Title: Anomalous weak values and contextuality: robustness, tightness, and imaginary parts

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Series: Quantum Foundations

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Abstract: Weak values are quantities accessed through quantum experiments involving weak measurements and post-selection. It has been shown that â€˜anomalousâ€™ weak values (those lying beyond the eigenvalue range of the corresponding operator) defy classical explanation in the sense of requiring contextuality [M. F. Pusey, Phys. Rev. Lett. 113, 200401, arXiv:1409.1535]. We elaborate on and extend that result in several directions. Firstly, the original theorem requires certain perfect correlations that can never be realised in any actual experiment. Hence, we provide new theorems that allow for a noise-robust experimental verification of contextuality from anomalous weak values. Secondly, the original theorem connects the anomaly to contextuality only in the presence of a whole set of extra operational constraints. Here we clarify the debate surrounding anomalous weak values by showing that these conditions are tight -- if any one of them is dropped, the anomaly can be reproduced classically. Thirdly, whereas the original result required the real part of the weak value to be anomalous, we also give a version for any weak value with nonzero imaginary part. Finally, we show that similar results hold if the weak measurement is performed through qubit pointers, rather than the traditional continuous system. All in all, we provide inequalities for witnessing nonclassicality using experimentally realistic measurements of any anomalous weak value, and clarify what ingredients of the quantum experiment must be missing in any classical model that can reproduce the anomaly.

Anomalous weak values and contextuality: robustness, tightness, and imaginary parts

Ravi Kunjwal,
Perimeter Institute Quantum Foundations Seminar

(joint work with M. F. Pusey and M. Lostaglio, arXiv:1812.06940)

September 24, 2019

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Outline

- Motivation
- Weak measurements: what are they?
- Anomalous Weak Value
- Pusey's original argument: its limitations
- Overcoming the limitations
- Some results
- Takeaway

Motivation

When is the observation of an anomalous weak value a signature of nonclassicality?

A debate

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

This paper will describe an experiment which measures a spin component of a spin-$\frac{1}{2}$ particle and yields a result which is far from the range of "allowed" values. We shall start with a brief description of the standard

a_i, the final probability distribution will be again close to a Gaussian with the spread $\Delta\pi$. The center of the Gaussian will be at the mean value of A: $\langle A\rangle = \sum_i |\alpha_i|^2 a_i$. One measurement like this will give no information be-

A debate

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PRL **113,** 120404 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

How the Result of a Single Coin Toss Can Turn Out to be 100 Heads

Christopher Ferrie and Joshua Combes

Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA

(Received 16 March 2014; revised manuscript received 18 July 2014; published 18 September 2014)

We show that the phenomenon of anomalous weak values is not limited to quantum theory. In particular, we show that the same features occur in a simple model of a coin subject to a form of classical backaction with pre- and postselection. This provides evidence that weak values are not inherently quantum but rather a purely statistical feature of pre- and postselection with disturbance.

DOI: 10.1103/PhysRevLett.113.120404 PACS numbers: 03.65.Ta, 02.50.Cw, 03.67.-a

In many quantum mechanical experiments, we observe a dissonance between what actually happens and what ought to happen given naïve classical intuition. For example, we would say that a particle cannot pass through a potential barrier—it is *not allowed* classically. In a quantum mechanical experiment, the "particle" can "tunnel" through a potential barrier—and a paradox is born. Most researchers spent the 20th century ignoring such paradoxes (that is, metrology [10] (but compare to Refs. [11–16]). One research program in the weak value community is to examine a paradoxical quantum effect or experiment and then calculate the weak value for that situation. Often, the calculated weak value is anomalous. From this, we are supposed to conclude that the paradox is resolved (see, for example, [17] for a recent review). So it would further seem, then, that anomalous weak values, if not *the* source

The "resolution"

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PRL 113, 200401 (2014) PHYSICAL REVIEW LETTERS week ending 14 NOVEMBER 2014

Anomalous Weak Values Are Proofs of Contextuality

Matthew F. Pusey*
Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada

(Received 5 September 2014; published 12 November 2014)

The average result of a weak measurement of some observable A can, under postselection of the measured quantum system, exceed the largest eigenvalue of A. The nature of weak measurements, as well as the presence of postselection and hence possible contribution of measurement disturbance, has led to a long-running debate about whether or not this is surprising. Here, it is shown that such "anomalous weak values" are nonclassical in a precise sense: a sufficiently weak measurement of one constitutes a proof of contextuality. This clarifies, for example, which features must be present (and in an experiment, verified) to demonstrate an effect with no satisfying classical explanation.

