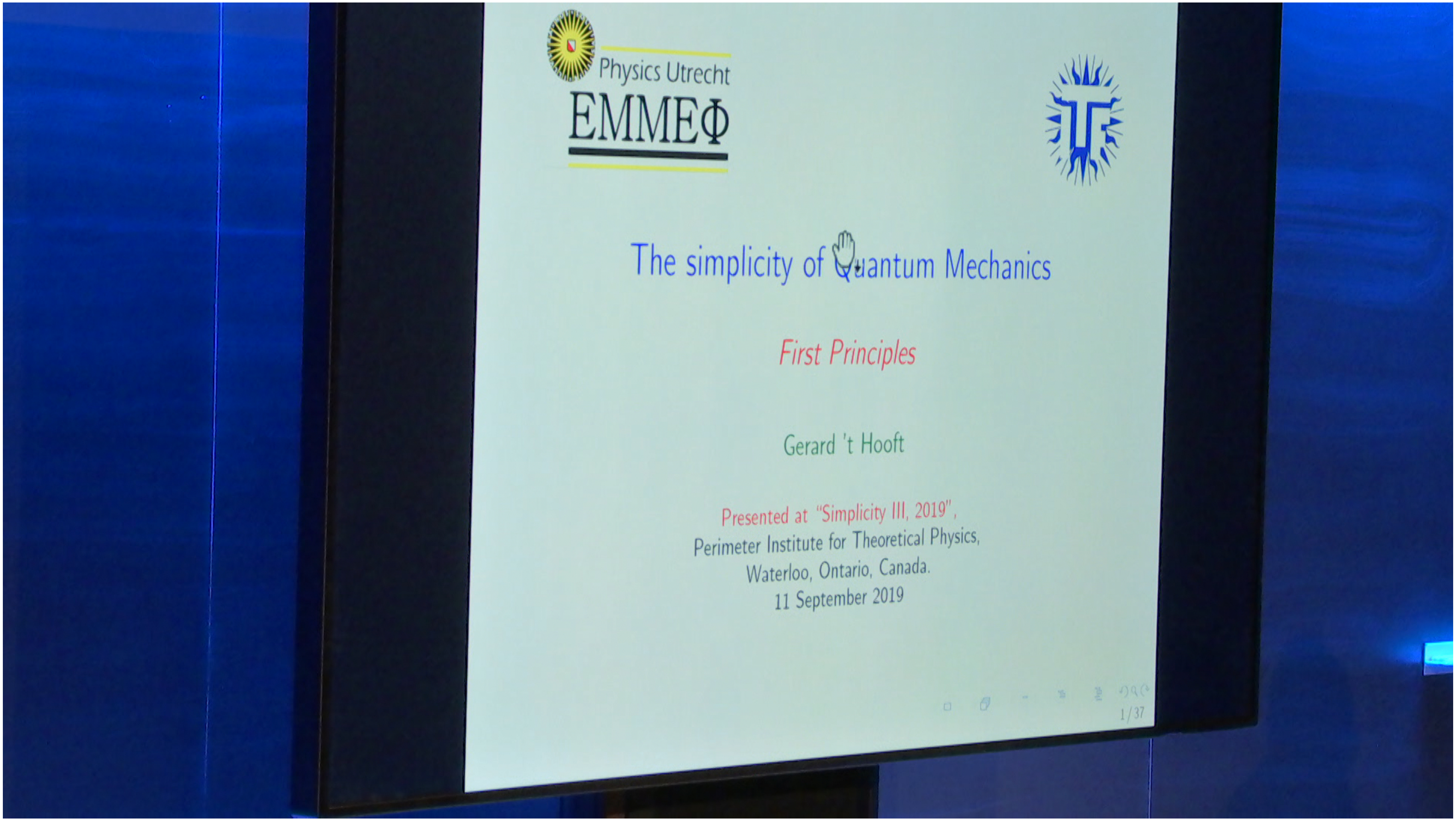
Title: The Simplicity of Quantum Mechanics

