Title: Discussion 6

Speakers:

Collection: Simplicity III

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Fun With Path Integrals II

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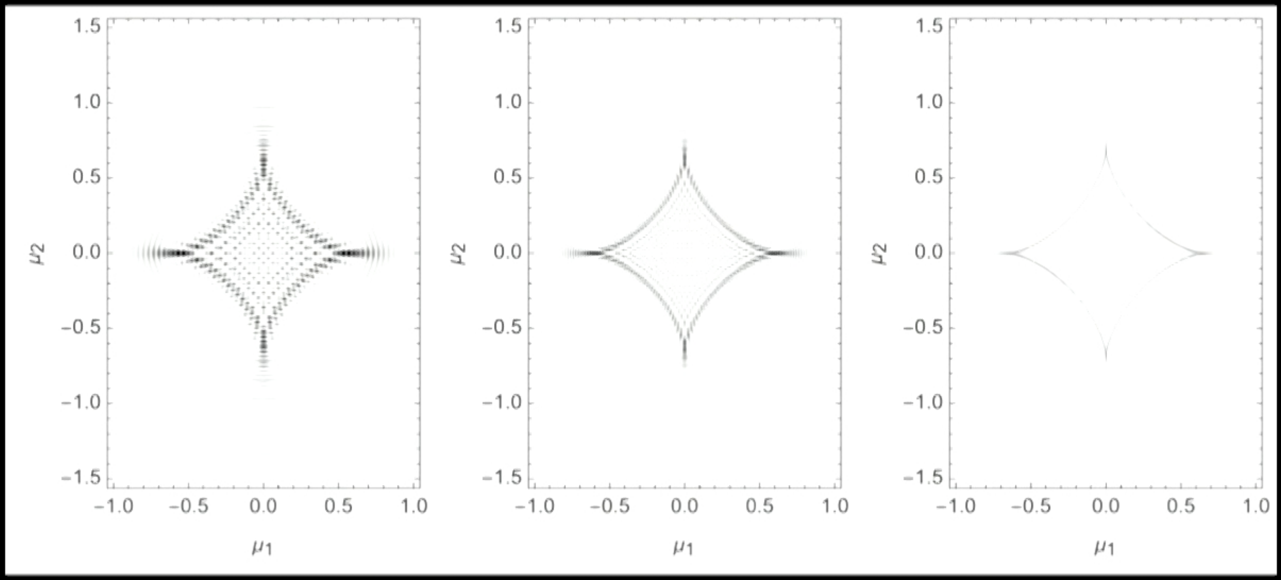
Interference

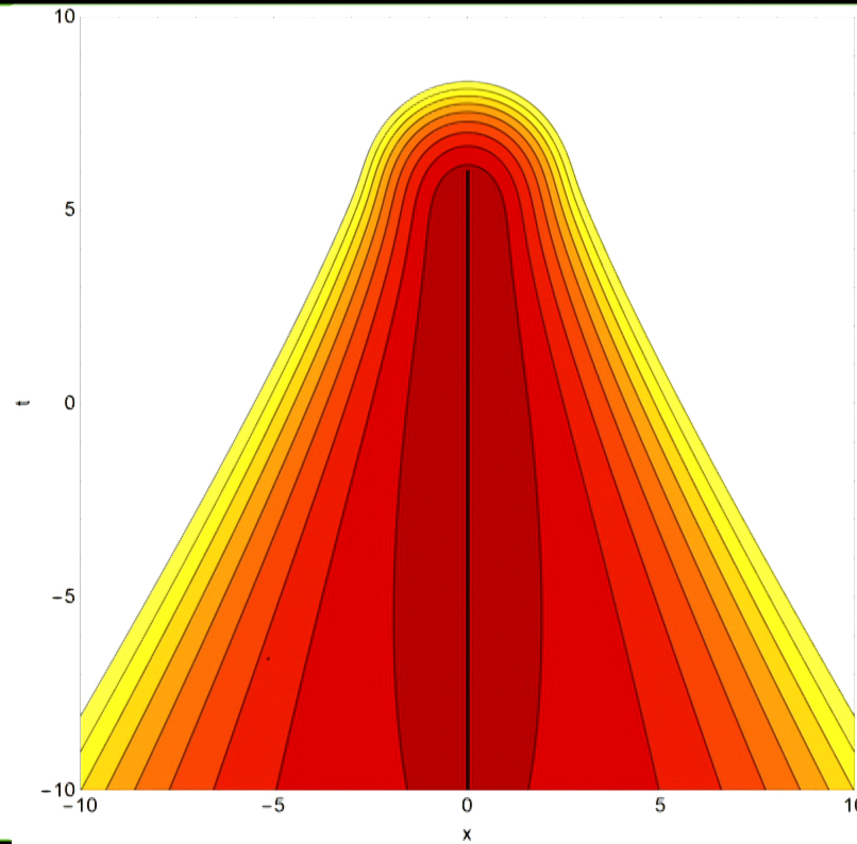
- Seed and trees in quantum field theory
- New perspective on quantum geometrodynamics



Schwinger effect

$$\varphi(x_1^\mu) = \int_{0^+}^{\infty} \mathrm{d}s \langle x_1^\mu | e^{-is\hat{H}/\hbar} | \psi_0 \rangle$$
$$= \int \mathrm{d}x_0^\mu G[x_1^\mu; x_0^\mu] \psi_0(x_0^\mu)$$

