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Pursuing simplicity with asymptotic safety

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SISSA, Trieste

Simplicity III,
PI, September 11, 2019

Asymptotic safety

Wilsonian RG in "theory space": couplings flow as functions of cutoff k

For most RG trajectories some couplings blow up for $k \to \infty$.

Sometimes they even blow up at finite k (Landau poles).

Generically leads to divergences in physical observables.

"UV safe" or *"renormalizable"* RG trajectories
= RG trajectories that reach a FP in the UV
= UV complete theories

$$\text{trajectory} \equiv \text{theory}$$

Use as criterion to select theories

Predictivity

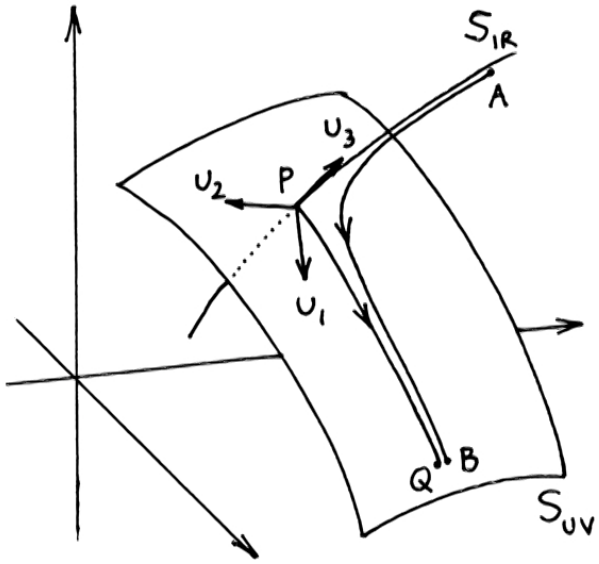
Perturbations around FP can be

- attracted to FP in UV = repelled by FP in IR =**relevant**
- repelled by FP in UV = attracted to in IR =**irrelevant**

Every irrelevant direction leads to a prediction (in either direction of the flow)

Generally expect only finitely many relevant directions.

General picture



Examples of AS

QCD:

- Gaußian Fixed Point at $\tilde{g}_{i*} = 0$.
- relevant couplings=renormalizable couplings

Non-AF examples?

General NLSM

$$\frac{1}{2g^2}\int d^d x\, \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta h_{\alpha\beta}(\varphi)$$

$g \approx \text{mass}^{\frac{2-d}{2}}$ nonrenormalizable in $d > 2$

Ricci flow:

$$\frac{d}{dt}\left(\frac{1}{g^2}h_{\alpha\beta}\right) = 2c_d k^{d-2} R_{\alpha\beta}$$

$c_d = \frac{1}{(4\pi)^{d/2}\Gamma(d/2+1)}$

$O(N)$ NLSM

In $d = 2$:

$$\beta_{g^2} = -2c_2(N-2)\tilde{g}^4$$

In $d = 2+\epsilon$: $\tilde{g}^2 = k^{d-2}g^2$ and

$$\beta_{g^2} = \epsilon\tilde{g}^2 - 2c_d(N-2)\tilde{g}^4$$

non gaussian FP at

$$\tilde{g}_*^2 = \frac{d-2}{2}\frac{1}{c_d(N-2)} .$$

$\left.\frac{d\beta}{d\tilde{g}}\right|_* = 2-d.$

Confirmed by Monte-Carlo calculations in $d = 3$.

Gauge theories in $d = 4$: one loop

$$L = -\frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

The beta function of $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$ is

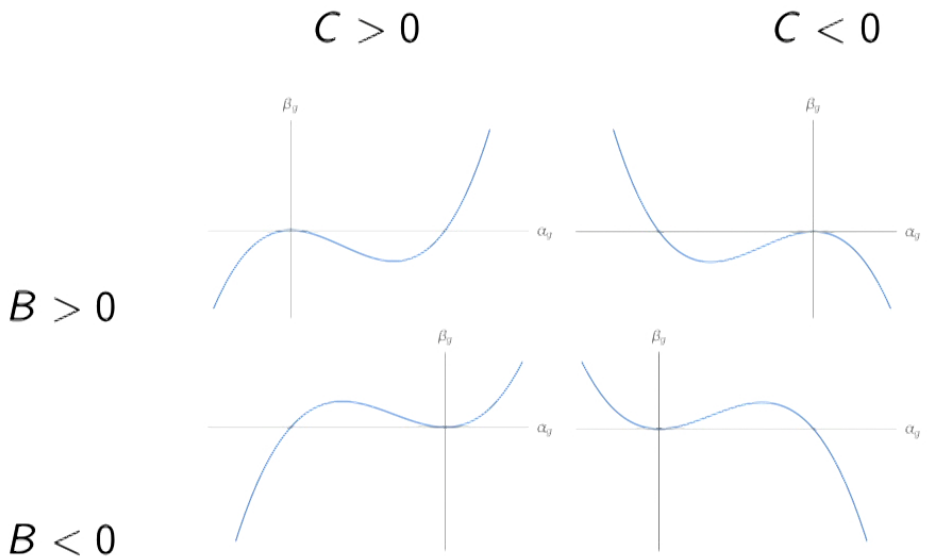
$$\beta_g = -B\alpha_g^2$$

$$B = -\frac{4}{3}\epsilon \; ; \qquad \epsilon = \frac{N_F}{N_c} - \frac{11}{2}$$

$$N_F < \tfrac{11}{2} N_c \Longrightarrow \epsilon < 0 \Longrightarrow B > 0 \Longrightarrow \mathrm{AF}$$