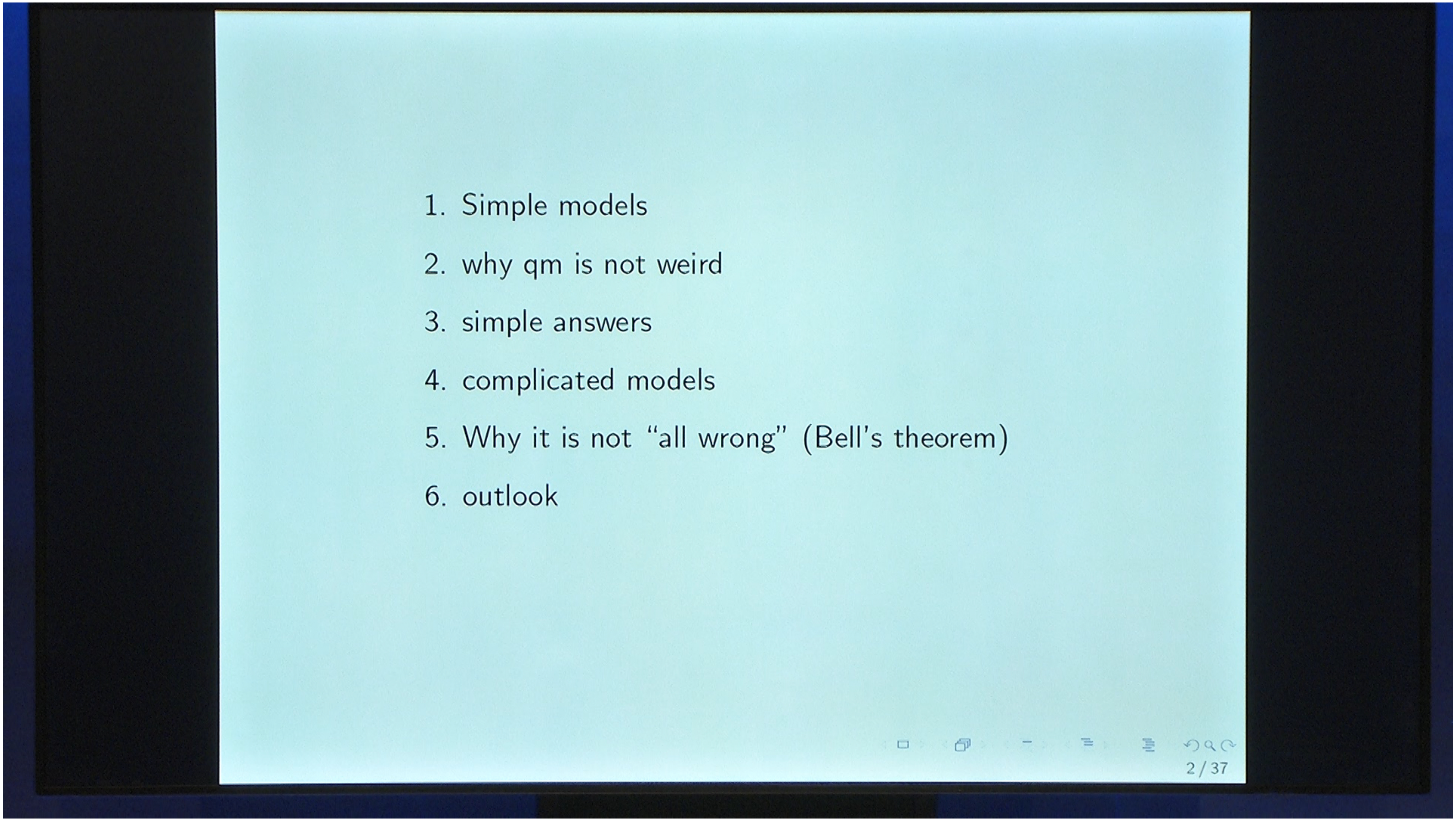
Speakers: Gerard 't Hooft

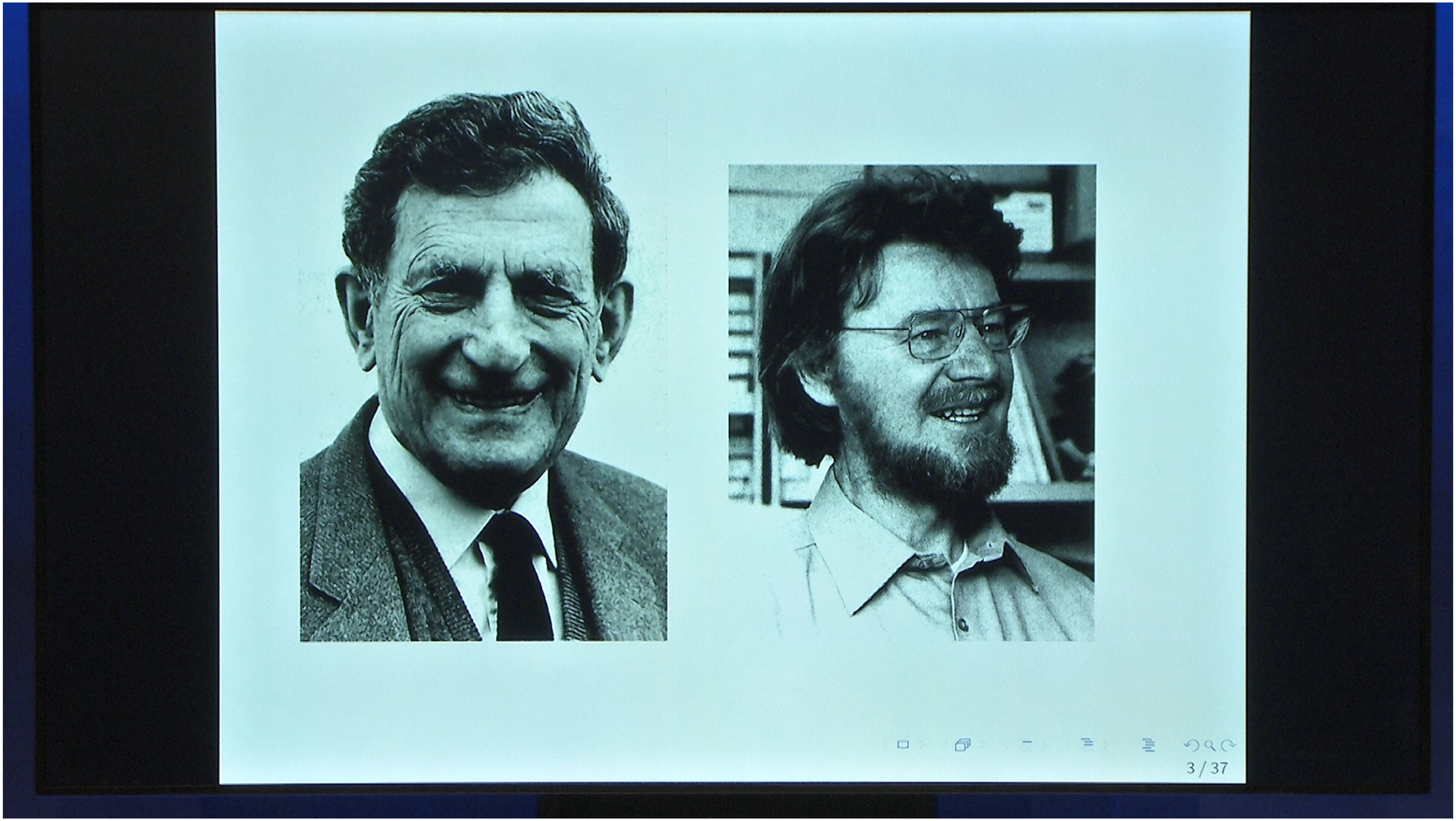
Collection: Simplicity III

Date: September 11, 2019 - 2:00 PM

URL: http://pirsa.org/19090083


Physics Utrecht
EMMEΦ
The simplicity of Quantum Mechanics
First Principles
Gerard 't Hooft
Presented at "Simplicity III, 2019",
Perimeter Institute for Theoretical Physics,
Waterloo, Ontario, Canada.
11 September 2019
1 / 37


1. Simple models
2. why qm is not weird
3. simple answers
4. complicated models
5. Why it is not "all wrong" (Bell's theorem)
6. outlook
2 / 37


3 / 37

Our first basic model is the simplest possible ‘automaton’: there are only N classical states in our ‘universe’.

