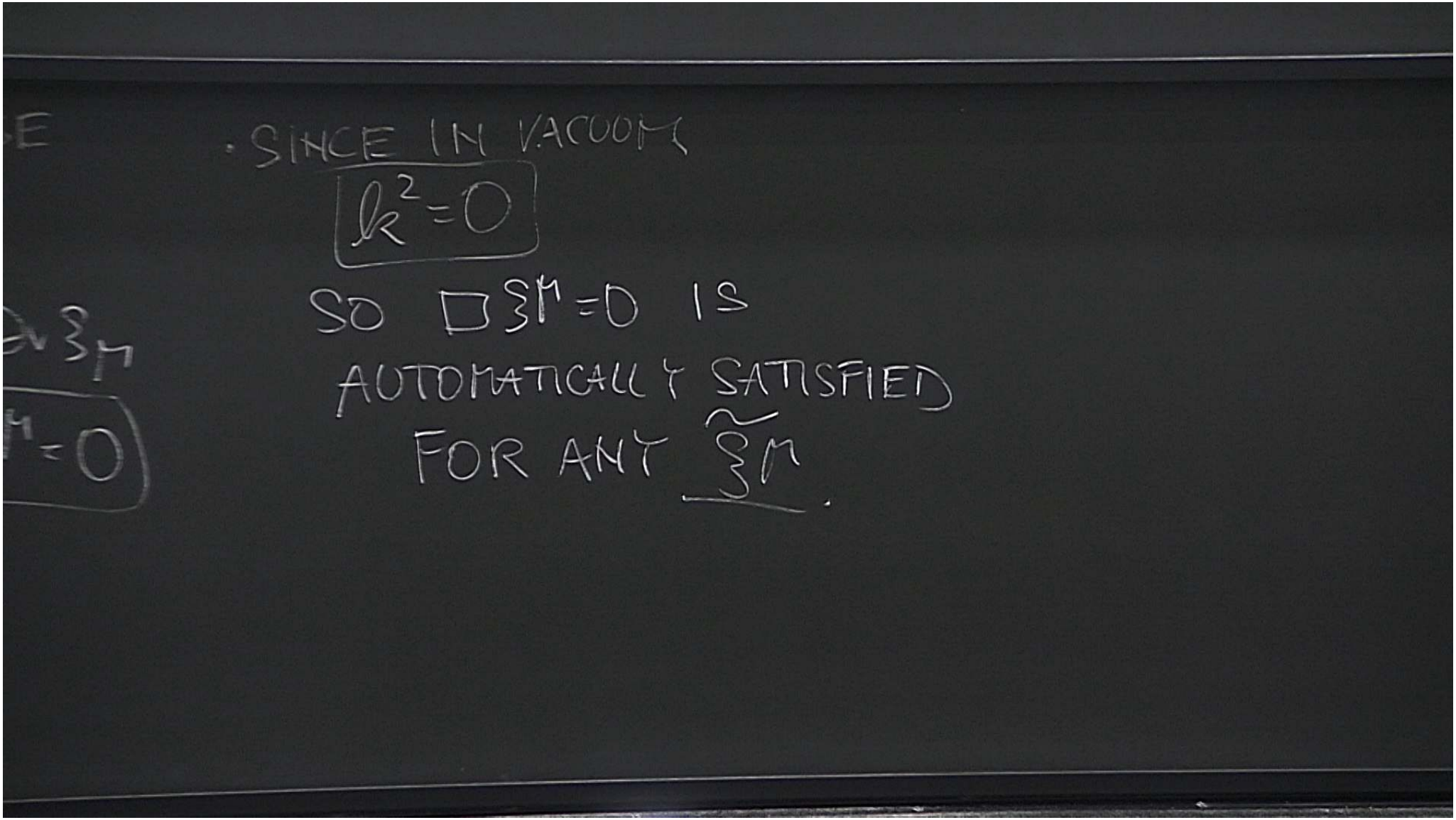
E VECTOR

- FIXING THE POLARIZATION TENSOR (OGLY WAY)
- HARMONIC GAUGE

$$k^\mu \varepsilon_{\mu\nu} - \frac{1}{2}\varepsilon k_\nu = 0 \,, \quad \varepsilon = \varepsilon^\alpha{}_\alpha$$

4 CONDITIONS FOR $\varepsilon_{\mu\nu}$.


• THERE IS STILL A RESIDUAL GAUGE FREEDOM
$x^\mu \to x^\mu - \xi^\mu$
$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
SO THAT $\Box x^\mu = 0 \leftrightarrow \Box \xi^\mu = 0$
$\xi^\mu = \tilde{\xi}^\mu e^{i k_\alpha x^\alpha}$


E
· SINCE IN VACUUM
$k^2 = 0$
SO $\Box \xi^\mu = 0$ IS
AUTOMATICALLY SATISFIED
FOR ANY $\tilde{\xi}^\mu$.
ξ_μ
$^\mu = 0$

SINCE IN VACUUM

$k^2 = 0$

SO $\Box \xi^\mu = 0$ IS

AUTOMATICALLY SATISFIED

FOR ANY $\tilde{\xi}^\mu$.

$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + ik_\mu \tilde{\xi}_\nu + ik_\nu \tilde{\xi}_\mu$$

KILL ANOTHER 4 COMPTS OF $\varepsilon_{\mu\nu}$

SPEC: FOR GW PROP IN X-DIRECTION:

$$\varepsilon_{\mu\nu} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & & \end{pmatrix} \end{array}$$

ATISFIED

$\hat{\xi}_\mu + ik_\nu \hat{\xi}_\nu$

COMPTS OF $\varepsilon_{\mu\nu}$

SPEC: FOR GW PROP IN X-DIRECTION:

$$\varepsilon_{\mu\nu} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_+ & \varepsilon_\times \\ 0 & 0 & \varepsilon_\times & -\varepsilon_+ \end{array}\right) \end{array}$$

ATISFIED

$\tilde{\xi}_\mu + ik_\nu \tilde{\xi}_\nu$

COMPTS OF $\varepsilon_{\mu\nu}$

SPEC: FOR GW PROP IN X-DIRECTION:

