In the framework of ontological models, the features of quantum theory that emerge as inherently nonclassical always involve properties that are fine tuned, i.e., properties that hold at the operational level but break at the ontological level (they only hold for fine tuned values of the ontic parameters). Famous examples of fine tuned properties are noncontextuality and locality. We here develop a precise theory-independent mathematical framework for characterizing operational fine tunings. These are distinct from causal fine tunings already introduced by Wood and Spekkens, as they do not involve any assumption on the underlying causal structure. We show how all the already known examples of operational fine tunings fit into our framework, we discuss possibly new fine tunings and we use the framework to shed new light on the relation between nonlocality and generalized contextuality. The framework is set in the language of functors in category theory and it aims at unifying the spooky properties of quantum theory as well as accounting for new ones.
A mathematical framework for operational fine tunings

Lorenzo Catani

Joint work with Matt Leifer
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  - Preparation noncontextuality
  - Parameter independence
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- Conclusion
Motivation

- We want a better understanding of QT.

  Single out the inherently non-classical features.
Motivation

- We want a better understanding of QT.
  
  Single out the inherently non-classical features.

- These features come from the comparison of QT and ontological models (no go theorems).

- These features unavoidably involve fine tunings.

  Fine tunings = properties that hold at the operational level, but cannot hold at the ontological level (they hold only by fine tuning of the ontic parameters).
Motivation

- Fine tunings characterize nature with a conspiratorial connotation. How to deal with them?
Motivation

• Fine tunings characterize nature with a conspiratorial connotation. How to deal with them?

  - Provide a physical explanation of why they arise.
Motivation

- Fine tunings characterize nature with a conspiratorial connotation. How to deal with them?
  - Provide a physical explanation of why they arise.
  - Develop new ontological framework for QT free of fine tunings.
Motivation

- Fine tunings characterize nature with a conspiratorial connotation. How to deal with them?
  - Provide a physical explanation of why they arise.
  - Develop new ontological framework for QT free of fine tunings.

- We first need to properly define them.
Motivation

Comparison with causal fine tunings

• Already discussed by Wood and Spekkens in the framework of causal models:

*The probability distribution induced by a causal model M is no fine tuned if its conditional independencies continue to hold for any variation of the causal-statistical parameters in M.*

Motivation

Comparison with causal fine tunings

• Already discussed by Wood and Spekkens in the framework of causal models:

*The probability distribution induced by a causal model M is no fine tuned if its conditional independencies continue to hold for any variation of the causal-statistical parameters in M.*


• Here we do not want to make any assumption on the causal structure. We refer to operational fine tunings.
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Examples

Operational approach to physical theories

- A **physical theory** is just a tool to predict the statistics of outcomes from experimental procedures.

\[ p(u|P, T, M) \]
Operational quantum theory

\[ P \]
\[ \rho \] (Density Operator)

\[ T \]
\[ \mathcal{E} \] (CPTP Map)

\[ M \]
\[ \{E_u\} \] (POVM)
Examples

Operational quantum theory

\[ P \]
\[ \rho \]  (Density Operator)
\[ T \]  (CPTP Map)
\[ M \]  (POVM)

\[ p(u|P, T, M) = \text{Tr}(\varepsilon(\rho)E_u) \]  (Born Rule)
Examples

Ontological model


• The system has definite properties even if no observer and no experiment.

• These are represented by the ontic states $\lambda \in \Lambda$.

• Experimental procedures:
Examples

Ontological model


- The system has definite properties even if no observer and no experiment.

- These are represented by the ontic states $\lambda \in \Lambda$.

- Experimental procedures:

  \[
  P \rightarrow \mu_P(\lambda) \\
  T \rightarrow \Gamma_T(\lambda, \lambda') \\
  M, u \rightarrow \xi_{M,u}(\lambda)
  \]
Examples

Ontological model

• The system has definite properties even if no observer and no experiment.

• These are represented by the ontic states $\lambda \in \Lambda$. 
Examples

Ontological model

• The system has definite properties even if no observer and no experiment.

• These are represented by the ontic states $\lambda \in \Lambda$.