‘Law of nature’: the system hops periodically through all these states.

No quantum mechanics, no Hilbert space:

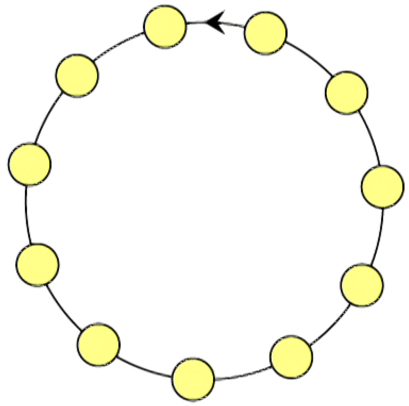
But we may use a **vector space notation:**

To find the matrix H, diagonalise $U(\delta t)$ by using the finite Fourier transform:

$$|k\rangle_H \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{2\pi i k n/N} |n\rangle_{\text{ont}} \ ,$$

$$|n\rangle_{\text{ont}} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{-2\pi i k n/N} |k\rangle_H \ .$$

$$n = 0, \cdots N-1 \ ; \quad k = 0, \cdots N-1 \ .$$



Ontological states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

Evolution law:

$$|n\rangle_{t+\delta t} = U(\delta t)\,|n\rangle_t$$

$$U(\delta t)\,|n\rangle = |n+1\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

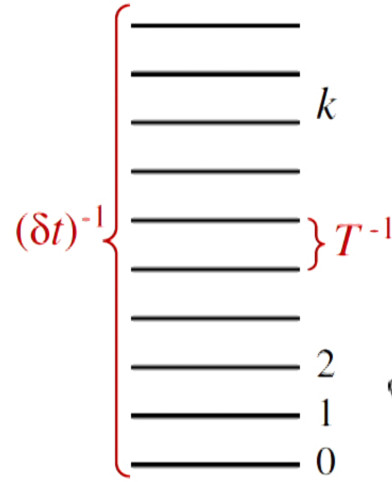
$$U(\delta t) = \begin{pmatrix} 0 & 0 & \cdots & & 1 \\ 1 & 0 & & & 0 \\ 0 & 1 & & & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & 1 & 0 \end{pmatrix} \equiv e^{-iH\,\delta t}$$

To find the matrix H, diagonalise $U(\delta t)$ by using the finite Fourier transform:

$$|k\rangle_H \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{2\pi i k n/N} |n\rangle_{\text{ont}} \ ,$$

$$|n\rangle_{\text{ont}} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{-2\pi i k n/N} |k\rangle_H \ .$$

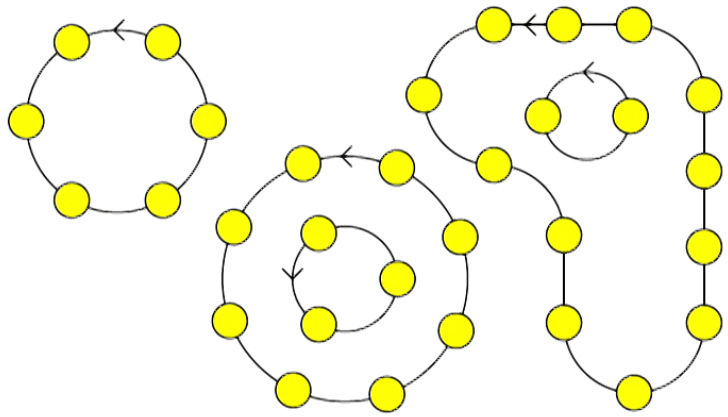
$$n = 0, \cdots N-1 \ ; \quad k = 0, \cdots N-1 \ .$$



$$U(\delta t)|k\rangle_H = e^{-2\pi i k/N}|k\rangle_H = e^{-iH\,\delta t}|k\rangle_H \ ; \qquad H = \frac{2\pi}{N\,\delta t}\,k$$

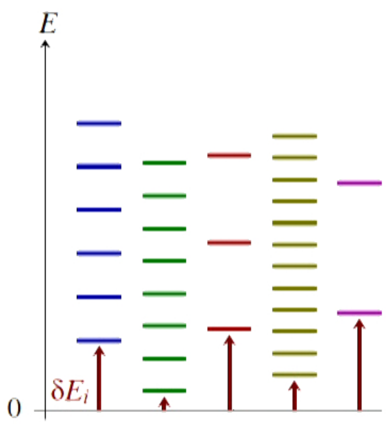
In this notation, the model becomes indistinguishable from real quantum mechanics.

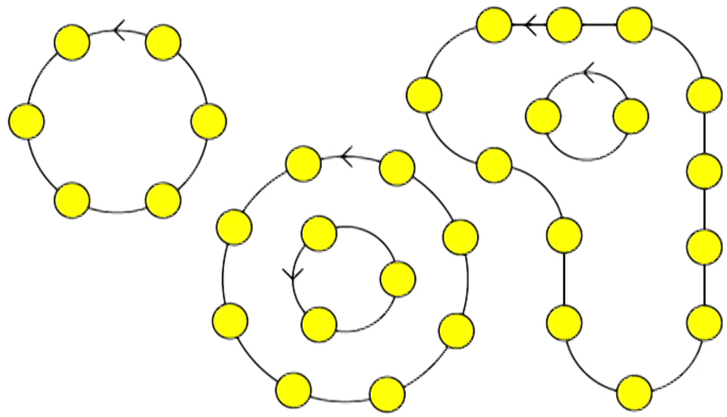
It is the quantum harmonic oscillator, also with period $T = N\,\delta t$.