Gauge theories in $d = 4$: two loops

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$



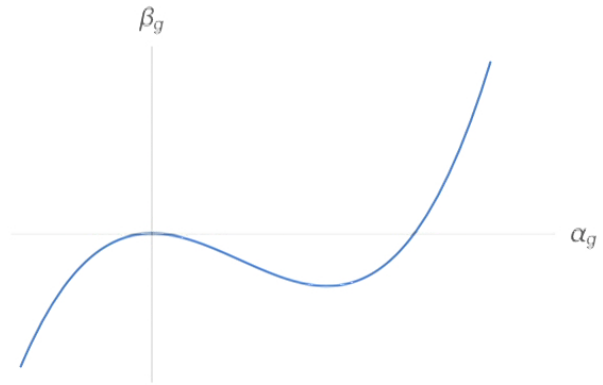
Gauge theories in $d = 4$: two loops

[W.E. Caswell, Phys. Rev. Lett. 33 (1974) 244]

found $C = 25$

Fixed point at $\alpha_{g*} = B/C$ with $\beta'(\alpha_{g*}) = B^2/C$

AF+IR FP if $B > 0$.



[T. Banks and A. Zaks, Nucl. Phys. B 196 (1982)]

From AF to AS: Yukawa couplings

$$L = \mathrm{tr}(\partial^\mu H)^\dagger(\partial_\mu H) + y\,\mathrm{tr}(\bar\psi_L H\psi_R + \bar\psi_R H\psi_L)$$

Beta functions for α_g and $\alpha_y = \frac{y^2 N_c}{(4\pi)^2}$

$$\begin{aligned}\beta_g &= \alpha_g^2\left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{3} + \epsilon\right)\alpha_y\right]\\ \beta_y &= \alpha_y\left[(13 + 2\epsilon)\alpha_y - 6\alpha_g\right]\end{aligned}$$

Fixed points

$$(\alpha_{g*}, \alpha_{y*}) = \left(-\frac{4\epsilon}{75+26\epsilon}, 0\right)$$

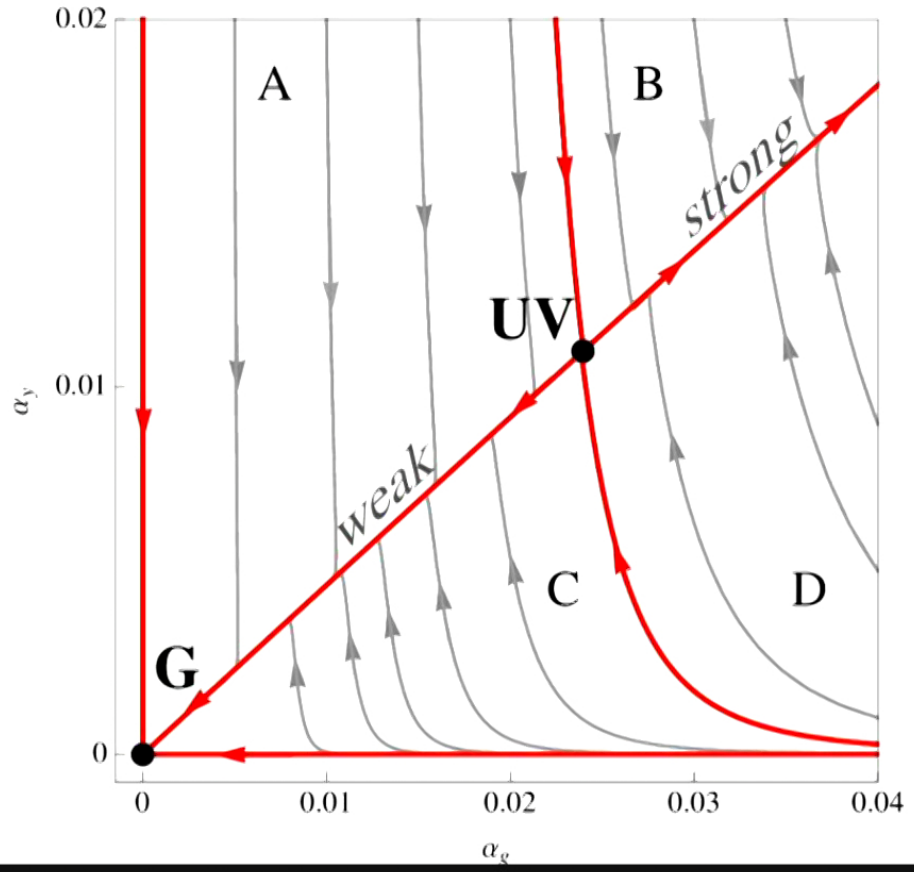
for $\epsilon < 0$, Banks-Zaks

$$(\alpha_{g*}, \alpha_{y*}) = \left(\frac{2\left(13\epsilon+2\epsilon^2\right)}{57-46\epsilon-8\epsilon^2}, \frac{12\epsilon}{57-46\epsilon-8\epsilon^2}\right)$$
$$\approx (0.456\epsilon + O(\epsilon^2), 0.211\epsilon + O(\epsilon^2))$$

for $\epsilon > 0$, Litim and Sannino

[D.F. Litim and F. Sannino, JHEP 1412 (2014) 178]