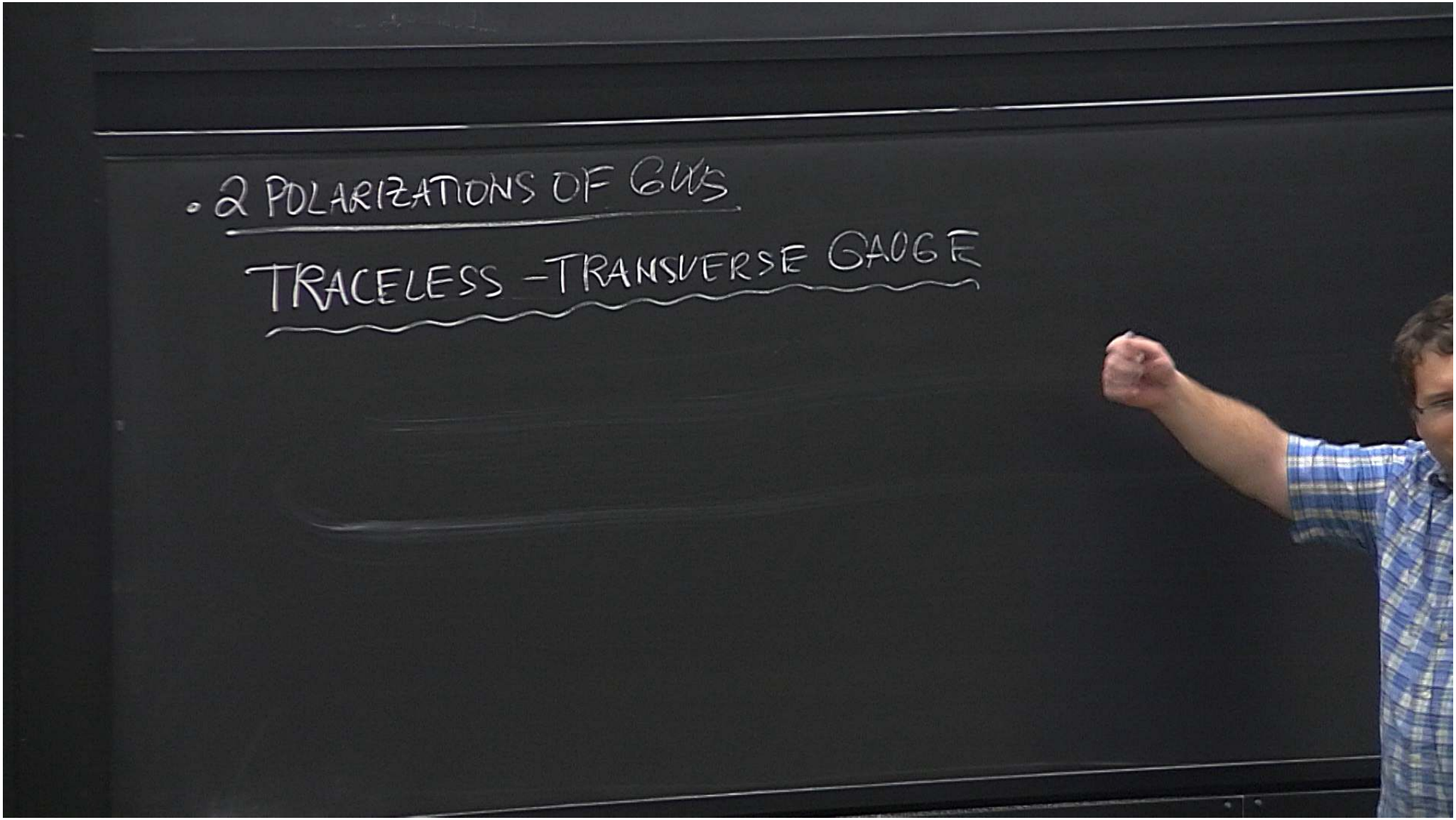
$$\varepsilon_{\mu\nu} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_+ & \varepsilon_\times \\ 0 & 0 & \varepsilon_\times & -\varepsilon_+ \end{array}\right) \end{array}$$

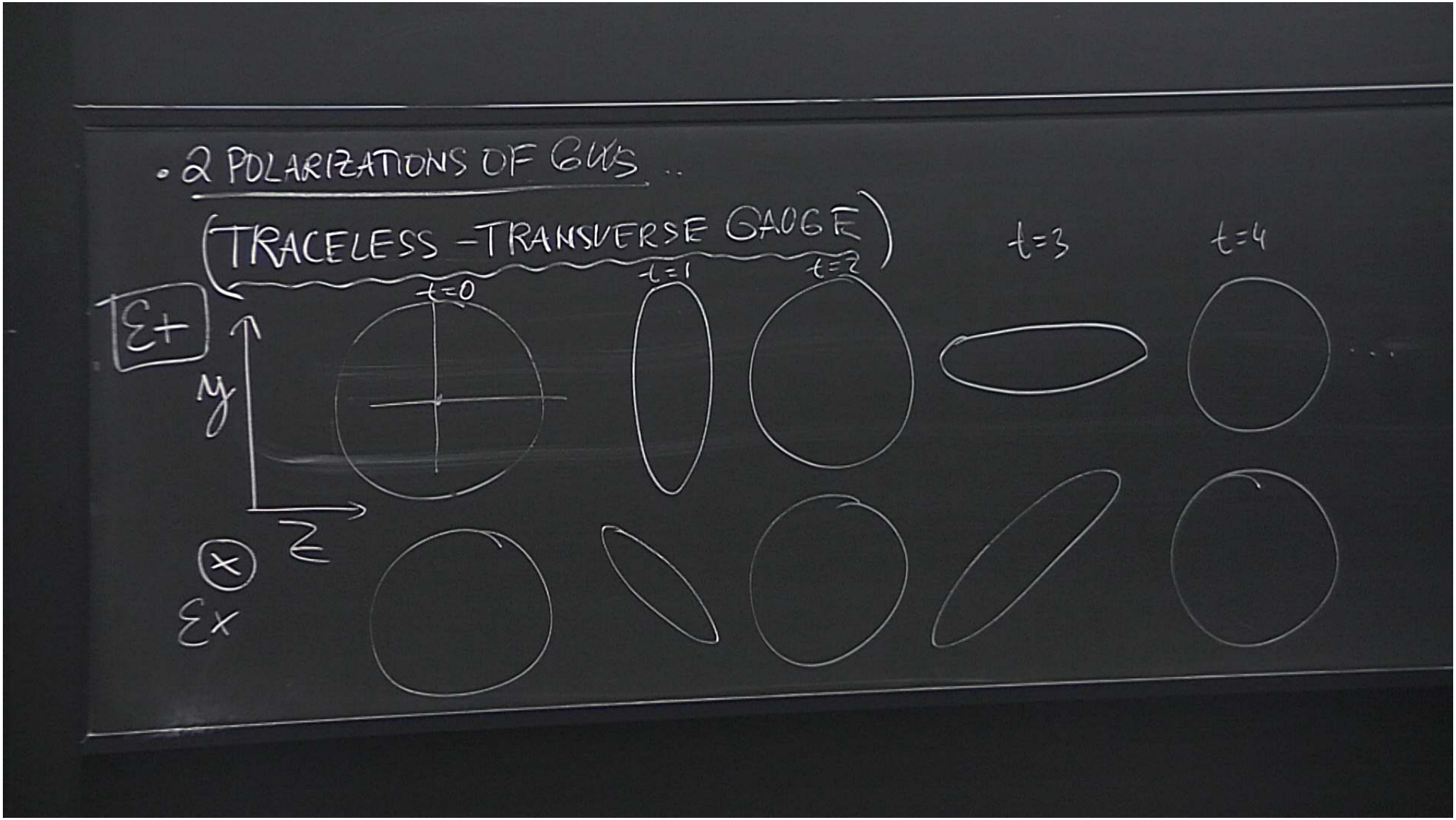
2 COMPTS $\varepsilon_+, \varepsilon_\times$ — 2 DOF

