Title: Talk 10

Speakers:

Collection: Simplicity III

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# The geometrical trinity of gravity

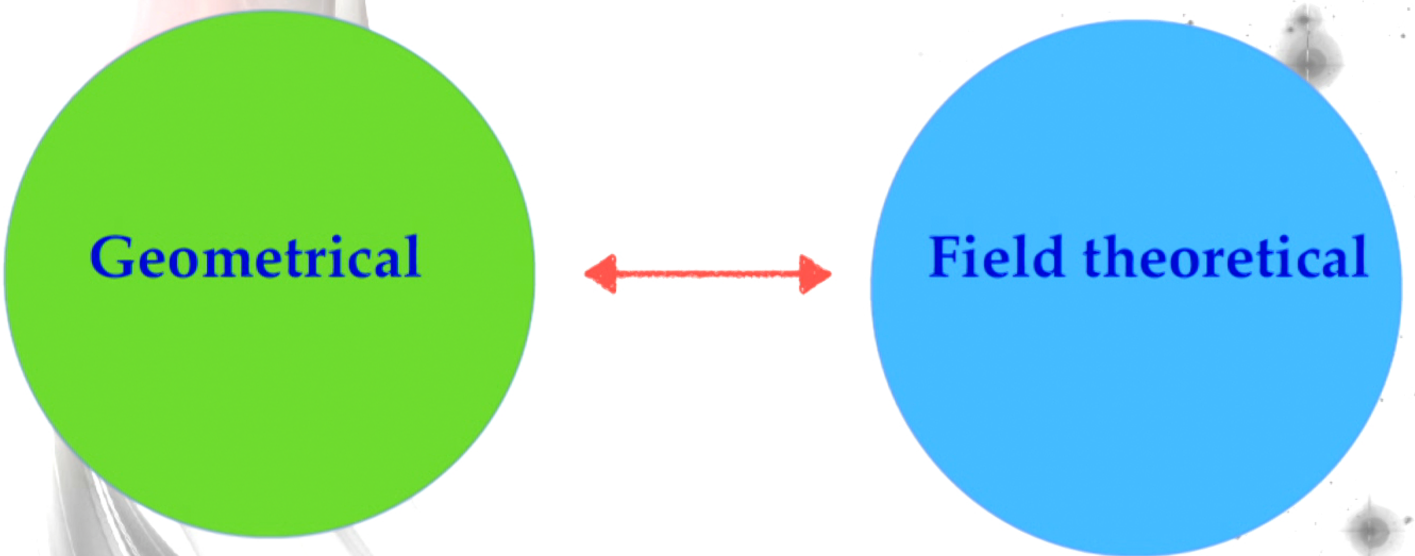

**Lavinia Heisenberg**
(ETH-ITP)