DOI: 10.1103/PhysRevLett.113.200401 PACS numbers: 03.65.Ta, 03.67.-a

In 1988 Aharonov, Albert and Vaidman explained "how the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100." [1] Defining the weak value of an observable A for a quantum system prepared in state $|\psi\rangle$ and postselected on giving the first outcome of $\{|\phi\rangle\langle\phi|, I - |\phi\rangle\langle\phi|\}$,

have been given that reproduce various aspects of the phenomena [16–18].

The question can be made precise by asking if anomalous weak values constitute proofs of the incompatibility of quantum theory with noncontextual ontological models [19], or equivalently [20] if anomalous weak values require negativity in all quasiprobability representations. This was

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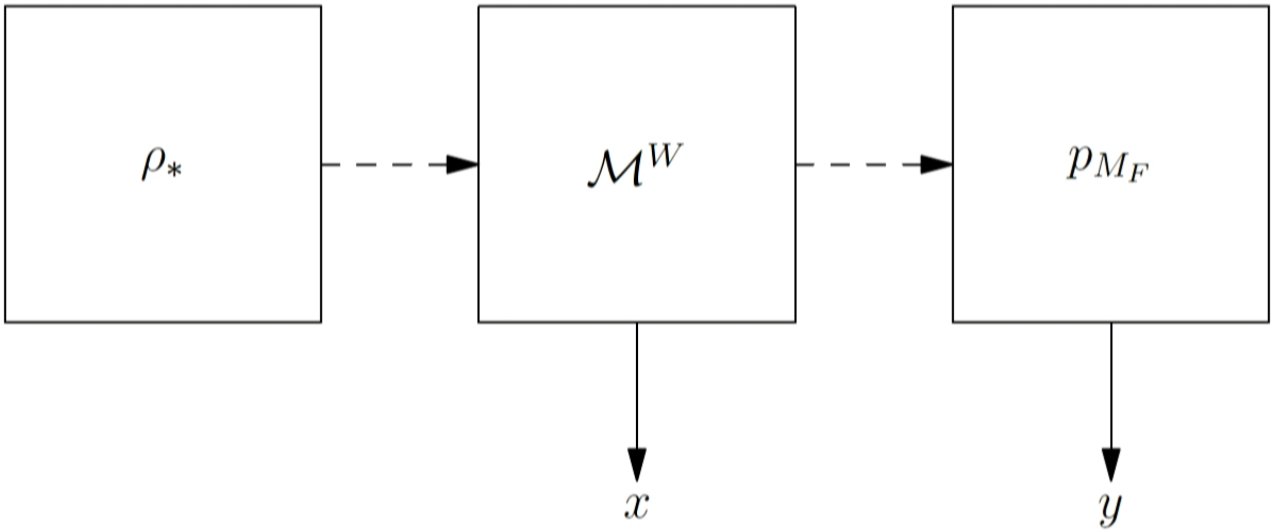
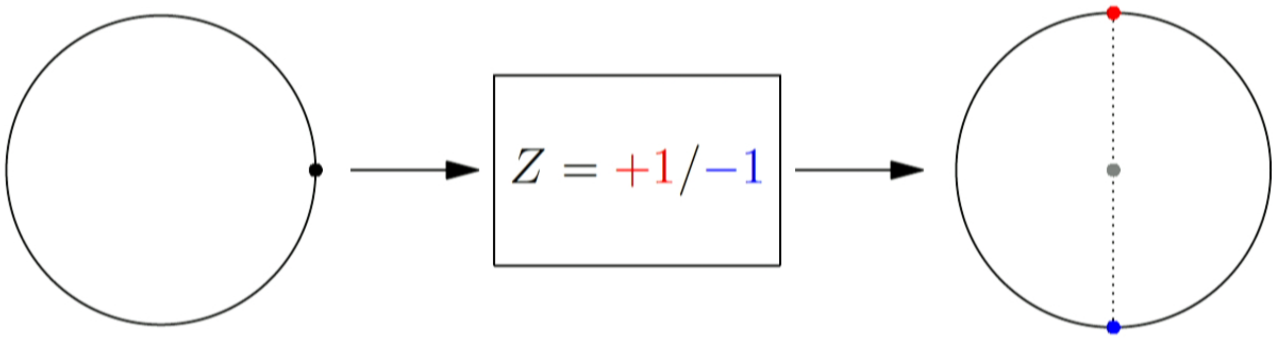
Weak value experiment