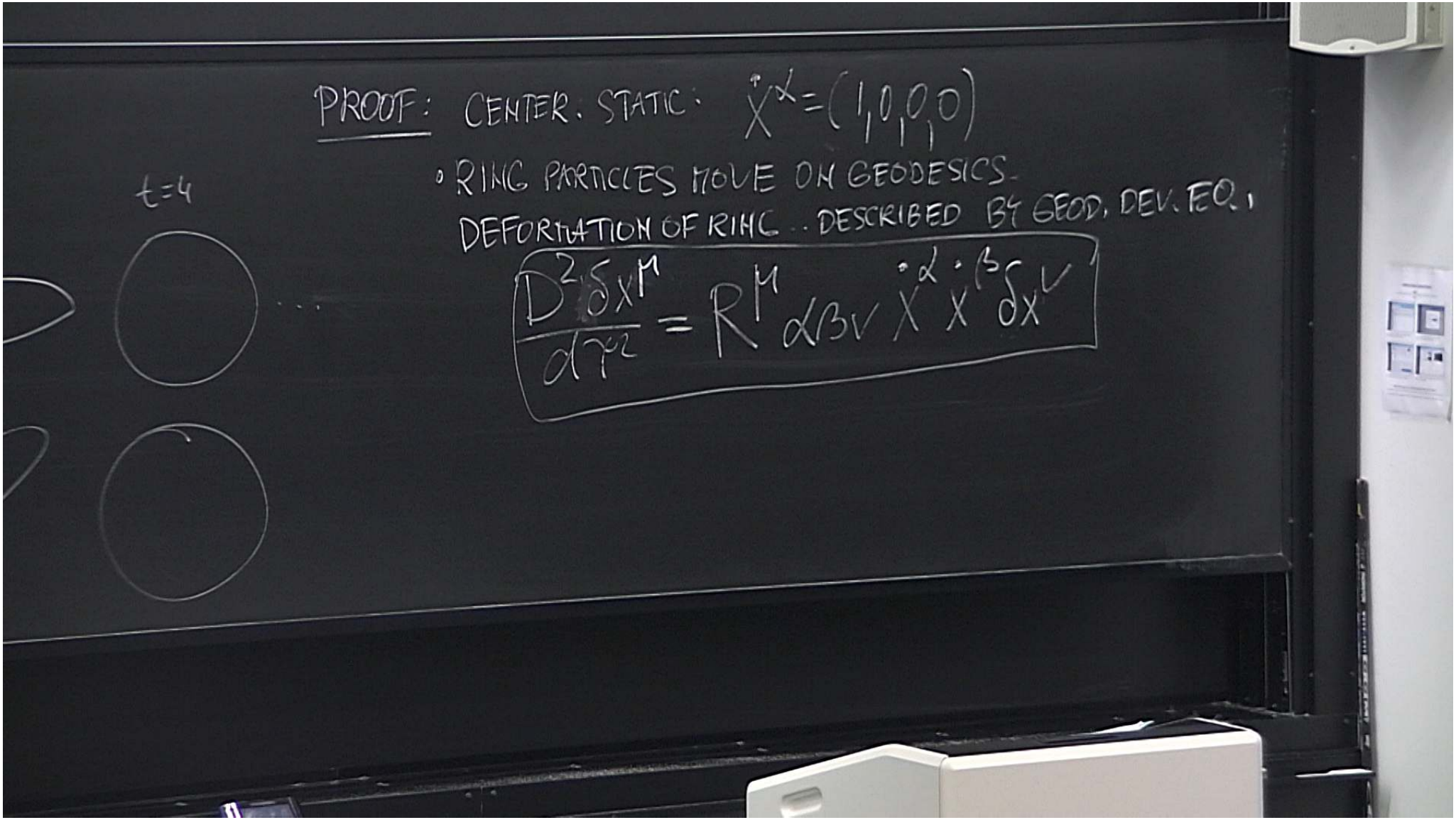
ATISFIED

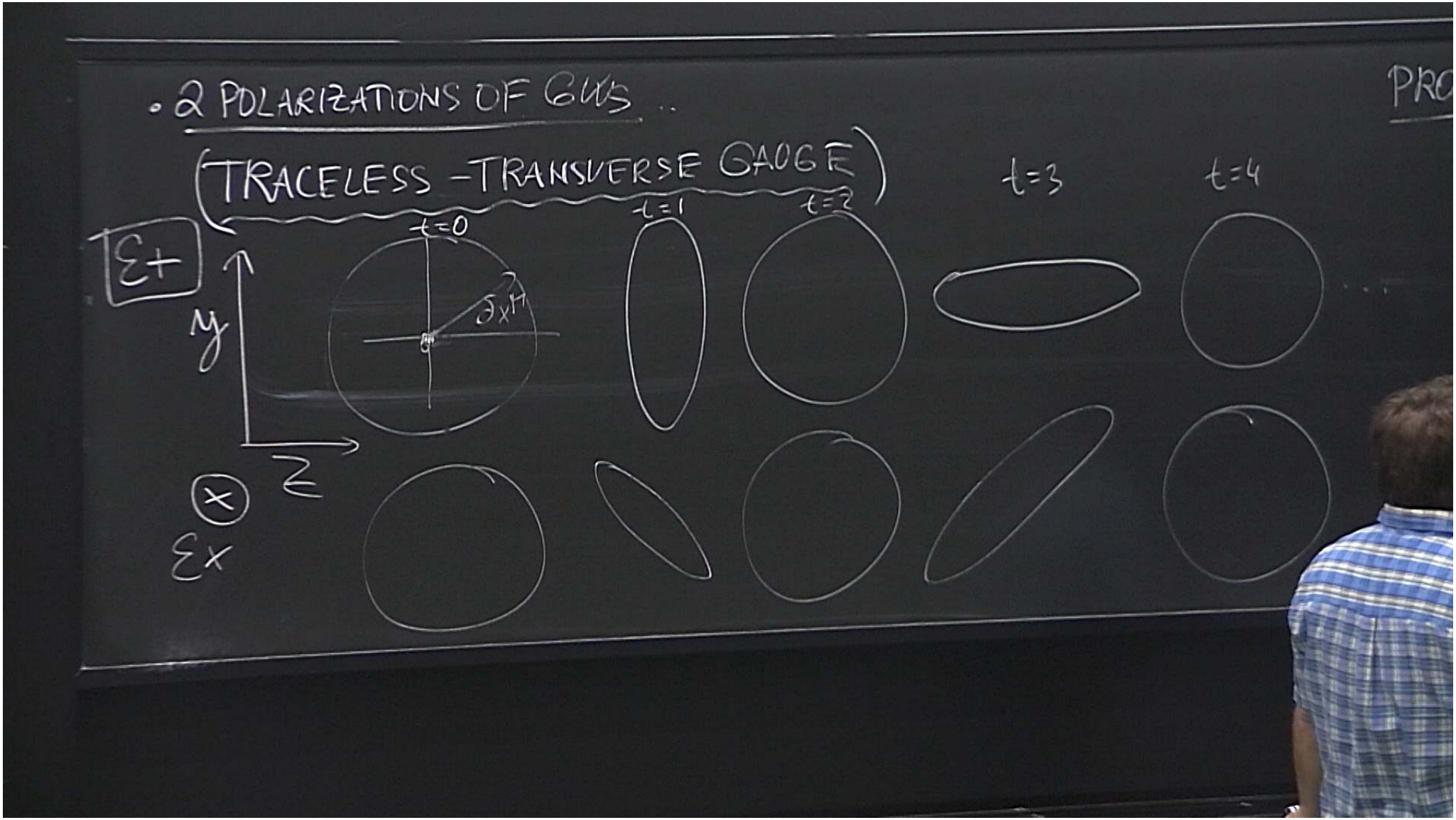
$\tilde{\xi}_\mu + ik_\nu \tilde{\xi}_\mu$

