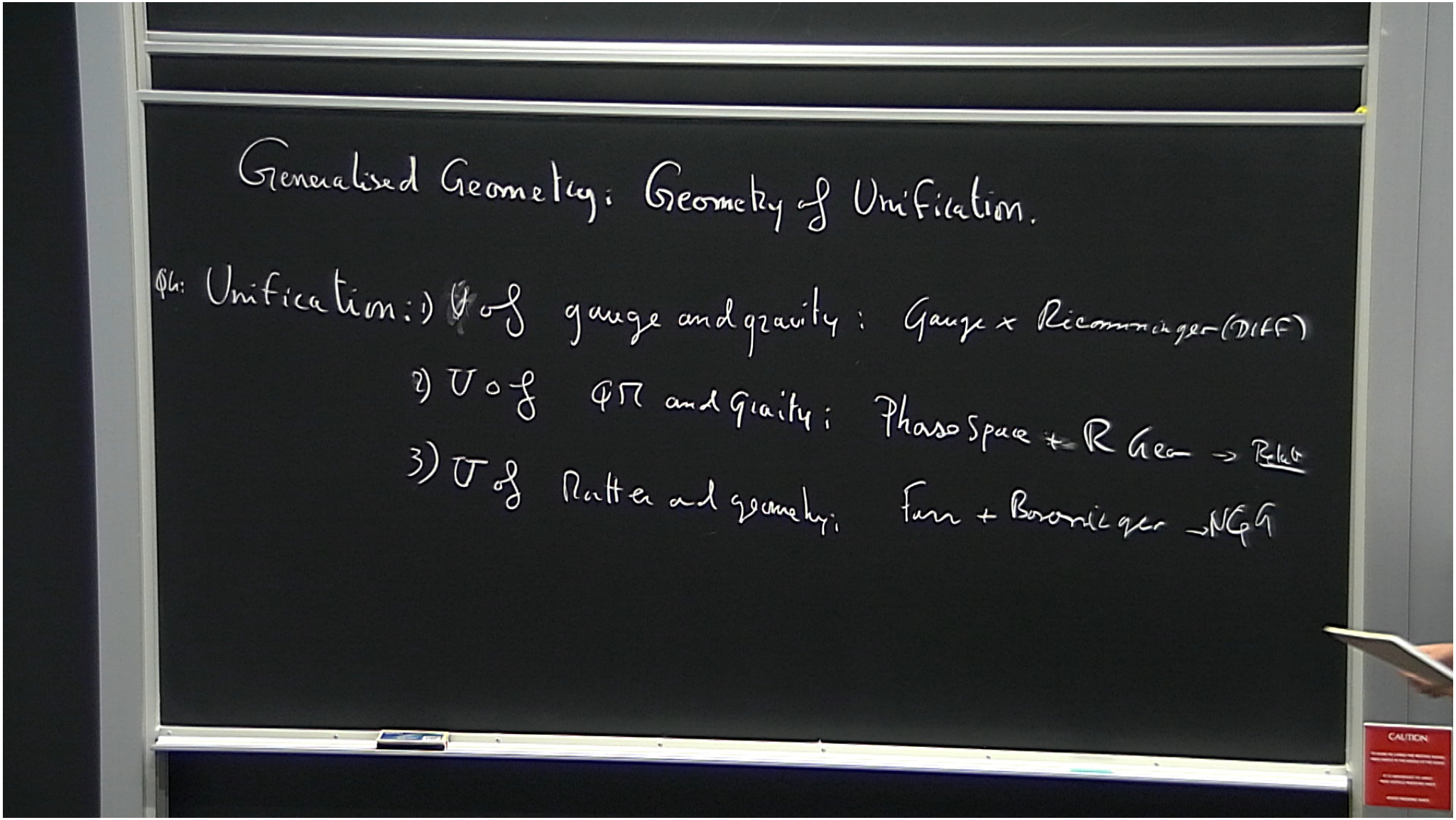
Title: Generalized geometry: geometry of unification

Speakers: Laurent Freidel

Collection: Simplicity III

Date: September 12, 2019 - 9:00 AM

URL: http://pirsa.org/19090087


Generalised Geometry: Geometry of Unification.
Phys: Unification: 1) U of gauge and gravity: Gauge + Riemannian (DIFF)
2) U of QFT and Gravity: Phase Space + R Geo → R Geo
3) U of Matter and geometry: Fermi + Bosonic geo → NCG
CAUTION

Lie algebroid

GR $P = (T\Pi, [,])$ over Π (Π, g)

Gauge: Atiyah bundle $B \to E = TB/G$

$$0 \to \underset{G \times \Pi}{V} \longrightarrow E \xrightarrow{\pi} T\Pi \to 0 \qquad (E, [,]_g, \pi)$$