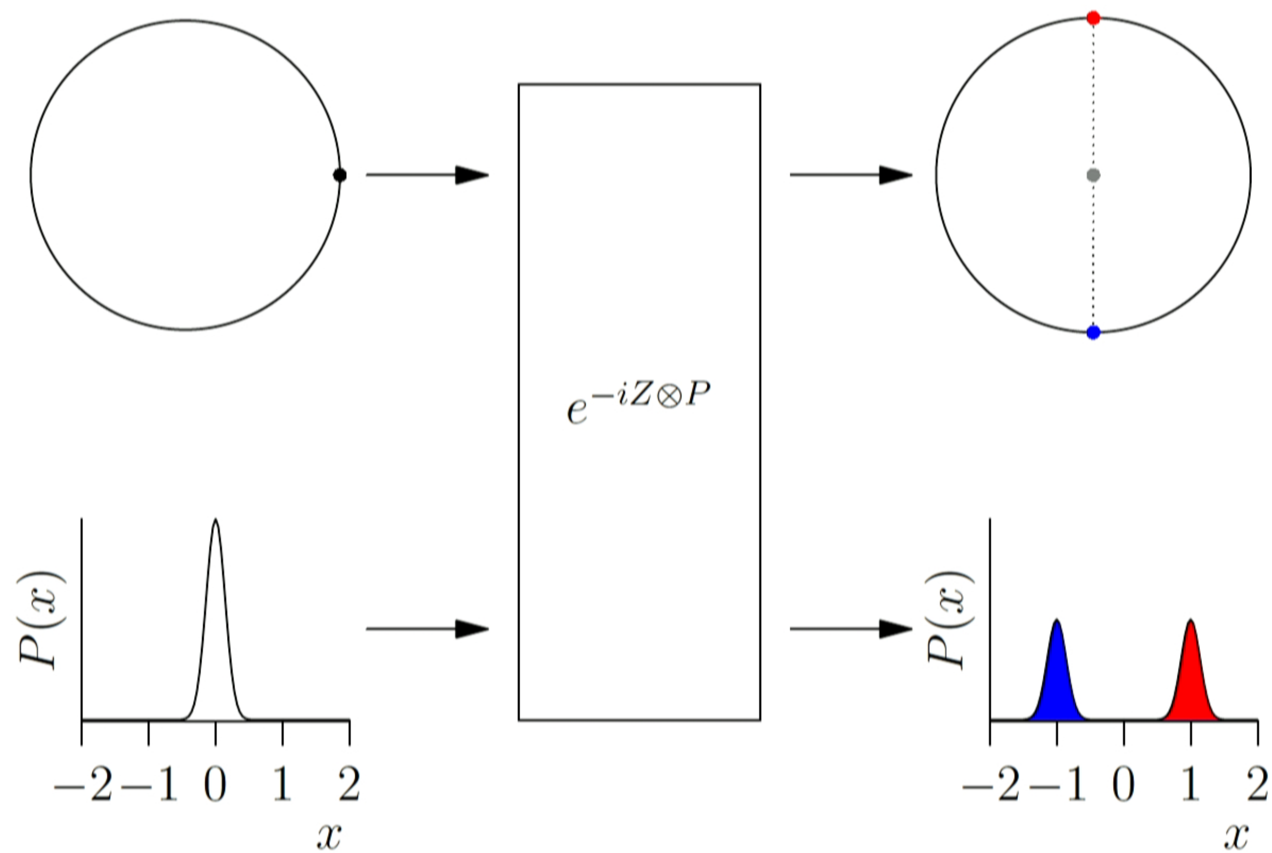


Strong measurements

