Title: Talk 11

Speakers:

Collection: Simplicity III

Date: September 11, 2019 - 11:00 AM

URL: http://pirsa.org/19090080

Macroscopic Gravity and the Conformal Anomaly

Scalar Gravitational Waves, Black `Holes' & Cosmology

E. M.

Los Alamos & Perimeter

- w. R. Vaulin, Phys. Rev. D 74, 064004 (2006)
- w. M. Giannotti, Phys. Rev. D 79, 045014 (2009)
- Dynamical Dark Energy & Non-Gaussianity CMB:
  w. I. Antoniadis & P. O. Mazur, N. Jour. Phys. 9, 11 (2007)
  JCAP 09, 024 (2012)
- Consmological Horizon Modes:
  w. P. R. Anderson & C. Molina-Paris, Phys. Rev. D 80, 084005 (2009)
- Review: Acta. Phys. Pol. B 41, 2031 (2010)
- w. D. Blaschke, R. Caballo-Rubio, JHEP 1412 (2014) 153
- Scalar Gravitational Waves: JHEP 07 (2017) 043
- w. A. Sadofyev, Nambu-Goldstone Mode of Anomaly: 1909.01974

Effective Field Theory & Quantum Anomalies

- Expansion of Effective Action in Local Invariants assumes Decoupling of UV from Long Distance Modes
- But Massless Modes do <u>not</u> decouple
- Chiral, Conformal Symmetries are Anomalous
- Special Non-local Additions to Local EFT
- **IR** Sensitivity to **UV** degrees of freedom
- Conformal Symmetry & its Breaking controlled by the

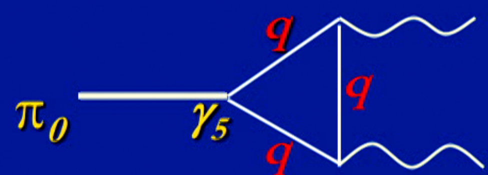
  Conformal Trace Anomaly
- Macroscopic Effects in Black Hole Physics, Cosmology

Chiral Anomaly and QCD

- **QCD** with N_f massless quarks: apparent $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$ (*not* N_f^2)

  massless pions at low energies

- **Low Energy** $\pi_0 \to 2\gamma$ dominated by the anomaly



$$\partial_\mu J^\mu{}_5 = e^2 N_c F_{\mu\nu} F^{\mu\nu} / 16\pi^2$$

- **Measured** decay rate verifies $N_c = 3$ in QCD
- **Non-Abelian Anomaly** gives rise to 0^- η' (938) meson mass
- *Additional* IR Relevant Operator violates naïve decoupling of UV

  Anomaly Matching of IR $\leftrightarrow$ UV

Anomalies provide an IR Window into UV Physics

Chiral Anomaly in QED$_4$

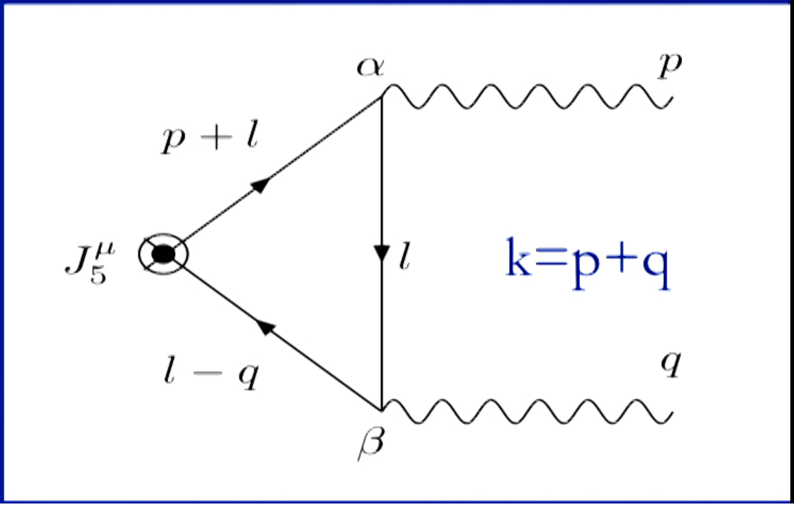
$$\Gamma_5^{\mu\alpha\beta}(p,q) = ie^2 \int d^4x \int d^4y \, e^{ip\cdot x + iq\cdot y} \left\langle \mathcal{T} J_5^{\mu}(0) J^{\alpha}(x) J^{\beta}(y) \right\rangle \Big|_{A=0}$$

Divergence on Axial Vertex

$$k_\mu \Gamma_5^{\mu\alpha\beta}(p,q) = \mathcal{A}\, \epsilon^{\alpha\beta\rho\sigma} p_\rho q_\sigma$$

$$\mathcal{A} \to \begin{cases} \dfrac{e^2}{2\pi^2} & m \to 0 \\ 0 & m \to \infty \end{cases}$$

$$\partial_\mu J_5^\mu = \frac{\alpha}{4\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = \frac{2\alpha}{\pi} \mathbf{E} \cdot \mathbf{B}$$

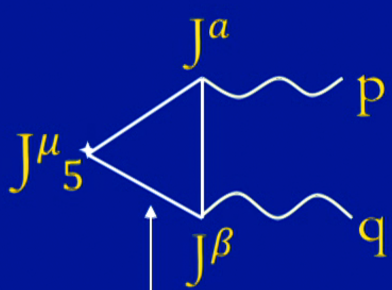


Massless Anomaly Pole in QFT

$$\Gamma_5^{\mu\alpha\beta}(p,q) = \frac{k^\mu}{k^2}\,\mathcal{A}\,\epsilon^{\alpha\beta\rho\sigma}p_\rho q_\sigma + \Gamma_{5\perp}^{\mu\alpha\beta}(p,q)$$

Longitudinal Part necessarily has a massless pole

k = p + q



$$\rho(s) \to \frac{2\alpha}{\pi}\,\delta(s)$$

Imaginary Part Spectral Fn. has a δ fn of propagator pole
Intermediate massless scalar degree of freedom in the two-particle correlated 0⁻ state

Effective Action of the Axial Anomaly

- Non-Local Form

$$\mathcal{S}_{anom} = \int d^4x \int d^4y \; (A^5_\lambda \, \partial^\lambda)_x \, \Box^{-1}_{xy} \, \mathcal{A}_y$$

- Local Form

$$\mathcal{S}_{anom}[\eta; A, A_5] = \int d^4x \Big\{ \left(\partial_\lambda \eta + A^5_\lambda\right) J^\lambda_5 + \eta \mathcal{A} \Big\}$$

- Local Field $\boldsymbol{\eta}$ Canonical Momentum $\quad \Pi_\eta = \dfrac{\delta \mathcal{S}}{\delta \dot{\eta}} = J^0_5$

$$\left[J^0_5(t, \vec{y}), \eta(t, \vec{x}) \right] = -i \, \delta^3(\vec{x} - \vec{y})$$