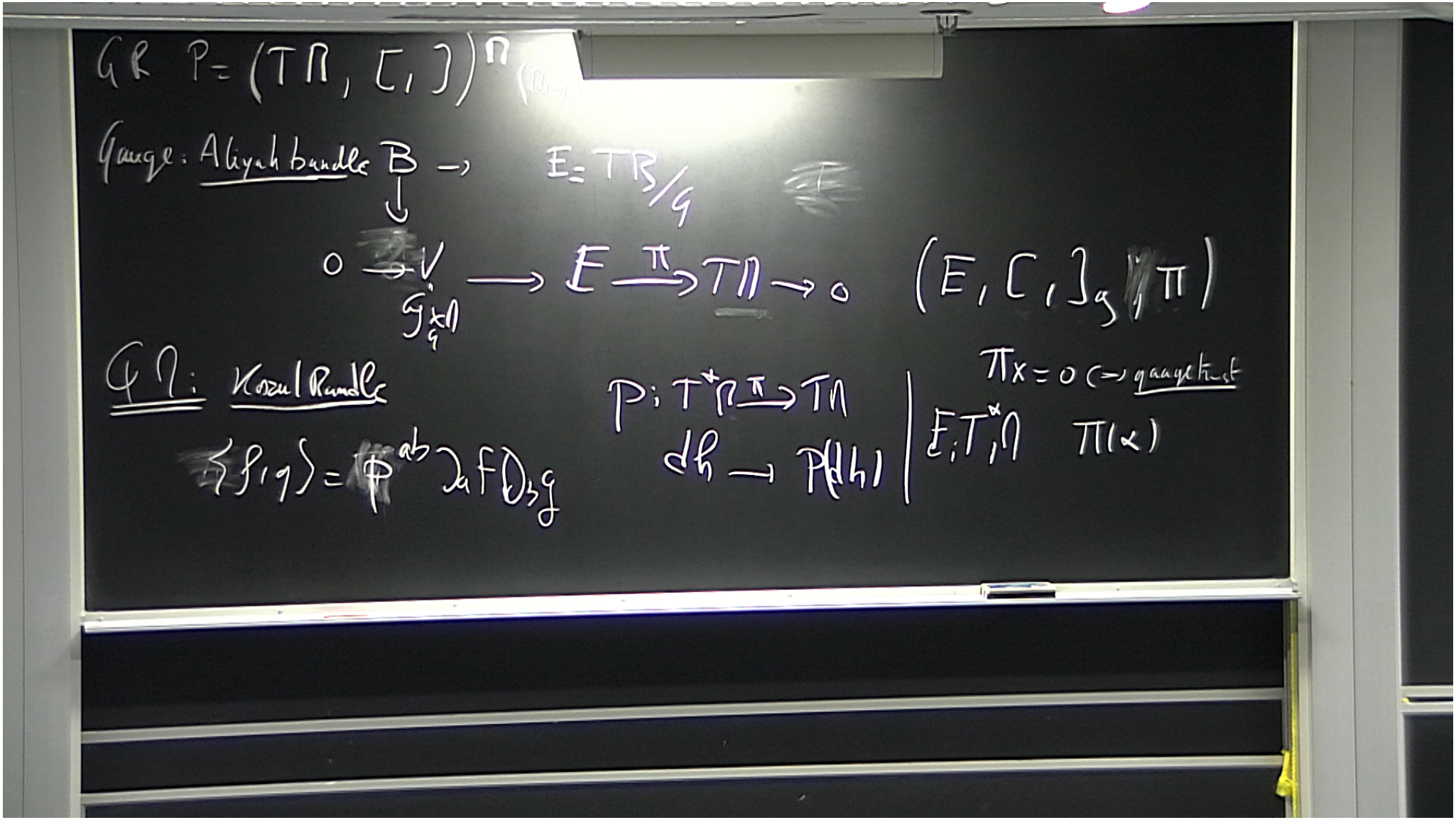
$\pi x = 0 \Rightarrow$ gauge invariant

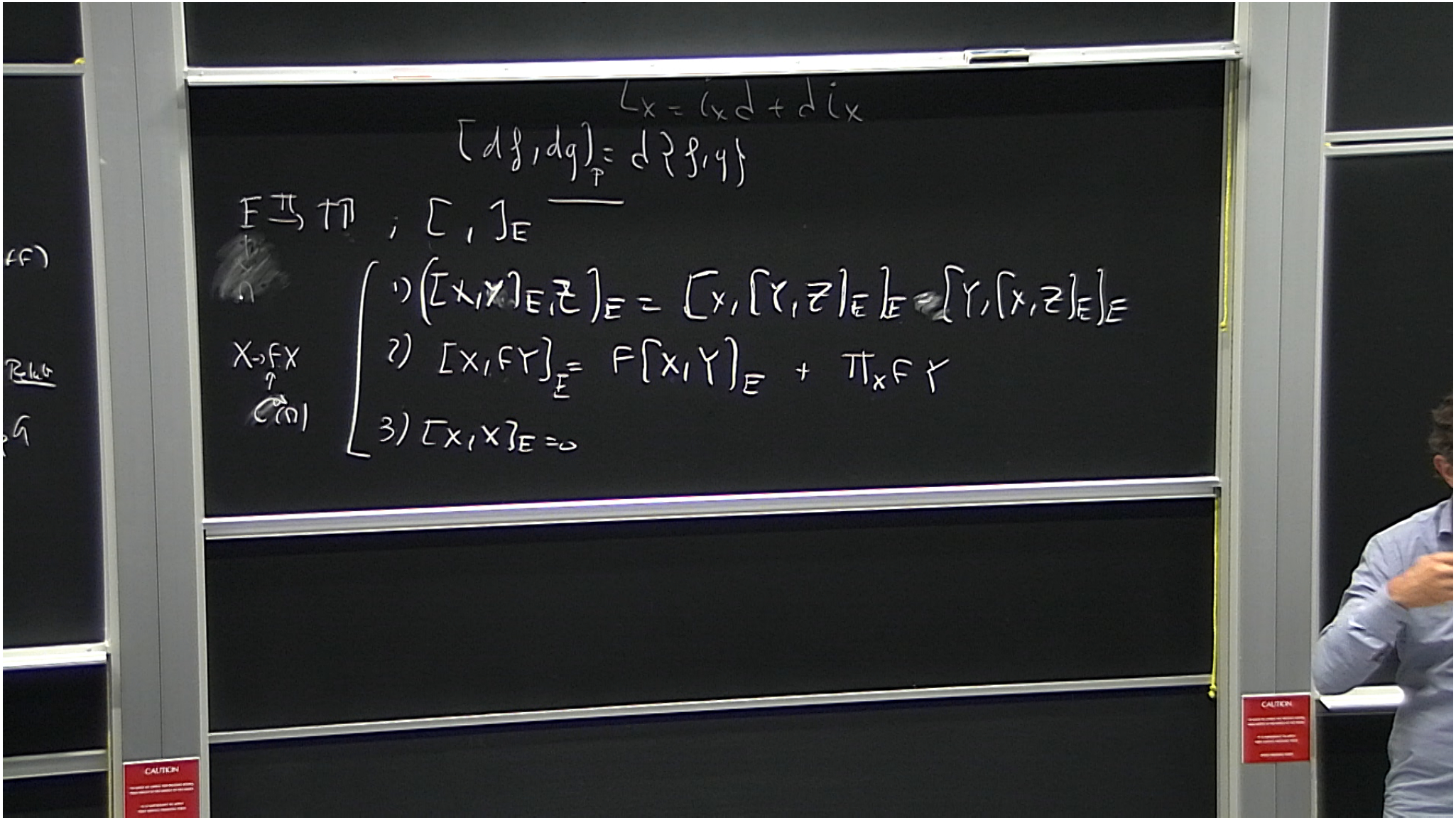
QG: Koszul Rundle

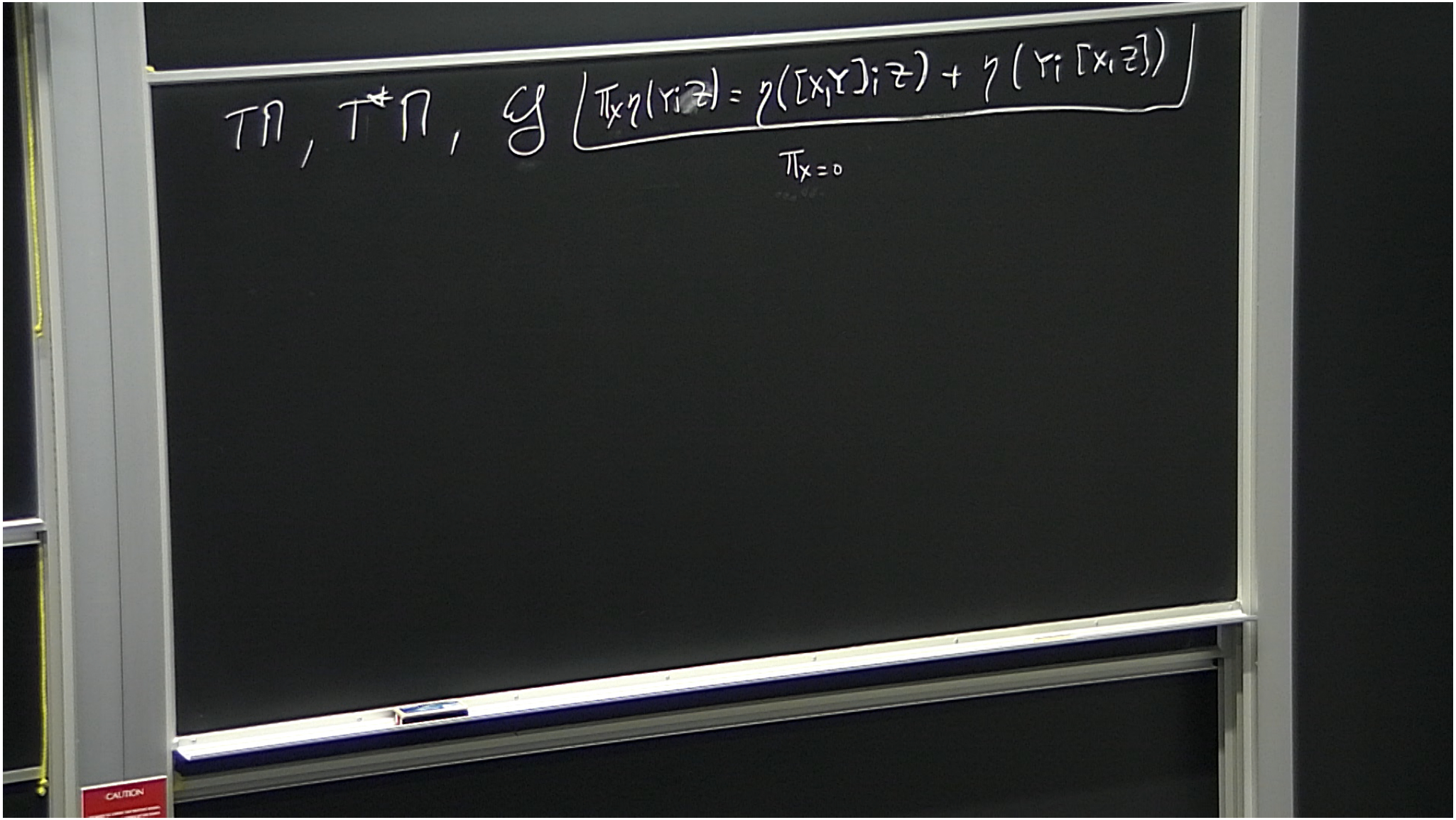
$P: T^*P \to T\Pi$

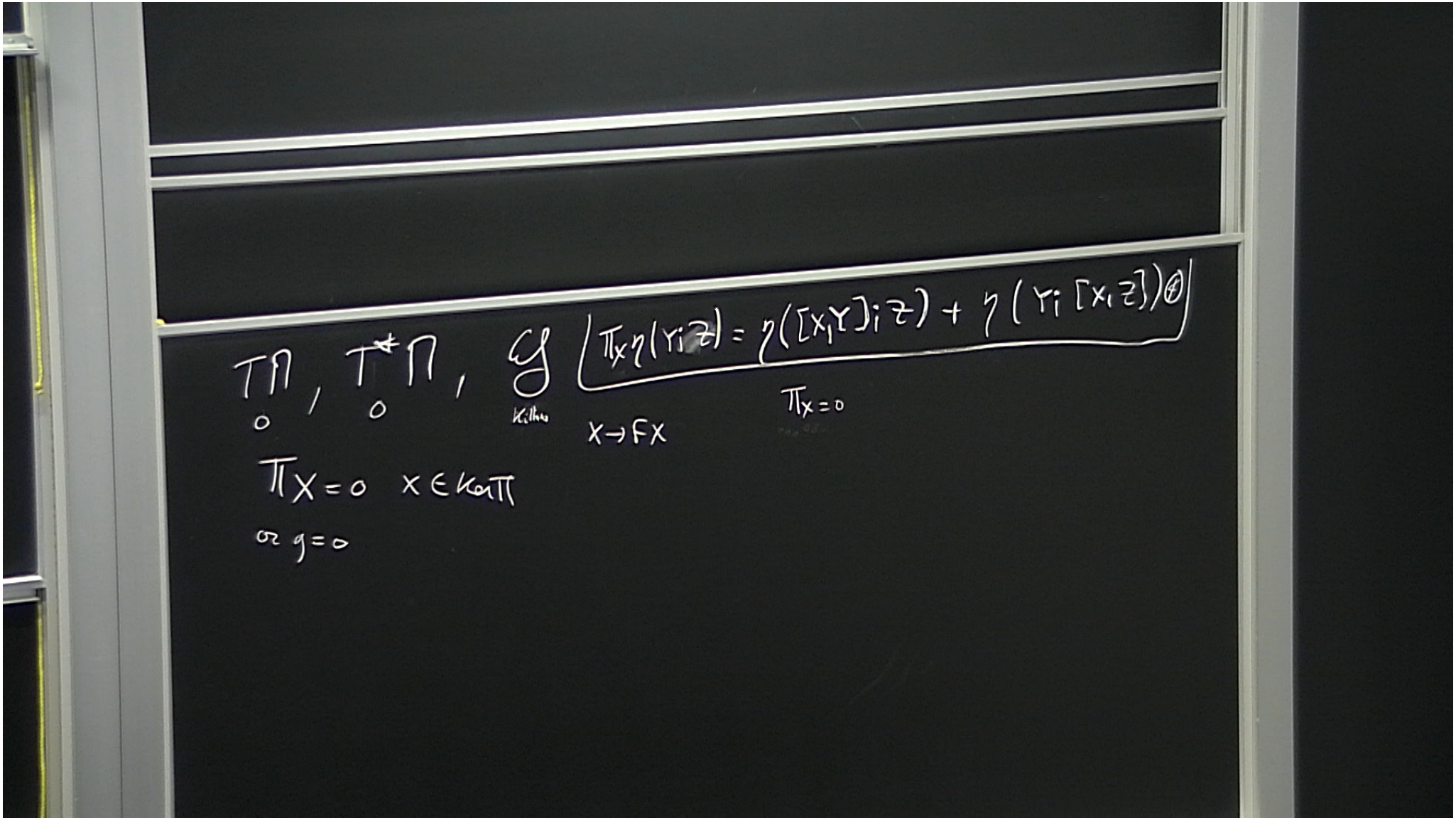
$dh \to P(dh)$

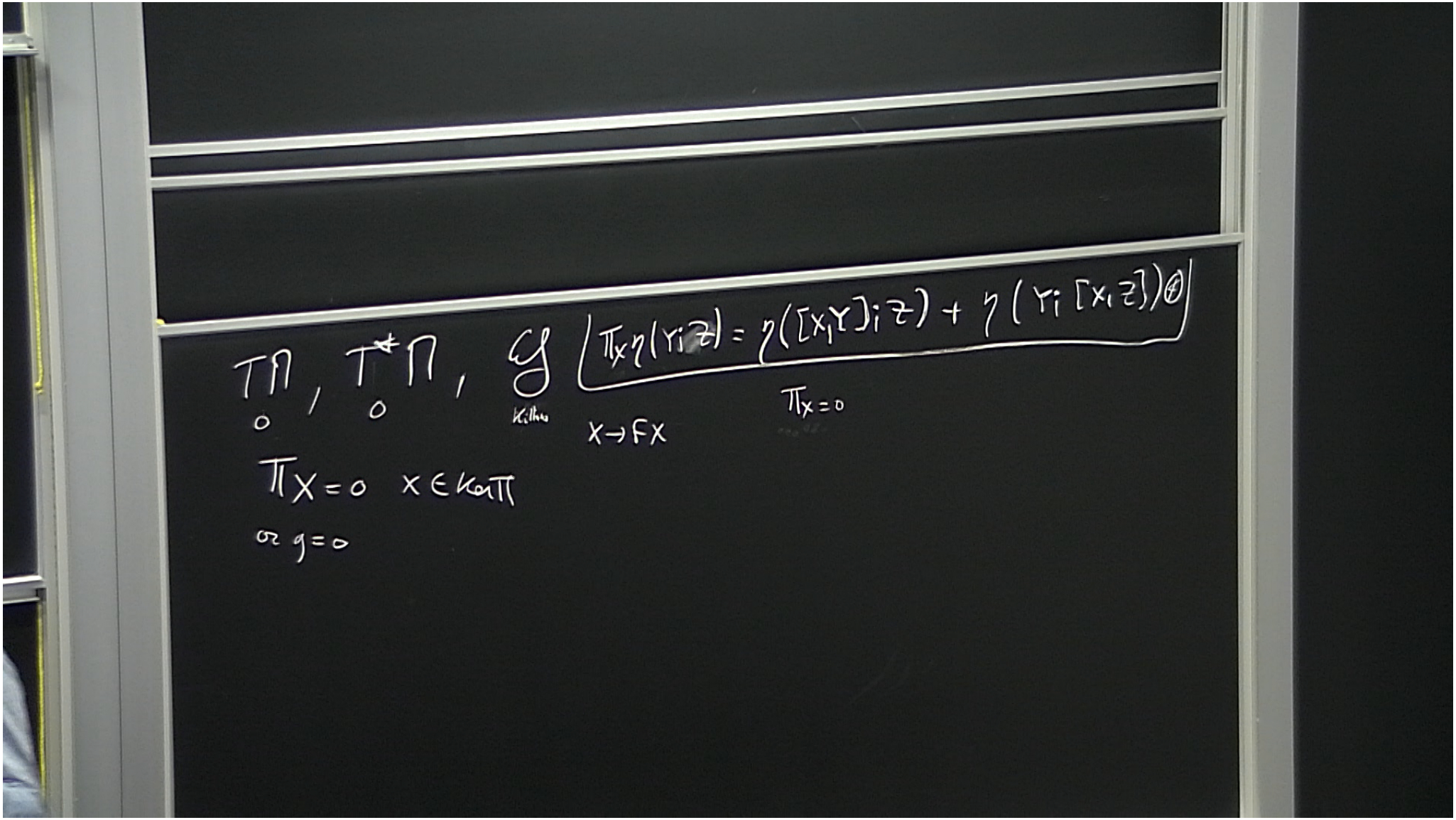
$\{g, g\} = \Phi^{ab} \partial_a F D_b g$