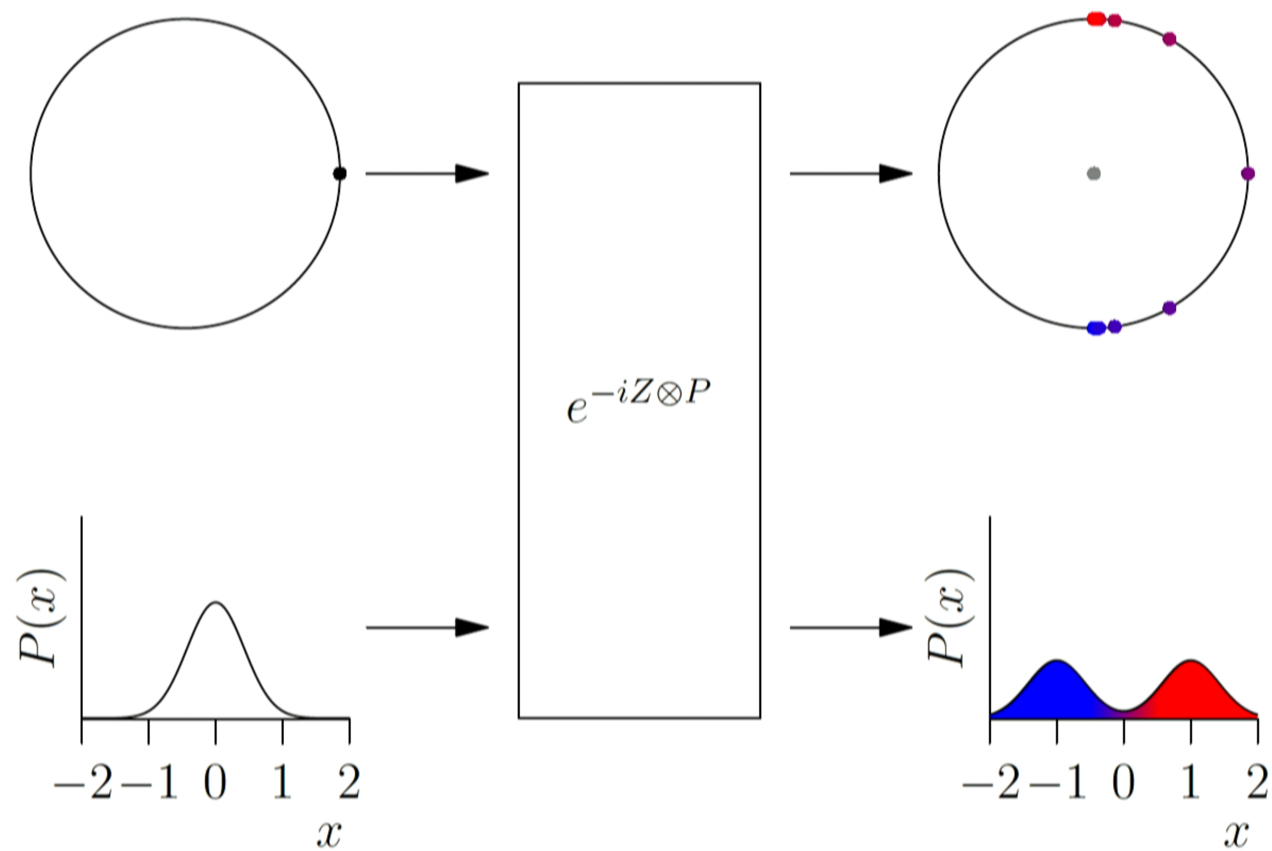


“Pointer”