Phase diagram



Higher Derivative Gravity

$$\Gamma_k = \int d^4x\sqrt{g}\left[2Z_N\Lambda - Z_N R + \left(\alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right)\right]$$

$$= \int d^4x\sqrt{g}\left[2Z_N\Lambda - Z_N R + \frac{1}{2\lambda}\left(C^2 - \frac{2\omega}{3}R^2 + 2\theta E\right)\right]$$

[K.S. Stelle, Phys. Rev. **D16**, 953 (1977).]

[J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).]

[E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).]

[I.G. Avramidi, A.O. Barvinski,Phys. Lett. **159 B**, 269 (1985).]

[G. de Berredo–Peixoto and I. Shapiro, Phys.Rev. **D71** 064005 (2005).]

Beta functions of HDG

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1+\lambda_0\frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

Unitarity

[A. Salam and J. Strathdee, Phys.Rev. D18 (1978) 4480]

[R. Floreanini and R.P. Phys. Rev. D52 (1995) 896]

[D. Anselmi, JHEP 1805 (2018) 027, arXiv:1803.07777 [hep-th]]

[J. Donoghue, arXiv:1908.02416 [hep-th]]

Asymptotic safety

Maybe gravity is renormalizable at a nontrivial FP
Need to take into account running of dimensionful couplings.

[Weinberg]

One loop beta functions

$$k\frac{d}{dk}\frac{1}{16\pi G(k)} = B_1 k^2$$

$$k\frac{dG}{dk} = -16\pi B_1 G^2 k^2$$

$$\tilde{G} = Gk^2$$

$$k\frac{d\tilde{G}}{dk} = 2\tilde{G} - 16\pi B_1 \tilde{G}^2$$

fixed point at $\tilde{G} = 2/16\pi B_1$

$O(N)$ NLSM

In $d = 2$:

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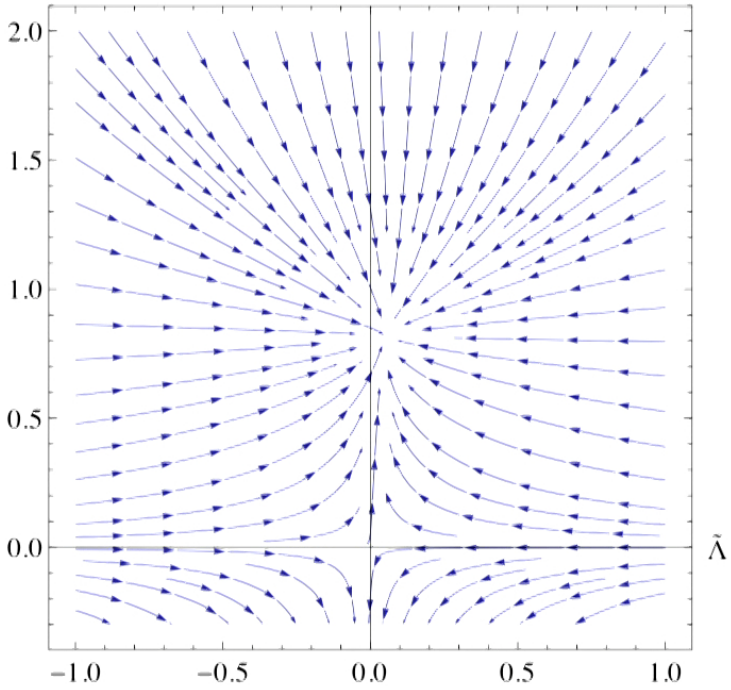
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fixed point at $\tilde{G} = 2/16\pi B_1$

One loop flow



Functional renormalization

Define the EAA $\Gamma_k(\phi)$. It satisfies

$$k\frac{d\Gamma_k}{dk} = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1} k\frac{dR_k}{dk}$$

[C. Wetterich, T. Morris]

Since $\lim_{k\to 0}\Gamma_k = \Gamma$, can use FRGE to calculate the EA.

For gravity $\Gamma_k(h_{\mu\nu}, \bar{g}_{\mu\nu})$

Most work deals with

$$\Gamma_k(g_{\mu\nu}) = \Gamma_k(0, g_{\mu\nu})$$

[M. Reuter, Phys. Rev. D **57** 971(1998)]

[D. Dou and R. Percacci, Class. and Quantum Grav. **15** 3449 (1998)]

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[M. Reuter, Phys. Rev. D **57** 971(1998)]

[D. Dou and R. Percacci, Class. and Quantum Grav. **15** 3449 (1998)]

Higher Derivative Gravity revisited

$$\Lambda + R + C^2 + R^2$$

[A. Codello and R. P., Phys.Rev.Lett. **97** 22 (2006).]