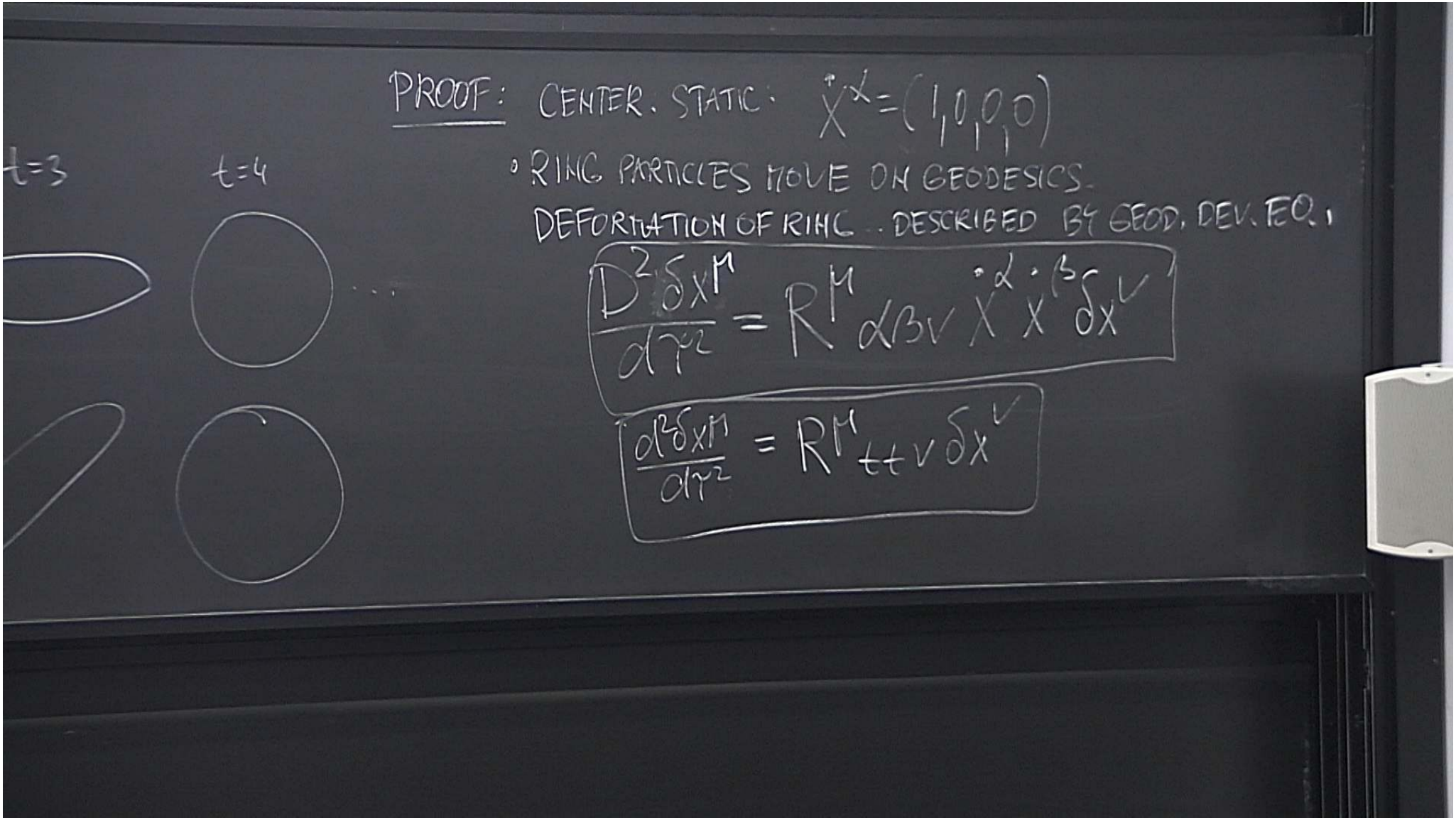
COMPTS OF $\varepsilon_{\mu\nu}$

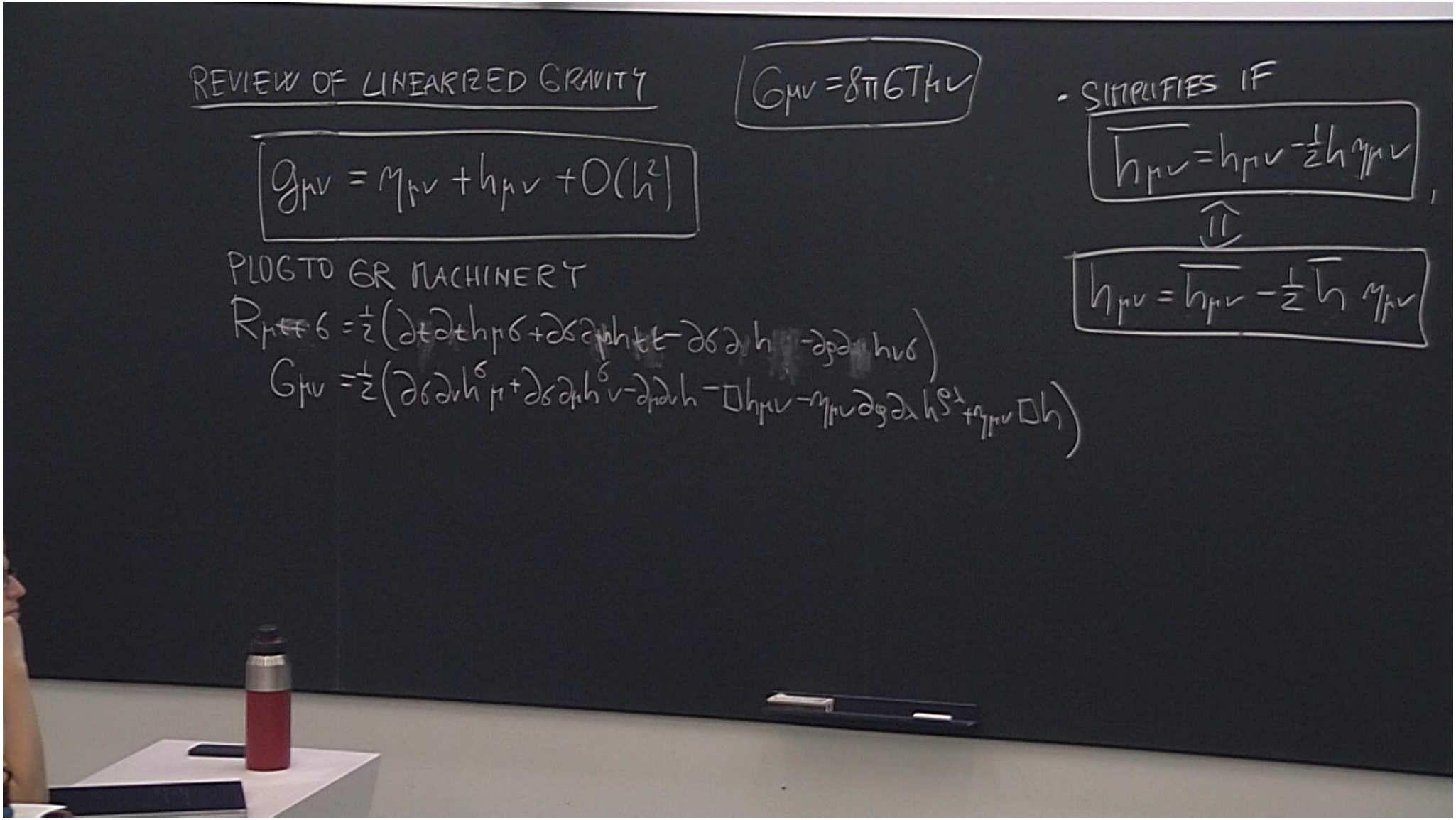

• 2 POLARIZATIONS OF GWS
TRACELESS - TRANSVERSE GAUGE

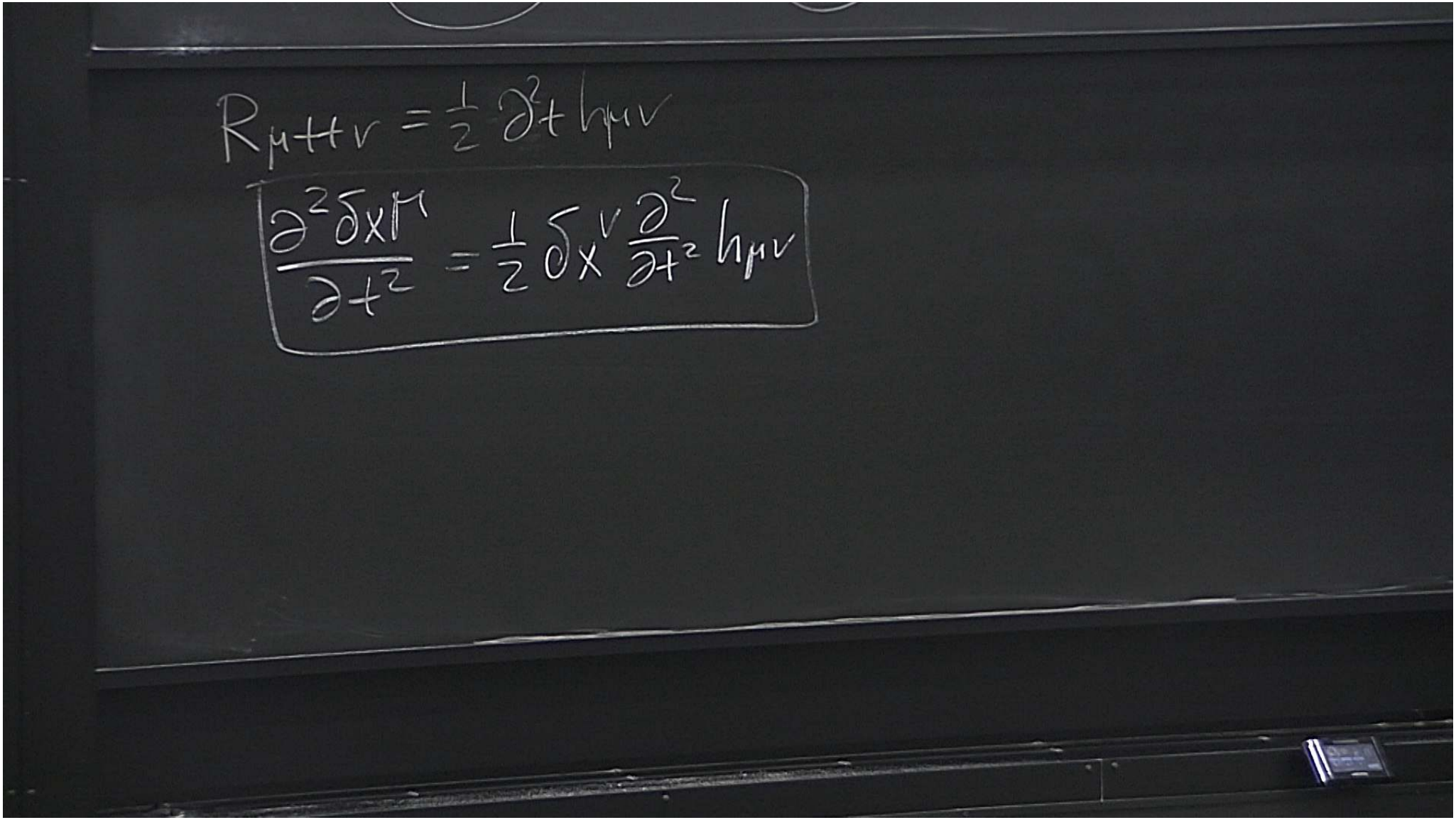

• 2 POLARIZATIONS OF GWS
(TRACELESS - TRANSVERSE GAUGE)
t=0
t=1
t=2
t=3
t=4
E+
y
z
x
Ex


PROOF: CENTER. STATIC: $\dot{X}^\alpha = (1,0,0,0)$
• RING PARTICLES MOVE ON GEODESICS.
DEFORMATION OF RING. DESCRIBED BY GEOD. DEV. EQ.:
$\frac{D^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\nu} \dot{X}^\alpha \dot{X}^\beta \delta x^\nu$
t=4


• 2 POLARIZATIONS OF GWS ..
(TRACELESS - TRANSVERSE GAUGE)
t=0
t=1
t=2
t=3
t=4
ε+
y
z
x
εx


PROOF: CENTER. STATIC: $\dot{X}^\mu = (1,0,0,0)$
t=3
t=4
• RING PARTICLES MOVE ON GEODESICS.
DEFORMATION OF RING. DESCRIBED BY GEOD. DEV. EQ.:
$\frac{D^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\nu} \dot{X}^\alpha \dot{X}^\beta \delta x^\nu$
$\frac{d^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{tt\nu} \delta x^\nu$


REVIEW OF LINEARIZED GRAVITY
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
• SIMPLIFIES IF
$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$
$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$
PLOG TO GR MACHINERY
$R_{\mu\nu\sigma\rho} = \frac{1}{2}(\partial_\nu\partial_\sigma h_{\mu\rho} + \partial_\rho\partial_\mu h_{\nu\sigma} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma})$
$G_{\mu\nu} = \frac{1}{2}(\partial_\sigma\partial_\nu h^\sigma_\mu + \partial_\sigma\partial_\mu h^\sigma_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \eta_{\mu\nu}\Box h)$


$R_{\mu tt\nu} = \frac{1}{2}\partial_t^2 h_{\mu\nu}$
$\frac{\partial^2 \delta x^\mu}{\partial t^2} = \frac{1}{2}\delta x^\nu \frac{\partial^2}{\partial t^2} h_{\mu\nu}$