Strong measurements



Fairly strong measurements



Weak Value

The weak value of an observable O under pre-selection ρ and post-selection $\Pi_\phi = |\phi\rangle\langle\phi|$ is defined as

$$ {}_\phi\langle O\rangle_\rho \equiv \frac{\mathrm{Tr}(\Pi_\phi O \rho)}{\mathrm{Tr}(\Pi_\phi \rho)}. \tag{1} $$

Anomalous Weak Value

Anomalous Weak Value (AWV)

Unlike the usual expectation value of O, its weak value can have both real and imaginary parts:

$$ {}_{\phi}\langle O\rangle_{\rho} = \sum_j o_j \, {}_{\phi}\langle \mathcal{E}_j\rangle_{\rho} \tag{2} $$

$$ = \sum_j o_j \mathrm{Re}_{\phi}\langle \mathcal{E}_j\rangle_{\rho} + i \sum_j o_j \mathrm{Im}_{\phi}\langle \mathcal{E}_j\rangle_{\rho}. \tag{3} $$

Anomaly: when ${}_{\phi}\langle O\rangle_{\rho}$ is outside the range of possible expectation values of O, i.e., its real part is outside this range or it has a non-zero imaginary part.

Anomalous Weak Value

Indeed, just the numerator of the above expression is enough to capture the anomaly. We define

$$\langle \Pi_\phi \mathcal{E} \rangle_\rho \equiv \mathrm{Tr}(\Pi_\phi \mathcal{E} \rho), \tag{7}$$

so that the anomaly corresponds to

$$\mathrm{Re}\langle \Pi_\phi \mathcal{E} \rangle_\rho < 0, \text{ or} \tag{8}$$

$$\mathrm{Im}\langle \Pi_\phi \mathcal{E} \rangle_\rho < 0. \tag{9}$$

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Operational theory and its Ontological model

- Operational theory:

$$p(k|M,T,P) \in [0,1], \quad \forall P \in \mathcal{P}, T \in \mathcal{T}, M \in \mathcal{M}, k \in \mathcal{K}_M. \tag{10}$$

- Ontological model:

$$p(k|M,T,P) = \int_{\lambda',\lambda} p_M(k|\lambda') p_T(\lambda'|\lambda) p_P(\lambda). \tag{11}$$

What is noncontextuality?

Identity of indiscernables.

- Operational equivalences:

$$P \simeq P' : p(k|M, T, P) = p(k|M, T, P'), \quad \forall k, M, T, \tag{12}$$

$$T \simeq T' : p(k|M, T, P) = p(k|M, T', P), \quad \forall k, M, P, \tag{13}$$

$$[k|M] \simeq [k'|M'] : p(k|M, T, P) = p(k'|M', T, P), \quad \forall T, P. \tag{14}$$

What is noncontextuality?