• Statistics (classical probability theory):

$$p(u|P, T, M) = \int d\lambda' d\lambda \, \xi_{M,u}(\lambda') \Gamma_T(\lambda', \lambda) \mu_P(\lambda)$$
Examples

Ontological model – Hidden assumptions

- **Free choice:** The experimenter is free to choose the input variables (associated to $(P, T, M)$) however she/he likes.
Examples

Ontological model – Hidden assumptions

• **Free choice:** The experimenter is free to choose the input variables (associated to \((P, T, M)\)) however she/he likes.

• **No-retrocausality:**

  1) no signaling from the future to the past
  2) measurement independence.
Examples

Ontological model – Hidden assumptions

• **Free choice:** The experimenter is free to choose the input variables (associated to \((P, T, M)\)) however she/he likes.

• **No-retrocausality:**
  1) no signaling from the future to the past
  2) measurement independence.

• **\(\lambda\)-mediation:** the ontic state of the system mediates any correlation between the preparations and the measurements
• **No-retrocausality:**

1) \( p(u_1|r_1, r_2) = p(u_1|r_1) \)

2) \( p(\lambda|u_1, r_1, r_2) = p(\lambda|u_1, r_1) \)
Examples

- **No-retrocausality:**

\[ p(u_1 | r_1, r_2) = p(u_1 | r_1) \]

\[ p(\lambda | u_1, r_1, r_2) = p(\lambda | u_1, r_1) \]

\[ p(u_1, u_2, \lambda | r_1, r_2) = p(u_2 | u_1, \lambda, r_1, r_2)p(\lambda | u_1, r_1)p(u_1 | r_1) \]
Examples

- No-retrocausality:

  1) \[ p(u_1 | r_1, r_2) = p(u_1 | r_1) \]

  2) \[ p(\lambda | u_1, r_1, r_2) = p(\lambda | u_1, r_1) \]

- \( \lambda \)-mediation: \[ p(u_2 | u_1, \lambda, r_1, r_2) = p(u_2 | \lambda, r_2) \]
Examples

• In general, we only assume the existence of an ontic extension for each experiment in the theory:

1) \((P, T, M) \Rightarrow (P, T, M, \Lambda)\)
Examples

• In general, we only assume the existence of an ontic extension for each experiment in the theory:

1) \((P, T, M) \Rightarrow (P, T, M, \Lambda)\)

2) \(p(u|P, T, M) \Rightarrow p(u, \lambda|P, T, M)\)

such that

\[
\int_{\Lambda} p(u, \lambda|P, T, M) = p(u|P, T, M)
\]
Examples

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Preparation noncontextual ontological model


Operational equivalences imply ontological equivalences.

\[ p(u|P, T, M) = p(u|P', T, M) \quad \forall \quad M, T, u \]
Examples

Preparation noncontextual ontological model


Operational equivalences imply ontological equivalences.

\[ p(u|P, T, M) = p(u|P', T, M) \quad \forall \ M, T, u \]

\[ \downarrow \]

\[ p(u, \lambda|P, T, M) = p(u, \lambda|P', T, M) \quad \forall \ \lambda \in \Lambda, \ \forall \ M, T, u \]
Examples

Preparation noncontextual ontological model


Operational equivalences imply ontological equivalences.

\[ p(u|P, T, M) = p(u|P', T, M) \ \forall \ M, T, u \]

\[ \downarrow \]

\[ p(u, \lambda|P, T, M) = p(u, \lambda|P', T, M) \ \forall \ \lambda \in \Lambda, \ \forall \ M, T, u \]

No preparation noncontextual ontological model is consistent with QT.
Examples

Preparation noncontextual ontological model
Examples

Preparation noncontextual ontological model

$P, Q$ are two experiments.

\[ P = Q \]
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Examples

Bell’s scenario

\[ \begin{align*}
& a \\
\quad & \uparrow \\
& x \\
& \rightarrow \\
\text{Alice} \\
& \lambda \\
& \rightarrow \\
& y \\
& \downarrow \\
& b \\
\end{align*} \]
Examples

Parameter Independence

Operational no signaling implies ontic no signaling.
Examples

Parameter Independence

\[ a \quad b \quad \quad a \quad b \]

\[ \begin{array}{c}
P \\
\hline
0 \\
Y_0 \\
\hline
x \\
y
\end{array} \quad = \quad \begin{array}{c}
P \\
\hline
1 \\
Y_1 \\
\hline
x \\
y
\end{array} \]
Examples

Parameter Independence

\[ a \quad b \quad \bar{a} \quad \bar{b} \]

\[ P \]

\[ x \quad y \quad Y_0 \quad Y_1 \]

\[ \lambda \]

\[ a \quad b \quad \omega \]

\[ P \]

\[ x \quad y \quad Y_0 \quad Y_1 \]

\[ \lambda \]
Examples

Bell’s local causality

Parameter independence + outcome independence.