Institute for Theoretical Physics, Zürich
Lavinia.Heisenberg@phys.ethz.ch


11th Sept 2019, Simplicity III, Perimeter Institute, Waterloo/Canada

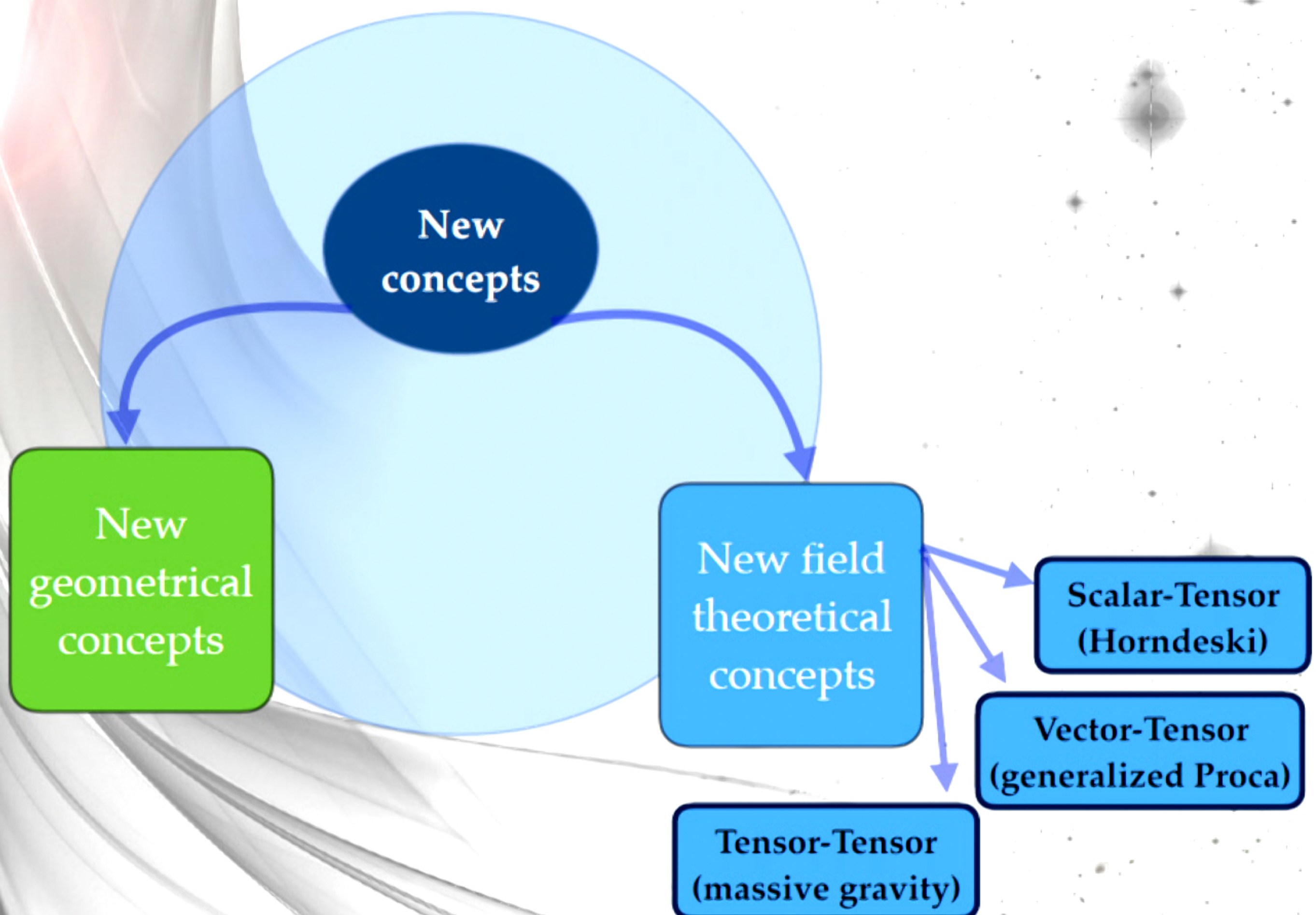
**Q:** **Are you related to Werner Heisenberg?**