[M. Niedermaier, Nucl. Phys. B833, 226-270 (2010).]

[N. Ohta and R.P. Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398]

[D. Benedetti, P.F. Machado, F. Saueressig, Mod. Phys. Lett. A24, 2233-2241 (2009) arXiv:0901.2984 [hep-th] Nucl. Phys. B824, 168-191 (2010), arXiv:0902.4630 [hep-th]]

Both trivial and nontrivial FP for all couplings

Other results

$R + C^3$

[H. Gies, B. Knorr, S. Lippoldt, F. Saueressig, Phys.Rev.Lett. 116 (2016) no.21, 211302, arXiv:1601.01800 [hep-th]]

$f(R)$ polynomial up to order 70

[K. Falls, D. Litim, K. Nikolakopulos and C. Rahmede (2013) Phys.Rev. D93 (2016) no.10, 104022, arXiv:1410.4815 [hep-th]]
[K. Falls, D. Litim, K. Nikolakopulos and J. Schroeder (2018) arXiv:1810.08550 [gr-qc]]

$f(R)$ nonpolynomial

[N. Ohta, R.P., G.P. Vacca, Eur. Phys. J. C (2016) 76:46, arXiv:1511.09393 [hep-th]]

$f_1(R) + Rf_2(R_{\mu\nu}R^{\mu\nu})$

[K. Falls, C.R. King, D.F. Litim, K. Nikolakopoulos and C. Rahmede (2018), Phys.Rev. D97 (2018) no.8, 086006, arXiv:1801.00162 [hep-ph]]

Vertex expansion

Flat space expansion.
Flow extracted from 3- and 4-point functions.
FP again present.

[N. Christiansen, J. Pawlowski, A. Rodigast,
Phys.Rev. D93 (2016) no.4, 044036 arXiv:1403.1232 [hep-th]]
[N. Christiansen, B. Knorr, J. Meibohm, J. Pawlowski, M. Reichert, Phys.Rev.
D92 (2015) no.12, 121501 arXiv:1506.07016 [hep-th]]
[T. Denz, J. Pawlowski and M. Reichert, arXiv:1612.07315 [hep-th]]

Split symmetry

Because

$$S = S(g_{\mu\nu}) = S(\bar{g}_{\mu\nu} + h_{\mu\nu})$$

the bare action is invariant under

$$\begin{aligned} \delta\bar{g}_{\mu\nu} &= \epsilon_{\mu\nu} \ , \\ \delta h_{\mu\nu} &= -\epsilon_{\mu\nu} \ . \end{aligned}$$

but the EAA $\Gamma_k(\mathbf{h};\bar{\mathbf{g}})$ is not.

$$\frac{\delta^{(n)}\Gamma_k(h;\bar{g})}{\delta h^n} \neq \frac{\delta^{(n)}\Gamma_k(h;\bar{g})}{\delta \bar{g}^n}$$

Split symmetry

Expanding the Hilbert action

$$S(g) = S(\bar{g}) + \int S'(\bar{g})h + \int S''(\bar{g})h^2 + \int S'''(\bar{g})h^3 + \int S''''(\bar{g})h^4 + \dots$$

all contain the same Newton constant.

Due to violation of split symmetry each term of the expansion gives a different "beta function"

Dealing with the split symmetry violation

- Write the anomalous Ward identity for the split symmetry or a subgroup thereof
- Solve it to eliminate from the EAA a number of fields equal to the number of parameters of the transformation
- Write the flow equation for the EAA depending on the remaining variables

Carried through for $\epsilon_{\mu\nu} = \epsilon \bar{g}_{\mu\nu}$

[T.R. Morris, JHEP 1611 (2016) 160 arXiv:1610.03081 [hep-th]]

[R.P., G.P. Vacca, Eur.Phys.J. C77 (2017) no.1, 52 arXiv:1611.07005 [hep-th]]

A class of BSM models

Are there phenomenologically viable AS gauge-Yukawa theories?

Finite N_f and N_c

[A. Bond, D. Litim, Eur.Phys.J. C77 (2017) no.6, 429, arXiv:1608.00519 [hep-th]]
[A. Bond, G. Hiller, K. Kowalska, D. Litim, JHEP 1708 (2017) 004, arXiv:1702.01727 [hep-ph]]
[G.M. Pelaggi, A.D. Plascencia, A. Salvio, F. Sannino, Y. Smirnov, A. Strumia, Phys.Rev. D97 (2018) no.9, 095013 arXiv:1708.00437 [hep-th]]
[R.B. Mann, J.R. Meffe, F. Sannino, T.G. Steele, Z.W. Wang and C. Zhang, Phys. Rev. Lett. 119, 261802 (2017), arXiv:1707.02942 [hep-th]]