Outcome independence:

\[
\begin{align*}
    p(a|b = 0, x, y) &= p(a|b = 1, x, y) \\
    p(b|a = 0, x, y) &= p(b|a = 1, x, y)
\end{align*}
\]
Examples

Bell’s local causality

Parameter independence + outcome independence.

Outcome independence:

\[
\begin{cases}
    p(a|b = 0, x, y) = p(a|b = 1, x, y) \\
    p(b|a = 0, x, y) = p(b|a = 1, x, y)
\end{cases}
\]

\[
\downarrow
\]

\[
\begin{cases}
    p(a|\lambda, b = 0, x, y) = p(a|\lambda, b = 1, x, y) \\
    p(b|\lambda, a = 0, x, y) = p(b|\lambda, a = 1, x, y).
\end{cases}
\]
Examples

Bell’s local causality

Parameter independence + outcome independence.

Outcome independence:

\[
\begin{align*}
\{ & \quad p(a|b = 0, x, y) = p(a|b = 1, x, y) \\
 & \quad p(b|a = 0, x, y) = p(b|a = 1, x, y) \\
\}
\downarrow
\{ & \quad p(a|\lambda, b = 0, x, y) = p(a|\lambda, b = 1, x, y) \\
 & \quad p(b|\lambda, a = 0, x, y) = p(b|\lambda, a = 1, x, y). \\
\}
\]

Local causality + ontological model framework inconsistent with QT.
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Examples

Time symmetry fine tuning


Operational time symmetry implies ontological time symmetry.

Operational time symmetry:
Examples

Time symmetry fine tuning


Operational time symmetry implies ontological time symmetry.

Operational time symmetry:

\[ \begin{align*}
& P \quad T \quad M \\
& \uparrow \quad \uparrow \quad \uparrow \\
& r_1 \quad u_1 \quad u_2 \\
& \downarrow \quad \downarrow \\
& \quad \quad r_2
\end{align*} \]
Examples

Time symmetry fine tuning


Operational time symmetry implies ontological time symmetry.

Operational time symmetry:

\[ p_{(P,T,M)}(u_1, u_2 | r_1, r_2) = p_{(P',T',M')} (u_2, u_1 | r_2, r_1) \]
Examples

Time symmetry fine tuning

Operational time symmetry implies ontological time symmetry.

\[ p(P, M, T)(u_1, u_2 | r_1, r_2) = p(P', M', T')(u_2, u_1 | r_2, r_1) \]

\[ \Downarrow \]

\[ p(P, M, T)(u_1, u_2, \lambda | r_1, r_2) = p(P', M', T')(u_2, u_1, k(\lambda) | r_2, r_1) \]

\[ k : \Lambda \rightarrow \Lambda' \]
Examples

Time symmetry fine tuning

Operational time symmetry implies ontological time symmetry.

\[ p(P, M, T)(u_1, u_2 | r_1, r_2) = p(P', M', T')(u_2, u_1 | r_2, r_1) \]

\[ \downarrow \]

\[ p(P, M, T)(u_1, u_2, \lambda | r_1, r_2) = p(P', M', T')(u_2, u_1, k(\lambda) | r_2, r_1) \]

\[ k : \Lambda \rightarrow \Lambda' \]

No time symmetric ontological model is consistent with QT.
Examples

Time symmetry fine tuning

\[ P \quad = \quad Q \]
Examples

Time symmetry fine tuning

\[ P = Q \]

\[ Q = P \]
The framework

No fine tuning condition

\[ f_0 \quad P \quad f_1 \quad Q \]
The framework

No fine tuning condition

\[ f_0 \quad P \quad = \quad f_1 \quad Q \quad \rightarrow \quad h_0 \quad P \quad = \quad h_1 \quad Q \]
The framework

No fine tuning condition

A property of an operational theory is no fine tuned if it is preserved at the ontological level:

$$\sum_{s,t,c} p_{f_0}(u|c,t)p_P(t|s)p_{f_0}(c,s|r) = \sum_{s,t,c} p_{f_1}(u|c,t)p_Q(t|s)p_{f_1}(c,s|r)$$
The framework