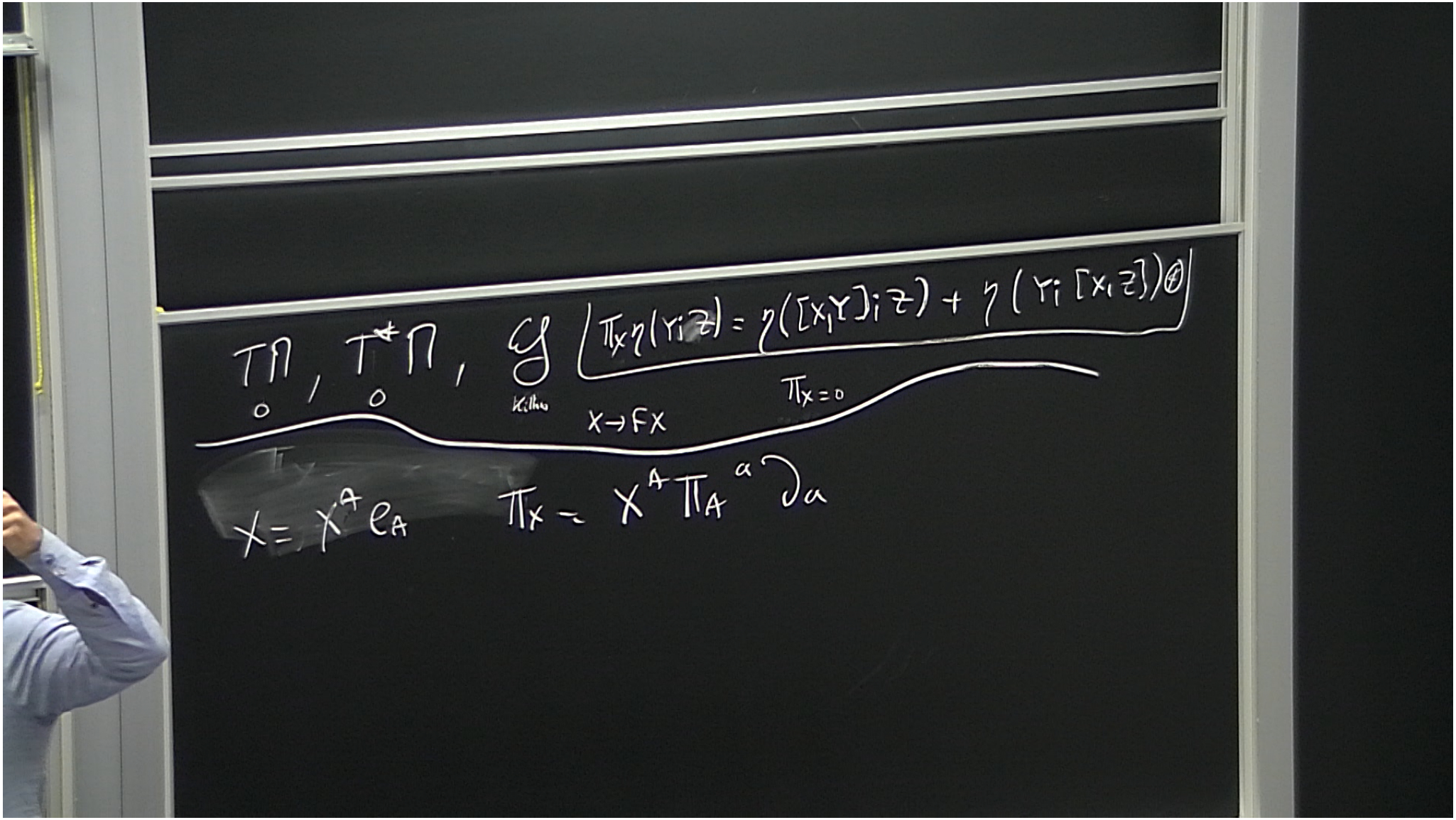

GR $P=(T\Pi, [,])^{\Pi}$
Gauge: Atiyah bundle $B \to E = TB/G$
$0 \to V \to E \xrightarrow{\pi} T\Pi \to 0$
$V \cong \mathfrak{g}\times\Pi$
$(E, [,]_g, \pi)$
GQ: Koszul Bundle
$\pi x = 0 \Rightarrow$ quantum
$P: T^*\Pi \xrightarrow{\pi} T\Pi$
$d\eta \to P(\eta)$
$E: T^*\Pi$ $\pi(\alpha)$

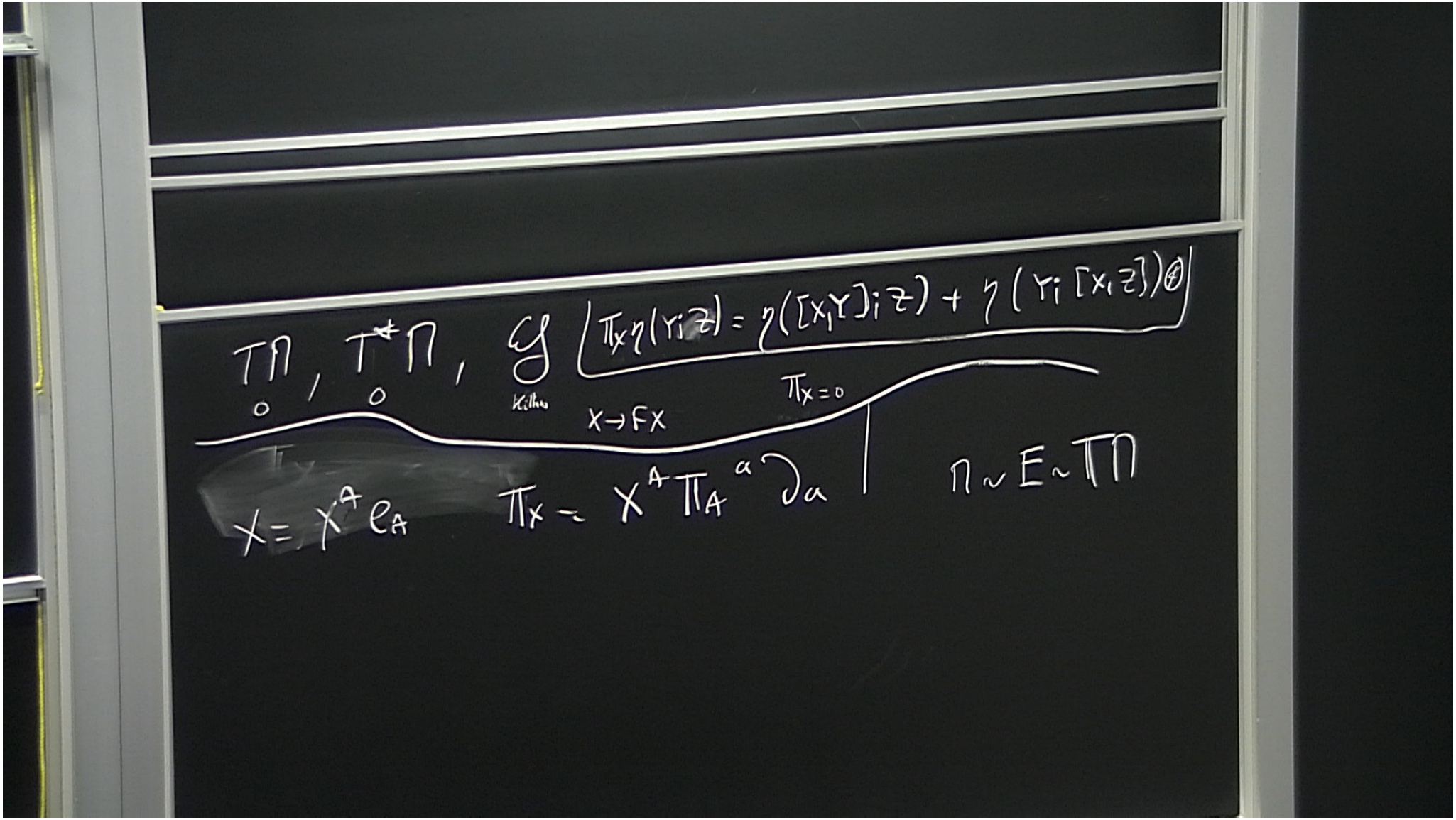

$\mathcal{L}_X = i_X d + d i_X$
$[df, dg]_P = d\{f, g\}$
$F \rightrightarrows \Pi$; $[,]_E$
$X \to FX$
$C^\infty(M)$
1) $([X,Y]_E, Z]_E = [X,[Y,Z]_E]_E - [Y,[X,Z]_E]_E$
2) $[X, FY]_E = F[X,Y]_E + \Pi_X F Y$
3) $[X,X]_E = 0$
CAUTION
CAUTION

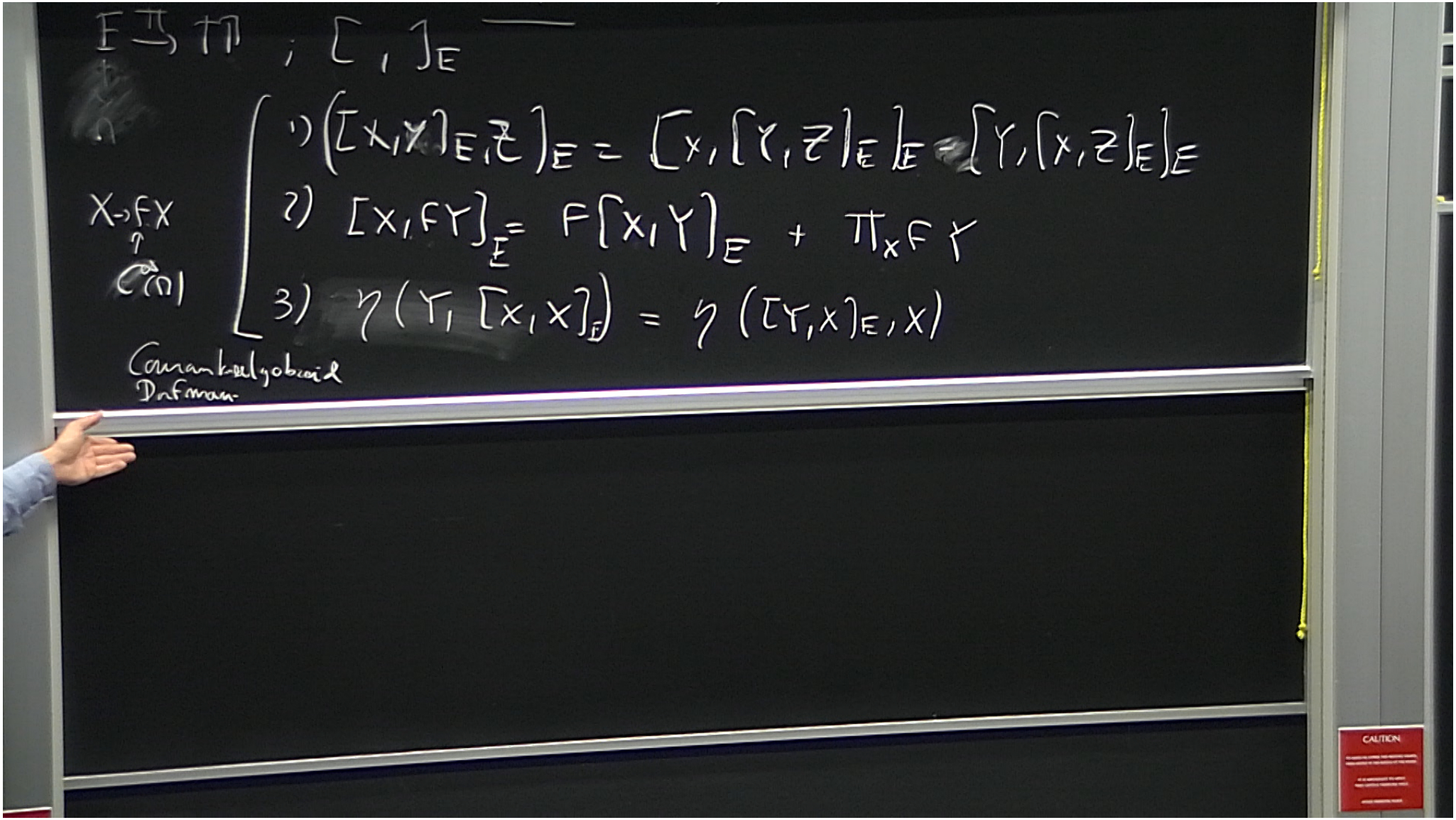

$T\Pi$, $T^*\Pi$, $\mathcal{G}$ $\pi_x \eta(Y_i z) = \eta([x,Y]_i z) + \eta(Y_i [x_i z])$
$\pi_x = 0$
CAUTION


$T\Pi_0$, $T^*\Pi_0$, $\mathcal{G}_{Killing}$
$\pi_X \eta(Y,Z) = \eta([X,Y],Z) + \eta(Y,[X,Z])$
$\pi_X = 0$
$X \to FX$
$\pi X = 0 \quad X \in \ker \pi$
or $g = 0$


$T\Pi_0$, $T^*\Pi_0$, $\mathcal{G}_{\text{Killing}}$
$\pi_X \eta(Y,Z) = \eta([X,Y],Z) + \eta(Y,[X,Z])$
$X \to FX$
$\pi_X = 0$
$\pi X = 0 \quad X \in \ker \pi$
or $g = 0$


$T\Pi$, $T^*\Pi$, $\mathcal{G}$
Killing
$\pi_X \eta(Y,Z) = \eta([X,Y],Z) + \eta(Y,[X,Z])$
$X \to FX$
$\pi_X = 0$
$X = X^A e_A$
$\pi_X = X^A \pi_A{}^a \partial_a$


$T\Pi_0$, $T^*\Pi_0$, $\mathfrak{g}$ Killing
$\pi_X \eta(Y,Z) = \eta([X,Y],Z) + \eta(Y,[X,Z])$
$X \to FX$
$\pi_X = 0$
$X = X^A e_A$
$\pi_X = X^A \pi_A{}^a \partial_a$
$\Pi \sim E \sim T\Pi$


$E \xrightarrow{\pi} M$; $[\,,\,]_E$
1) $([X,Y]_E, Z]_E = [X,[Y,Z]_E]_E - [Y,[X,Z]_E]_E$
2) $[X, fY]_E = f[X,Y]_E + \pi_X f\, Y$
3) $\eta([Y,[X,X]_E]) = \eta([[Y,X]_E, X])$
$X \to fX$
$C^\infty(M)$
Courant algebroid
Dorfman

Koszul bracket $[\alpha,\beta]_P = \mathcal{L}_{\pi\alpha}\beta - i_{\pi\beta}d\alpha$, $\alpha,\beta \in \Omega^1(M)$