$$\hat{H}\varphi(x^\mu) = -i\hbar\psi_0(x^\mu)$$



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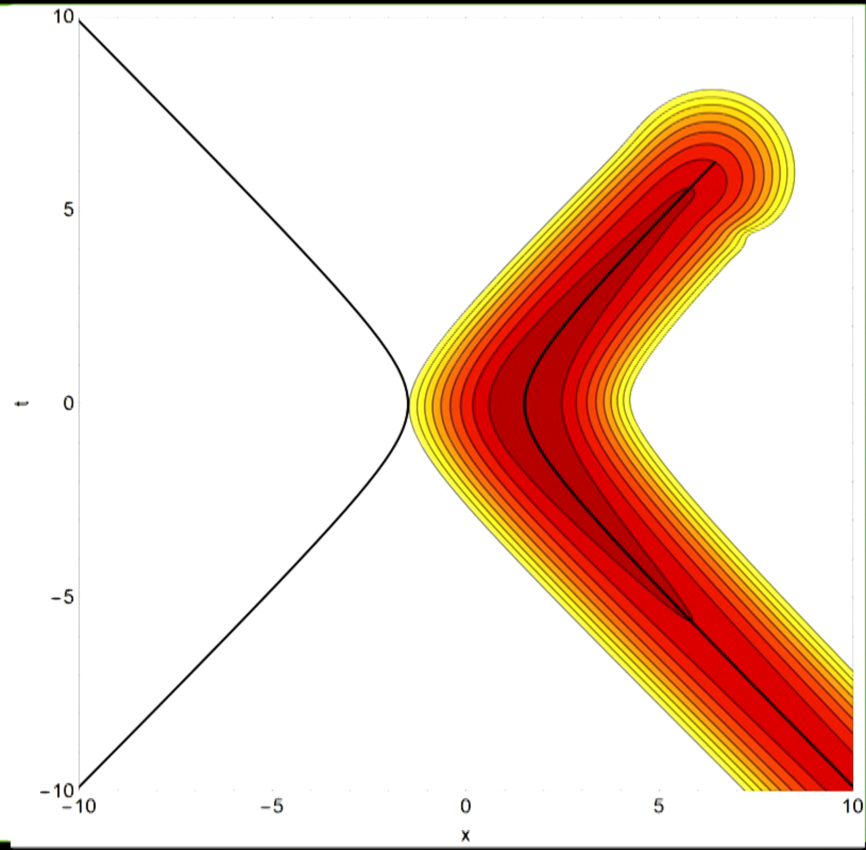

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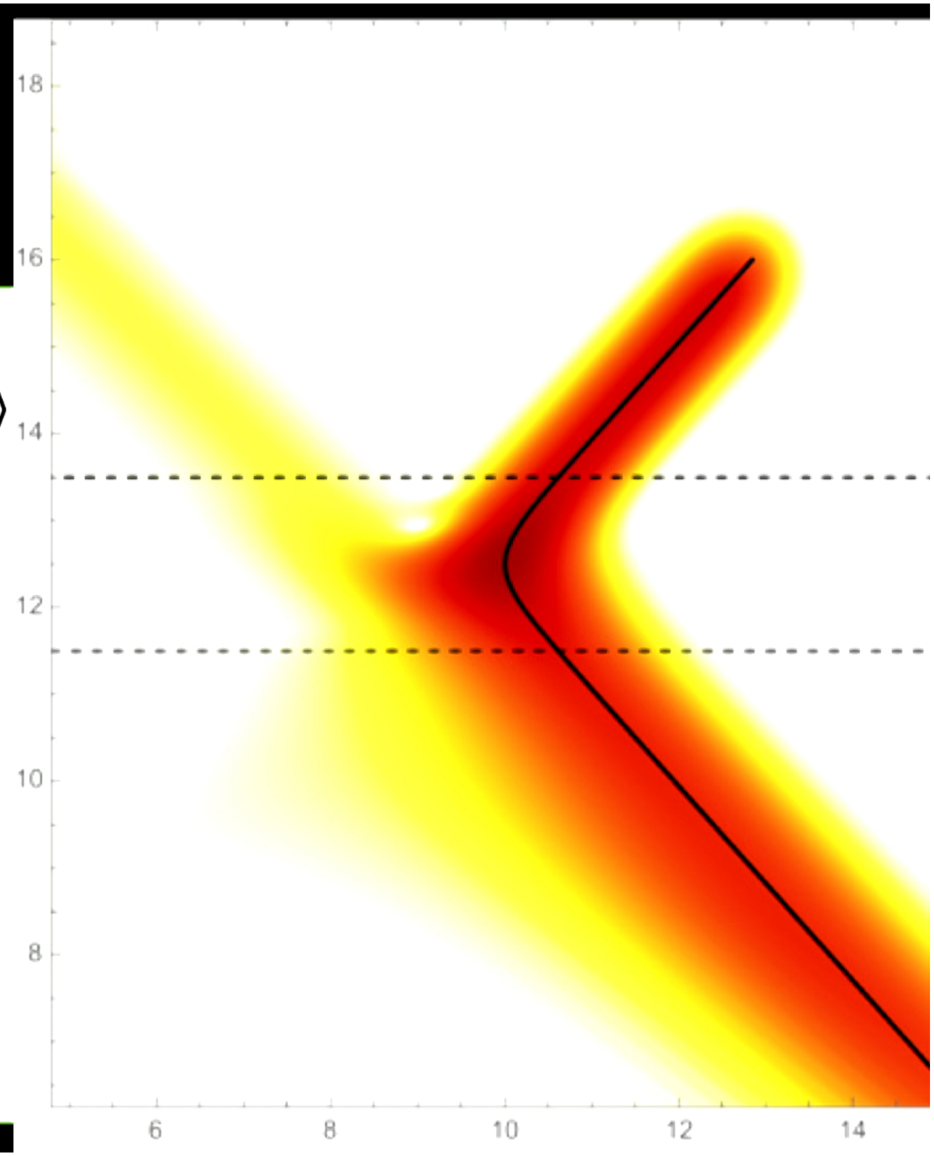
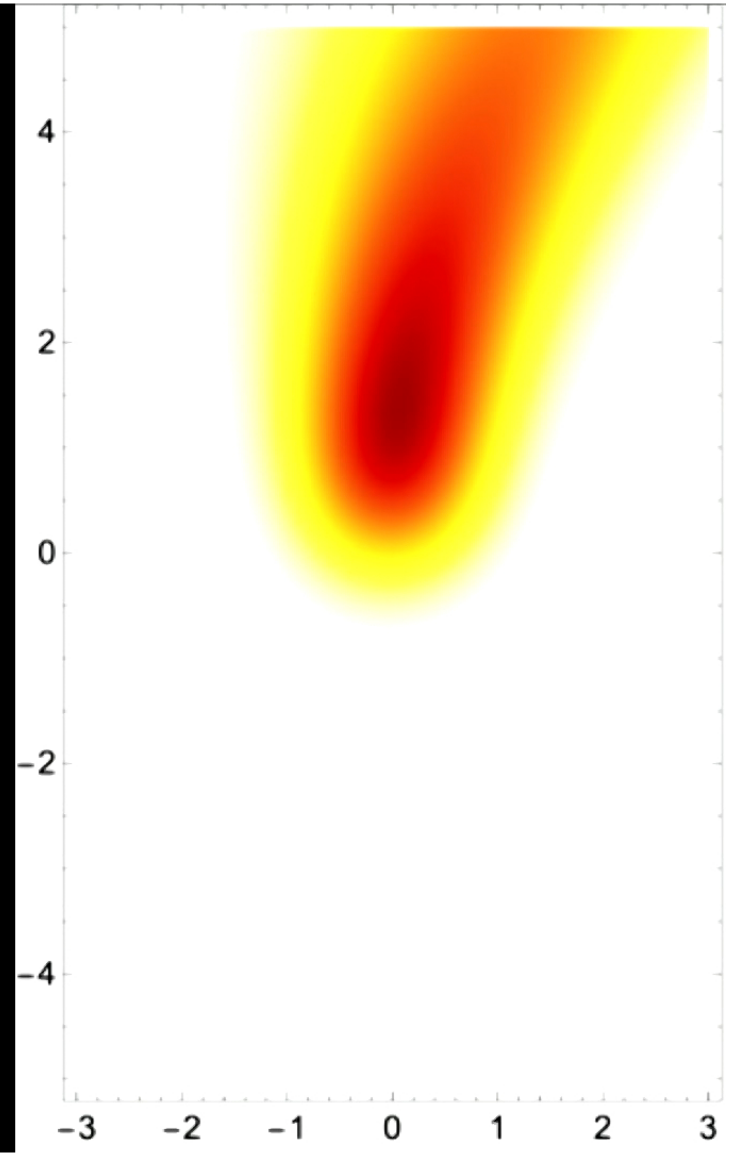