- **Canonical** Commutator of Generates **Anomalous** Current Commutators (Schwinger Terms)

- Chiral Rotation of $\boldsymbol{\eta}$ Phase $\left[Q_5(t), e^{2i\eta(t,\vec{x})} \right] = 2 \, e^{2i\eta(t,\vec{x})}$

Nambu-Goldstone Anomaly Collective Mode

- Consider $\displaystyle\int d^4y\, e^{ik\cdot(x-y)} \langle \mathcal{T} J_5^\lambda(x)\, e^{2i\eta(y)}\rangle = ik^\lambda F(k^2)$
- Commutator of Anomaly Effective 0- Scalar ***η*** Phase Field implies

$$\partial_\lambda \langle \mathcal{T} J_5^\lambda(x) e^{2i\eta(y)}\rangle = \delta(x^0 - y^0)\Big\langle \big[J_5^0(x), e^{2i\eta(y)}\big]\Big\rangle = \delta^4(x-y)\langle e^{2i\eta}\rangle$$

- If $\langle e^{2i\eta}\rangle = e^{2i\eta_0} \equiv z_0 \neq 0$ Fourier Transforming

$$k^2 F(k^2) = z_0 \Longrightarrow F(k^2) = \frac{z_0}{k^2}$$

Massless Nambu-Goldstone Mode
(IR Collective Degree of freedom)
is a General Consequence of the Anomaly

Conformal Trace Anomaly in QED_4

Massless QED in an External E&M Field

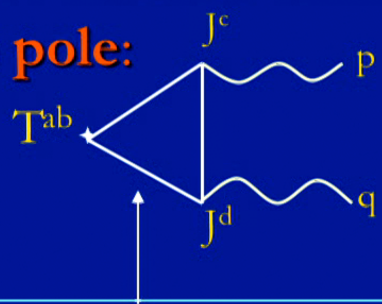
$$\langle T_a{}^a \rangle = e^2 F_{\mu\nu} F^{\mu\nu} / 24\pi^2$$

Triangle Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\bullet q - q^c p^d)\, F_1(k^2) + \ldots$$

In the limit of massless fermions, $F_1(k^2)$ must have a **massless pole:**

$k = p + q$



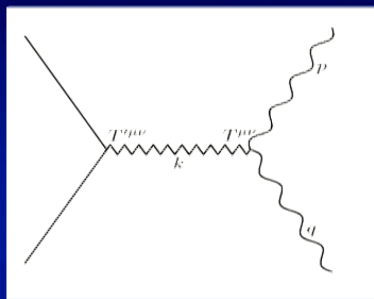
$$F_1 = \frac{e^2}{18\pi^2 k^2}$$

$$\rho(s) = \frac{e^2}{18\pi^2}\delta(s)$$

M. Giannotti & E. M. (2009)

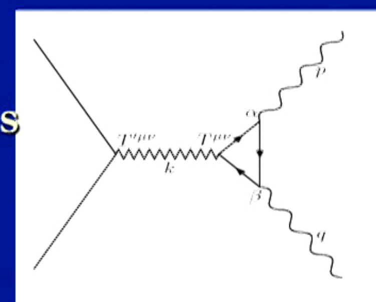
Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

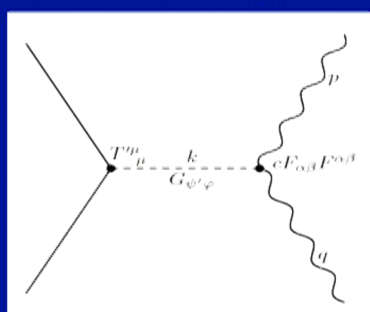
Scalar Pole in Gravitational Scattering



- In Einstein's GR only transverse, tracefree polarized waves (<u>spin-2</u>) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)

- But for $m_e = 0$ there is a scalar pole in the <TJJ> triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$





- Couples to quantum trace anomaly $T'^{\mu}_{\ \mu}$
- <TTT> triangle of massless photons has pole
- <u>At least one</u> new scalar degree of freedom in EFT

2D Gravity

$$S_{cl}[g] = \int d^2x\sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}

Quantum Trace or Conformal Anomaly

$$\langle T^a{}_a \rangle = -\frac{c_m}{24\pi}R$$

$c_m = N_S + N_F$ for massless scalars or fermions

Linearity in σ in the variational eq.

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\,\langle T^a{}_a \rangle$$

determines the Wess-Zumino Action by *inspection*

2D Anomaly Action

- Integrating the anomaly linear in **σ** gives

$$\Gamma_{WZ}[\bar{g},\sigma] = \frac{c_m}{24\pi}\int d^2x\sqrt{\bar{g}}\,\left(-\sigma\,\bar{\Box}\,\sigma + \bar{R}\,\sigma\right)$$

- This is local but **non-covariant**. Note kinetic term for **σ**
- By solving for **σ** the WZ action can be also written

$$\Gamma_{WZ}[\bar{g},\sigma] = S_{anom}[g = e^{2\sigma}\bar{g}] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but **non-local**

$$S_{anom}[g] = -\frac{c}{96\pi}\int d^2x\sqrt{g}\int d^2x'\sqrt{g'}\;R_x\left(\Box^{-1}\right)_{x,x'}R_{x'}$$

- A covariant local form implies a **dynamical scalar** field

$$S_{anom}[g;\varphi] = \frac{c}{96\pi}\int d^2x\sqrt{g}\left[g^{ab}(\nabla_a\varphi)(\nabla_b\varphi) + 2R\varphi\right]$$

$$-\Box\varphi = R \qquad\qquad \varphi \leftrightarrow 2\sigma$$

Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

$$\Pi_{abcd}(x,x') = \langle T_{ab}(x) T_{cd}(x') \rangle$$

Conservation of T_{ab} Ward Identity in 2D implies

$$\Pi_{abcd}(k) = (\eta_{ab}k^2 - k_a k_b)(\eta_{cd}k^2 - k_c k_d)\,\Pi(k^2)$$

Anomalous Trace Ward Identity in 2D implies

$k^2\,\Pi(k^2) \neq 0$ at $k^2 = 0$ **massless pole**

Quantum Effects of 2D Anomaly Action

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: long distance/IR macroscopic physics
- Topological Properties, Large Effects near Horizons
- Additional non-local Infrared Relevant Operator in S_{EFT}

New Massless Scalar Degree of Freedom at low energy

Constructing the EFT of Gravity

- Assume ***Equivalence Principle*** (Symmetry)
- Metric Order Parameter Field $\mathbf{g_{ab}}$
- Only two strictly relevant operators (R, Λ)
- Einstein's General Relativity is an EFT
- But EFT = General Relativity + Quantum Corrections
- Semi-classical Einstein Eqs. $(k << M_{pl})$:

$$\mathbf{G_{ab} + \Lambda\, g_{ab} = 8\pi\, G\, \langle T_{ab} \rangle}$$

- But there is also a **conformal anomaly**:

$$\langle T_a^{\ a} \rangle \equiv \mathcal{A} = b\, C^2 + b'\left(E - \frac{2}{3}\Box R\right) + b''\, \Box R + \Sigma_i\, c_i\, Fi^2$$