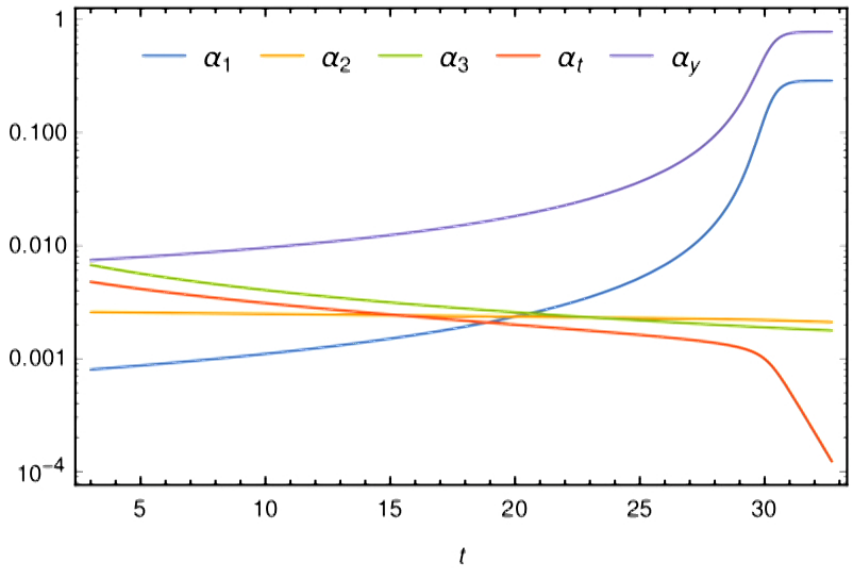
Simplest models use additional vector-like fermions.

A class of BSM models

- $SU_c(3) \times SU_L(2) \times U_Y(1)$ gauge fields+SM fermions
- N_f copies of vector-like fermions ψ_i minimally coupled to SM gauge fields, Yukawa interactions new scalars S.
- $\mathcal{L} = \mathcal{L}_{SM} + \mathrm{Tr}(\bar{\psi} i \not{D} \psi) + \mathrm{Tr}(\partial_\mu S^\dagger \partial_\mu S) - y \mathrm{Tr}(\bar{\psi}_L S \psi_R + \bar{\psi}_R S^\dagger \psi_L)$.
- Representation labels (p, q), ℓ, Y.
- Couplings: $(\alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_y, \alpha_\lambda)$, where $\alpha_i \equiv \left(\frac{g_i}{4\pi}\right)^2$ and $\alpha_\lambda \equiv \frac{\lambda}{(4\pi)^2}$.

A promising model

$$N_f = 3,\ \ell = 1/2,\ Y = 3/2,\ p = q = 0$$
$$\alpha_1^* = 0.188,\ \alpha_2^* = 0,\ \alpha_3^* = 0,\ \alpha_t^* = 0,\ \alpha_y^* = 0.778$$
$$\vartheta_1 = 33.2,\ \vartheta_2 = -3.36,\ \vartheta_3 = -0.817,\ \vartheta_4 = 0,\ \vartheta_5 = 0.$$



at 2 loops in gauge fields

Requirements

- Small couplings: $\alpha_i^* \lesssim 1$
- Small scaling exponents: $\theta_i \lesssim 1$
- Hierarchy in the loop contributions
  At the FP $0 = \beta_i = A_*^{(i)} + B_*^{(i)} + C_*^{(i)}$,
  demand $|C_*^{(i)}| < |A_*^{(i)}|$ and $|B_*^{(i)}| < |A_*^{(i)}|$.
- CFT data [Dondi, Sannino & Prochazka, 2017]
  $\langle T_\mu^\mu \rangle = cW^2 - aE_4 + \cdots$, where $a = a_{free} + a_q$ and $c = c_{free} + c_q$.
  Demand $|a_q/a_{free}| < 1$ and $|c_q/c_{free}| < 1$.
- SM matching at Fermi scale

Systematic scan of 378.000 models

[D. Barducci, M. Fabbrichesi, C.M. Nieto , R.P., V. Skrinjar, JHEP 1811 (2018) 057 arXiv:1807.05584 [hep-ph]]

$\ell = 1/2, 1, \ldots 10$
$Y = 0, 1/2, 1 \ldots 10$
$(p, q) = (0, 0), (0, 1), (1, 1)$

Many models give promising FPs in 210 scheme but fail in 321 scheme.

Those that are stable have the $U(1)$ triviality problem

$\Rightarrow$ no UV perturbative FPs with realistic IR physics

More recent work

Use GUT to solve the $U(1)$ triviality problem

Use resummation for large N_f

[E. Molinaro, F. Sannino, Z.W. Wang, Asymptotically safe Pati-Salam theory, Phys.Rev. D98 (2018) no.11, 115007 , arXiv:1807.03669 [hep-ph]]
[S. Abel, E. Mølgaard, F. Sannino, Complete asymptotically safe embedding of the standard model Phys.Rev. D99 (2019) no.3, 035030, arXiv:1812.04856 [hep-ph]]
[F. Sannino, J. Smirnov, Z.-W. Wang, Safe Clockwork e-Print: arXiv:1902.05958 [hep-ph]]

Not especially simple

Gravity plus matter

Gravity modifies the beta functions of matter couplings by universal (flavor-independent) terms.