The most general, finite, deterministic, time reversible models,

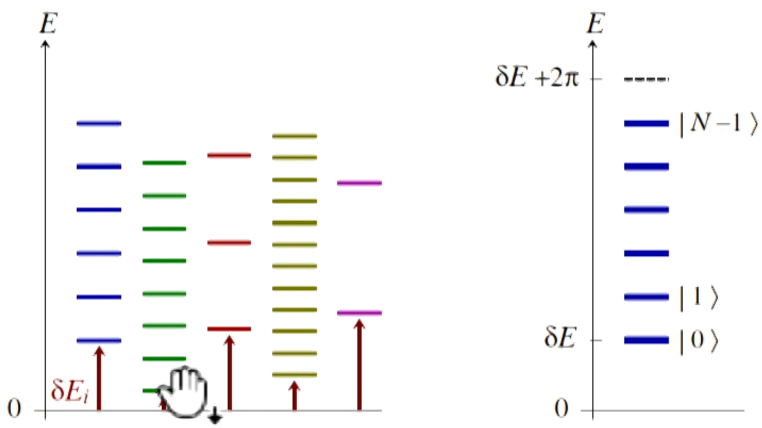
without information loss

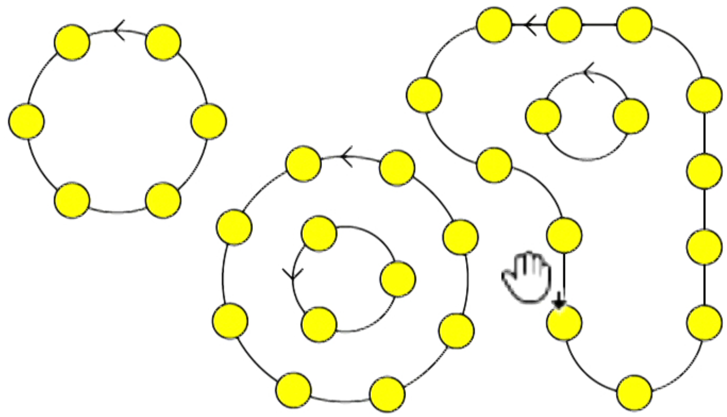




The most general, finite, deterministic, time reversible models,

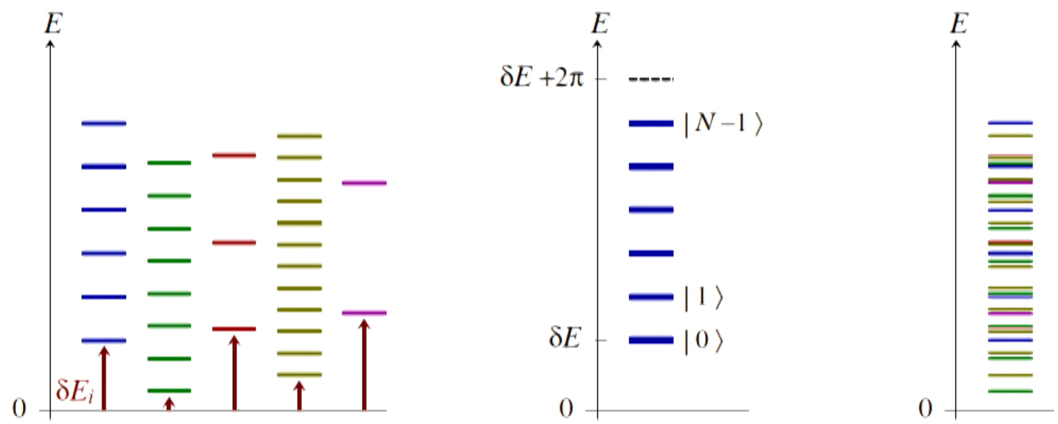
without information loss



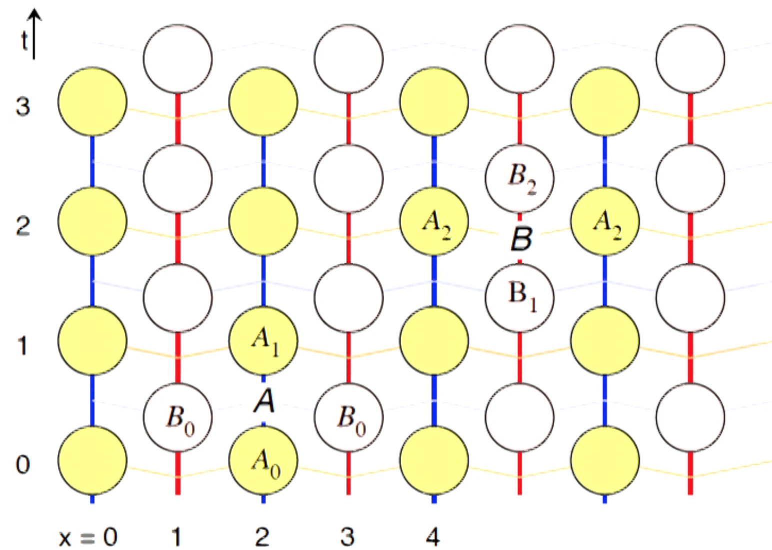


The most general, finite, deterministic, time reversible models,

without information loss



The cellular automaton



Since it is also an element of the class of time-reversible, finite and deterministic systems, the cellular automaton will also feature interlocking sequences of equally-spaced energy levels. These automata, with their data locally defined, are very similar to (quantum) field theories, discretised to fit on a lattice. Studying such lattice models may lead to systems whose energy levels are of this general type.