Schwinger effect

$$\phi(x_1^\mu) = \int_{-\infty}^{\infty} \mathrm{d}s \langle x_1^\mu | e^{-is\hat{H}/\hbar} | \psi_0 \rangle$$

$$= \int \mathrm{d}x_0^\mu G_H[x_1^\mu; x_0^\mu] \psi_0(x_0^\mu)$$

$$\hat{H}\phi = 0$$



Schwinger effect

- Relative probability

$$P[\psi_1|\psi_0] = \left| \int_{0+}^{\infty} \mathrm{d}s \langle \psi_1 | e^{-is\hat{H}/\hbar} | \psi_0 \rangle \right|^2$$

- Given that the particle propagates between two states, what is the expected properties of the phenomenon?
  - Relativistic weak value theory

Weak value theory

- Couple observable to a von Neumann pointer

$$S_p[X, P] = \int_0^T \mathrm{d}t \left[-X\dot{P} - \mathcal{H}_p \right] \qquad \mathcal{H}_p = \frac{P^2}{2M} + gPO[x^\mu]$$

$$\langle X_1 \rangle = \langle X_0 \rangle + gT \ \mathrm{Re}[O_w]$$

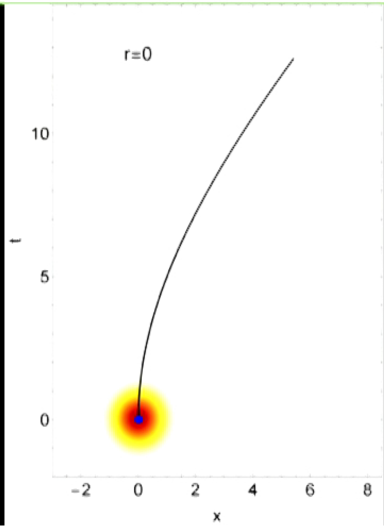
$$\langle P_1 \rangle = \langle P_0 \rangle + \frac{gT\hbar}{2\Delta_i^2} \ \mathrm{Im}[O_w]$$

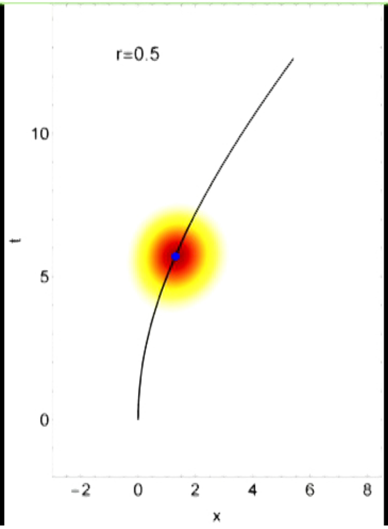
$$O_w = \frac{\int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS/\hbar} O[x^\mu]}{\int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS/\hbar}}$$