$\mathcal{L}_X = i_X d + d i_X$

$[df, dg]_P = d\{f,g\}$

$E \xrightarrow{\pi} TM$; $[\,,\,]_E$

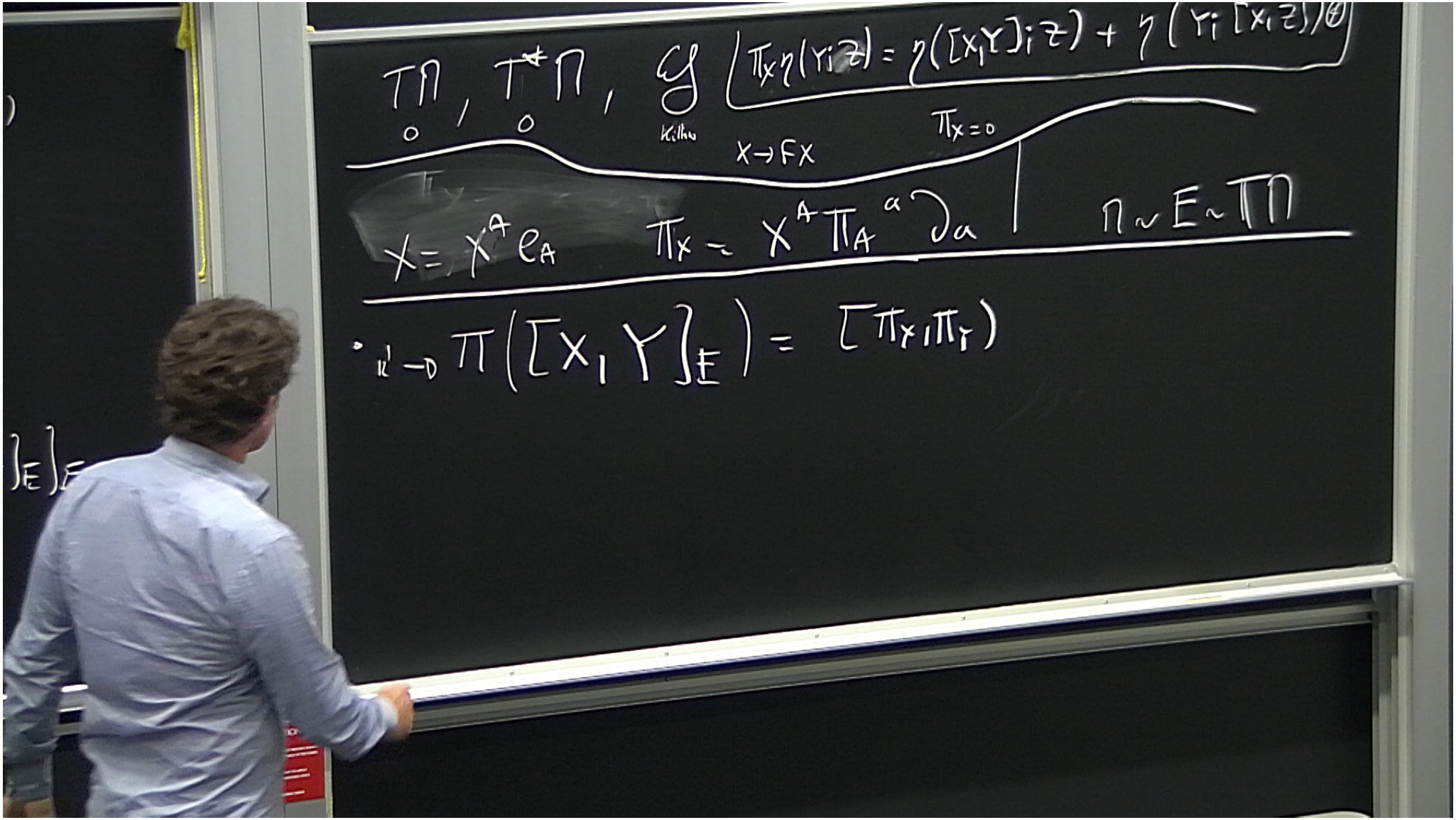
$X \to fX$ ($C^\infty(M)$)

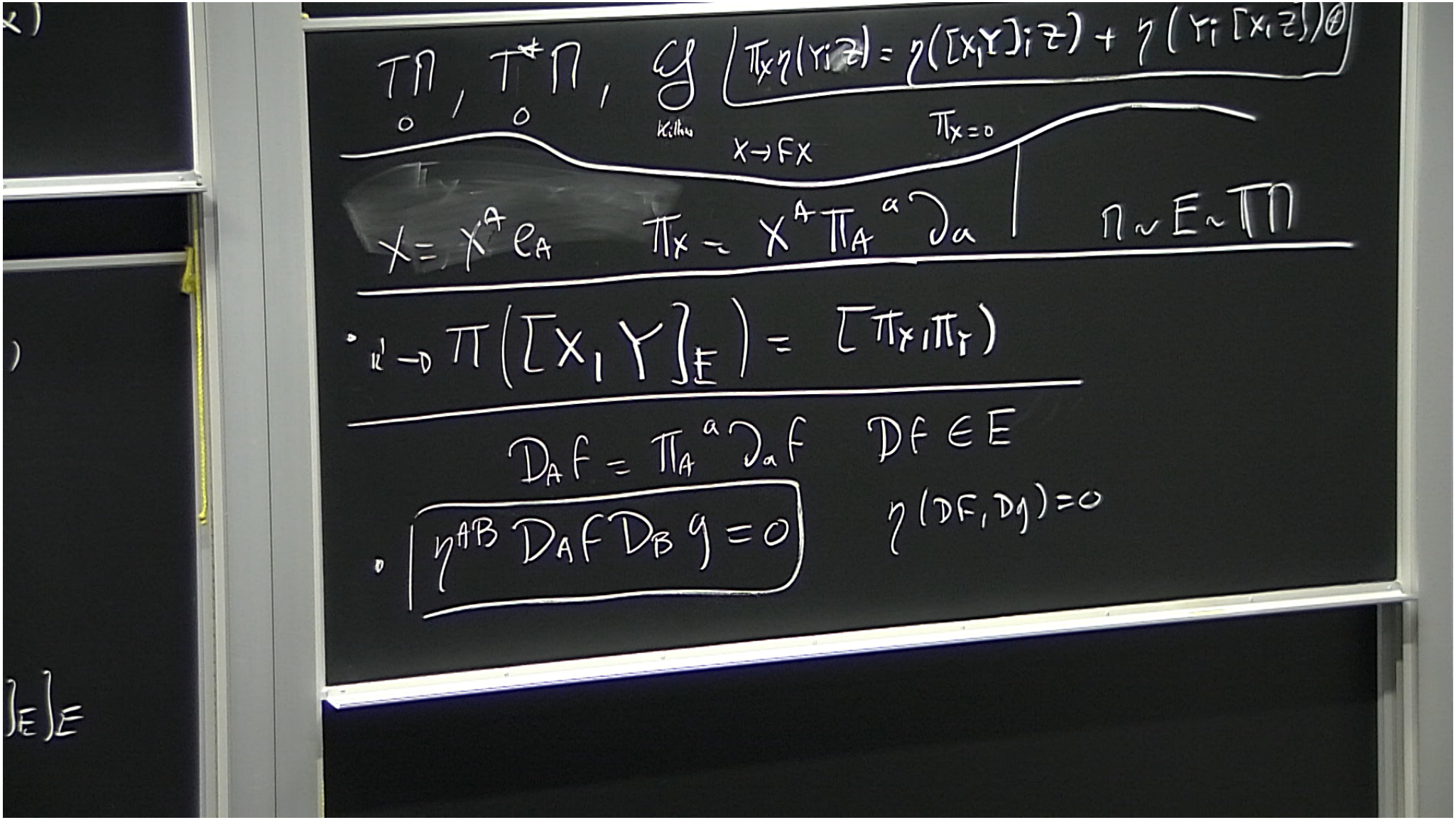
1) $([X,Y]_E, Z]_E = [X,[Y,Z]_E]_E - [Y,[X,Z]_E]_E$