These can generate nontrivial fixed points for matter couplings

Corrections to running of λ

$$L = \frac{1}{2}(\partial\phi)^2 + F(\phi^2)R - V(\phi^2)$$

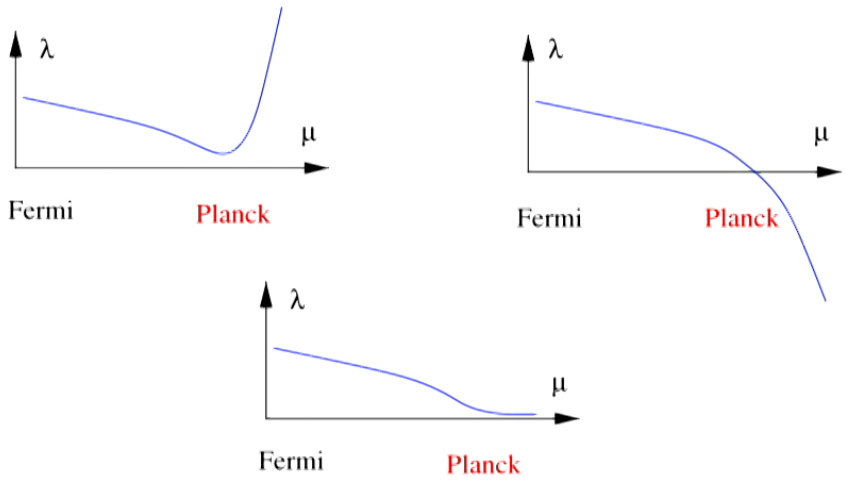
$$\begin{aligned} V(\phi^2) &= \lambda_0 + \lambda_2\phi^2 + \lambda_4\phi^4 + \dots \\ F(\phi^2) &= \xi_0 + \xi_2\phi^2 + \dots \end{aligned}$$

$$\partial_t\lambda_4 = \frac{9\lambda_4^2}{2\pi^2} + f\tilde{G}\lambda_4 + \dots$$

$f > 0$

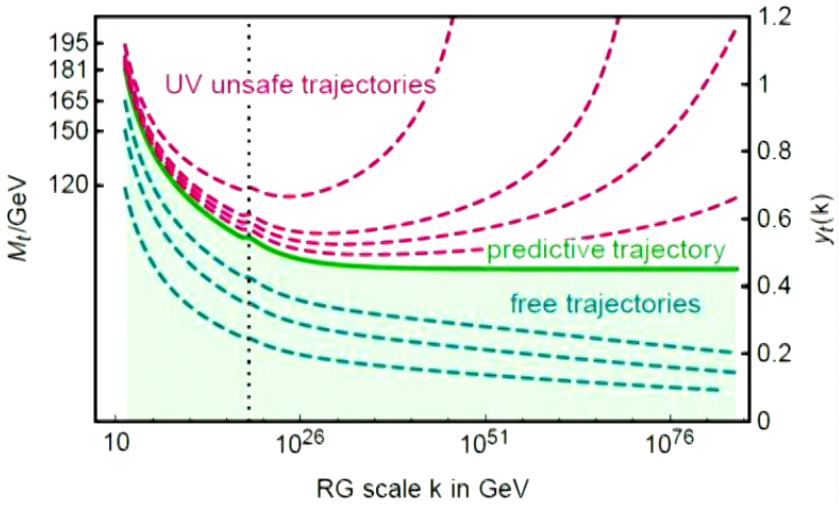
[G. Narain., R.P., Class.Quant.Grav. 27 (2010) 075001, e-Print: arXiv:0911.0386 [hep-th]]

Prediction of the Higgs mass



[M. Shaposhnikov and C. Wetterich, Phys.Lett. B683, 196 (2010)]

Calculation of top mass



[A. Eichhorn and A. Held, arXiv:1707.01107 [hep-th]]

Calculation of top mass

Quantum gravity effects on gauge and Yukawa interactions

$$\begin{aligned}\beta_{g_i} &= b_i g_i^3/(4\pi)^2 - f_g g_i \\ \beta_{y_{t(b)}} &= \frac{y_{t(b)}}{(4\pi)^2}\left(\frac{3y_{b(t)}}{2} + \frac{9y_{t(b)}}{2} - 3(Y_Q^2 + Y_{t(b)}^2)g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right) - f_y y_{t(b)}\end{aligned}$$

Non-trivial fixed-point

$$y_{t*}^2 - y_{b*}^2 = \frac{1}{3}g_{1*}^2$$

Top mass prediction $M_t = 178 GeV$

[A. Eichhorn and A. Held, Phys. Rev. Lett. **121** (2018) 151302]

Extend to full SM

[R. Alkofer, A. Eichhorn, A. Held, C.Nieto, R.P., M. Schroefl, in preparation]

$$\frac{dg_i}{dt} = \frac{1}{16\pi^2} b_i g_i - f_g g_i.$$

$$\beta_{Y_U} = -\frac{1}{16\pi^2}\left[\frac{3}{2} Y_U Y_U^\dagger Y_U - \frac{3}{2} Y_D Y_D^\dagger Y_U + 3\,\mathrm{Tr}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) Y_U - \left(\frac{17}{12} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) Y_U\right] - f_y\, Y_U.$$

$$\beta_{Y_U} = -\frac{1}{16\pi^2}\left[\frac{3}{2} Y_D Y_D^\dagger Y_D - \frac{3}{2} Y_U Y_U^\dagger Y_D + 3\,\mathrm{Tr}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) Y_D - \left(\frac{5}{12} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) Y_D\right] - f_y\, Y_D.$$