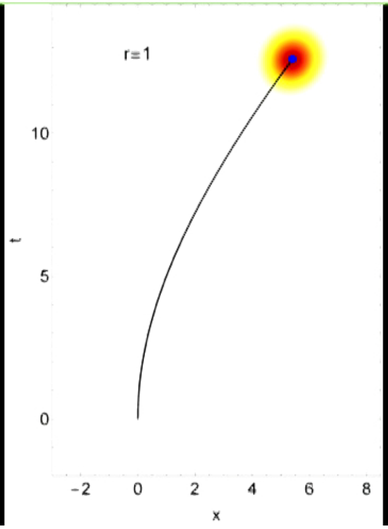
Schwinger effect

- Relativistic Aharonov-Bergmann-Lebowitz formula

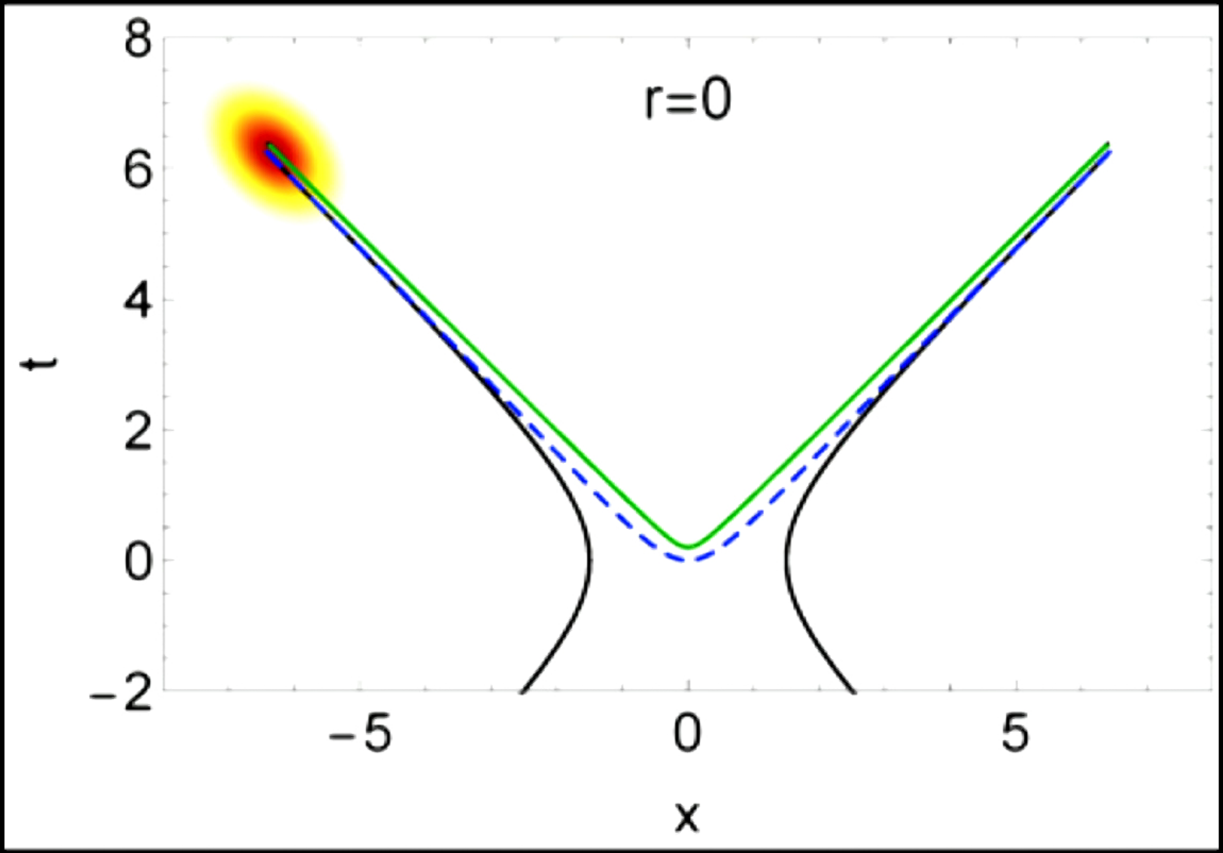
$$P_{ABL}[x_m^\mu; r] = \frac{\left| \int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS[x^\mu]/\hbar} \delta(x_m^\mu - x^\mu(rs)) \right|^2}{\int \left| \int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS[x^\mu]/\hbar} \delta(x_m^\mu - x^\mu(rs)) \right|^2 \mathrm{d}x_m^\mu}$$







Schwinger effect



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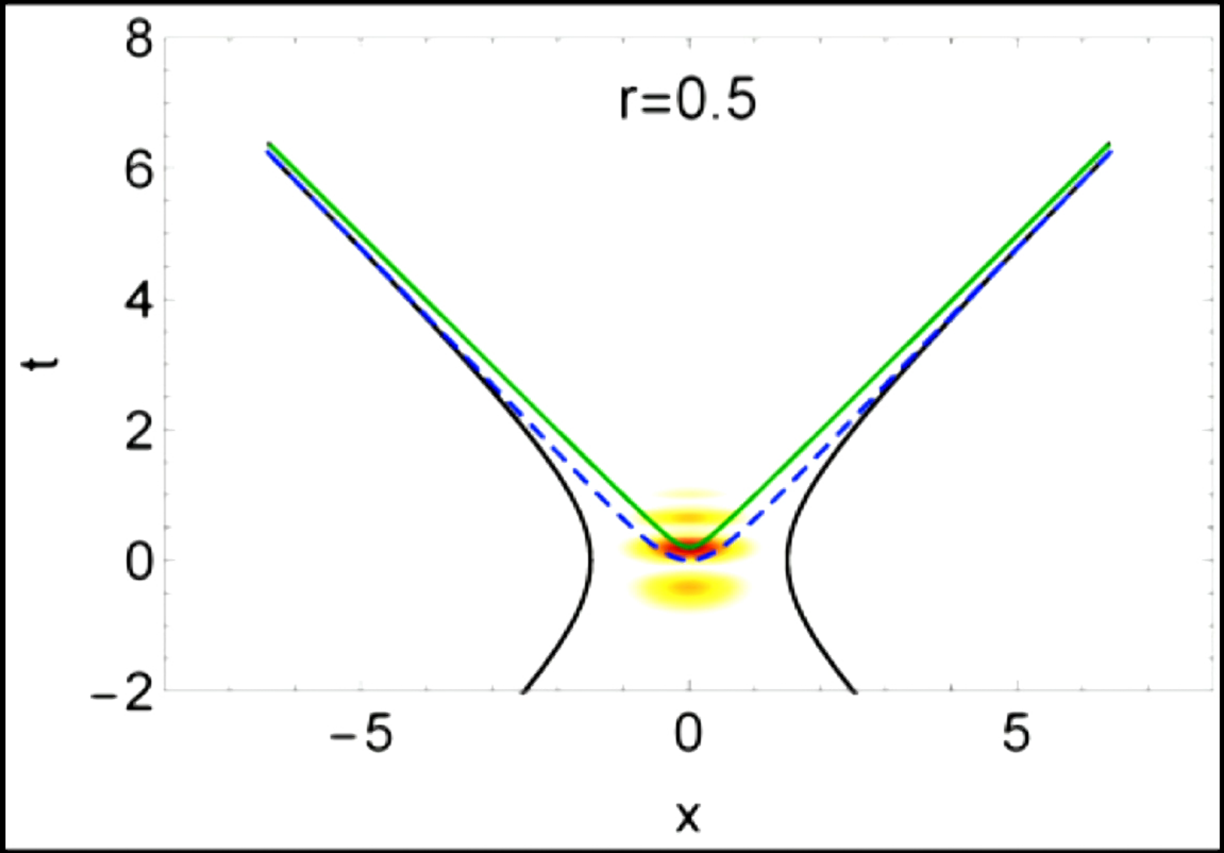