New relevant operator(s) appear in EFT of Gravity

Effective Action for the Trace Anomaly

- Non-Local Covariant Form (logarithmic propagator)

$$S_{anom}[g] = \frac{1}{8}\int d^4x\sqrt{g_x}\left(E - \frac{2}{3}\Box R\right)_x \int d^4x'\sqrt{g_{x'}}\,(\Delta_4)^{-1}_{x,x'}\,\mathcal{A}_{x'}$$

$$\mathcal{A} = b'\left(E - \frac{2}{3}\Box R\right) + b\,C^2 + c\,F^2 + c_s\,\mathrm{tr}\,G^2$$

- Local Covariant Form in Terms of New Scalar Field

$$S_{anom}[g;\varphi] = -\frac{b'}{2}\int d^4x\sqrt{g}\,\varphi\,\Delta_4\,\varphi + \frac{1}{2}\int d^4x\sqrt{g}\,\mathcal{A}\,\varphi$$

- Dynamical Scalar in Conformal Sector: 'Conformalon'

$$\Delta_4\varphi = \frac{1}{2b'}\mathcal{A}$$

$$b = \frac{\hbar}{120(4\pi)^2}\left(N_s + 6N_f + 12N_v\right)$$

$$b' = -\frac{\hbar}{360(4\pi)^2}\left(N_s + 11N_f + 62N_v\right)$$

IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

Not given in terms of Local Curvature

This is a non-trivial modification of classical General Relativity required by quantum effects

$$S_{eff}[g;\varphi] = S_{EH}[g] + S_{anom}[g;\varphi]$$

Fluctuations of conformal scalar degree of freedom couples to Λ_{eff} allowing it to change, and can generate a Quantum Conformal Phase of 4D Gravity where $\Lambda_{\mathrm{eff}} = 0$

Scalar Gravitational Waves in EFT

- Linearize Vacuum Einstein Eqs. around Flat Space with $T^{(\varphi)}_{\mu\nu}$ source

$$R_{ab} - \tfrac{1}{2} g_{ab} R = 8\pi G\, T^{(\varphi)}_{ab}$$

- Linear Metric Decomposition (Covariant)

$$h_{ab} = h^{\perp}_{ab} + \nabla_a v^{\perp}_b + \nabla_b v^{\perp}_a + \left(\nabla_a \nabla_b - \tfrac{1}{4}\eta_{ab}\Box\right) w + \tfrac{1}{4}\eta_{ab} h$$

- Usual Einstein Constraints now Dynamical

$$\delta G^{(S)}_{tt} = -\frac{1}{4}\,\vec{\nabla}^2\,(h - \Box w) = -\frac{16\pi G b'}{3}\,\vec{\nabla}^2(\Box\varphi)$$

$$\delta G^{(S)}_{ti} = -\frac{1}{4}\,\partial_t \vec{\nabla}_i\,(h - \Box w) = -\frac{16\pi G b'}{3}\,\partial_t \vec{\nabla}_i(\Box\varphi)$$

- Solved by $\frac{1}{4}(h - \Box w) = \frac{16\pi G b'}{3}\,\Box\varphi$ gauge invariant

$$\Box^2 \varphi = 0 \qquad \Box\varphi \sim \exp(-i\omega_k t + i\vec{k}\cdot\vec{x})$$

- Scalar 'Breathing' Mode GW: Only half of Solns. Couple to metric

Gauge Invariant Space + Time Split

- Linear Metric Decomposition

$$h_{tt} = -2A \qquad h_{ti} = \mathcal{B}_i^{\perp} + \vec{\nabla}_i B$$

$$h_{ij} = \mathcal{H}_{ij}^{\perp} + \vec{\nabla}_i \mathcal{E}_j^{\perp} + \vec{\nabla}_j \mathcal{E}_i^{\perp} + 2\,\eta_{ij}\,C + 2\left(\vec{\nabla}_i \vec{\nabla}_j - \tfrac{1}{3}\,\eta_{ij}\vec{\nabla}^2\right) D$$

- Gauge Invariant Scalar Potentials $\Upsilon_A \equiv A + \partial_t B - \partial_t^2 D$

- Linearized Einstein Eqs. $\Upsilon_C \equiv C - \frac{1}{3}\vec{\nabla}^2 D$

$$\vec{\nabla}^2 \Upsilon_C = \frac{8\pi G b'}{3}\,\vec{\nabla}^2(\Box\varphi)$$

$$\partial_t \vec{\nabla}_i \Upsilon_C = \frac{8\pi G b'}{3}\,\partial_t \vec{\nabla}_i(\Box\varphi)$$

$$(\eta_{ij}\vec{\nabla}^2 - \vec{\nabla}_i\vec{\nabla}_j)(\Upsilon_A + \Upsilon_C) - 2\eta_{ij}\,\partial_t^2 \Upsilon_C = -\frac{16\pi G b'}{3}\,\vec{\nabla}_i\vec{\nabla}_j(\Box\varphi)$$

- Solved by

$$\Upsilon_A = \Upsilon_C = \frac{8\pi G b'}{3}\,\Box\varphi$$

$$\Box\Upsilon_A = \Box\Upsilon_C = \frac{8\pi G b'}{3}\,\Box^2\varphi = 0 = \delta R$$

Potentials obey <u>2nd order</u> wave eq.

Localized Sources of Scalar Gravitational Waves

- **Retarded Green's Fn.** $\square^2 D_R(t-t';\vec{x}-\vec{x}') = \delta(t-t')\,\delta(\vec{x}-\vec{x}')$

$$D_R(t-t';\vec{x}-\vec{x}') = \frac{1}{8\pi}\,\theta(t-t'-|\vec{x}-\vec{x}'|)\,\theta(t-t')$$

- **Anomaly Scalar Conformalon Field**

$$\varphi(t,\vec{x}) = \frac{1}{16\pi b'}\int d^3\vec{x}' \int_{-\infty}^{t-|\vec{x}-\vec{x}'|} dt'\,\mathcal{A}(t',\vec{x}')$$

- **Scalar Metric Peturbation in Far (Radiation) Zone**

$$\frac{\delta L}{L} = \frac{16\pi G b'}{3}\,\square\varphi \to -\frac{G}{3r}\int d^3\vec{x}\,\mathcal{A}(t_{ret},\vec{x})$$