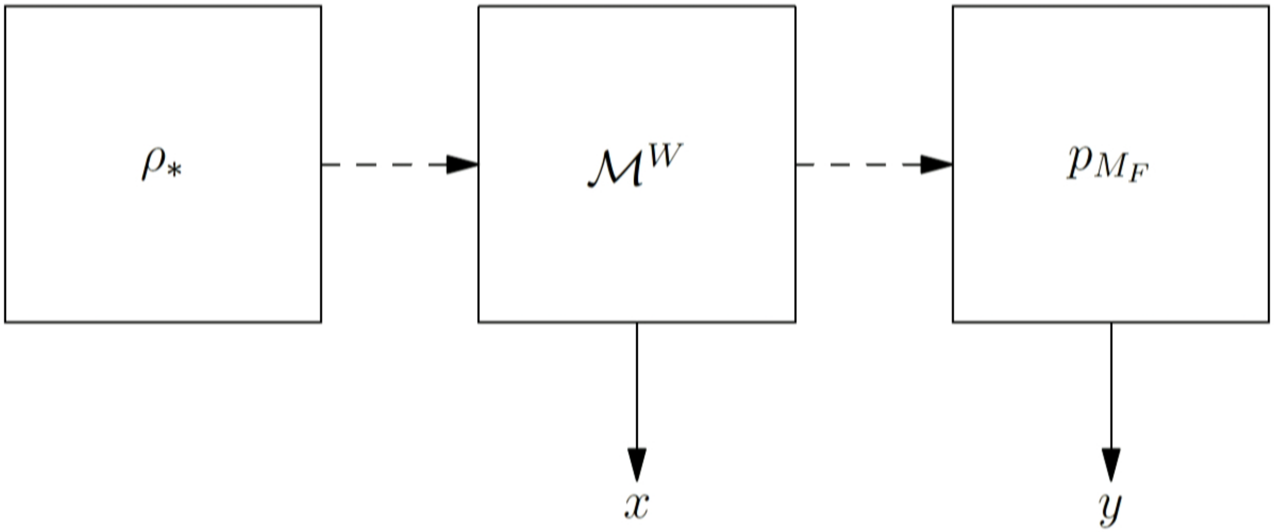
- Noncontextual ontological model:

$$P \simeq P' \Rightarrow p_P(\lambda) = p_{P'}(\lambda), \quad \forall \lambda \in \Lambda, \tag{15}$$

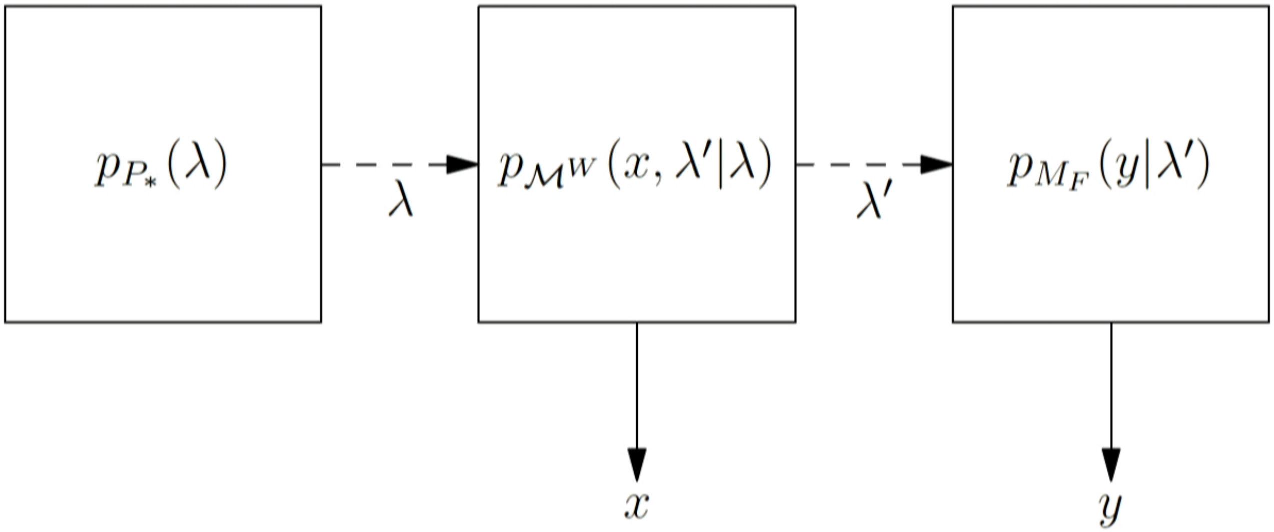
$$T \simeq T' \Rightarrow p_T(\lambda'|\lambda) = p_{T'}(\lambda'|\lambda), \quad \forall \lambda', \lambda \in \Lambda, \tag{16}$$

$$[k|M] \simeq [k'|M'] \Rightarrow p_M(k|\lambda) = p_{M'}(k'|\lambda), \quad \forall \lambda \in \Lambda. \tag{17}$$