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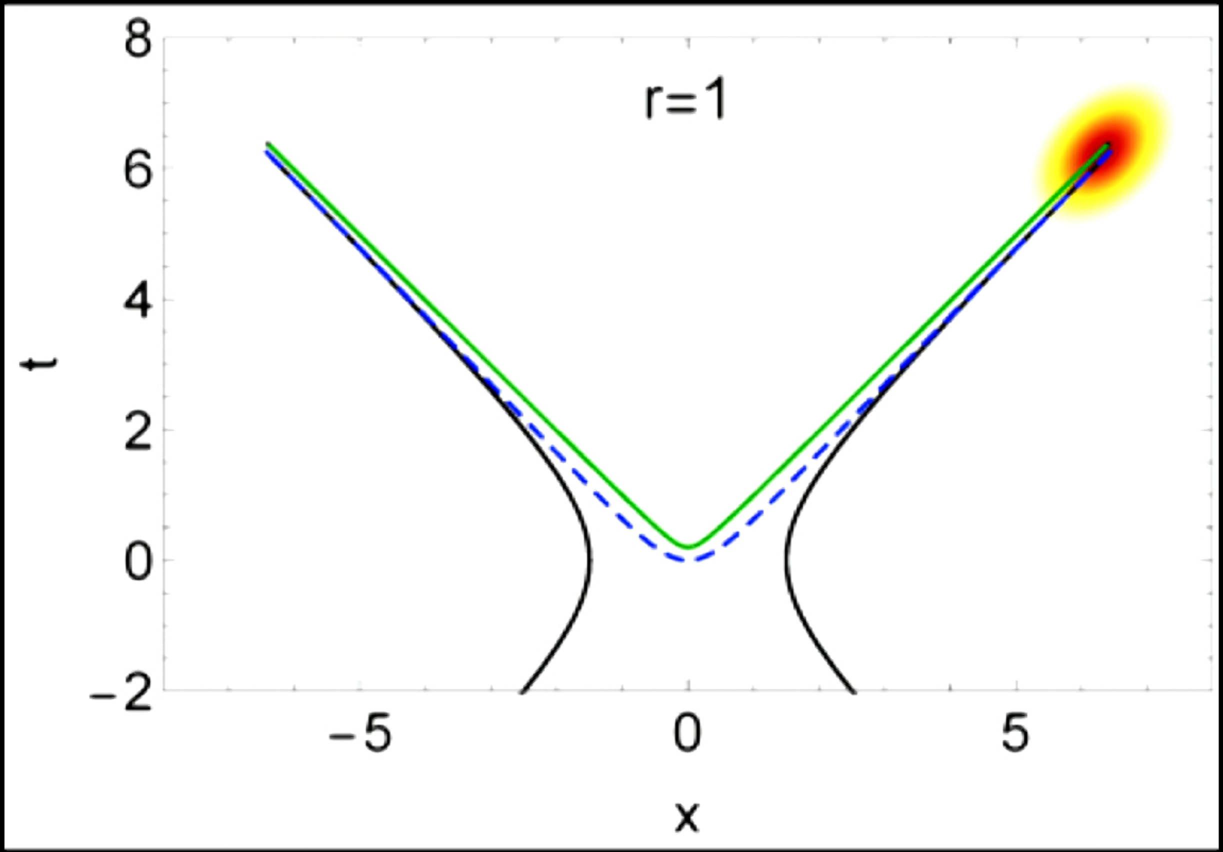
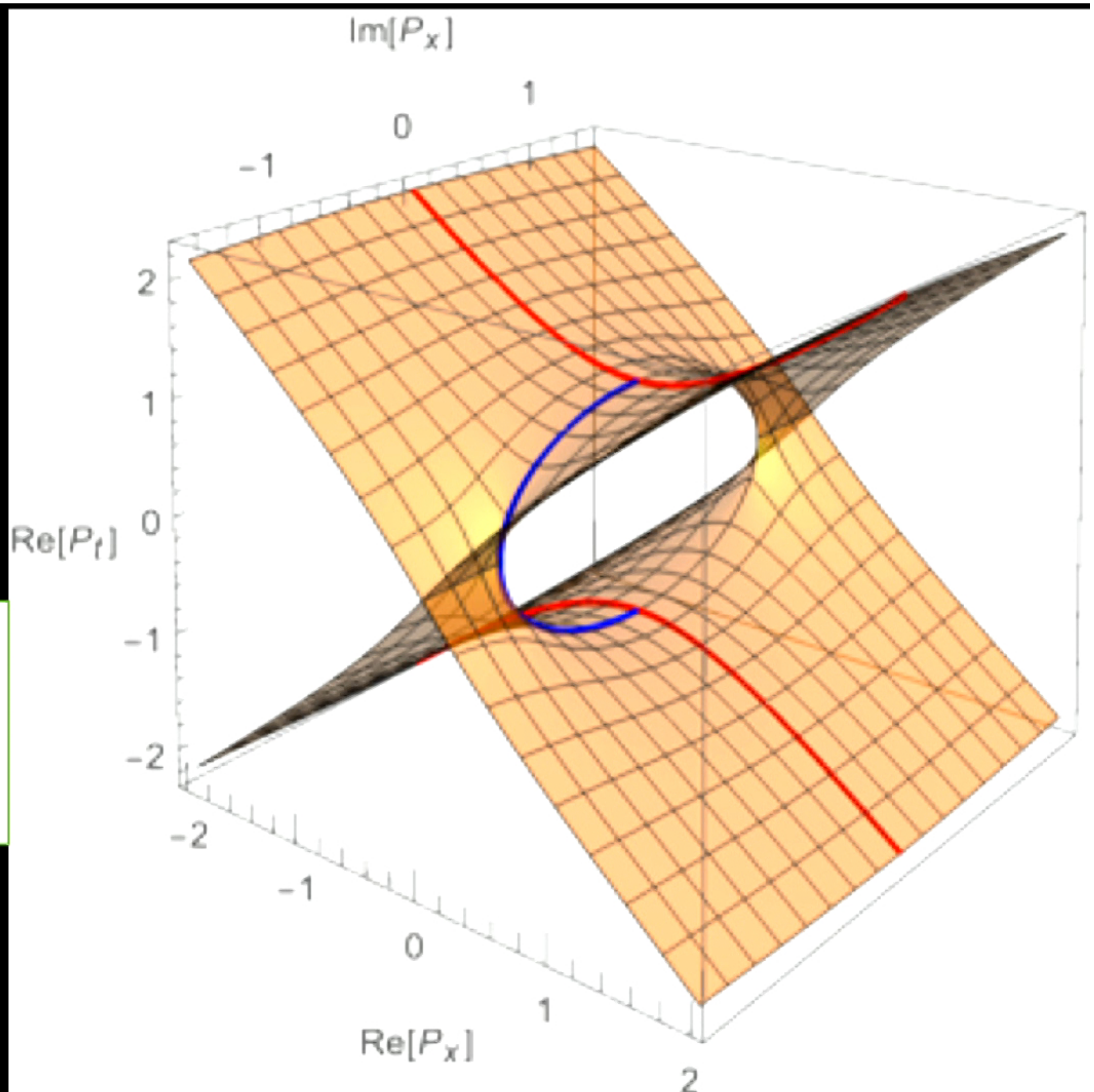