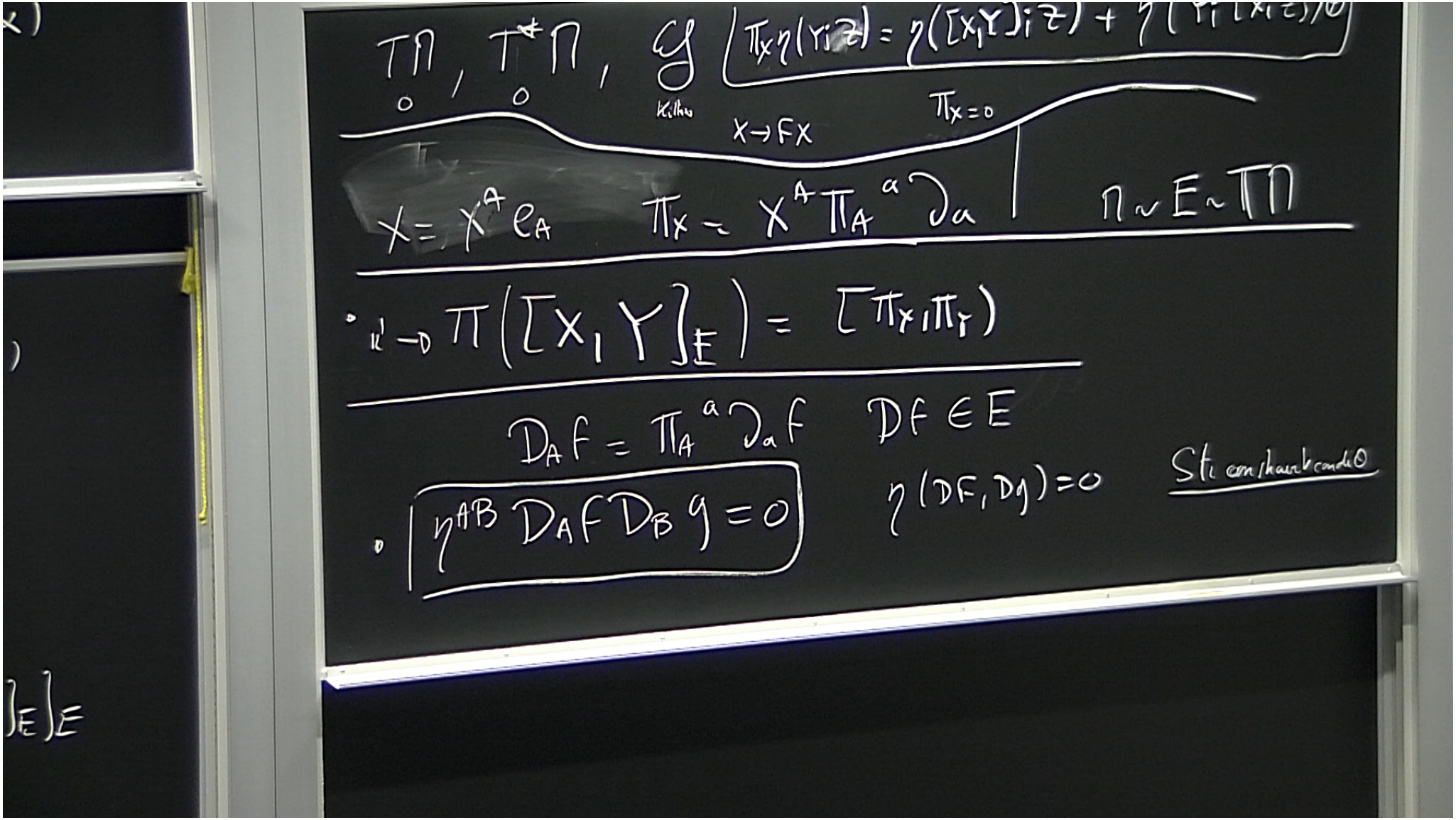
2) $[X, fY]_E = f[X,Y]_E + \pi_X f\, Y$

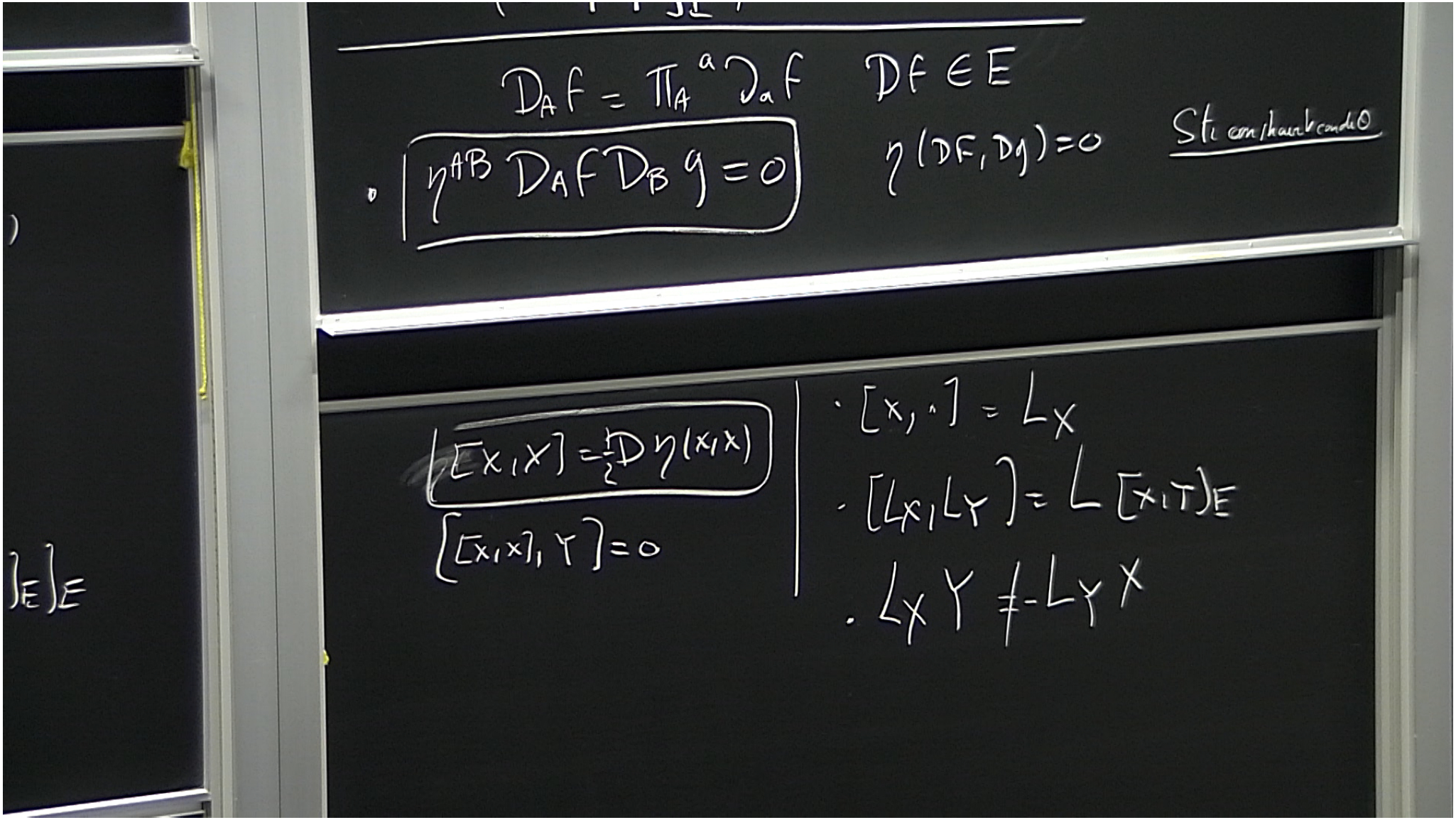
3) $\eta([Y,X]_E) = \eta([Y,X]_E, X)$

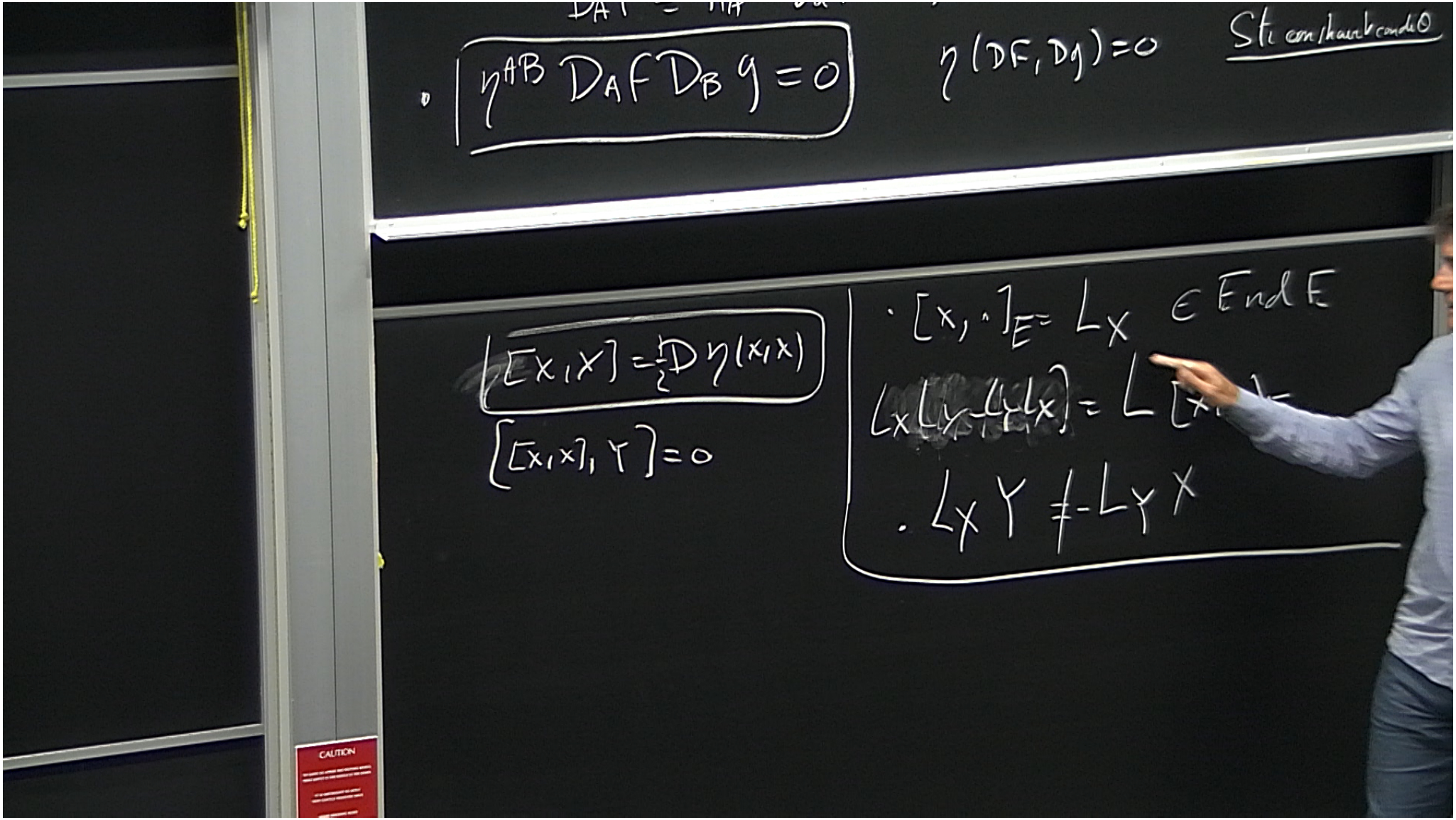
Courant-algebroid
Dorfman

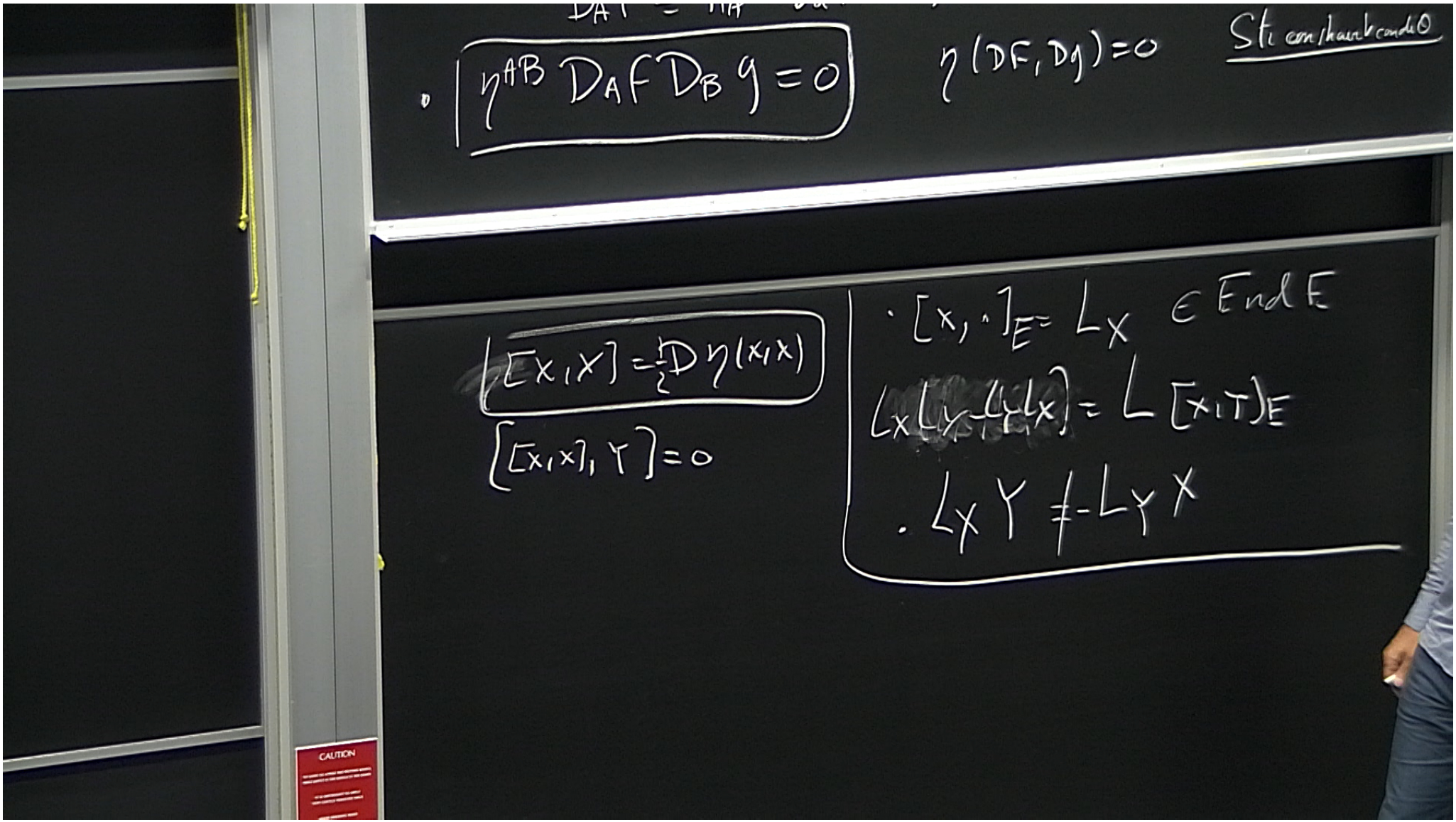

$T\Pi$, $T^*\Pi$, $\mathcal{G}$
Killing
$\pi_X \eta(Y,Z) = \eta([X,Y],Z) + \eta(Y,[X,Z])$
$X \to FX$
$\pi_X = 0$
$X = X^A e_A$
$\pi_X = X^A \pi_A{}^a \partial_a$
$\Pi \sim E \sim T\Pi$
$\pi([X,Y]_E) = [\pi_X, \pi_Y]$

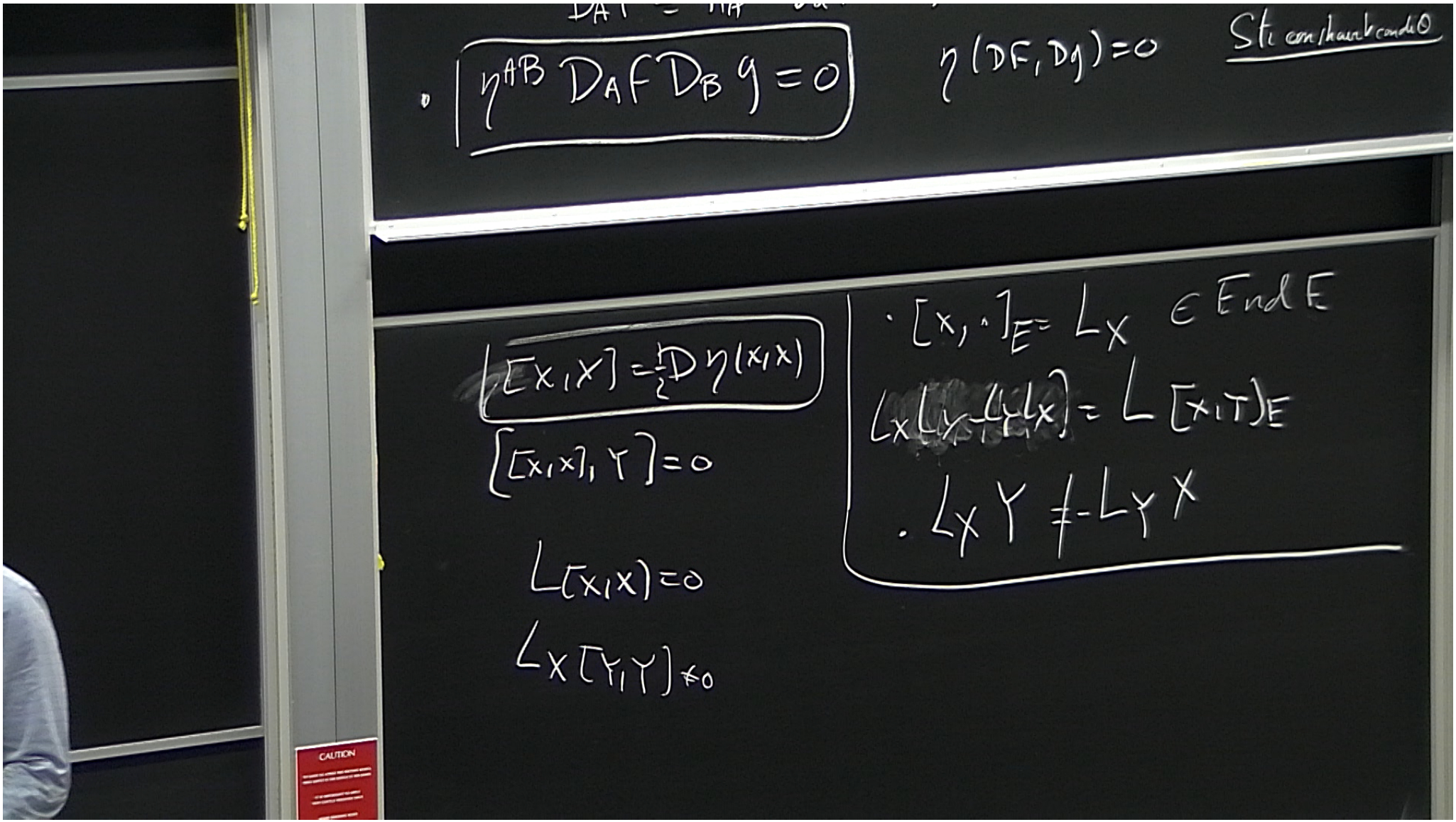

$T\Pi$, $T^*\Pi$, g
Killing
$\pi_X \eta(Y,Z) = \eta([X,Y],Z) + \eta(Y,[X,Z])$
$X \to FX$
$\pi_X = 0$
$X = X^A e_A \quad \pi_X = X^A \pi_A{}^a \partial_a$
$\Pi \sim E \sim T\Pi$
$\pi([X,Y]_E) = [\pi_X, \pi_Y]$
$D_A f = \pi_A{}^a \partial_a f \quad Df \in E$
$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$