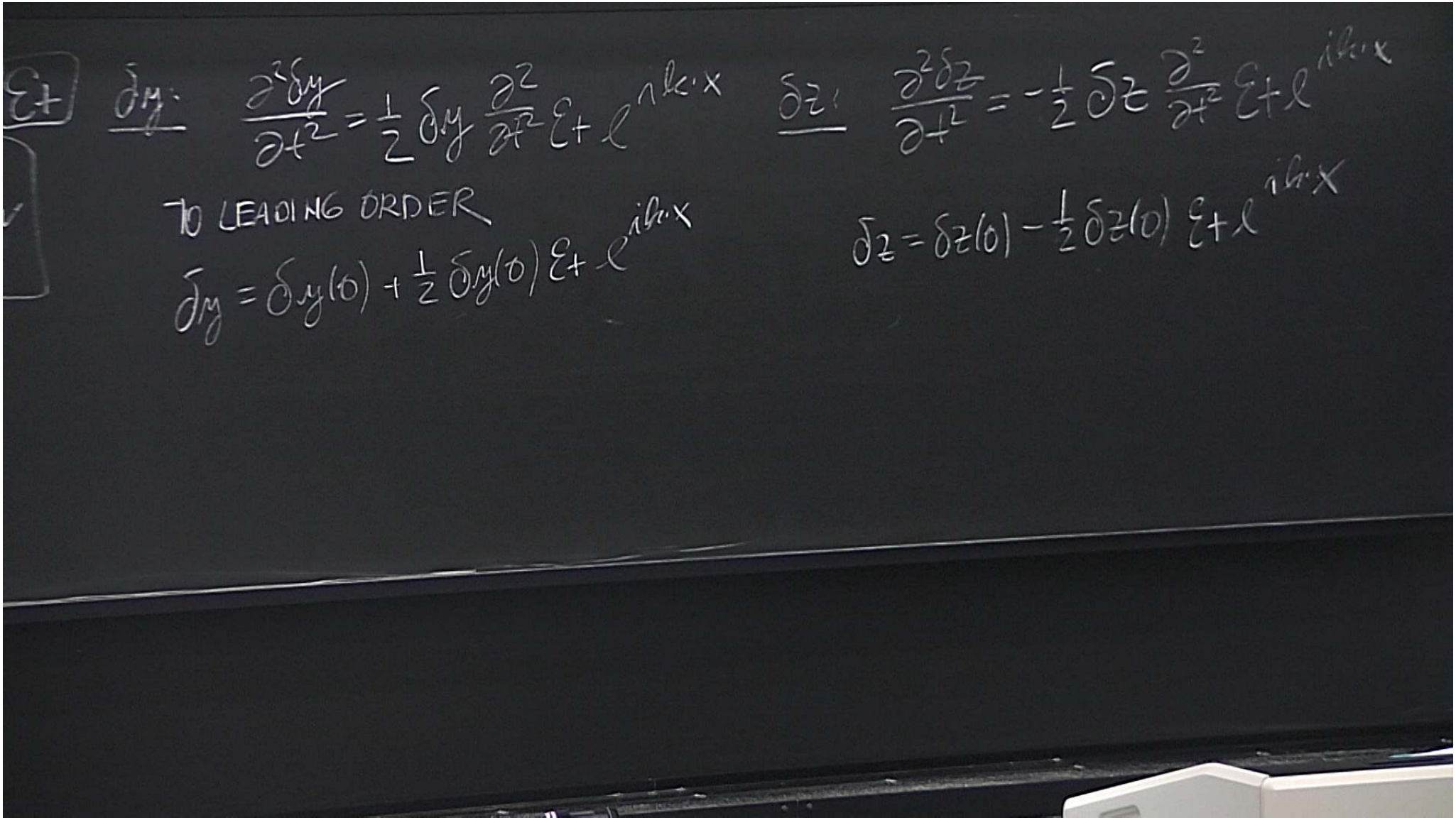
$$R_{\mu tt\nu} = \frac{1}{2}\partial_t^2 h_{\mu\nu}$$

$\mathcal{E}_+$ δ_y:

$$\frac{\partial^2 \delta_y}{\partial t^2} = \frac{1}{2}\delta_y \frac{\partial^2}{\partial t^2}\mathcal{E}_+ e^{i k\cdot x}$$

$$\boxed{\frac{\partial^2 \delta x^\mu}{\partial t^2} = \frac{1}{2}\delta x^\nu \frac{\partial^2}{\partial t^2} h_{\mu\nu}}$$

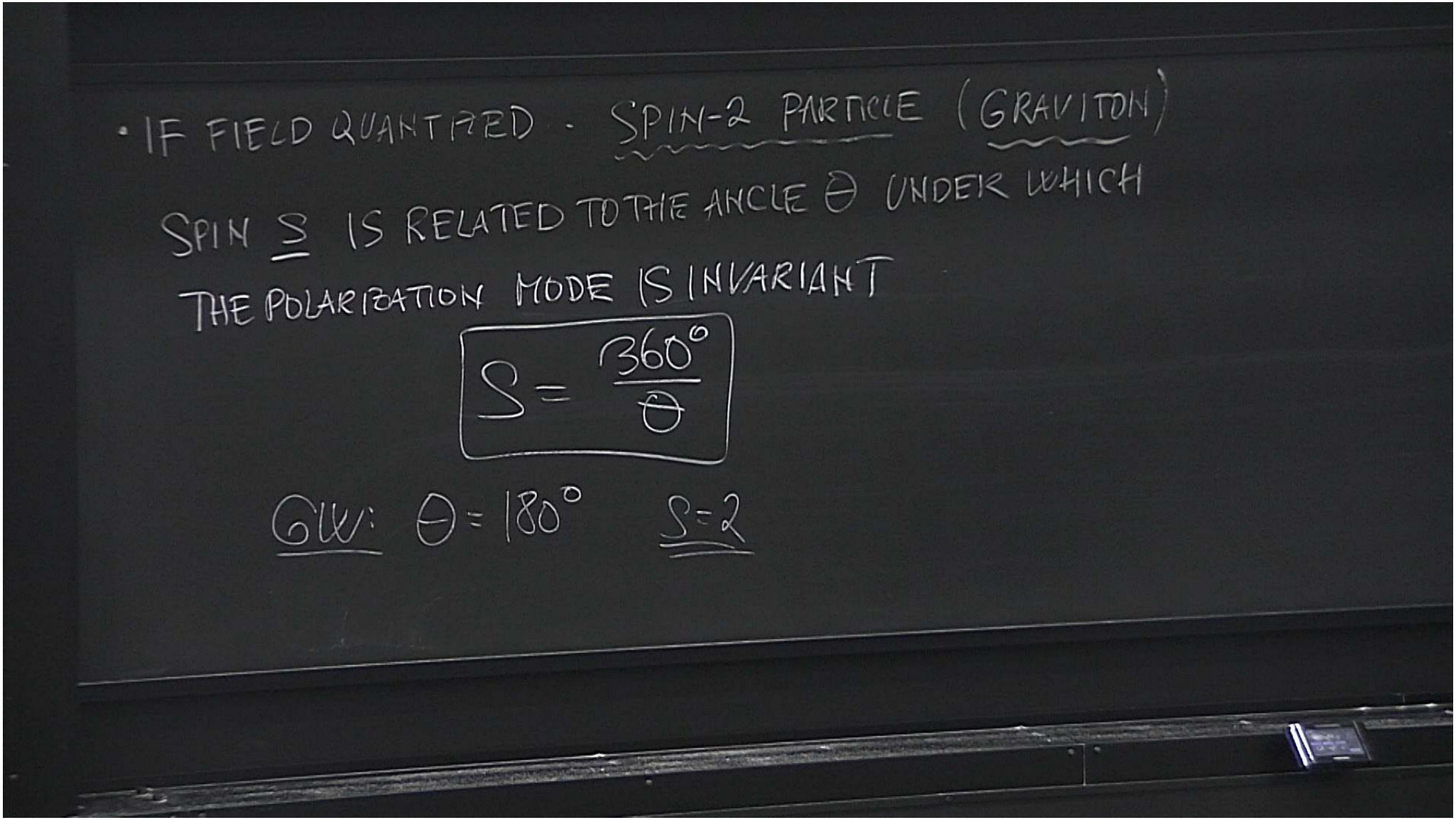
$$\delta x^\mu = (\ ,0,\delta_y,\delta_z)$$