Schwinger effect

- Constraint surface
- Geodesic on complexified Lorentz group

$$H[x^\mu; P_\mu] = -P_t^2 + P_x^2 + m^2 = 0$$



Schwinger effect

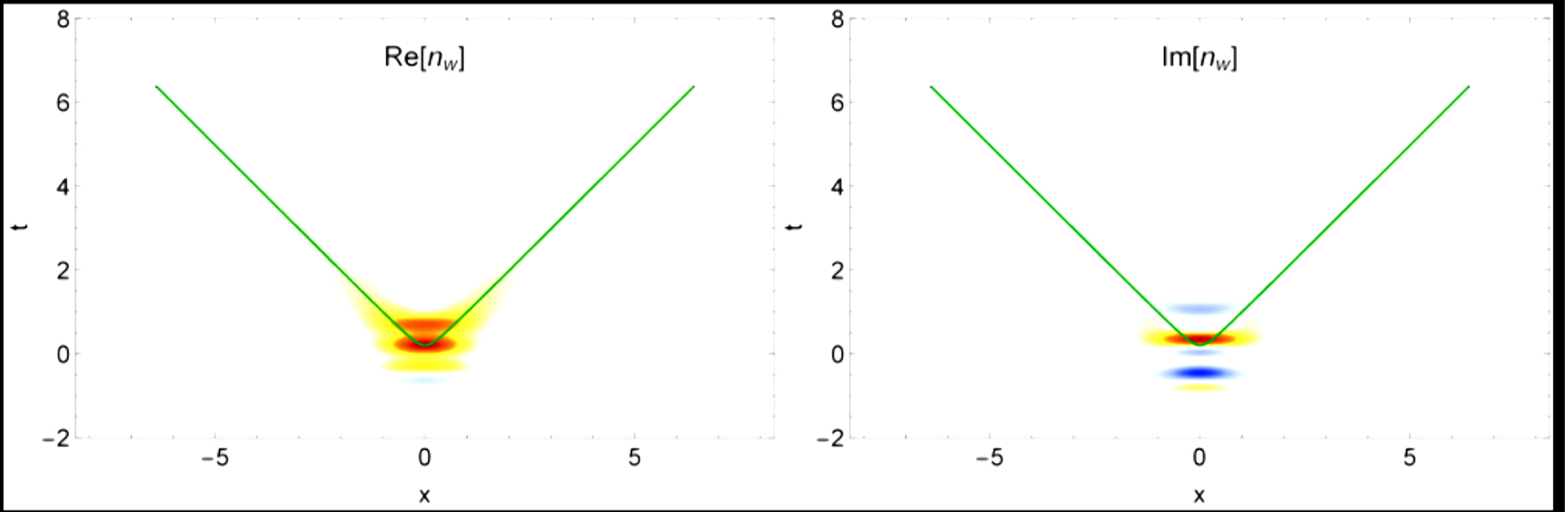
- Weak density

$$n_w(x_m^\mu) = \frac{\int_0^1 \mathrm{d}r \int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS/\hbar} \delta(x^\mu(rs) - x_m^\mu)}{\int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS/\hbar}}$$

$$\rho_w(x_m^\mu) = \frac{e}{m} \frac{\int_0^1 \mathrm{d}r \int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS/\hbar} P_0 \delta(x^\mu(rs) - x_m^\mu)}{\int_{0^+}^{\infty} \mathrm{d}s \int_{\psi_0}^{\psi_1} \mathcal{D}x^\mu e^{iS/\hbar}}$$

Schwinger effect

- Weak density



Schwinger effect

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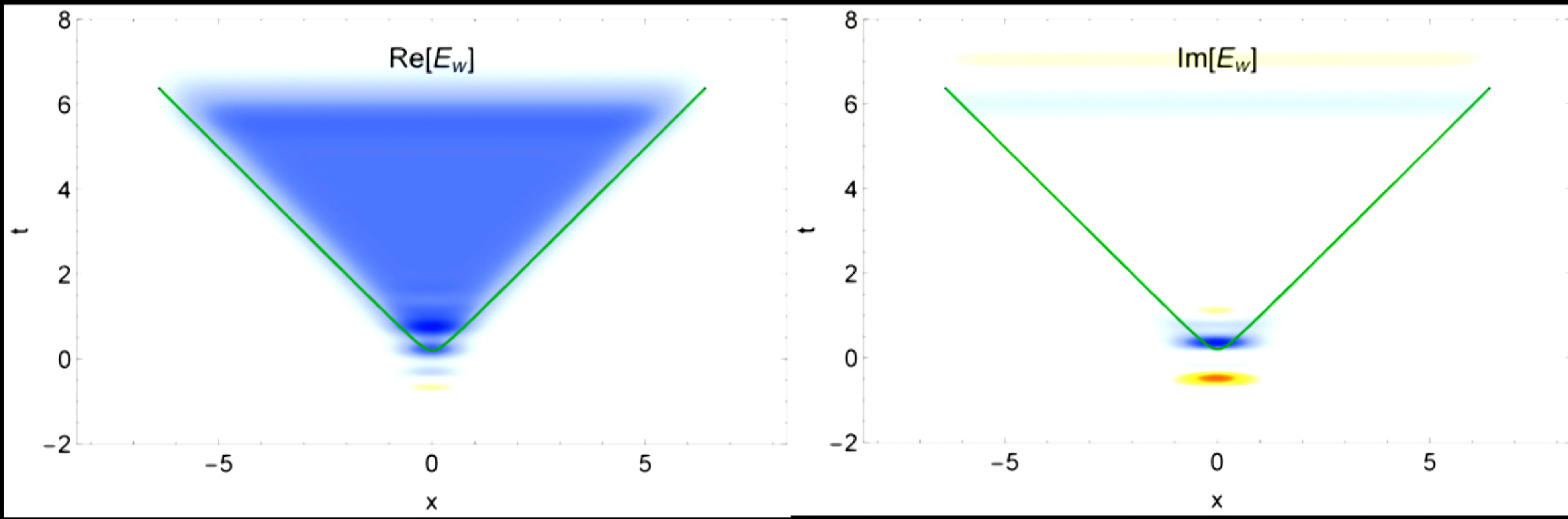
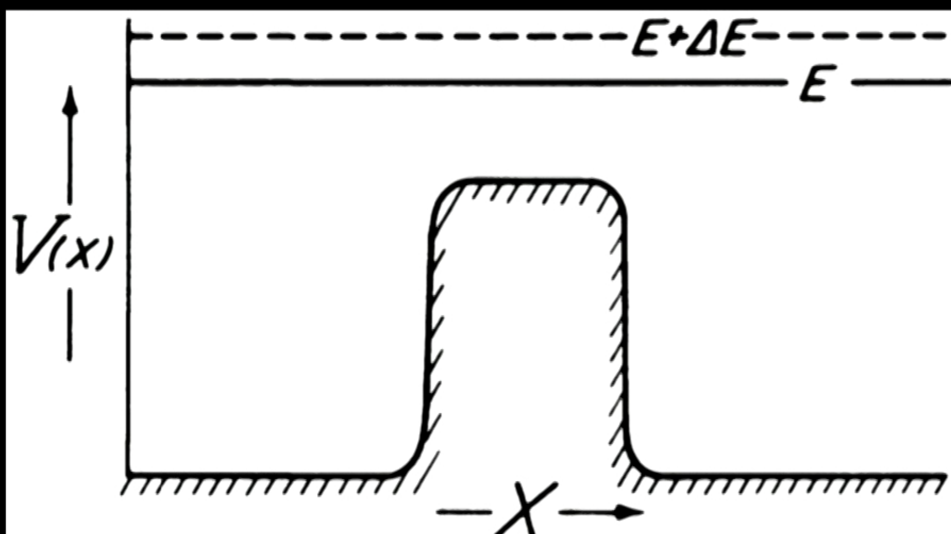