- **Power Radiated for Time Harmonic Source** $\mathcal{A}(t,\mathbf{x}) = e^{-i\omega t}\mathcal{A}_\omega(\mathbf{x})$

$$\left(\frac{dP}{d\Omega}\right)_{SGW}(\hat{\mathbf{r}}) = \frac{G\,\omega^2}{72\pi c^5}\left|\int d^3\mathbf{x}\,e^{-i\omega\hat{\mathbf{r}}\cdot\mathbf{x}/c}\mathcal{A}_\omega(\mathbf{x})\right|^2$$

$$P_{SGW}\big|_{monopole} = \frac{G\omega^2}{18c^5}\left|\int d^3\mathbf{x}\,\mathcal{A}_\omega(\mathbf{x})\right|^2$$

Scalar GWaves Carry Positive Energy

Sources of Scalar Gravitational Waves

- Sources of φ are all the trace anomaly terms

$$\Delta_4\,\varphi = \frac{E}{2} - \frac{\Box R}{3} + \frac{1}{2b'}\Big(b\,C^2 + cF_{\mu\nu}F^{\mu\nu} + \dots\Big)$$

- Curvature Invariants are extremely small
- QED and QCD Gauge Field Anomalies are much larger
- Magnetar Field **B ~ 10^{15} Gauss**

$$\mathcal{A}_{mag} = -\frac{e^2}{24\pi^2}F_{\mu\nu}F^{\mu\nu} = -\frac{\alpha B^2}{3\pi} \simeq -8\times 10^{26}\left(\frac{B}{10^{15}\,\text{Gauss}}\right)^2 \text{erg/cm}^3$$

$$\frac{\delta L}{L} \simeq -\frac{G}{3r}\int d^3x\,\mathcal{A}_{mag} \simeq 5\times 10^{-26}\left(\frac{\text{kpc}}{r}\right)$$

- Still not large enough to be observable by aLIGO
- No effects on solar system or direct terrestrial tests of GR

QCD Source of Scalar Gravitational Waves

- QCD Trace Anomaly is also a Source for φ

$$\Box^2 \varphi = \frac{1}{2b'} \mathcal{A}_{QCD}$$

- Gluonic Condensate much larger than $\mathcal{A}_{mag}$ (10 Orders of Magnitude)

$$\mathcal{A}_{QCD} = -(11N_c - 2N_f)\,\frac{\alpha_s}{24\pi}\,\langle G^a_{\mu\nu} G^{a\mu\nu}\rangle \simeq -5.6 \times 10^{36}\,\mathrm{erg/cm}^3$$

- In a Neutron Star Merger with another Compact Object this Gluonic Condensate ('Bag Constant') is disturbed

$$\frac{\delta L}{L} = \Upsilon_A = \Upsilon_C \simeq -\frac{G}{3rc^4}\int d^3\mathbf{x}\,\mathcal{A}_{NS} \lesssim 3 \times 10^{-21}\left(\frac{100\,\mathrm{Mpc}}{r}\right)$$

- Scalar GW Mode most likely excited in Neutron Star Mergers
- Requires quantitative control of nuclear physics in NS mergers
- Condensate excited also in Gravastar Alternative to BH's

Stress Tensor of the Anomaly Action

Variation of the Effective Action with respect to the metric gives conserved stress-energy tensor

$$T_{\mu\nu}[\varphi] \equiv -\frac{2}{\sqrt{g}}\,\frac{\delta}{\delta g^{\mu\nu}}\, S_{anom}[g;\varphi]$$

- Quantum Vacuum Polarization in Terms of (Semi-)Classical Scalar Conformalon
- φ depends upon the global Topology of spacetime and its boundaries, horizons

Anomaly Effective Field Theory

Stress Tensor

$$T_{\mu\nu}[\varphi] = b'\, E_{\mu\nu} + b\, C_{\mu\nu} + \sum_i \beta_i\, T^{(i)}_{\mu\nu}[\varphi]$$

Euler-Gauss-Bonnet—Quadratic & Linear in φ

$$\begin{aligned}
E_{\mu\nu} \equiv\; & -2\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\Box\varphi) + 2\nabla^{\alpha}\left[(\nabla_{\alpha}\varphi)(\nabla_{\mu}\nabla_{\nu}\varphi)\right] - \tfrac{2}{3}\,\nabla_{\mu}\nabla_{\nu}\left[(\nabla_{\alpha}\varphi)(\nabla^{\alpha}\varphi)\right] \\
& + \tfrac{2}{3}\,R_{\mu\nu}\,(\nabla_{\alpha}\varphi)(\nabla^{\alpha}\varphi) - 4\,R^{\alpha}_{\ (\mu}\left[(\nabla_{\nu)}\varphi)(\nabla_{\alpha}\varphi)\right] + \tfrac{2}{3}\,R\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\varphi) \\
& + \tfrac{1}{6}\,g_{\mu\nu}\left\{-3\,(\Box\varphi)^2 + \Box\left[(\nabla_{\alpha}\varphi)(\nabla^{\alpha}\varphi)\right] + 2\left(3R^{\alpha\beta} - Rg^{\alpha\beta}\right)(\nabla_{\alpha}\varphi)(\nabla_{\beta}\varphi)\right\} \\
& - \tfrac{2}{3}\,\nabla_{\mu}\nabla_{\nu}\Box\varphi - 4\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,\nabla_{\alpha}\nabla_{\beta}\varphi - 4\,R^{\alpha}_{(\mu}\nabla_{\nu)}\nabla_{\alpha}\varphi + \tfrac{8}{3}\,R_{\mu\nu}\,\Box\varphi + \tfrac{4}{3}\,R\,\nabla_{\mu}\nabla_{\nu}\varphi \\
& - \tfrac{2}{3}\,(\nabla_{(\mu}R)\,\nabla_{\nu)}\varphi + \tfrac{1}{3}\,g_{\mu\nu}\left[2\,\Box^2\varphi + 6\,R^{\alpha\beta}\,\nabla_{\alpha}\nabla_{\beta}\varphi - 4\,R\,\Box\varphi + (\nabla^{\alpha}R)\nabla_{\alpha}\varphi\right]
\end{aligned}$$

Weyl—Purely Linear in φ

$$C_{\mu\nu} = -4\,\nabla_{\alpha}\nabla_{\beta}\left(C_{(\mu\ \ \nu)}^{\ \ \alpha\ \ \beta}\,\varphi\right) - 2\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,R_{\alpha\beta}\,\varphi$$

Trace recovered using Eq. of Motion

$$\Delta_4\,\varphi = \nabla_{\mu}\left(\nabla^{\mu}\nabla^{\nu} + 2R^{\mu\nu} - \tfrac{2}{3}Rg^{\mu\nu}\right)\nabla_{\nu}\varphi = \frac{E}{2} - \frac{\Box R}{3} + \frac{1}{2b'}\left(b\,C^2 + \sum_i \beta_i \mathcal{L}_i\right)$$