Mass basis

- We introduce the unitary matrices V_L^U and V_L^D such that

$$V_L^U M_U V_L^{U\dagger} = D_U^2 = diag[y_u^2, y_c^2, y_t^2],$$
$$V_L^D M_D V_L^{D\dagger} = D_D^2 = diag[y_d^2, y_s^2, y_b^2].$$

- Thus, we get the CKM-matrix

$$V = V_L^U V_L^{D\dagger} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}.$$

- Set of variables: $y_i = (y_u, y_c, y_t)$, $y_\rho = (y_d, y_s, y_b)$ and 4 CKM elements $|V_{i\rho}|^2$

Diagonalized System

Up-type Yukawas

$$\frac{dy_i}{dt} = \frac{y_i}{16\pi^2}\left(3\sum_j y_j^2 + 3\sum_\rho y_\rho^2 - \left(\tfrac{17}{12}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2\right) + \tfrac{3}{2}y_i^2 - \tfrac{3}{2}\sum_\rho y_\rho^2 |V_{i\rho}|^2\right) - f_y\, y_i.$$

Down-type Yukawas

$$\frac{dy_\rho}{dt} = \frac{y_\rho}{16\pi^2}\left(3\sum_j y_j^2 + 3\sum_\alpha y_\alpha^2 - \left(\tfrac{5}{12}g_Y^2 + \tfrac{9}{4}g_2^2 + 8g_3^2\right) + \tfrac{3}{2}y_\rho^2 - \tfrac{3}{2}\sum_i y_i^2 |V_{i\rho}|^2\right) - f_y\, y_\rho.$$

CKM elements

$$\beta_{|V_{i\rho}|^2} = -\frac{3}{2}\left(\sum_{\sigma, j\neq i}\frac{y_i^2+y_j^2}{y_i^2-y_j^2}y_\sigma^2\left(V_{i\sigma}V_{j\sigma}^*V_{j\rho}V_{i\rho}^* + V_{i\sigma}^*V_{j\sigma}V_{j\rho}^*V_{i\rho}\right)\right.$$
$$\left. + \sum_{j,\sigma\neq\rho}\frac{y_\rho^2+y_\sigma^2}{y_\rho^2-y_\sigma^2}y_j^2\left(V_{j\sigma}^*V_{j\rho}V_{i\sigma}V_{i\rho}^* + V_{j\sigma}V_{j\rho}^*V_{i\sigma}^*V_{i\rho}\right)\right).$$

Three generations

Yukawa couplings $y_i = (y_u, y_c, y_t)$, $y_\rho = (y_d, y_s, y_b)$.

CKM matrix

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}.$$

Choose $|V_{ud}|^2 = X$, $|V_{us}|^2 = Y$, $|V_{cd}|^2 = Z$, $\left[|V_{ij}|^2\right] = W$

$$|V|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}.$$

Three generations

Fixed points at

$$M_{123} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_{132} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_{321} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$M_{213} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_{312} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_{231} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

We seek for solutions of $\beta_{y_i} = 0 = \beta_{y_\rho}$ for each M_{abc} as functions of f_g and f_y.

We find that they can all be obtained from the solutions for M_{123} by permuting the quarks.