No fine tuning condition

A property of an operational theory is no fine tuned if it is preserved at the ontological level:

$$\sum_{s,t,c} p_{f_0}(u|c,t)p_P(t|s)p_{f_0}(c,s|r) = \sum_{s,t,c} p_{f_1}(u|c,t)p_Q(t|s)p_{f_1}(c,s|r)$$

$$\sum_{\lambda,s,t,c} p_{h_0}(u,\omega|\lambda,c,t)p_P(\lambda,t|s)p_{h_0}(c,s|r) = \sum_{s,t,c} p_{h_1}(u,\omega|\lambda,c,t)p_Q(\lambda,t|s)p_{h_1}(c,s|r)$$
The framework

Extra conditions

• Ontic extension

\[ \sum_\omega p_{h_i}(u, \omega|\lambda, r, c) = p_{f_i}(u|r, c) \]

• Structure preservation

i. Identity: \( f_i = \mathbb{I} \Rightarrow h_i = \mathbb{I}_\Lambda \)

ii. Composition: \( f_0 \circ f_1 \Rightarrow h_0 \circ_\Omega h_1 \)
Extra conditions

- Ontic extension

\[ \sum_{\omega} p_{h_i}(u, \omega|\lambda, r, c) = p_{f_i}(u|r, c) \]

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  i. Identity: \( f_i = \mathbb{I} \Rightarrow h_i = \mathbb{I}_\Lambda \)

  ii. Composition: \( f_0 \circ f_1 \Rightarrow h_0 \circ_\Omega h_1 \)

  iii. Convexity: \( \sigma f_0 + (1 - \sigma)f_1 \Rightarrow \sigma h_0 + (1 - \sigma)h_1 \), \( \sigma \in [0, 1] \)
The framework

Extra conditions

- Ontic extension

\[ \sum_{\omega} p_{h_i}(u, \omega|\lambda, r, c) = p_{f_i}(u|r, c) \]

- Structure preservation
  
  i. Identity: \( f_i = \mathbb{I} \Rightarrow h_i = \mathbb{I}_\lambda \)

  ii. Composition: \( f_0 \circ f_1 \Rightarrow h_0 \circ_\Omega h_1 \)

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- \( \omega \) sufficient statistics for \( \lambda \)
The framework

Extra conditions

- Ontic extension
  \[ \sum_{\omega} p_{h_i}(u, \omega|\lambda, r, c) = p_{f_i}(u|r, c) \]

- Structure preservation
  
  i. Identity: \( f_i = \mathbb{I} \Rightarrow h_i = \mathbb{I}_{\Lambda} \)

  ii. Composition: \( f_0 \circ f_1 \Rightarrow h_0 \circ \Omega h_1 \)

  iii. Convexity: \( \sigma f_0 + (1 - \sigma)f_1 \Rightarrow \sigma h_0 + (1 - \sigma)h_1, \ \sigma \in [0, 1] \)

- \( \omega \) sufficient statistics for \( \lambda \)
  \[ p(u, t|\omega, r, s, c) = p(u, t|\lambda, r, s, c) \]
The framework

Preparation noncontextuality

- \[ \sum_{s,t,c} p_{f_0}(u|c,t)p_P(t|s)p_{f_0}(c,s|r) = \sum_{s,t,c} p_{f_1}(u|c,t)p_Q(t|s)p_{f_1}(c,s|r) \]

- \[ f_0 = f_1 \equiv \mathbb{I}, \text{ so } p_{\mathbb{I}}(u|c,t) = \delta_{u,t} \frac{1}{N_c}, \quad p_{\mathbb{I}}(c,s|r) = \delta_{s,r} \frac{1}{N_c}, \]

where \( \sum_{c} N_c = 1. \)

Operational equation restored: \( p_P(u|r) = p_Q(u|r). \)
The framework

Preparation noncontextuality

1. \[ \sum_{s,t,c} p_{f_0}(u|c,t)p_P(t|s)p_{f_0}(c,s|r) = \sum_{s,t,c} p_{f_1}(u|c,t)p_Q(t|s)p_{f_1}(c,s|r) \]