Anomaly Stress Tensor Near Horizons

- An apparent horizon is a null surface, where outgoing null rays are first marginally trapped
- Near horizon region is conformal to $EAdS_3 \otimes$ time
- Fields become effectively massless there, CFT
- Conformal Anomaly becomes the dominant term in effective action in the near horizon region
- Stress Tensor from S_{anom} determines $\langle T_{ab} \rangle$
- Stress Tensor is generally singular there
- Singular behavior has invariant meaning in terms of conformalon φ scalar degree of freedom on horizon

Conformal Behavior on Horizon

Wave Eq. $\left(-\Box + \mu^2\right)\Phi = 0$

Mode decomposition $\Phi_{\omega\ell m} = e^{-i\omega t}\, Y_{\ell m}(\theta,\phi)\, \dfrac{\psi_{\omega\ell}(r)}{r}$

Radial Mode Eq. in Wheeler coordinate r* $\left(-\dfrac{d^2}{dr^{*2}} + V_\ell\right)\psi_{\omega\ell} = \omega^2\, \psi_{\omega\ell}$

$$V_\ell = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2\right] \xrightarrow{r\to 2M} 0$$

Becomes Conformal at Horizon
All masses become irrelevant

$$ds^2 = f(r)\left(-dt^2 + ds^2_{opt}\right)$$

$$ds^2_{opt} \xrightarrow{r\to r_H} d\mathrm{Ł}^2 = \frac{r_H^2}{z^2}\left(dx^2 + dy^2 + dz^2\right)$$

3D Lobachevsky Space (Euclidean AdS_3)

Symmetry Group SO(3,1) is Conformal Group of Horizon S^2

Powers of $z \propto \sqrt{f(r)} = e^\sigma \to 0$ measure Conformal Weights

Schwarzschild Spacetime

$$ds^2 = -\left(1 - \frac{2M}{r}\right)\, dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2\, d\Omega^2$$

$$\varphi = \sigma = \ln\sqrt{f} = \frac{1}{2}\,\ln\left(1 - \frac{2M}{r}\right) \to \infty$$

solves homogeneous $\Delta_4\varphi = 0$

Timelike Killing field is Invariant of Geometry

$K^a = (1, 0, 0, 0)$ $\quad e^{\sigma} = (-K_a K^a)^{\frac{1}{2}} = \sqrt{f}$

Energy density scales like $e^{-4\sigma} = f^{-2}$

Scalar Potential φ gives Geometric (Coordinate Invariant) Meaning to Non-Local Quantum correlations becoming Large on Horizon

Conformalon Scalar in Schwarzschild Space

- General solution for $\varphi(r)$ for Schwarzschild is

$$\left.\frac{d\varphi}{dr}\right|_S = -\frac{1}{3M} - \frac{1}{r} + \frac{2Mc_H}{r(r-2M)} + \frac{c_\infty}{2M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right) + \frac{q-2}{6M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right)\ln\left(1 - \frac{2M}{r}\right) - \frac{q}{6r}\left[\frac{4M}{r-2M}\ln\left(\frac{r}{2M}\right) + \frac{r}{2M} + 3\right]$$

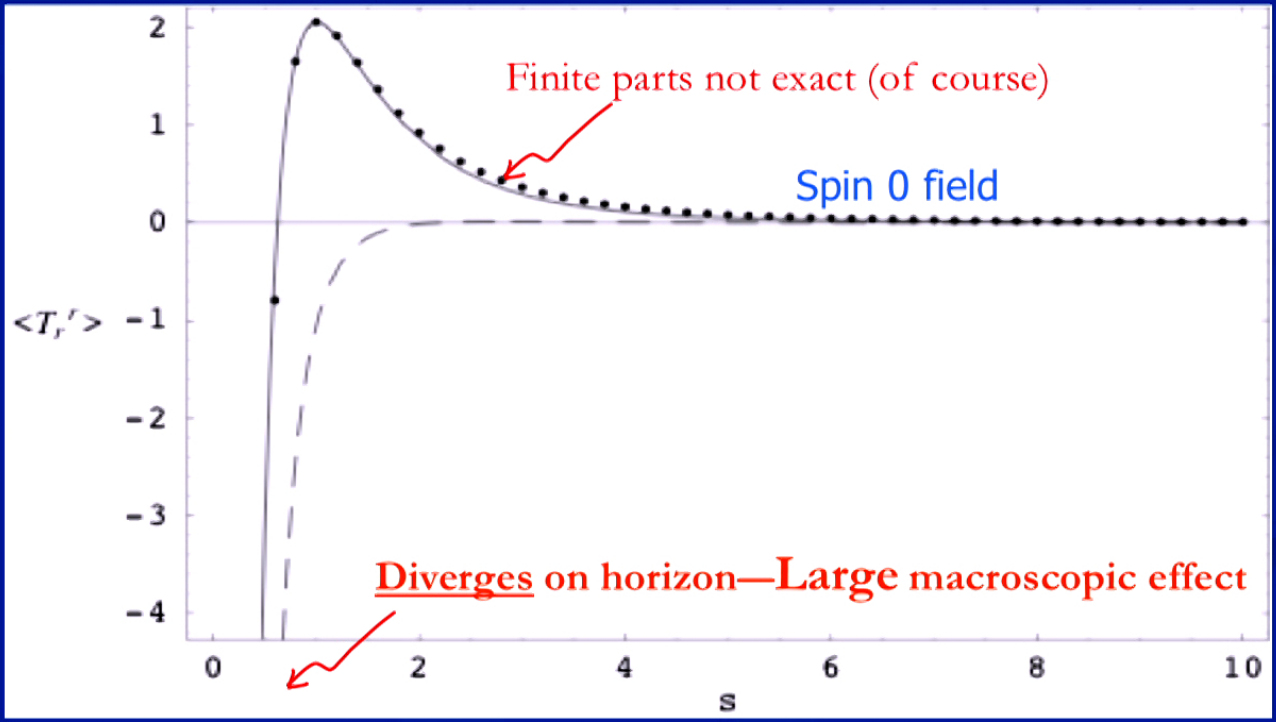
- q, c_H, c_∞ are state dependent integration constants
- Linear time dependence can be added
- Only way to have φ fall off as $r \to \infty$ is $c_\infty = q = 0$
- But only way to have finiteness on the horizon is

$$c_H = 0,\ q = 2$$

- Topological obstruction to finiteness vs. falloff as $r \to \infty$
- 2 conditions on 3 integration constants for horizon finiteness

Stress-Energy Tensor in Boulware State Radial Component

Dots – Direct Numerical Evaluation of $<T_a^{\ b}>$ (Jensen et. al. 1992)
Solid – Stress Tensor from the Anomaly (E.M. & R. Vaulin 2006)
Dashed – Page, Brown and Ottewill approximation (1982-1986)



Anomaly Stress Tensor in de Sitter Space

- Conformally Flat

$$ds^2 = -d\tau^2 + a^2(\tau)\, d\vec{x}^2 = a^2\left(-d\eta^2 + d\vec{x}^2\right) \qquad a(\tau) = e^{H\tau}$$