δ_y: $\frac{\partial^2 \delta_y}{\partial t^2} = \frac{1}{2}\delta_y \frac{\partial^2}{\partial t^2} \mathcal{E}_+ e^{i k\cdot x}$
δ_z: $\frac{\partial^2 \delta_z}{\partial t^2} = -\frac{1}{2}\delta_z \frac{\partial^2}{\partial t^2} \mathcal{E}_+ e^{i k\cdot x}$
TO LEADING ORDER
$\delta_y = \delta_y(0) + \frac{1}{2}\delta_y(0)\,\mathcal{E}_+ e^{i k\cdot x}$
$\delta_z = \delta_z(0) - \frac{1}{2}\delta_z(0)\,\mathcal{E}_+ e^{i k\cdot x}$

- IF FIELD QUANTIZED · SPIN-2 PARTICLE (GRAVITON)

SPIN S IS RELATED TO THE ANGLE θ UNDER WHICH THE POLARIZATION MODE IS INVARIANT

$$S = \frac{360^\circ}{\theta}$$


• IF FIELD QUANTIZED · SPIN-2 PARTICLE (GRAVITON)
SPIN S IS RELATED TO THE ANGLE θ UNDER WHICH
THE POLARIZATION MODE IS INVARIANT
S = 360°/θ
GW: θ = 180° S = 2

- IF FIELD QUANTIZED · SPIN-2 PARTICLE (GRAVITON)

SPIN S IS RELATED TO THE ANGLE θ UNDER WHICH THE POLARIZATION MODE IS INVARIANT

$$S = \frac{360^\circ}{\theta}$$

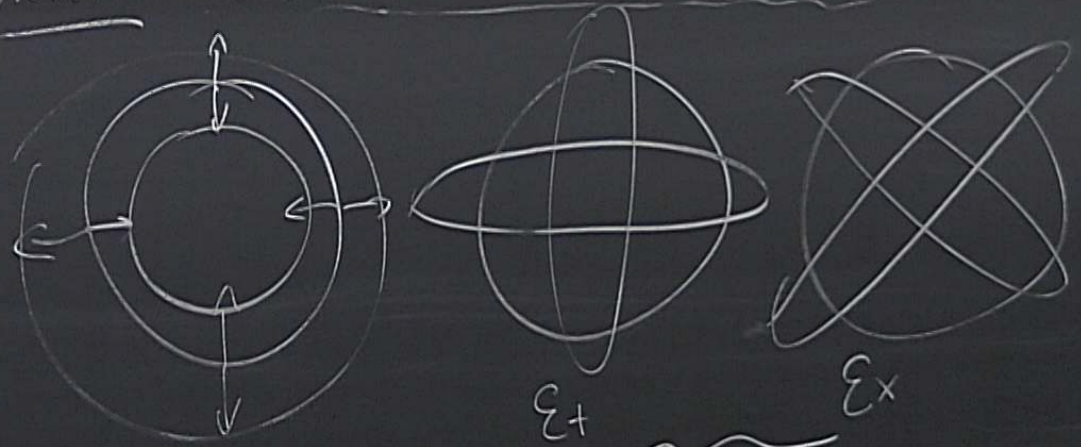
GW: $\theta = 180^\circ$ $S = 2$

EM: $\theta = 360^\circ$ $S = 1$

GRAVITON)

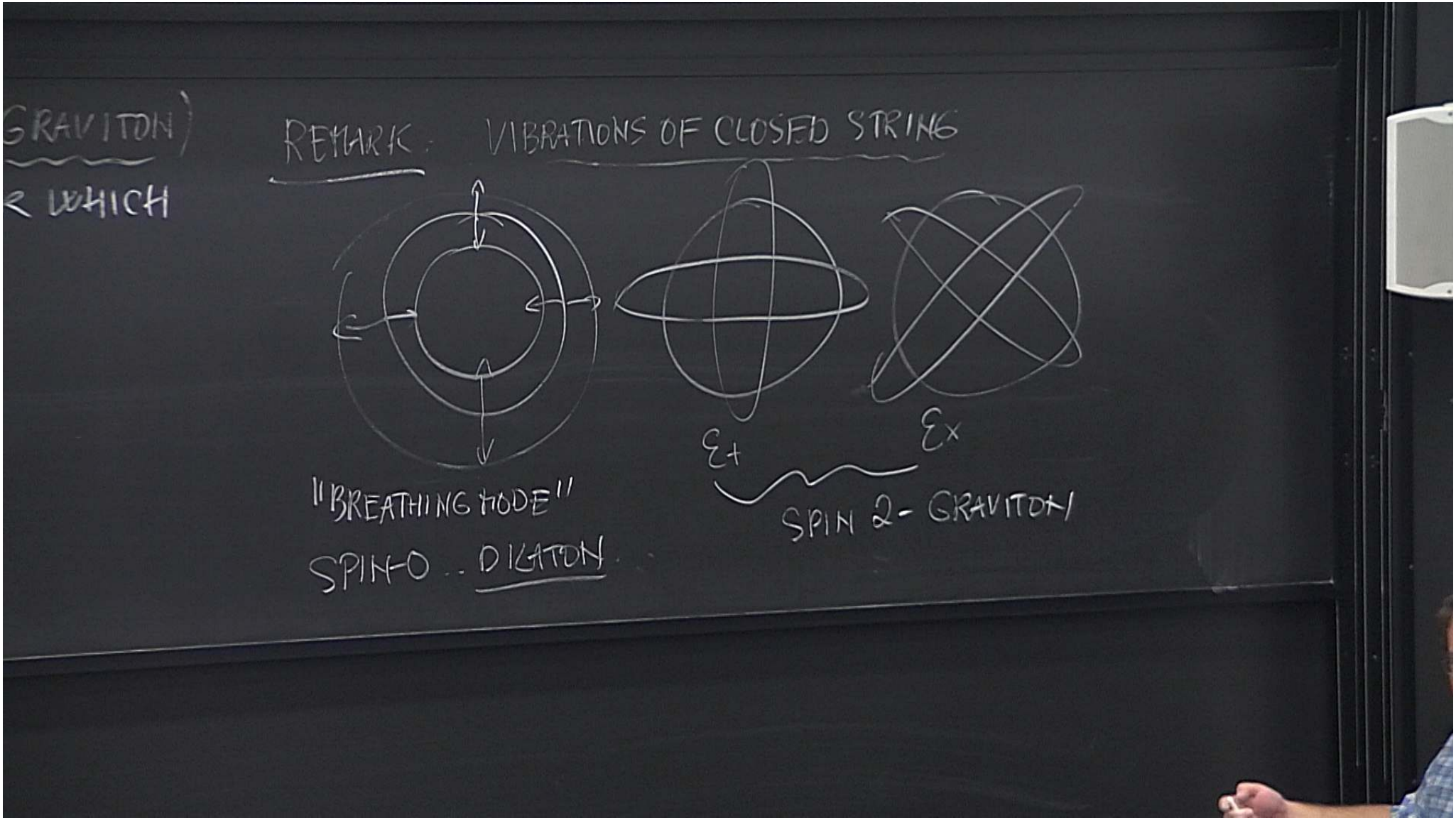
R WHICH

REMARK: VIBRATIONS OF CLOSED STRING



"BREATHING MODE"