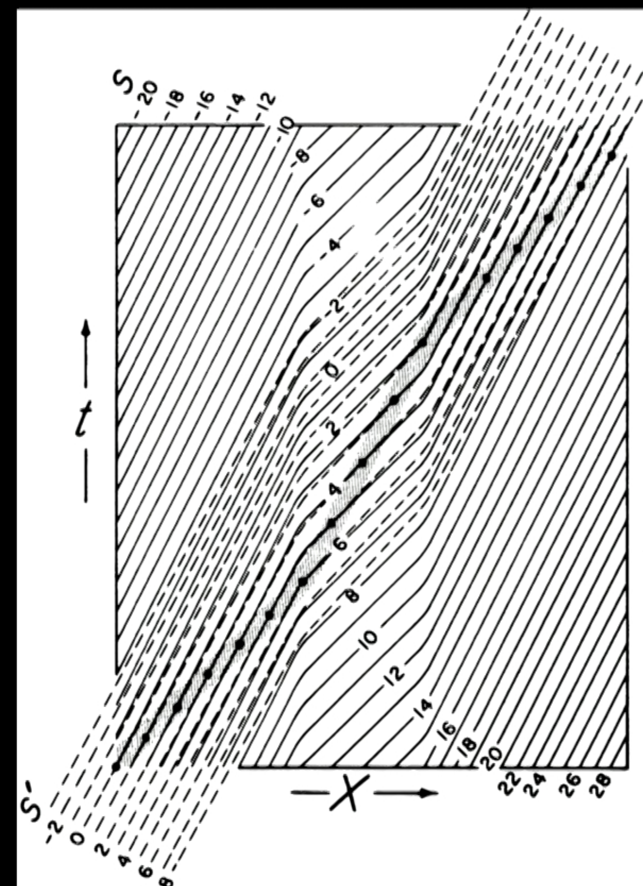
Schwinger effect

- In Minkowski spacetime, the Feynman propagator reproduces the S-matrix
- When imposing unitarity, we can recover the Schwinger decay rate (Feynman/Nikishov)
- On curved spacetime, without asymptotic regions, the geometric formalism is still predictive
- Weak value theory enables us to recover evolution

Quantum geometrodynamics

- In the '60s Wheeler started to quantize gravity
- Quantum mechanics is interference
- Wavefunction on the space of three-geometries





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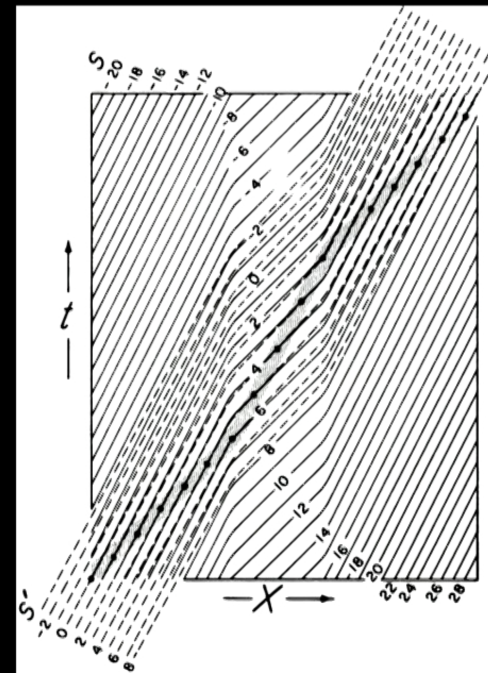


	Quantum particle	Quantum universe
Domain	spacetime: M	superspace: $\mathcal{S}$
Point	spacetime event: $x^\mu = (t, x)$	three-geometry: ${}^3\mathcal{G}$
wave function	of trajectory: $\psi(x^\mu)$	of universe: $\psi({}^3\mathcal{G})$
Ordering	ordering of space points x	interlocking foliations
Dynamics	Schrödinger equation	Wheeler-DeWitt equation