The Cellular Automaton Theory (CAT) suggests that there is a deterministic system underlying the Standard Model with the gravitational forces included.

The Cellular Automaton Interpretation (CAI) suggests to assume such an ontological system and draw conclusions as to what it is that is really happening when we analyse EPR experiments, observe interference, *etc.*

CAI

The Cellular Automaton Interpretation of Quantum Mechanics

If the Hamiltonian of the world happens to be that of a cellular automaton, then we can identify observables called *Beables*.

beables $\mathcal{B}_i(t)$ are ordinary quantum operators with the special property that, at all t, t', i and j, we have $[\mathcal{B}_i(t), \mathcal{B}_j(t')] = 0$.

If the eigen states of $\mathcal{B}_i(t)$ for a chosen set of values for i and a given time t form a *basis*, this is called an *ontological basis*.

In a given quantum theory, we may adopt strategies to construct an ontic basis, starting from the ordinary quantum states.

The CAI *assumes* that an ontic basis exists.

If the beables can be constructed *more or less locally* from the known states, we have a classical, "hidden variable theory".

The states we normally use to do quantum mechanics are called *template states.* They form a basis of the kind normally used. This is a unitary transformation. Templates are quantum superpositions of ontic states and *vice versa*.

They all obey the same Schrödinger equation!

In a quantum calculation, we may assume the initial state to be an ontic state, $|\psi\rangle_{\mathrm{ont}}$. This state will be some *superposition of template states* $|k\rangle_{\mathrm{template}}$:

$$|\psi\rangle_{\mathrm{ont}} = \sum_k \alpha_k |k\rangle_{\mathrm{template}}$$

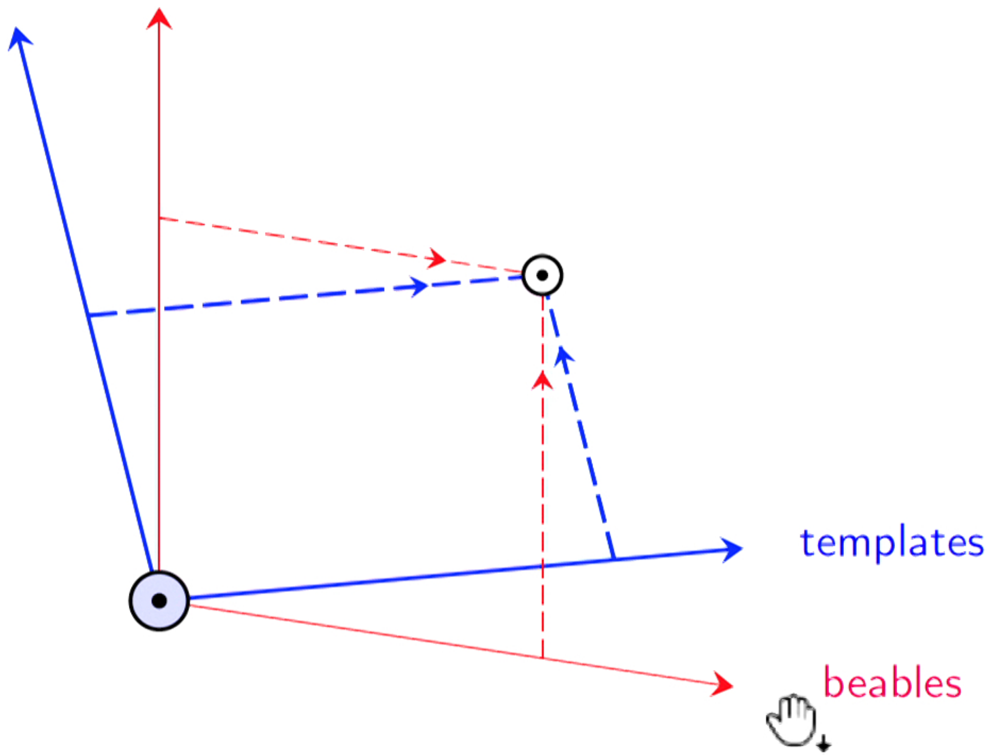
In practice, we use some given template state of our choice. It will be related to the ontic states by

$$|k\rangle_{\mathrm{template}} = \sum_n \lambda_n |n\rangle_{\mathrm{ont}} \ ,$$

where $|\lambda_n|^2$ may be postulated to represent the probabilities that we actually have ontic state $|n\rangle_{\mathrm{ont}}$.

The use of Templates

Hydrogen atom, plane waves of in- or out-particles, *etc.*



Classical states

How are the *classical* states related to the *ontic* states?

Imagine a *planet*. The interior is very different from the local vacuum state. Vacuum state has *vacuum fluctuations*.

Take 1 mm^3 of matter inside the planet. Using statistics, looking at the ontic states, we may establish, with some probability, that the fluctuations are different from vacuum.

Combining the statistics of billions of small regions inside the planet, we can establish *with certainty* that there is a planet, by looking at the ontic state.

But what holds for a planet should then be true for all classical configurations, hence:

All classical states are ontological states! Classical states do not superimpose.

Measurements

Paraphrase a simple "experiment":

First, make the initial state. We take a *template* for that (such as plane in-going waves). Remember:

$$|\,\rangle_{\rm template} = \sum_n \lambda_n |n\rangle_{\rm ont} \ ,$$

Here, $P_n = |\lambda_n|^2$. Absolute squares *defined* as probabilities

Compute the final state, using Schrödinger equation, or Scattering matrix. The final state template is associated to some definite classical state. Compute for all template states $|k\rangle_{\rm template}$:

$${}^{\rm classical}_{\rm template}\langle \ell \mid k\rangle_{\rm template} = \sum_n \lambda_{kn} \,{}^{\rm classical}\langle \ell \mid n\rangle_{\rm ont} \ .$$

<u>Ontic States evolve into Ontic States</u>, and the classical states are ontological $\rightarrow$ $\langle \ell | n\rangle_{\rm ont} = \delta_{kn}$. Therefore:

$P_n = |\lambda_{kn}|^2$ are the Born probabilities.