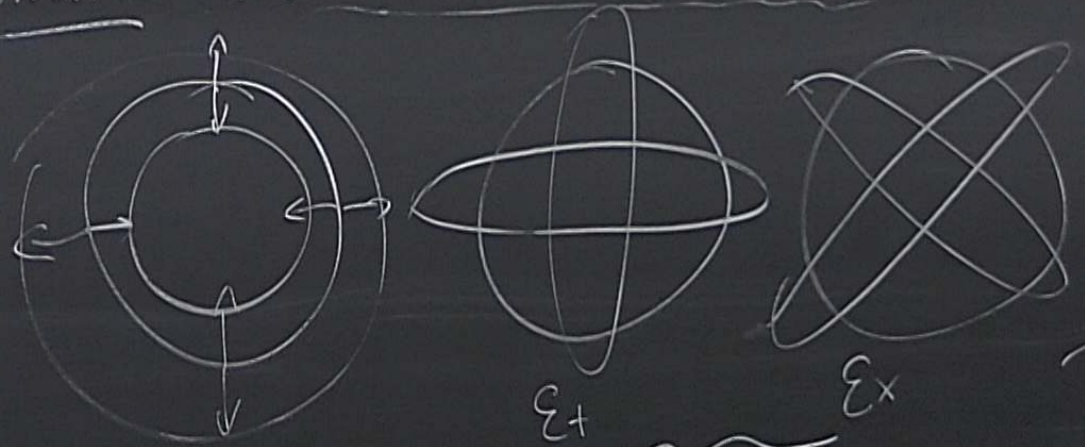
SPIN 2 - GRAVITON


GRAVITON)
R WHICH
REMARK: VIBRATIONS OF CLOSED STRING
"BREATHING MODE"
SPIN-0 . DILATON
ε+
εx
SPIN 2 - GRAVITON

GRAVITON)

R WHICH

REMARK: VIBRATIONS OF CLOSED STRING



"BREATHING MODE"

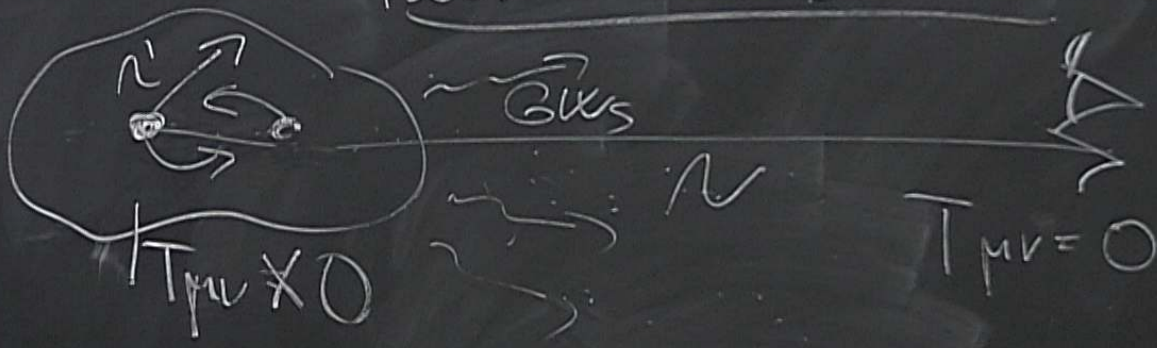
SPIN-0 .. DILATON

SPIN 2 - GRAVITON

A THEORY OF QG.

GRAVITATIONAL RADIATION FROM AN ISOLATED SYSTEM

ISOLATED SOURCE OF GWS



$|\vec{x}'| \ll |\vec{x}|$

$\omega \ll \frac{1}{|\vec{x}'|}$ SLOW MOTION

- HAVE TO SOLVE $\Box \overline{h^{\mu\nu}} = -16\pi G T^{\mu\nu}$

SPECIAL SOL. .. GIVEN BY RETARDED POTENTIAL.

$$\overline{h^{\mu\nu}}(t,\vec{x}) = 4G \int d^3x' \, \frac{T^{\mu\nu}(t_R,\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$t_R = t - |\vec{x}-\vec{x}'|$$

$\omega \ll \frac{1}{|\vec{x}'|}$ SLOW MOTION

$$\tilde{h}_{\mu\nu}(\omega,\vec{x}) = \frac{1}{\sqrt{2\pi}}\int dt\, e^{i\omega t} h_{\mu\nu} = \frac{4G}{\sqrt{2\pi}}\int dt\, d^3x'\, e^{i\omega t}\frac{T_{\mu\nu}(t_R,\vec{x}')}{|x-x'|}$$

$|\vec{x}'| \ll |\vec{x}|$

$\omega \ll \frac{1}{|\vec{x}'|}$ SLOW MOTION

$t_R = t - |\vec{x} - \vec{x}'|$

$$\tilde{h}_{\mu\nu}(\omega, \vec{x}) = \frac{1}{\sqrt{2\pi}} \int dt\, e^{i\omega t} h_{\mu\nu} = \frac{4G}{\sqrt{2\pi}} \int dt\, d^3x'\, e^{i\omega t} \frac{T_{\mu\nu}(t_R, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

. CHANGE $t \to$

$$= \frac{4G}{\sqrt{2\pi}} \int dt_R\, d^3x'\, e^{i\omega(t_R + |\vec{x} - \vec{x}'|)} \frac{T_{\mu\nu}(t_R, \vec{x}')}{|\vec{x} - \vec{x}'|} = 4G \int d^3x'\, e^{i\omega|\vec{x} - \vec{x}'|} \frac{\tilde{T}_{\mu\nu}(\omega, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

$|\vec{x}'| \ll |\vec{x}|$

$\omega \ll \frac{1}{|\vec{x}'|}$ SLOW MOTION

$t_R = t - |\vec{x} - \vec{x}'|$

$$\tilde{h}_{\mu\nu}(\omega, \vec{x}) = \frac{1}{\sqrt{2\pi}} \int dt\, e^{i\omega t} h_{\mu\nu} = \frac{4G}{\sqrt{2\pi}} \int dt\, d^3x'\, e^{i\omega t} \frac{T_{\mu\nu}(t_R, x')}{|x - x'|}$$

. CHANGE $t \to$

$$= \frac{4G}{\sqrt{2\pi}} \int dt_R\, d^3x'\, e^{i\omega(t_R + |x - x'|)} \frac{T_{\mu\nu}(t_R, x')}{|x - x'|} = 4G \int d^3x'\, e^{i\omega|x - x'|} \frac{\tilde{T}_{\mu\nu}(\omega, x')}{|x - x'|}$$

$$\boxed{\tilde{h}_{\mu\nu} \simeq 4G \frac{e^{i\omega r}}{r} \int d^3x'\, T_{\mu\nu}(\omega, x')}$$