- Eq. of Motion Operator factorizes

$$\Delta_4 \varphi = \Box(\Box - 2H^2)\varphi = 12H^4$$

- Inhomogeneous Soln. $\varphi_{BD} = 2\,\ln a = 2H\tau$ gives

$$T_{ab}\Big|_{BD,dS} = 6b' H^4 g_{ab} = -\frac{H^4}{960\pi^2}\, g_{ab}\,(N_s + 11N_f + 62N_v)$$

- This is the soln. for conformal map to flat spacetime

$$ds^2 = e^{\varphi_{BD}} (ds^2)_{\rm flat}$$

Otherwise $T^a_{\ b}$ is generally divergent at the static horizon $r=H^{-1}$ behaving like $(1-H^2r^2)^{-2}$ Conformal Weight= 4

- Fluctuations? Since BD is a self-consistent de Sitter solution we may linearize the EFT Stress Tensor & Einstein Eqs. about it

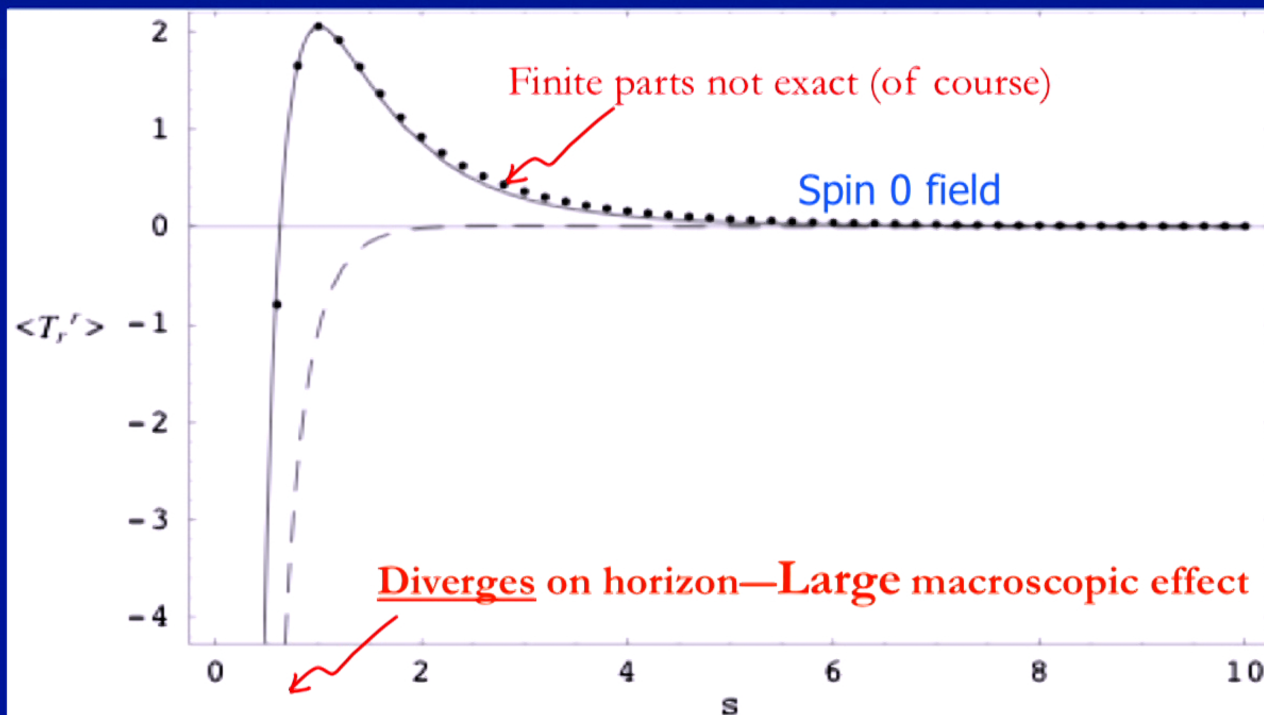
$$\langle T_a{}^b \rangle = \frac{\hbar}{M^4}\left(-1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)\left(1-\frac{2M}{r}\right)^{-2}$$