2. \[ f_0 = f_1 \equiv \mathbb{I}, \text{ so } p_\mathbb{I}(u|c,t) = \delta_{u,t} \frac{1}{N_c}, p_\mathbb{I}(c,s|r) = \delta_{s,r} \frac{1}{N_c}, \]
   where \[ \sum_c N_c = 1. \]

   Operational equation restored: \[ p_P(u|r) = p_Q(u|r). \]

3. No fine tuning condition implies \[ \sum_\lambda p_{h_0}(\omega|\lambda)p_P(u,\lambda|r) = \sum_\lambda p_{h_1}(\omega|\lambda)p_Q(u,\lambda|r). \]
Preparation noncontextuality

\[
\sum_{s,t,c} p_{f_0}(u|c,t)p_P(t|s)p_{f_0}(c,s|r) = \sum_{s,t,c} p_{f_1}(u|c,t)p_Q(t|s)p_{f_1}(c,s|r)
\]

\[
f_0 = f_1 \equiv \mathbb{1}, \text{ so } p_{\chi}(u|c,t) = \delta_{u,t} \frac{1}{N_c}, p_{\chi}(c,s|r) = \delta_{s,r} \frac{1}{N_c},
\]

where \( \sum_c N_c = 1. \)

Operational equation restored: \( p_P(u|r) = p_Q(u|r). \)

- No fine tuning condition implies \( \sum_\lambda p_{h_0}(\omega|\lambda)p_P(u,\lambda|r) = \sum_\lambda p_{h_1}(\omega|\lambda)p_Q(u,\lambda|r). \)

- Condition i. implies \( h_0 = h_1 = \mathbb{1}_\Lambda, \text{ i.e. } p_{h_i}(\omega|\lambda) = \delta_{\omega,\lambda} \ \forall \ i \in \{0, 1\} \) and \( \omega \equiv \lambda. \)
The framework

Preparation noncontextuality

- \[ \sum_{s,t,c} p_{f_0}(u|c,t)p_P(t|s)p_{f_0}(c,s|r) = \sum_{s,t,c} p_{f_1}(u|c,t)p_Q(t|s)p_{f_1}(c,s|r) \]

- \[ f_0 = f_1 \equiv 1, \text{ so } p_1(u|c,t) = \delta_{u,t} \frac{1}{N_c}, p_1(c,s|r) = \delta_{s,r} \frac{1}{N_c}, \]
  where \[ \sum_c N_c = 1. \]

  Operational equation restored: \[ p_P(u|r) = p_Q(u|r). \]

- No fine tuning condition implies \[ \sum_{\lambda} p_{h_0}(\omega|\lambda)p_P(u,\lambda|r) = \sum_{\lambda} p_{h_1}(\omega|\lambda)p_Q(u,\lambda|r). \]

- Condition i. implies \[ h_0 = h_1 = \Pi_{\lambda}, \text{ i.e. } p_{h_i}(\omega|\lambda) = \delta_{\omega,\lambda} \forall i \in \{0,1\} \text{ and } \omega \equiv \lambda. \]

  Ontological equation restored: \[ p_P(u,\lambda|r) = p_Q(u,\lambda|r). \]
The framework

Categorical framework

Operational Category $\mathcal{O}_p = (\mathcal{P}, \mathcal{F})$
Categorical framework

Operational Category \( \mathcal{O}_P = (\mathcal{P}, \mathcal{F}) \)

\[ \mathcal{P} = \{ p(u_1, u_2, \ldots, u_n \mid r_1, r_2, \ldots, r_m) \} \]

\[ \mathcal{F} = \{ f_i : p(u_1, \ldots, u_n \mid r_1, \ldots, r_m) \rightarrow p(u'_1, \ldots, u'_n \mid r'_1, \ldots, r'_m) \} \]
The framework

Categorical framework

Operational Category $\mathcal{O}_p = (\mathcal{P}, \mathcal{F})$

$\mathcal{P} = \{ p(u_1, u_2, \ldots, u_n | r_1, r_2, \ldots, r_m) \}$

$\mathcal{F} = \{ f_i : p(u_1, \ldots, u_n | r_1, \ldots, r_m) \rightarrow p(u'_1, \ldots, u'_n | r'_1, \ldots, r'_m) \}$

Ontological Category $\mathcal{O}_n = (\mathcal{P}_\Lambda, \mathcal{H})$

$\mathcal{P}_\Lambda = \{ p(u_1, u_2, \ldots, u_n, \lambda_1, \lambda_2, \ldots, \lambda_l | r_1, r_2, \ldots, r_m) \}$
The framework