Weak value experiment



Ontological description of the weak value experiment



Operational quantities of interest

- Probability of weak measurement outcome $[x|\mathcal{M}^W]$ such that $x < 0$ and successful post-selection $[y = 1|M_F]$, given pre-selection P_*:

$$p_- \equiv \int_{-\infty}^{0} p(x, y = 1|P_*, \mathcal{M}^W, M_F)dx \tag{18}$$

$$= \int_{-\infty}^{0} p(x, y = 1|P_*, M_F \circ M_W)dx. \tag{19}$$

- Probability of successful post-selection $[y = 1|M_F]$ immediately following the pre-selection P_*:

$$p_F \equiv p(y = 1|P_*, M_F). \tag{20}$$

What is the connection between AWV and contextuality?

"Anomalous weak values are proofs of contextuality."[1]

That is, the noncontextuality inequality $p_- \leq \frac{p_F}{2} + p_d$ is violated for a sufficiently large anomaly since $p_- = \frac{p_F}{2} - \frac{\mathrm{Re}(\langle \Pi_\phi \mathcal{E} \rangle_{\rho *}}{\sqrt{\pi s}} + o(\frac{1}{s})$.

[1] M. Pusey, Phys. Rev. Lett. 113, 200401 (2014).

Subject to the following limitations:

1. The postselection is projective.
2. Not clear if the operational equivalences used are really necessary to show the connection between AWV and contextuality.
3. Doesn't show the connection between AWV and contextuality due to imaginary part of the weak value.
4. Requires an infinite number of operational equivalences to be verified: continuum of outcomes.
5. Uniqueness and tightness of the noncontextuality inequality unclear.

AWV vis-à-vis contextuality: new and improved!

1. Projective postselection not required: noncontextuality inequality robust to noise in the postselection. Two different proofs.
2. Necessity of the operational equivalences used: relaxing any one of them renders a noncontextual ontological model possible.
3. Contextuality from imaginary part of the weak value.
4. Extension to a discrete pointer.
5. Algorithmic approach to examine tightness and uniqueness of the noncontextuality inequality.

Some results

An inequality from measurement and transformation noncontextuality

Suppose we have a noncontextual ontological model and that:

1. There exists a 2-outcome measurement $M_{\mathcal{E}}$ and a probability distribution $q(x)$ with median $x = 0$ such that, for all $x \in \mathbb{R}$,

$$[x|M_W] \simeq q(x-1)[y=1|M_{\mathcal{E}}] + q(x)[y=0|M_{\mathcal{E}}]. \qquad (21)$$

2. If $\mathcal{M} := \int \mathcal{M}_x^W dx$, there exists $p_d \in [0,1]$ such that

$$\mathcal{M} \simeq (1-p_d)\mathcal{I} + p_d \mathcal{M}^D, \qquad (22)$$

   where $\mathcal{I}$ denotes the identity transformation and $\mathcal{M}^D$ some other transformation.

Then,

$$p_- \leq p_-^{NC} \equiv p_F \frac{1}{2} + (1-p_F)p_d. \qquad (23)$$

An inequality from preparation and measurement noncontextuality

We introduce an ensemble of preparations

$$\{(q_0, [b = 0|S]), (q_1, [b = 1|S])\},$$

ideally picked in a way that its correlation with the post-selection outcomes, given by

$$C_S \equiv p(b = 0, y = 0|S, M_F) + p(b = 1, y = 1|S, M_F) \qquad (24)$$

is maximized. This provides a proxy for the quality of the measurements, e.g., any projective post-selection admits an ensemble of preparations for which $C_S = 1$.