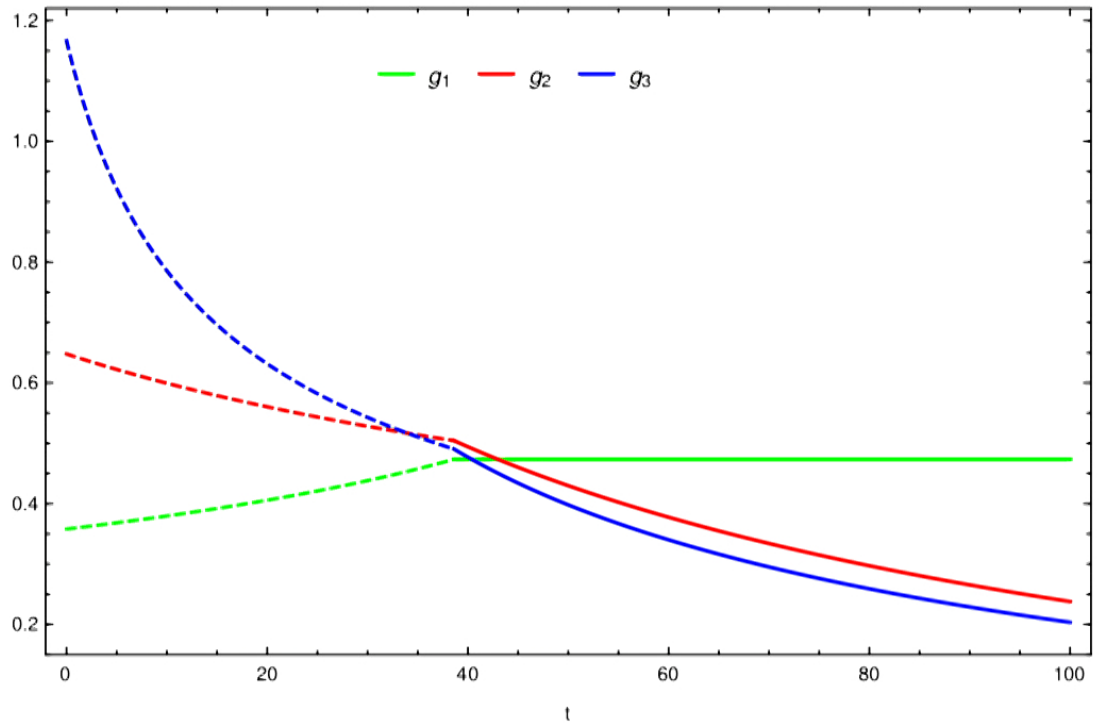
Three generations

We select the positive solutions having at most two (one up, one down) AF Yukawa couplings.

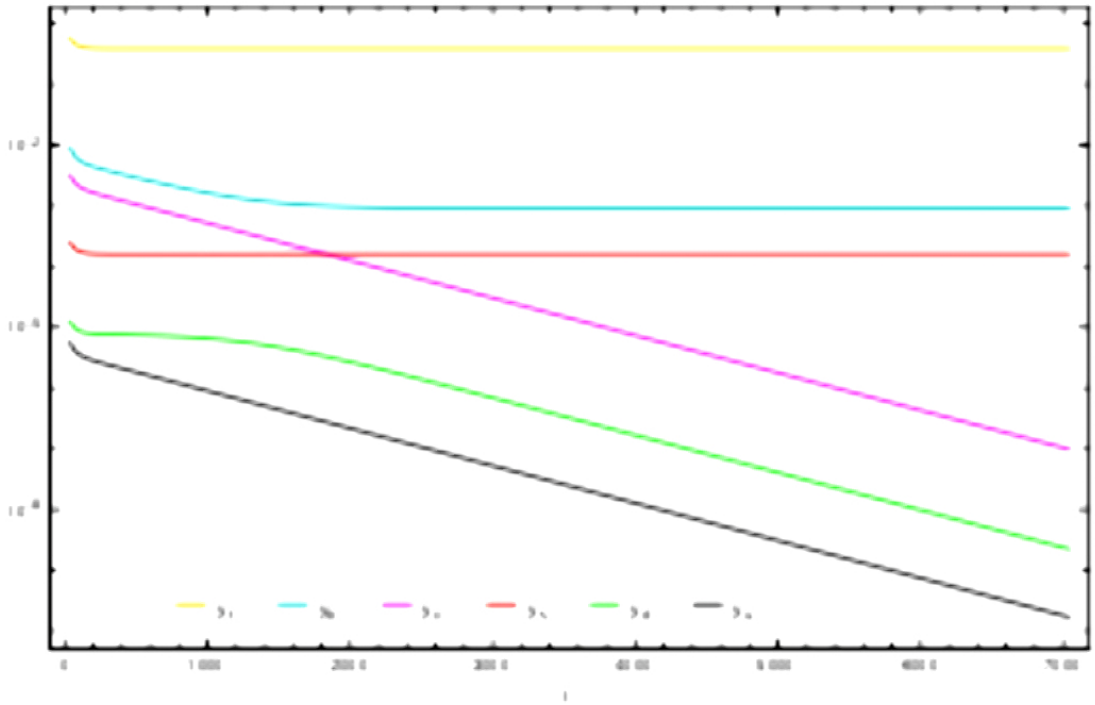
For fixed CKM FP, 392 solutions for the Yukawa couplings (1 plane, 6 lines of FPs). All can be ruled out.
Lines and planes of FPs disappear at 2 loops.

Realistic trajectory can be found with four (u, d, s, c) AF Yukawas.

Three generations - Gauge running



Three generations - Yukawa running



Realistic RG trajectory

use the FP at
$g_{1*} = 4\pi\sqrt{6f_g/41}$, $g_{2*} = g_{3*} = 0$
$X = Y = Z = 0, W = 1,$
$y_{t*} = \frac{4\pi}{\sqrt{15}}\sqrt{f_g + 2f_y}$, $y_{b*} = \frac{4\pi}{\sqrt{615}}\sqrt{-19f_g + 82f_y}$
$y_{c*} = y_{s*} = y_{u*} = y_{d*} = 0.$

Fix f_g f_y to achieve IR values of g_1 and y_b. Then get
$m_t = 186\text{GeV}$ (173), $m_b = 4.34\text{GeV}$ (4.2),
$m_c = 1.26\text{GeV}$ (1.3), $m_s = 96\text{MeV}$ (96),
$m_u = 2\text{MeV}$ (2.3), $m_d = 4.7\text{MeV}$ (4.2),
$X = 0.932$ (0.949), $Y = 0.05$ (0.05)
$Z = 0.05$ (0.05), $W = 0.949$ (0.948).

S & W

- use only the d.o.f. that we know exist, or little more
- powerful QFT tools (large N, ϵ expansion, functional RG, CFT...)
- bottom up approach (guaranteed to give correct IR limit)
- as predictive as SM or more

but

- complicated structure of the FP action
- for BSM applications simplest models don't work
- for QG, theoretical uncertainties
- in QG, issue of physical observables