The Born probabilities coincide with the probabilistic distributions reflecting the unknown details of the initial states.

And that's exactly how probabilities arise in an "ordinary" classical deterministic theory.

Ontological states form an orthonormal set: superpositions of ontological states are never ontological states themselves.
The universe is in an ontological state.

Collapse of the Wave function

When we use a template, we find the final state to be

$$\alpha_1|n_1\rangle + \alpha_2|n_2\rangle + \cdots$$

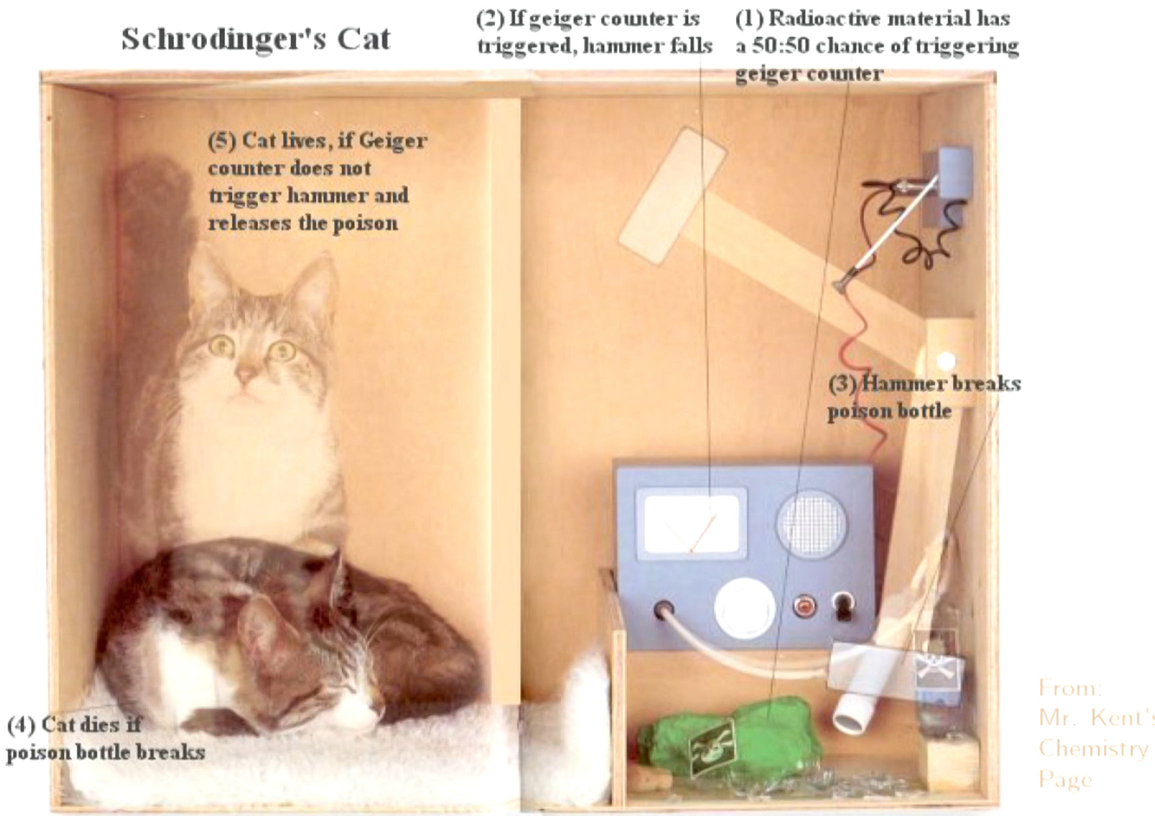
According to "Copenhagen", $P_1 = |\alpha_1|^2,\ P_2 = |\alpha_2|^2,\ \cdots$

Why is the final state only one of these states? Why are P_i probabilities?

The CAI gives the answer: $|n_1\rangle$ is a possible ontic final state,

and so is $n_2\rangle$, but $\alpha_1|n_1\rangle + \alpha_2|n_2\rangle$ is *not* an ontic state. That's why it never occurs in the real world.

Schrödinger's cat is ontic when it is dead, also when it is alive, but *not* when it is in a superposition.



The CAI has the advantage that the reasoning simplifies a lot:

A (set of) state(s) is quantum mechanical if it is phrased in terms of templates, which all are superpositions of ontic states. As of today, we have been unable to identify the basis of ontic states of our universe.

A (set of) state(s) is classical if we have found how to express it in terms of ontic states. Superpositions of these never arise.

Example: the *planetary system* is classical.

The **Earth - Mars exchange operator**, X_{EM} puts Mars where Earth is and Earth where Mars is, while also exchanging the velocity vectors. Leave the Moon and other planets where they are.

$$X_{EM}^2 = 1 \quad \to \quad X_{EM} = \pm 1.$$

X_{EM} is hermitian, and therefore, according to the Copenhagen doctrine, it should be observable.

But we cannot measure X_{EM}. It is *counter-factual*. This is because we know that the coordinates of Earth and Mars are ontological. How does X_{EM} evolve?

Counter-factual reality : you can't measure Earth's and Mars' positions *and* X_{EM} at the same time.

Some interesting models that allow for the construction of an ontological basis:

1. Massless free particles in 1 space + 1 time dimension. *This includes string theories, which turn their target space-time into a lattice!*
2. Massive, non-relativistic free particles in 2+1 dimensions

Both these models allow for the identification of equally spaced energy levels, which can be subjected to discrete Fourier transformations.