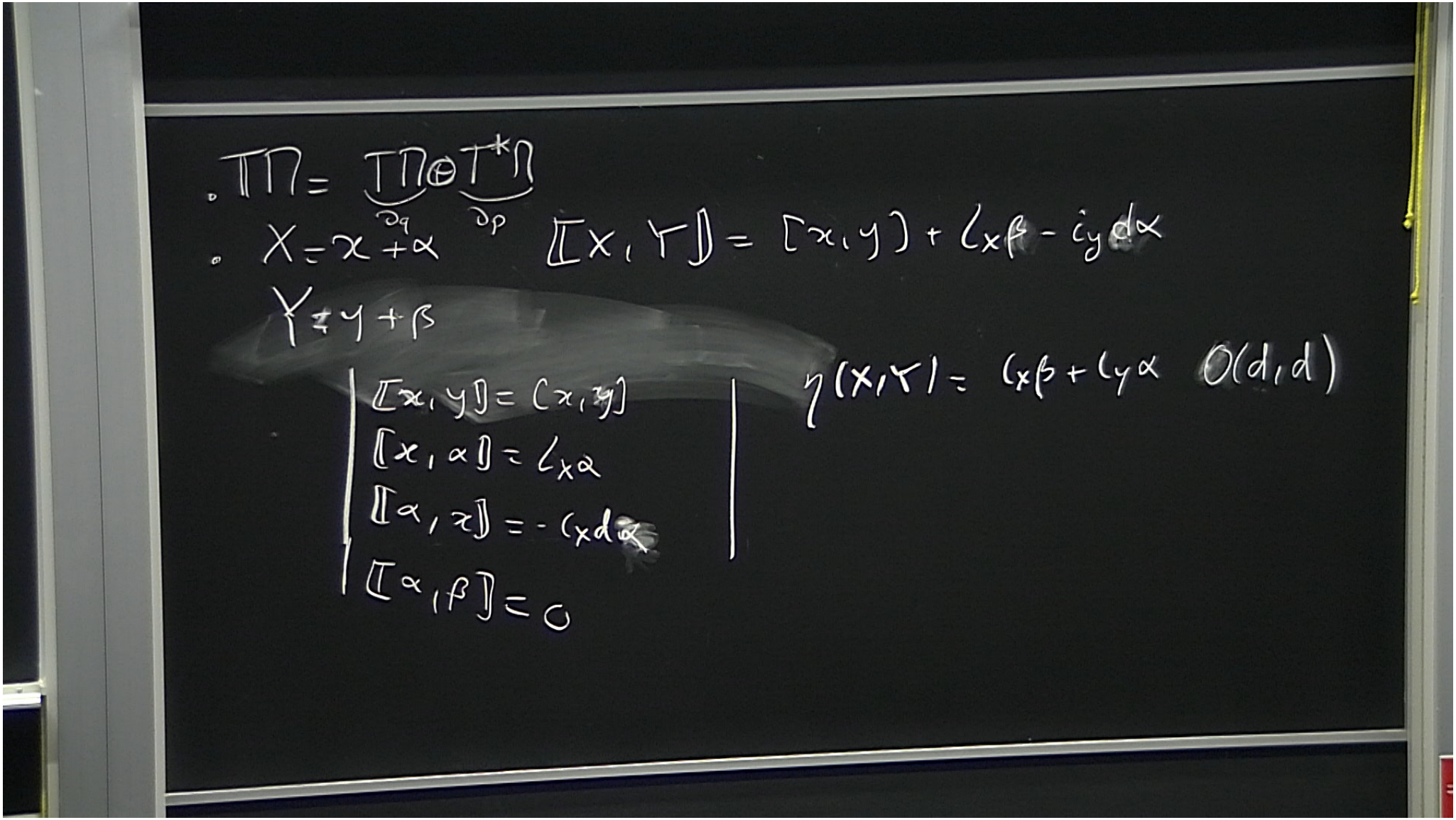

$T\Pi$, $T^*\Pi$, g
Killing
$X \to FX$
$\pi_X = 0$
$\Pi \sim E \sim T\Pi$
$X = X^A e_A \quad \pi_X \sim X^A \pi_A{}^a \partial_a$
$\pi([X,Y]_E) = [\pi_X, \pi_Y]$
$D_A f = \pi_A{}^a \partial_a f \quad Df \in E$
$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Stronger hamiltonian


$D_A f = \pi_A{}^a \partial_a f$
$Df \in E$
$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Sì, con hamiltoniano 0
$[X, X] = \frac{1}{2} D\eta(X, X)$
$[[X, X], Y] = 0$
$[X, \cdot] = L_X$
$[L_X, L_Y] = L_{[X,Y]_E}$
$L_X Y \neq -L_Y X$


$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Sti cm/hamb cond 0
$[X, X] = \frac{1}{2} D\eta(X, X)$
$[[X, X], Y] = 0$
$[X, \cdot]_E = L_X \in End\, E$
$L_X L_Y - L_Y L_X = L_{[X,Y]}$
$L_X Y \neq -L_Y X$
CAUTION


$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Sti cem/hamb cond 0
$[X, X] = \frac{1}{2} D\eta(X, X)$
$[[X, X], Y] = 0$
$\cdot [X, \cdot]_E = L_X \in End\, E$
$L_X L_Y - L_Y L_X = L_{[X,Y]_E}$
$\cdot L_X Y \neq -L_Y X$
CAUTION


$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Sti cem/hamb cond 0
$[X, X] = \frac{1}{2} D\eta(X, X)$
$[[X, X], Y] = 0$
$L_{[X,X]} = 0$
$L_X [Y, Y] \neq 0$
$[X, \cdot]_E = L_X \in End\, E$
$L_X L_Y - L_Y L_X = L_{[X,Y]_E}$
$L_X Y \neq -L_Y X$
CAUTION


$T\Pi = T\Pi \oplus T^*\Pi$
∂q ∂p
$X = x + \alpha$
$Y = y + \beta$
$[\![X, Y]\!] = [x, y] + \mathcal{L}_x\beta - i_y d\alpha$
$[\![x, y]\!] = [x, y]$
$[\![x, \alpha]\!] = \mathcal{L}_x\alpha$
$[\![\alpha, x]\!] = -\mathcal{L}_x d\alpha$
$[\![\alpha, \beta]\!] = 0$
$\eta(X, Y) = \mathcal{L}_x\beta + \mathcal{L}_y\alpha$ $O(d, d)$

Generalised Geometry: Geometry of Unification.

Qu: Unification: 1) U of gauge and gravity: Gauge + Riemannian geo (DIFF)

2) U of QM and Gravity: Phase Space + R Geo → Relativ

3) U of Matter and geometry: Fermion + Bosonic geo → NCG

$$\frac{c^2}{G} = \frac{\Omega}{R}$$ Tensor

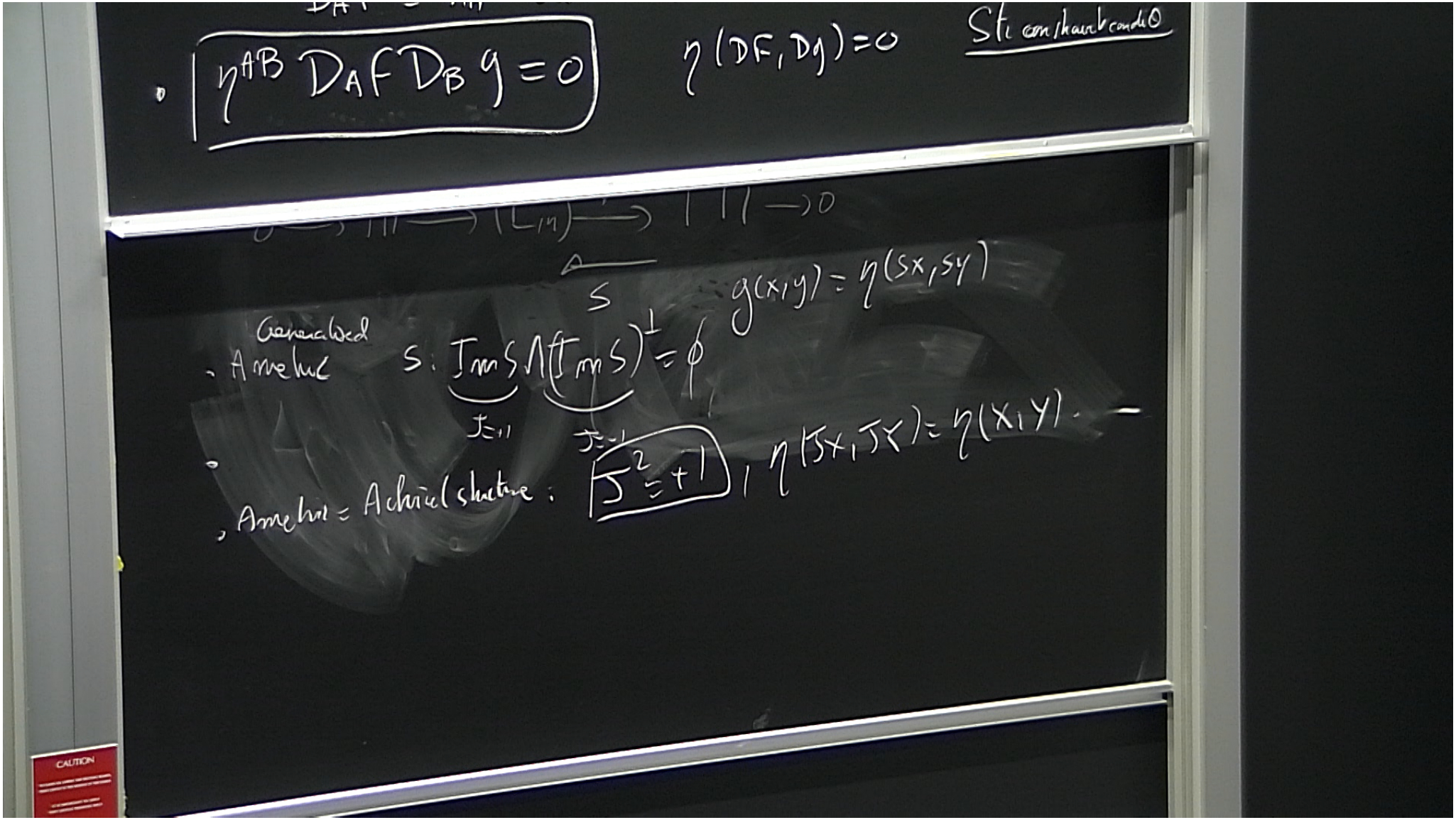
Generalised Geometry: Geometry of Unification.