Categorical framework

Operational Category $\mathcal{O}_P = (\mathcal{P}, \mathcal{F})$

$\mathcal{P} = \{ p(u_1, u_2, \ldots, u_n | r_1, r_2, \ldots, r_m) \}$

$\mathcal{F} = \{ f_i : p(u_1, \ldots, u_n | r_1, \ldots, r_m) \rightarrow p(u'_1, \ldots, u'_n | r'_1, \ldots, r'_m) \}$

Ontological Category $\mathcal{O}_N = (\mathcal{P}_\Lambda, \mathcal{H})$

$\mathcal{P}_\Lambda = \{ p(u_1, u_2, \ldots, u_n, \lambda_1, \lambda_2, \ldots, \lambda_l | r_1, r_2, \ldots, r_m) \}$

$\mathcal{H} = \{ h_i : p(u_1, \ldots, u_n, \lambda_1, \ldots, \lambda_k | r_1, \ldots, r_m) \rightarrow p(u'_1, \ldots, u'_n, \omega_1, \ldots, \omega_l | r'_1, \ldots, r'_m) \}$

$h_i \text{ s.t. } p(u_1, \ldots, u_n | \omega_1, \ldots, \omega_l, r_1, \ldots, r_m) = p(u'_1, \ldots, u'_n | \lambda_1, \ldots, \lambda_k, r'_1, \ldots, r'_m)$
The framework

No fine tuning condition - Categorical framework

A property of an operational category $\mathcal{O}_p = (\mathcal{P}, \mathcal{F})$ is no fine tuned if there exists a convexity preserving functor $\mathcal{G}$ that preserves it in an ontological category $\mathcal{O}_n = (\mathcal{P}_\Lambda, \mathcal{H})$. 
The framework

No fine tuning examples - Categorical framework

\[ O_P = (\mathcal{P}, \mathcal{F}) \xrightarrow{G} O_n = (\mathcal{P}_\Lambda, \mathcal{H}) \]

- Preparation Noncontextuality

\[ P = Q \xrightarrow{G_{NC}} \omega P = \omega Q \]
The framework

No fine tuning examples - Categorical framework

\[ \mathcal{O}_p = (\mathcal{P}, \mathcal{F}) \xrightarrow{\mathcal{G}} \mathcal{O}_n = (\mathcal{P}_\Lambda, \mathcal{H}) \]

- Preparation Noncontextuality

\[ P = Q \xrightarrow{\mathcal{G}_{NC}} \omega P = \omega Q \]

- Parameter Independence

\[ Y_0(P) = Y_1(P) \xrightarrow{\mathcal{G}_{PI}} \omega Y_0(P) = \omega Y_1(P) \]
The framework

No fine tuning examples - Categorical framework

\[ \mathcal{O}_P = (\mathcal{P}, \mathcal{F}) \quad \xrightarrow{\mathcal{G}} \quad \mathcal{O}_n = (\mathcal{P}_\Lambda, \mathcal{H}) \]

• Preparation Noncontextuality

\[ P = Q \quad \xrightarrow{\mathcal{G}_{NC}} \quad \omega P = \omega Q \]

• Parameter Independence

\[ Y_0(P) = Y_1(P) \quad \xrightarrow{\mathcal{G}_{PI}} \quad \omega Y_0(P) = \omega Y_1(P) \]

• Time Symmetry

\[ P = \Pi(Q) \quad \xrightarrow{\mathcal{G}_{TS}} \quad \omega P = \omega \Pi(Q) \]
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Applications

Relation between nonlocality and contextuality

Kochen-Specker contextuality* is considered to be a generalization of nonlocality.
Applications

Relation between nonlocality and contextuality

Generalized Contextuality \rightarrow \text{Involves operational fine tuning.}
Applications

Relation between nonlocality and contextuality

Nonlocality = failure of Bell’s local causality

Parameter independence

\[
\begin{aligned}
&\{ p(a|x, y = 0) = p(a|x, y = 1) \\
&p(b|x = 0, y) = p(b|x = 1, y) \\
&\downarrow \\
&\{ p(a|\lambda, x, y = 0) = p(a|\lambda, x, y = 1) \\
&p(b|\lambda, x = 0, y) = p(b|\lambda, x = 1, y)
\end{aligned}
\]
Applications