An inequality from preparation and measurement noncontextuality

Suppose we have a noncontextual ontological model and:

1. $\exists$ a 2-outcome measurement $M_{\mathcal{E}}$ and a probability distribution $q(x)$ with median $x = 0$ such that, for all $x \in \mathbb{R}$,

$$[x|M_W] \simeq q(x-1)[y=1|M_{\mathcal{E}}] + q(x)[y=0|M_{\mathcal{E}}]. \qquad (25)$$

2. Given the sequential measurement $[x, y|M_F \circ M_W]$, define $[y|\tilde{M}_F] \equiv \int dx[x, y|M_F \circ M_W]$. Then there exists $p_d \in [0, 1]$ such that

$$[y|\tilde{M}_F] \simeq (1 - p_d)[y|M_F] + p_d[y|M_D], \qquad (26)$$

for some 2-outcome measurement M_D.

3. There exists an ensemble

$$\{(q_*, P_*), (q_\perp, P_\perp)\}$$

such that

$$q_0[b=0|S] + q_1[b=1|S] \simeq q_* P_* + q_\perp P_\perp. \qquad (27)$$

An inequality from preparation and measurement noncontextuality

Then,

$$p_{-} \leq p_F \frac{1}{2} + (1 - p_F) p_d + \frac{1 - C_S}{2q_*}. \quad (28)$$

Tightness of the inequality[2]

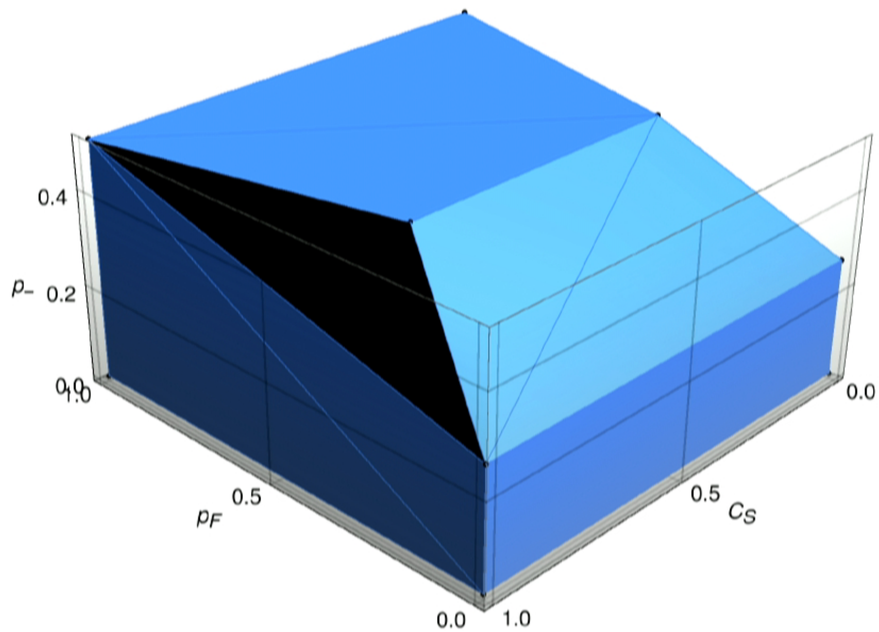


Figure: The noncontextuality tradeoff between p_-, p_F and C_S for $p_d = 1/4, \tilde{p} = 1/2, q_0 = q_* = 1/2$. The facet corresponding to Eq. (28) is shown in black.

[2]Schmid et al., Phys. Rev. A 97, 062103 (2018).

Necessity of operational equivalences

If any of the operational equivalences in our Theorem(s) fails, then the anomaly can be reproduced by a noncontextual ontological model.

In summary: what makes anomalous weak values contextual?

Two facts about weak measurements:

1. Ignoring the post-measurement state, they are like projective measurements with unbiased noise.
2. Ignoring the outcome, they approximate an identity channel much better than the size of the anomaly.

Takeaway

- Sufficiently anomalous weak values signal contextuality as long as the operational equivalences assumed in our theorems are satisfied.
- The postselection need not be projective, one can work with a discrete pointer, the obtained noncontextuality inequality is tight, and imaginary part of the weak value can also witness contextuality.

Thank you!