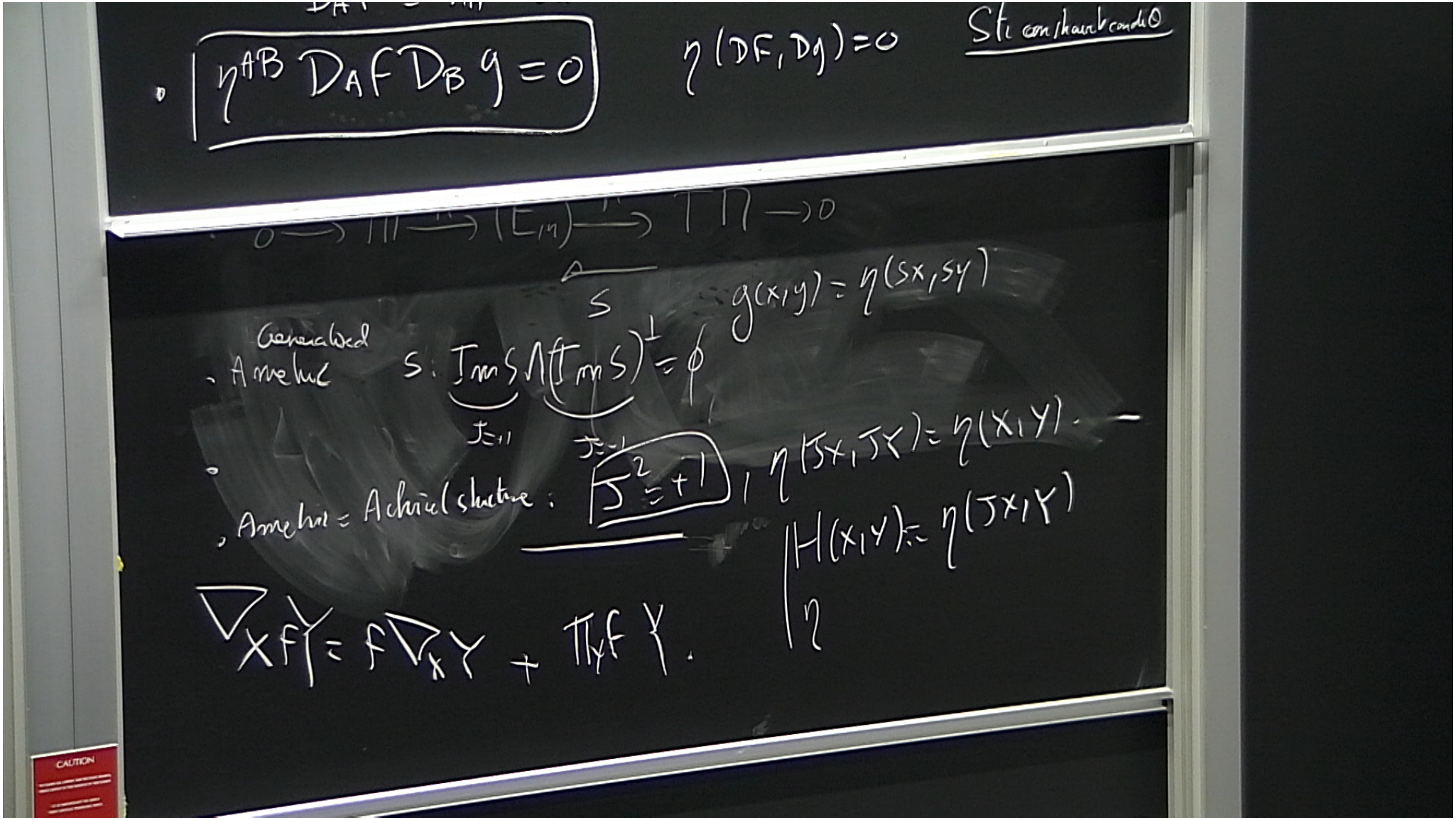
Ph: Unification: 1) U of gauge and gravity: Gauge + Riemannian geo (DIFF)

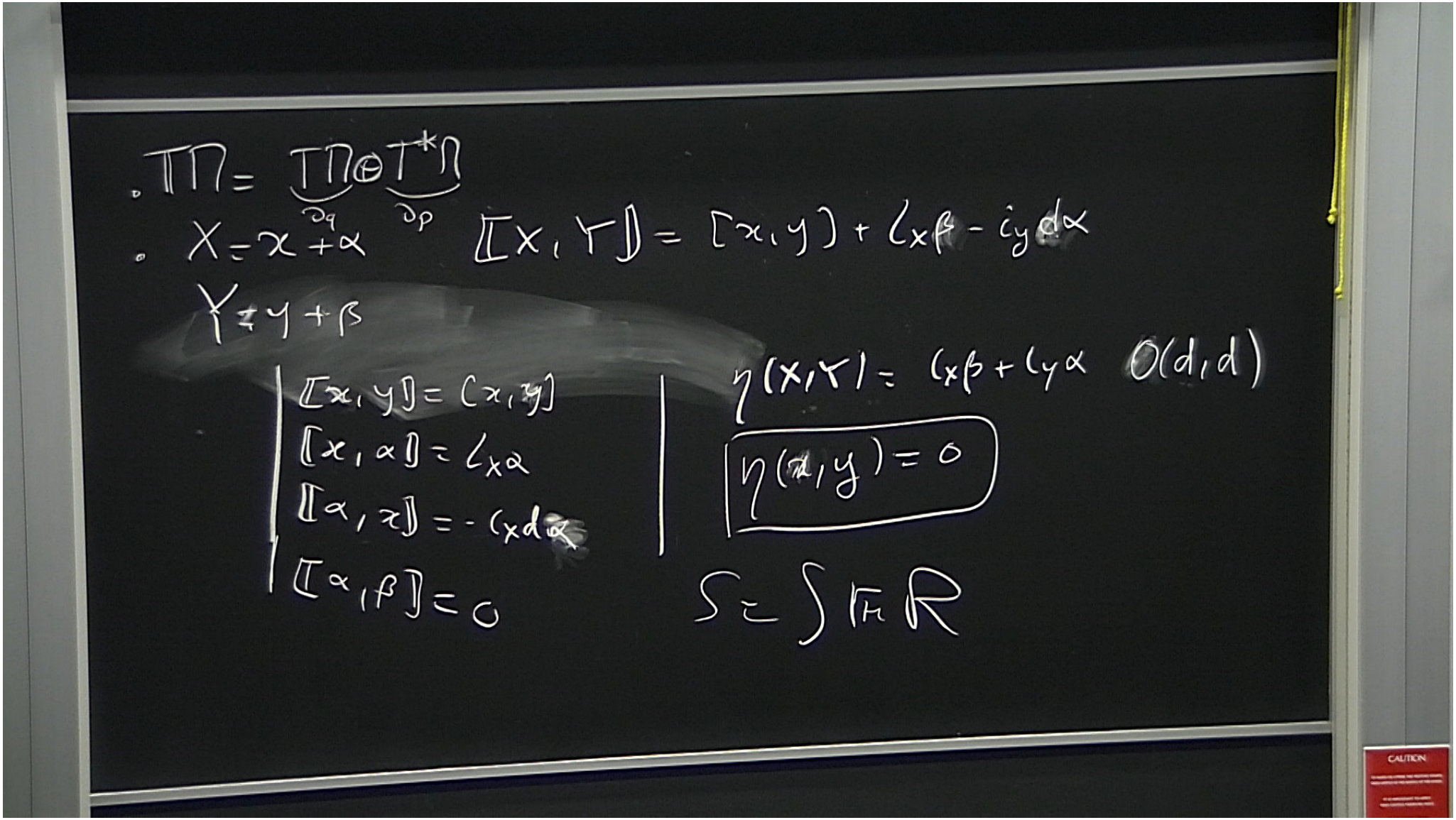
2) U of QM and Gravity: Phase Space + R Geo → Relativity

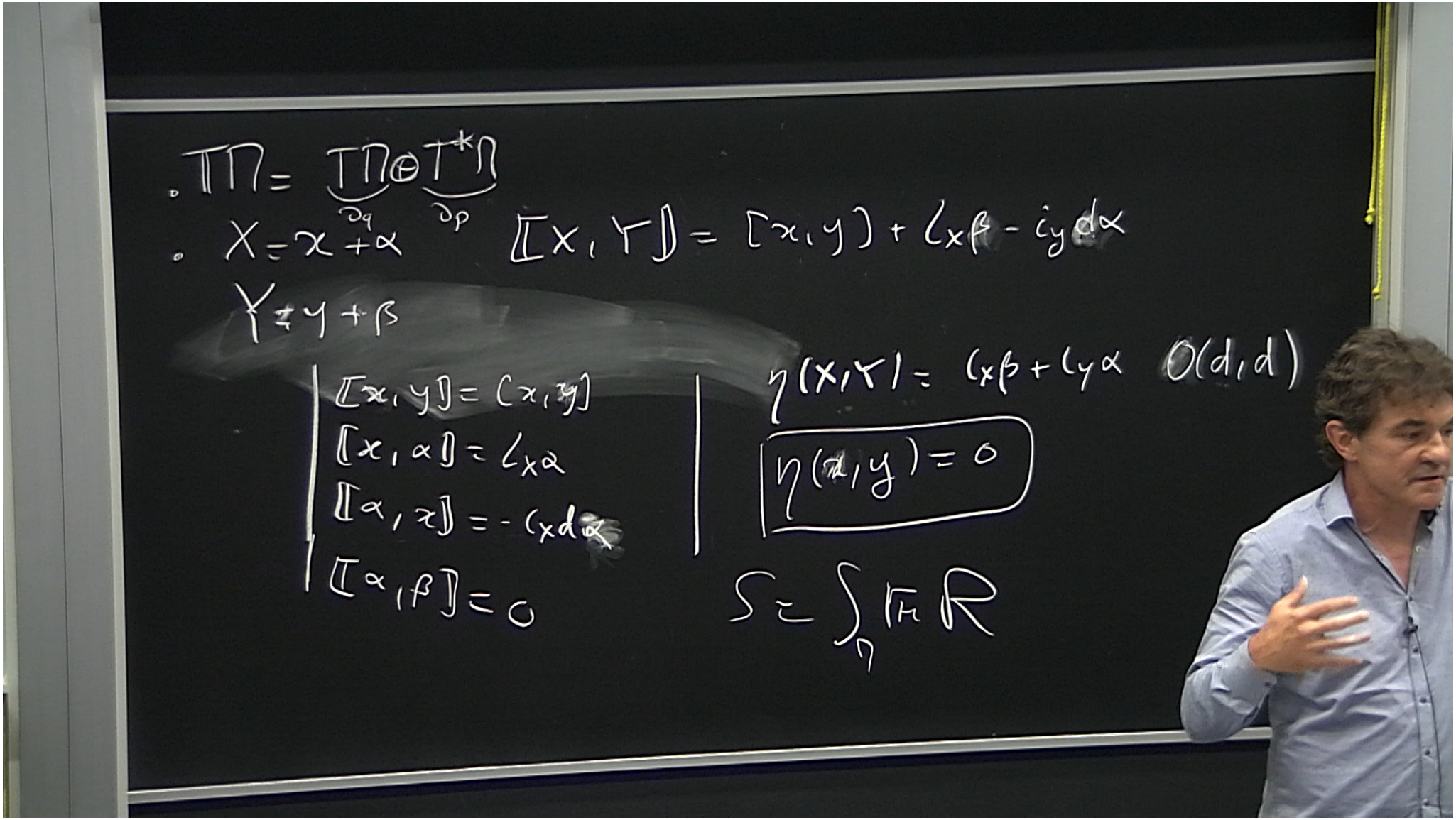
3) U of Matter and geometry: Fermi + Bosonic geo → NCG

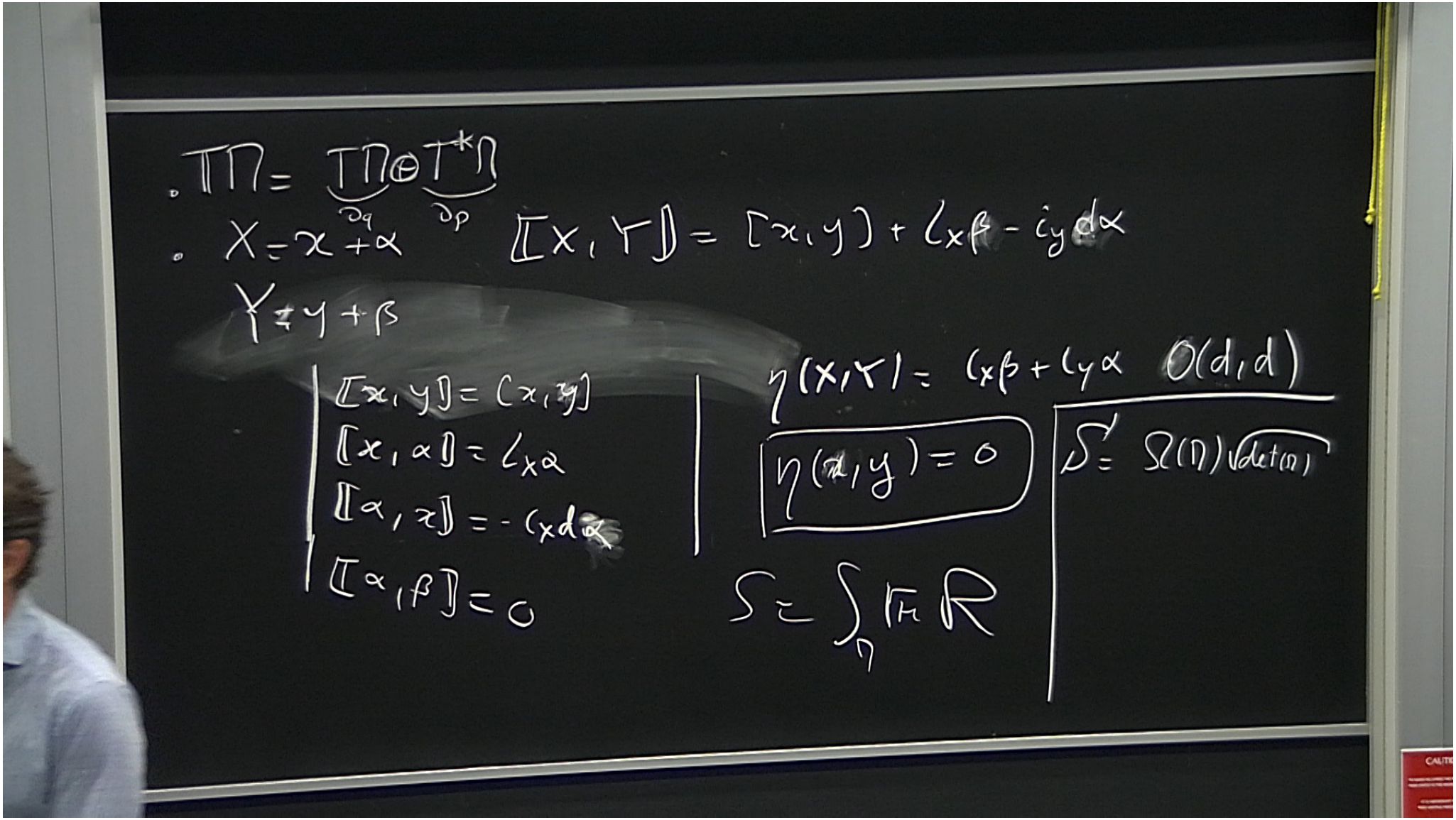
$\frac{c^2}{G} = \frac{\Omega}{R}$ Tensor ; $\Delta u = \Delta p f(\alpha)$