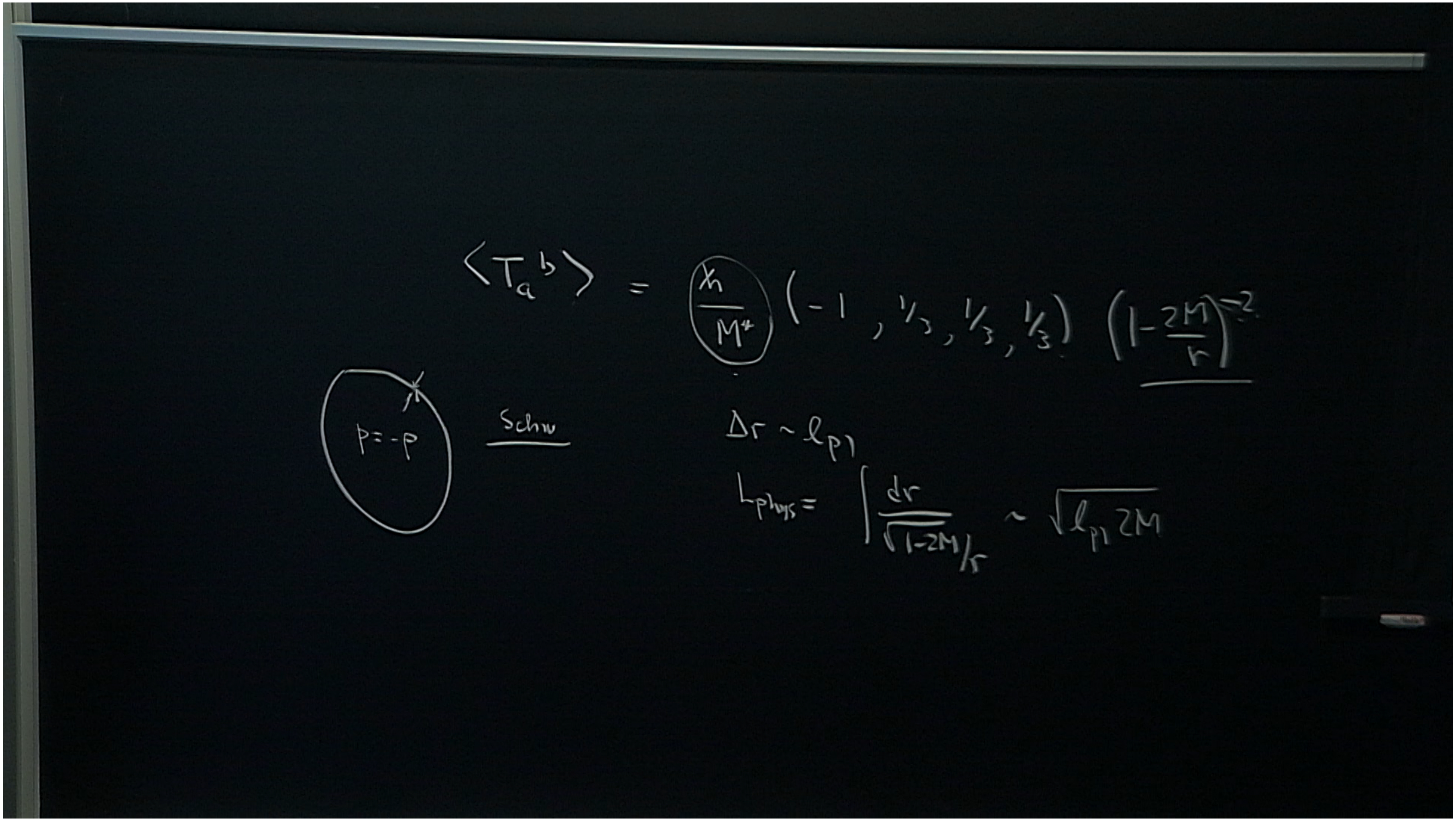
$$\Delta r \sim l_{pl}$$

$$L_{phys} = \int \frac{dr}{\sqrt{1-2M/r}} \sim \sqrt{l_{pl}\, 2M}$$

Stress-Energy Tensor in Boulware State Radial Component

Dots – Direct Numerical Evaluation of $<T_a^{\ b}>$ (Jensen et. al. 1992)
Solid – Stress Tensor from the Anomaly (E.M. & R. Vaulin 2006)
Dashed – Page, Brown and Ottewill approximation (1982-1986)




$\langle T_a{}^b \rangle = \frac{\hbar}{M^4}(-1, 1/3, 1/3, 1/3)\left(1-\frac{2M}{r}\right)^{-2}$
$p = -\rho$
Schw
$\Delta r \sim l_{pl}$
$L_{phys} = \int \frac{dr}{\sqrt{1-2M/r}} \sim \sqrt{l_{pl} 2M}$

Scalar Waves in de Sitter Space

- In de Sitter space there is factorization of the scalar wave operator

$$\Delta_4\Big|_{dS} = \left(\frac{\partial^2}{\partial\tau^2} + 5H\frac{\partial}{\partial\tau} + 6H^2 - \frac{\vec{\nabla}^2}{a^2}\right)\left(\frac{\partial^2}{\partial\tau^2} + H\frac{\partial}{\partial\tau} - \frac{\vec{\nabla}^2}{a^2}\right)$$

- And Linearized Einstein Eqs. of the Anomaly EFT depend only upon

$$u \equiv \left(\frac{\partial^2}{\partial\tau^2} + H\frac{\partial}{\partial\tau} - \frac{\vec{\nabla}^2}{a^2}\right)\delta\varphi \neq 0$$

- Again **only half** of the solns. obeying the 2nd order wave eq. survive

$$\Delta_4\,\delta\varphi = \left(\frac{\partial^2}{\partial\tau^2} + 5H\frac{\partial}{\partial\tau} + 6H^2 - \frac{\vec{\nabla}^2}{a^2}\right)u = 0$$

- Couple to the Gauge Invariant Scalar Metric Potentials

$$\Upsilon_{\mathcal{A}} + \Upsilon_{\mathcal{C}} = -\frac{8\pi G b'}{3}\,u$$

$$\frac{\vec{\nabla}^2}{a^2}\left(\Upsilon_{\mathcal{A}} - \Upsilon_{\mathcal{C}}\right) = 8\pi G H b'\left(\frac{\partial}{\partial\tau} + 2H\right)u$$

Scalar GW Solns in de Sitter

- Solns. are easily found in FRW flat coordinates

$$\Upsilon_{\mathcal{A}} + \Upsilon_{\mathcal{C}} = \frac{8\pi G b'}{3}\, u \;\propto\; \frac{e^{i\vec{k}\cdot\vec{x}} e^{\mp ik\eta}}{a^2}$$

reduce to the previous SGWs found in flat space

- But in dS the Second Linear Combination of Scalar Potentials

$$\Upsilon_{\mathcal{A}} - \Upsilon_{\mathcal{C}} \;\propto\; \frac{\partial}{\partial\tau}(a^2 u) \;\propto\; \frac{e^{i\vec{k}\cdot\vec{x}} e^{\mp ik\eta}}{a}$$

is non-vanishing & obeys the 2nd order wave eq. of a conformal scalar

$$\big(-\Box + 2H^2\big)\big(\Upsilon_{\mathcal{A}} - \Upsilon_{\mathcal{C}}\big) = 0$$

which can also be easily solved in de Sitter static time coordinates

$$ds^2 = -(1 - H^2 r^2)\, dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2$$

- de Sitter Static Horizon at $r = H^{-1}$ similar to Schwarzschild Case

Cosmological Horizon Modes

- In de Sitter static time coordinates

$$\Upsilon_A - \Upsilon_C \xrightarrow{Hr\to 1} e^{-i\omega t}\, Y_{\ell m}(\hat{n})\, (1 - Hr)^{\pm \frac{i\omega}{2H}}$$

oscillate rapidly on the cosmological horizon

- General soln. for as fn. of r

$$\Upsilon_A - \Upsilon_C \;\propto\; \frac{c_1}{r}\,\ln\left(\frac{1-Hr}{1+Hr}\right) + \frac{c_2}{r}\,\ln\left(1 - H^2 r^2\right)$$

- Behaves logarithmically---Conformal Weight Zero Field
- Correct Conformal Weight to give Large Scale CMB Anisotropy

$$\Big\langle (\Upsilon_A - \Upsilon_C)(\hat{n})\, (\Upsilon_A - \Upsilon_C)(\hat{n}')\Big\rangle \;\propto\; \frac{1}{4\pi}\ln\left(1 - \hat{n}\cdot\hat{n}'\right) = \sum_{\ell\neq 0} \frac{Y_{\ell m}(\hat{n}) Y_{\ell m}(\hat{n}')}{\ell(\ell+1)}$$

$$c_\ell\, \ell(\ell+1) = const.$$

Fluctuations of Conformalon Scalar in de Sitter Space drive Fluctuations in the Scalar Gravitational Potentials with the same spectrum of CMB Anisotropy for large angles as inflation

CFT Behavior also predicts Different Non-Gaussianity than Slow Roll

New Cosmological Scalar Fluctuations

- Linear Variation of $\langle T_{\mu\nu} \rangle$ in de Sitter space contains contributions from S_{anom} of scalar conformalon field φ
- Relevant gauge inv. scalar modes satisfy second order wave eqs. (Scalar Field without an inflaton)
- New conformalon scalar degree of freedom in cosmology
- In static de Sitter coordinates the modes grow large ln $(1 - H^2r^2)$ on the horizon
- Corresponding stress tensor perturbation

$$\delta \langle T^a{}_b \rangle \sim H^4 (1 - H^2r^2)^{-2} \text{ diag } (-3, 1, 1, 1)$$

diverges on the horizon—Suggests inhomogeneous Cosmology

Correct log scaling for scale invariant Harrison spectrum
Fluctuations of Anomaly Fields can generate CMB w/o inflaton