Quantum geometrodynamics

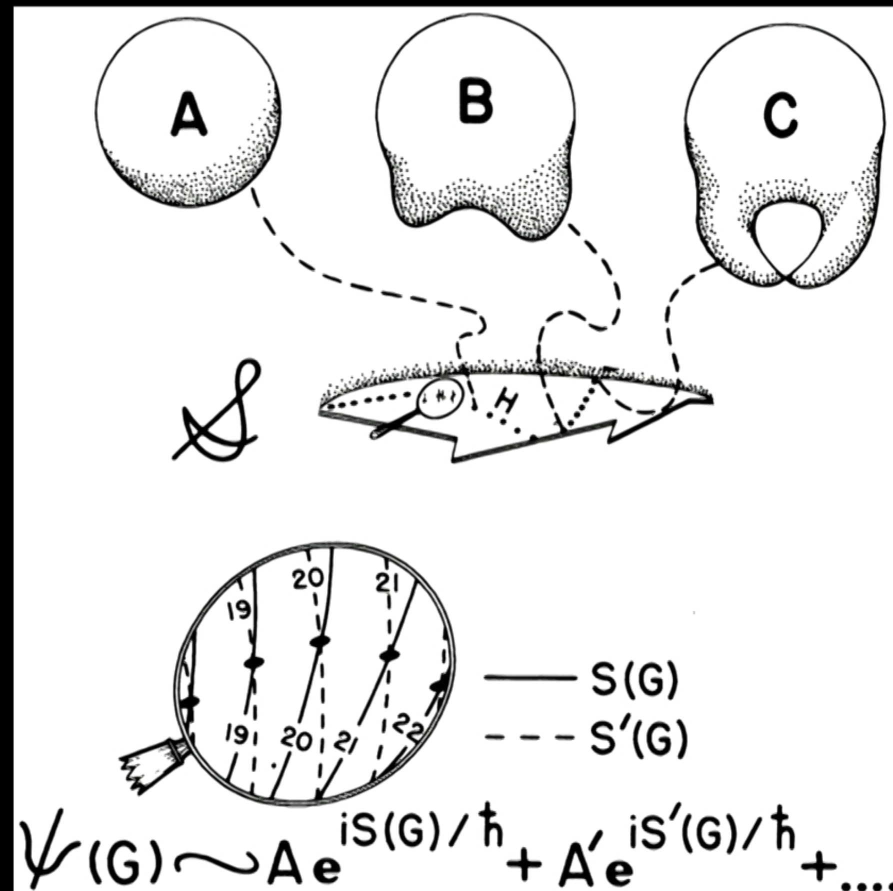
- Wheeler-DeWitt equation

$$\left[(2\kappa)\Delta^{(6)} + (2\kappa)^{-1}h({}^3R - 2\Lambda) + \xi\,{}^6R\right]\psi[{}^3\mathcal{G}] = 0$$

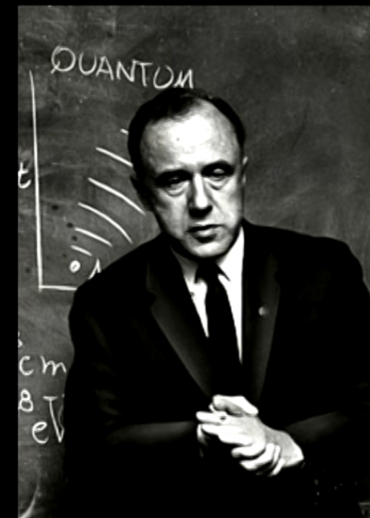
- Path integral of gravity

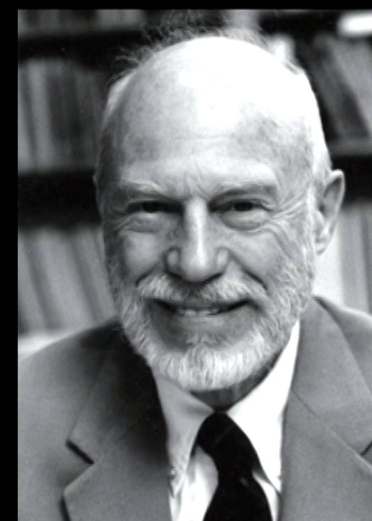
$$G[{}^3\mathcal{G}_1, X_1; {}^3\mathcal{G}_0, X_0] = \int_{{}^3\mathcal{G}_0, X_0}^{{}^3\mathcal{G}_1, X_1} \mathcal{D}g_{\mu\nu}\mathcal{D}X\, e^{\frac{i}{\hbar}S[g_{\mu\nu}, X]}$$

- Not complete without interpretation
  - Emergence of classical spacetime
  - Emergence of time
  - Boundary conditions at big bang



- **John Wheeler:** "… so today it is called the Wheeler-DeWitt equation. But is is one thing to have an equation, another to solve it and still another to interpret the solution."
- **Bryce DeWitt:** "This equation should be confined to the dustbin of history for the following reasons:"
  - **Violat sprit of relativity**
  - **No natural time parameter**
  - **Constraint Hamiltonian system**
  - **Path integral**

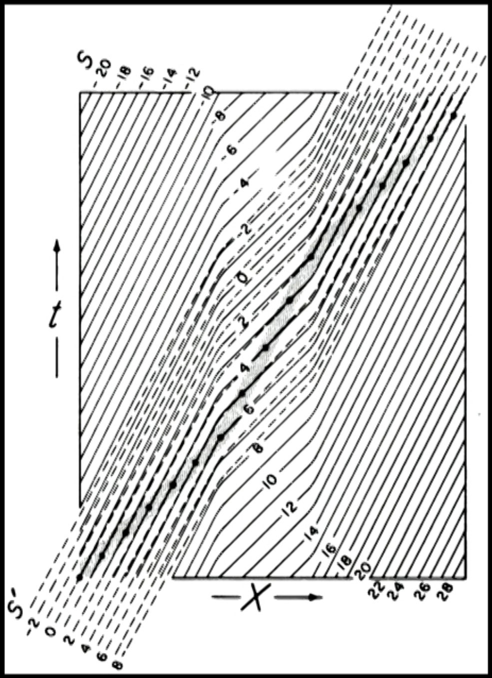




Quantum geometrodynamics

- We propose to implement Wheeler and DeWitt's vision by evaluating the path integral

$$\varphi[\mathcal{G}_1] = \int_{0^+}^{\infty} \mathrm{d}N \int_{\psi_0}^{\mathcal{G}_1} \mathcal{D}h_{ij} e^{iS_{EH}[h_{ij};N]/\hbar}$$



- Probed with weak relativistic measurements:
  - Weak spacetime, solving Einstein equations in semi-classical limit
  - Weak measurement of time between two states

Summary

- Schwinger effect can be understood from the world-line quantization
- Relativistic weak value theory allow us to probe the evolution and recover classical evolution
- Quantum geometrodynamics can be given a new interpretation using the path integral and relativistic weak value theory