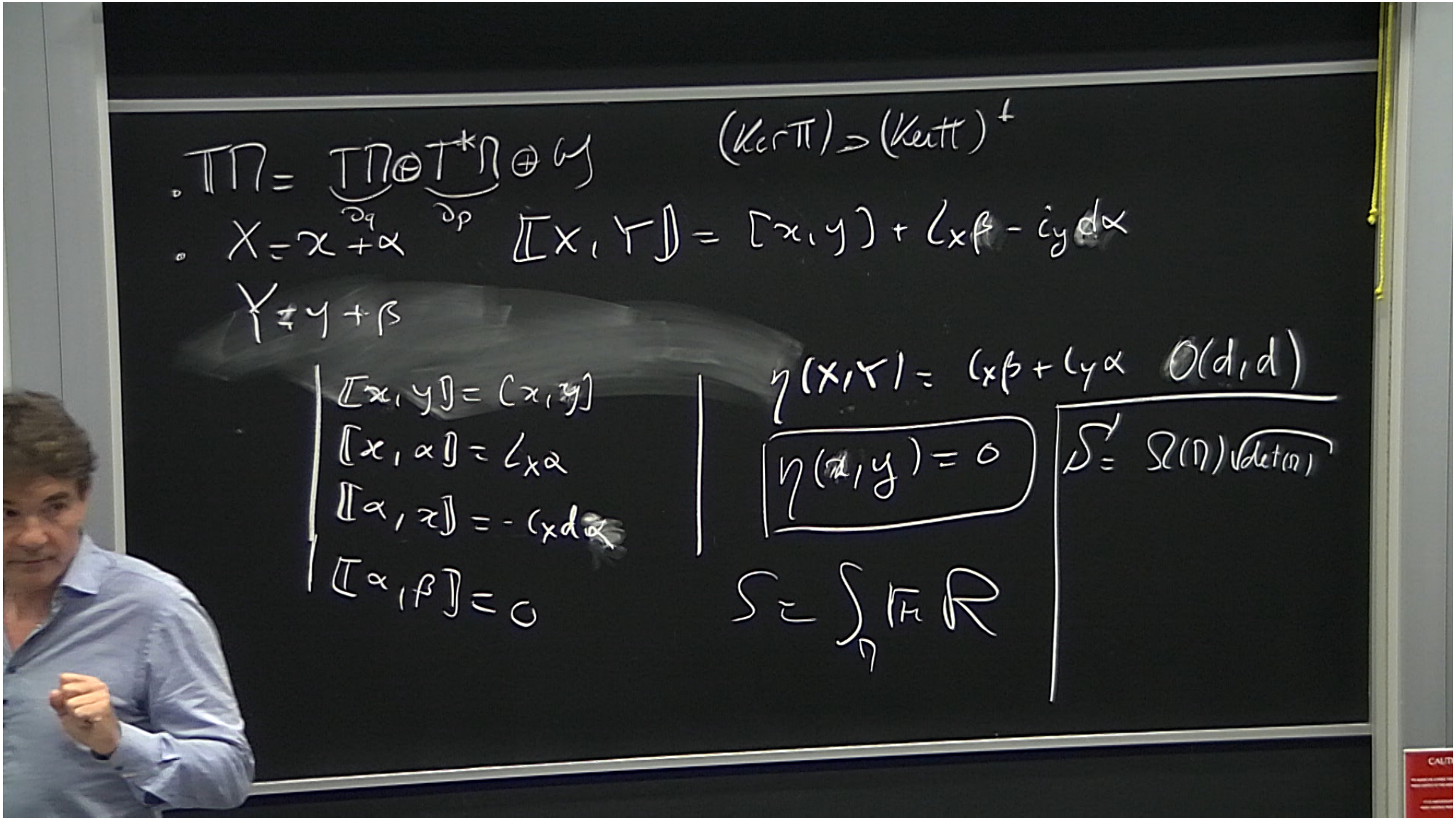

$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Sti con/hausdorff condition
$g(x,y) = \eta(Sx, Sy)$
Generalized
Amelino
$S: \underbrace{Im\,S} \cap (\underbrace{Im\,S})^{\perp} = \phi$
$j=1$
$j=-1$
$S^2 = +1$
$\eta(Sx, Sx) = \eta(x, y)$
Amelino = Achiral structure:

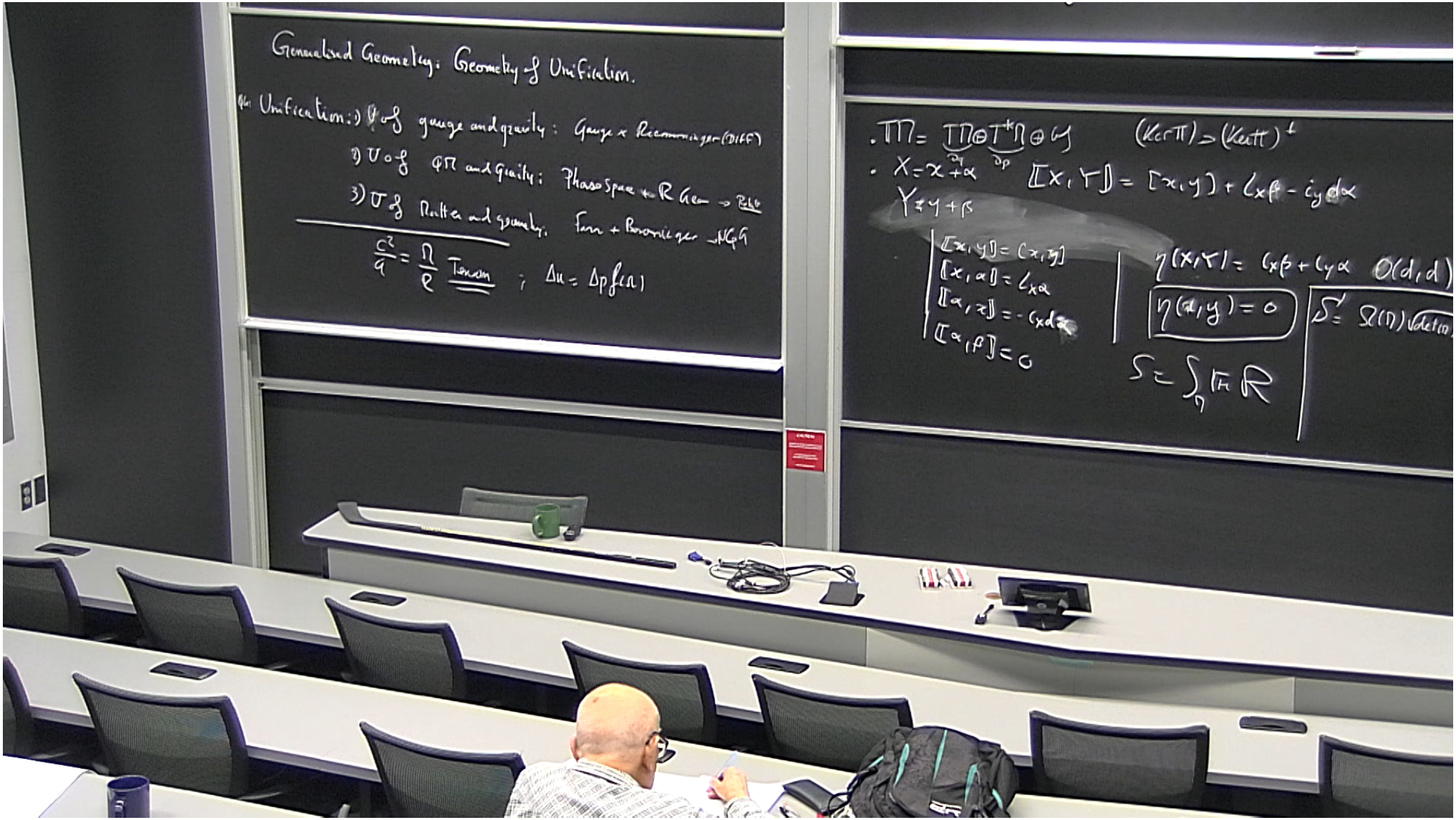

$\eta^{AB} D_A f D_B g = 0$
$\eta(Df, Dg) = 0$
Sti con/hamilt cond 0
$0 \to \Pi \to (E, \eta) \to T\Pi \to 0$
S
Generalized
Amelie
$S: \underbrace{Im\,S}_{J=1} \cap \underbrace{(Im\,S)}_{J=-1}^{\perp} = \phi$
$g(X,Y) = \eta(SX, SY)$
$J^2 = +1$
, Amelie = Achiral structure :
$\eta(JX, JX) = \eta(X, Y)$
$H(X,Y) := \eta(JX, Y)$
$\nabla_X fY = f \nabla_X Y + \Pi_X f\, Y.$
CAUTION


$T\Pi = \underbrace{T\Pi}_{\partial q} \oplus \underbrace{T^*\Pi}_{\partial p}$
$X = x + \alpha$
$Y = y + \beta$
$[[X, Y]] = [x, y] + \mathcal{L}_x\beta - i_y d\alpha$
$[[x, y]] = [x, y]$
$[[x, \alpha]] = \mathcal{L}_x\alpha$
$[[\alpha, x]] = -\mathcal{L}_x d\alpha$
$[[\alpha, \beta]] = 0$
$\eta(X, Y) = \mathcal{L}_x\beta + \mathcal{L}_y\alpha$ $O(d, d)$
$\eta(x, y) = 0$
$S = \int_\Pi R$


$T\Pi = T\Pi \oplus T^*\Pi$
∂_q ∂_p
$X = x + \alpha$
$Y = y + \beta$
$[[X, Y]] = [x, y] + \mathcal{L}_x\beta - i_y d\alpha$
$[[x, y]] = [x, y]$
$[[x, \alpha]] = \mathcal{L}_x\alpha$
$[[\alpha, x]] = -\mathcal{L}_x d\alpha$
$[[\alpha, \beta]] = 0$
$\eta(X, Y) = \mathcal{L}_x\beta + \mathcal{L}_y\alpha$ $O(d,d)$
$\eta(x, y) = 0$
$S = \int_\eta F_1 R$


$T\Pi = T\Pi \oplus T^*\Pi$
$\partial_q \quad \partial_p$
$X = x + \alpha$
$Y = y + \beta$
$[[X, Y]] = [x, y] + \mathcal{L}_x\beta - i_y d\alpha$
$[[x, y]] = [x, y]$
$[[x, \alpha]] = \mathcal{L}_x \alpha$
$[[\alpha, x]] = -\mathcal{L}_x d\alpha$
$[[\alpha, \beta]] = 0$
$\eta(X, Y) = \mathcal{L}_x\beta + \mathcal{L}_y\alpha$
$\eta(x, y) = 0$
$S = \int_\eta \Gamma R$
$O(d, d)$
$S = \Omega(\eta)\sqrt{\det(\eta)}$


$\mathbb{T}\Pi = T\Pi \oplus T^*\Pi \oplus \mathcal{O}_M$
$(Ker\,\Pi) \supset (Ker\,\Pi)^\perp$
$X = x + \alpha$
$[\![X, Y]\!] = [x, y] + \mathcal{L}_x\beta - i_y d\alpha$
$Y = y + \beta$
$[\![x, y]\!] = [x, y]$
$[\![x, \alpha]\!] = \mathcal{L}_x\alpha$
$[\![\alpha, x]\!] = -\mathcal{L}_x d\alpha$
$[\![\alpha, \beta]\!] = 0$
$\eta(X, Y) = \mathcal{L}_x\beta + \mathcal{L}_y\alpha$
$O(d,d)$
$\eta(x, y) = 0$
$S = \int_\eta \mathcal{L}_R R$
$S = \Omega(\eta)\sqrt{\det(\eta)}$


Generalised Geometry: Geometry of Unification.
Unification: 1) of gauge and gravity: Gauge + Riemannian (DIFF)
2) of QM and Gravity: Phase Space + R Geo → Relativity
3) of Particle and Gravity: Fermi + Riemannian → NCG
Torsion
[x, y] = [x, y]
[x, α] = ℒ_x α
[α, x] = -ℒ_x α
[α, β] = 0
η(X, Y) = ℒ_x β + ℒ_y α
η(α, γ) = 0
S = ∫_η √η R