Relation between nonlocality and contextuality

Nonlocality = failure of Bell’s local causality

Parameter independence

\[
\begin{align*}
&\{ p(a|x, y = 0) = p(a|x, y = 1) \\
&p(b|x = 0, y) = p(b|x = 1, y) \\
\} \\
\downarrow \\
&\{ p(a|\lambda, x, y = 0) = p(a|\lambda, x, y = 1) \\
&p(b|\lambda, x = 0, y) = p(b|\lambda, x = 1, y) \\
\}
\]

Outcome independence

\[
\begin{align*}
&\{ p(a|b = 0, x, y) = p(a|b = 1, x, y) \\
&p(b|a = 0, x, y) = p(b|a = 1, x, y) \\
\} \\
\downarrow \\
&\{ p(a|\lambda, b = 0, x, y) = p(a|\lambda, b = 1, x, y) \\
&p(b|\lambda, a = 0, x, y) = p(b|\lambda, a = 1, x, y) \\
\}
\]
Applications

Relation between nonlocality and contextuality

Nonlocality = failure of Bell’s local causality

Parameter independence
\[
\begin{align*}
p(a|b = 0) &= p(a|b = 1) \\
p(b|x = 0) &= p(b|x = 1)
\end{align*}
\]

Outcome independence
\[
\begin{align*}
p(a|b = 0, x, y) &= p(a|b = 1, x, y) \\
p(b|a = 0, x, y) &= p(b|a = 1, x, y)
\end{align*}
\]
Applications

Relation between nonlocality and contextuality

Nonlocality = failure of Bell’s local causality

Parameter independence
\[
\begin{align*}
  p(a | b = 0, y = 0) &= p(a | b = 0, y = 1) \\
  p(b | x = 0, y = 0) &= p(b | x = 1, y)
\end{align*}
\]

Outcome independence
\[
\begin{align*}
  p(a | b = 0, x, y) &= p(a | b = 1, x, y) \\
  p(b | a = 0, x, y) &= p(b | a = 1, x, y)
\end{align*}
\]

Operational fine tuning
Applications

Relation between nonlocality and contextuality

Nonlocality = failure of Bell’s local causality

Parameter independence
\[
\begin{align*}
p(a|x, y = 0) &= p(a|x, y = 1) \\
p(b|x = 0, y) &= p(b|x = 1, y)
\end{align*}
\]

\[\downarrow\]
\[
\begin{align*}
p(a|\lambda, x, y = 0) &= p(a|\lambda, x, y = 1) \\
p(b|\lambda, x = 0, y) &= p(b|\lambda, x = 1, y)
\end{align*}
\]

Outcome independence
\[
\begin{align*}
p(a|x, y = 0) &= p(a|x, y = 1) \\
p(b|x = 0, y) &= p(b|x = 1, y)
\end{align*}
\]

\[
\begin{align*}
p(a|\lambda, b = 0, x, y) &= p(a|\lambda, b = 1, x, y) \\
p(b|\lambda, x = 0, y) &= p(b|\lambda, x = 1, y)
\end{align*}
\]
Relation between nonlocality and contextuality

Generalized Contextuality $\rightarrow$ Involves operational fine tuning.

Nonlocality $\rightarrow$
Applications

Relation between nonlocality and contextuality

Generalized Contextuality $\rightarrow$ Involves operational fine tuning.

Nonlocality $\rightarrow$ Involves operational or causal fine tuning.
Conclusion
Conclusion

- Mathematical framework for operational fine tunings.
Conclusion

• Mathematical framework for operational fine tunings.

• It accommodates existing examples and provide ground for new ones.
Conclusion

Future challenges
Conclusion

Future challenges

• Study novel fine tunings, in particular group symmetry fine tunings (e.g. Lorentz symmetry group,...).
Conclusion

Future challenges

- Study novel fine tunings, in particular group symmetry fine tunings (e.g. Lorentz symmetry group,...).

- Resource theory of fine tunings and look for applications in quantum computation.
Conclusion

Future challenges

• Study novel fine tunings, in particular group symmetry fine tunings (e.g. Lorentz symmetry group,...).

• Resource theory of fine tunings and look for applications in quantum computation.

• Develop new ontological model for QT free of fine tunings.