3. Massless, non-interacting fermions such as (idealised) neutrinos

A massless particle in 1 space-, 1 time-dimension either moves to the right or to the left, with a fixed velocity $v = \pm c$.
In this case, the space coordinate $x(t)$ itself is an ontic variable, or beable.

Because of the fixed velocity,

$$[x(t_1), x(t_2)] = 0 \ .$$ Also note:

$$H = |p| \ , \qquad v = \partial H / \partial p = \pm 1 \ .$$

In a box with length L, momentum is quantised: $p = 2\pi n / L$.

Therefore, energy levels are equally spaced.

A non-relativistic particle in 2 space, 1 time dimension
has its energy levels almost equally spaced:

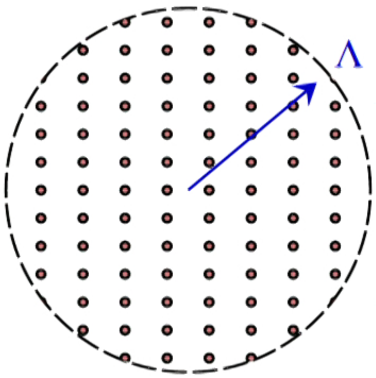
In a rectangular box, sides L_1 and L_2, both p_x and p_y are equally spaced.

In a region $(\delta p_x,\ \delta p_y)$ the number of momentum states is

$$\delta N = L_1 L_2\, \delta p_x\, \delta p_y / 4\pi^2\ .$$

Therefore, the total number of states in a circle of radius Λ in momentum space is $N = \pi\Lambda^2 L_1 L_2 / 4\pi^2$,
while the total energy is bounded by

$$E_{\rm max} = \Lambda^2/2m.$$



Thus the total number of states grows linearly with energy E.
The energy eigen values are on average equally spaced; in practice there is a subtle, unimportant fluctuation, depending on the shape of the box.

But we can also design a perfect box:

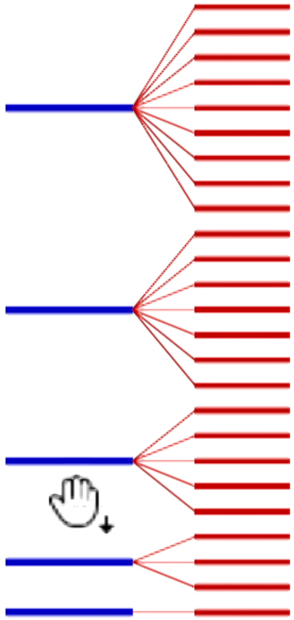
Take space to be the surface of a 2-sphere, and add a weak magnetic field such that, apart from constants,

$$H \;=\; L^2 + L_3 \;=\; \ell(\ell+1) + m \;=\; n \; .$$

Identify the states $|n\rangle \;=\; |\ell,\, m\rangle$.

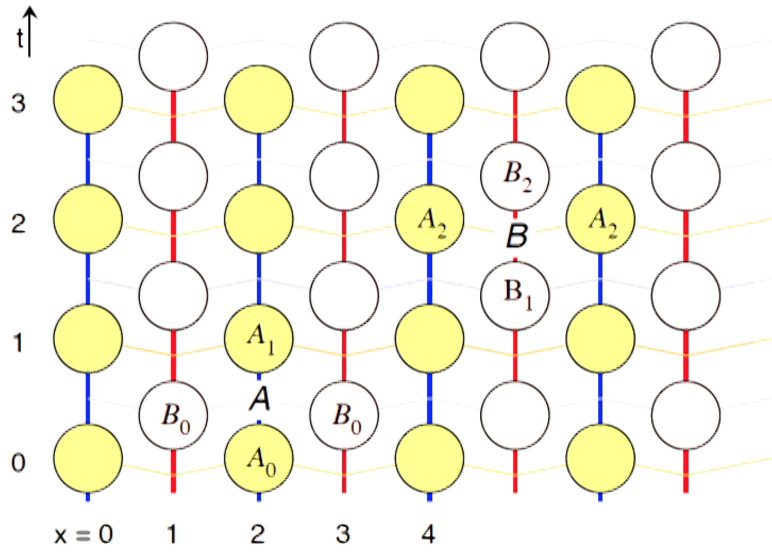
The discrete Fourier transforms of these states form a continuous, periodic variable $\alpha \in [0, 2\pi]$,

$$|\alpha\rangle = \frac{1}{\sqrt{2\pi}} \sum_{\ell,m} e^{i(\ell(\ell+1)+m)\,\alpha}\, |\ell,\, m\rangle \; .$$

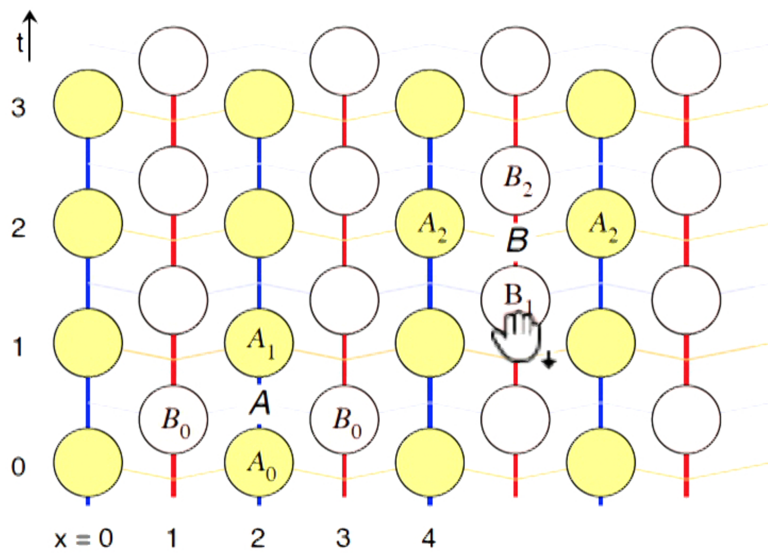


α is a beable.