Additional Implication:
Vacuum Energy is Dynamical

- Conformal part of the metric, $g_{ab} = e^{2\sigma}\ \bar{g}_{ab}$ constrained --frozen--by trace of Einstein's eq. $R=4\Lambda$ becomes dynamical and can fluctuate due to φ
- Fluctuations of φ describe a conformally invariant phase of gravity in 4D $\Rightarrow$ non-Gaussian statistics of CMB

I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770; Phys. Rev. Lett. 79 (1997) 14; N. Jour. Phys. 9, 11 (2007)

- Λ a dynamical state dependent condensate generated by SSB of global Conformal Invariance
- The Quantum Phase Transition to this phase characterized by the 'melting' of the scalar condensate Λ

Fluctuations of conformalon scalar d. of f. allow Λ_{eff} to vary dynamically, and can generate a Quantum Conformal Phase of 4D Gravity where $\Lambda_{eff} \to 0$

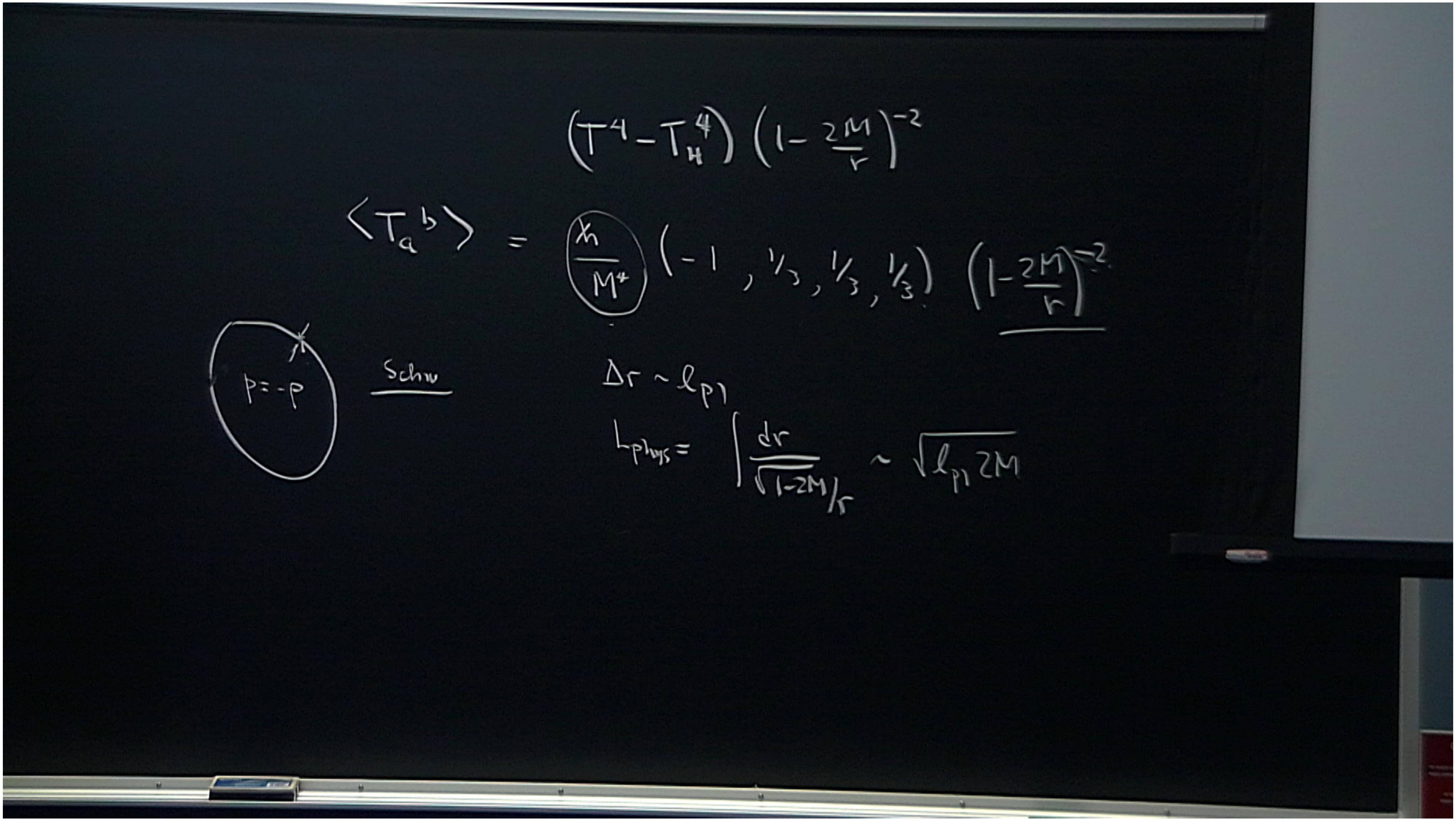
Summary

- Einstein's classical GR receives Quantum Corrections relevant at macroscopic Distances from Trace Anomaly
- This is a necessary quantum modification of classical GR
- Scalar 'Conformalon' φ degree of freedom in the EFT of Low Energy Gravity derived from Conformal Anomaly
- Scalar-Tensor Theory quite different from Brans-Dicke
- Does not couple to Classical Matter directly—only thru Conformal/Trace Anomaly
- IR Modification of GR passes all Observational Tests
- Form of Effective Action & Stress Tensor fixed

New Horizons in Astrophysics/Cosmology

- **Anomaly EFT of Gravity predicts the existence of Scalar GWs**
- Most significant astrophysical source of Scalar Gravitational Waves are disturbances in gluonic vacuum energy 'bag constant' of dense nuclear matter in Neutron Star Mergers with other compact objects, and **Gravastars**
- Conformalon Scalar shows macroscopically large quantum effects are generic near **BH & de Sitter** horizons—large backreaction
- Scalar Mode in Cosmology without inflaton
- Couples to Spacetime Dependent Dark Energy (Discussion)
- Possible Spatially Inhomogeneous Cosmologies

Conformal Anomaly Essential IR Window into UV Physics of Gravity


$(T^4 - T_H^4)\left(1 - \frac{2M}{r}\right)^{-2}$
$\langle T_a{}^b \rangle = \frac{\hbar}{M^4}\left(-1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\left(1 - \frac{2M}{r}\right)^{-2}$
$p = -\rho$
Schw
$\Delta r \sim \ell_{pl}$
$L_{phys} = \int \frac{dr}{\sqrt{1 - 2M/r}} \sim \sqrt{\ell_{pl} 2M}$