The cellular automaton



The cellular automaton



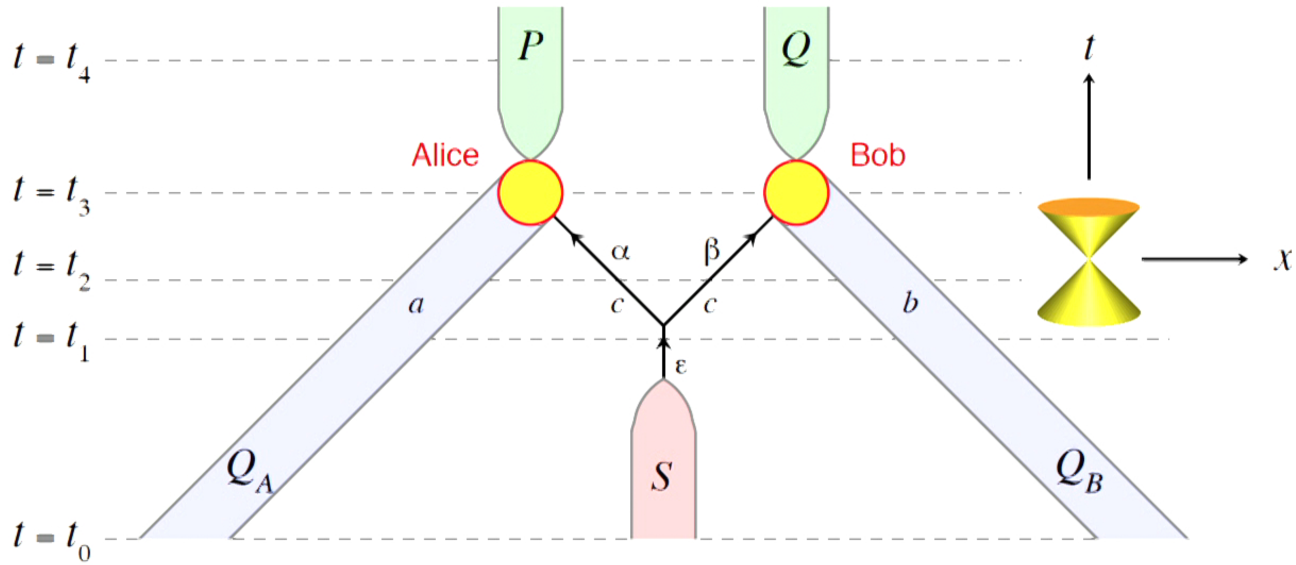
$$U = e^{-iH} \;=\; e^{-iA}\, e^{-iB} \;; \quad A = \sum_x A(x) \;, \quad B = \sum_x B(x)$$

where $[A(x),\, A(x')] = 0$, $[B(x),\, B(x')] = 0$; $[A(x),\, B(x')] \neq 0$ only if x and x' are neighbours.

Baker Campbell Hausdorff :

$$H = A + B - \tfrac{1}{2} i [A, B] - \tfrac{1}{12}([A, [A, B]] + [[A, B], B]) + \cdots .$$

Why it is all wrong: Bell's theorem



In the Bell experiment, at $t = t_0$, one must demand that those degrees of freedom that later force Alice and Bob to make their decisions, and the source that emits two entangled particles, need to have *3 - body correlations of the form*

$$W(a,\ b,\ c) \propto |\sin(2(a+b)-4c)| \quad \text{(the Mousedropping Function)}$$

But Alice and Bob have *free will*. How can their actions be correlated with what the decaying atom did, at time $t = t_2 \ll t_3$?

Answer: they don't have free will: *superdeterminism*.

What happened according to the CAI ?

We have the *ontology conservation law* :

Ontic states evolve into ontic states.

$$ {}^{\text{classical}}_{\text{template}}\langle \ell | k \rangle_{\text{template}} = \sum_n \lambda_{\ell n} \, {}^{\text{classical}}\langle \ell | n \rangle_{\text{ont}} $$

If Alice makes an infinitesimal modification of her settings, the *classical* state will change $\rightarrow$ all ontic states will change:

$$ {}^{\text{classical}}_{\text{template}}\langle \ell + \delta\ell | k \rangle_{\text{template}} = \sum_{\ell} \lambda_{\ell' m} \, {}^{\text{classical}}\langle \ell + \delta\ell | m \rangle_{\text{ont}} $$

All Alice's ontological states $|m\rangle_{\text{ont}}$ are now different from all $|n\rangle_{\text{ont}}$ that she had before.

All Alice's ontological states $|m\rangle_{\rm ont}$ are now different from all $|n\rangle_{\rm ont}$ that she had before.

So, both her past light-cone and her future light-cone are now entirely different. These light-cones do overlap with Bob's. Does this affect Bob's world, and that of the decaying atom S?

If all ontological states had equal probabilities, the answer would be no. But one can easily imagine that some ontic states are more probable than others.
In that case, the *counterfactual* experiment $\ell \rightarrow \ell + \delta\ell$ would lead to drastically different probabilities. So it is easy to generate non-vanishing correlation functions that disobey Bell.
All ontic states in the universe are associated with strong space-like correlations. These correlations obey the ontology conservation law.
The photons c then automatically align in such a way that, after detection by Alice and Bob, they are still in an ontic state.

The law of ontology conservation

Is this *conspiracy* ? Not if the ontological nature of a physical state is *conserved in time*. If, at late times, a photon is observed to be in a given polarisation state, it has been in *exactly the same state* from the very moment it was emitted by the source (omniscient photons).

These are future-past correlations. The conspiracy argument now demands that the "ontological basis" be *unobservable!*

Non-observable hidden variables?

Shut up and calculate!