**LH:** **It is uncertain!**

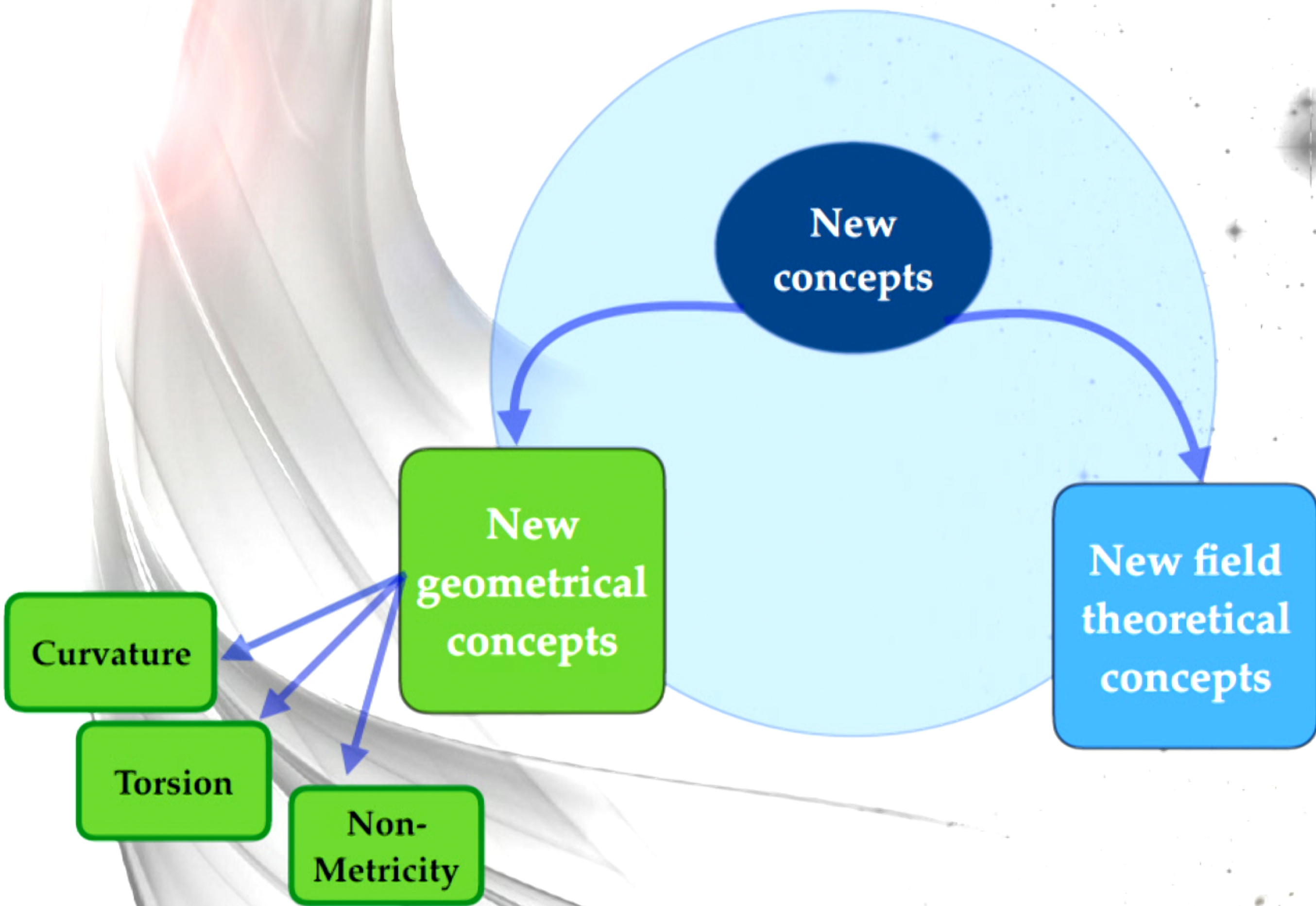
# Different Interpretations of Gravity



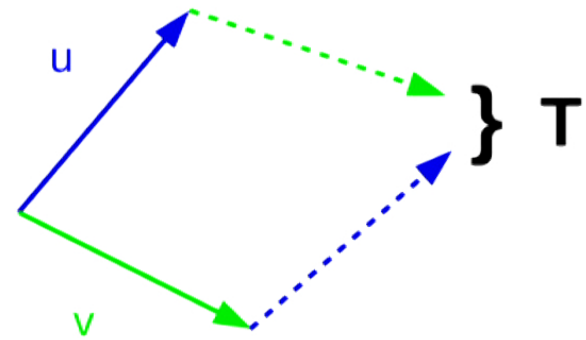
# Gravity theories



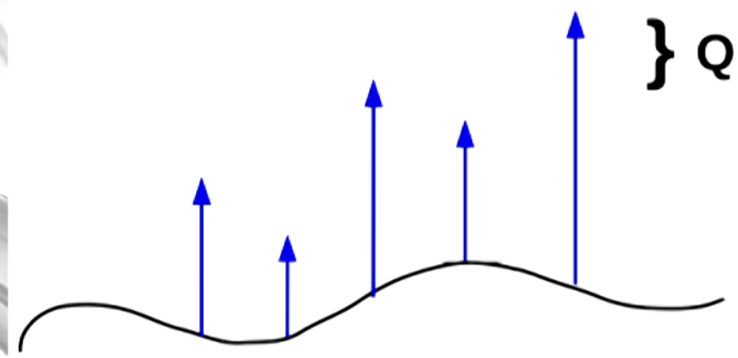
# Gravity theories



# Geometrical objects

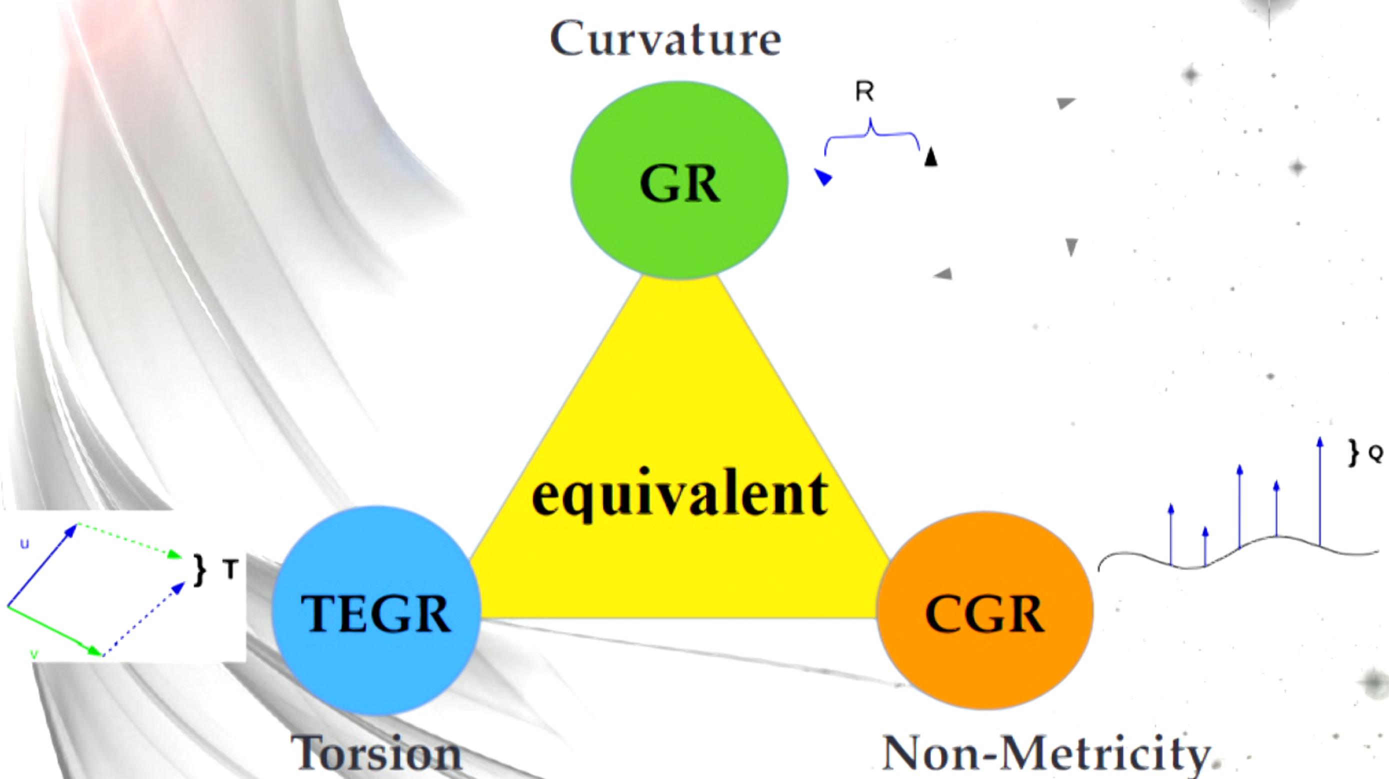

- Riemann: $R^{\alpha}_{\beta\mu\nu} \neq 0$
- Torsion: $T^{\alpha}_{\mu\nu} \neq 0$




- Non-metricity: $Q^{\alpha}_{\mu\nu} \neq 0$

# Geometrical Trinity of GR!

L.H & J.Beltran, T.Koivisto
Universe 5 (2019) 7, 173,
arXiv:1903.06830



# General Relativity (Curvature)

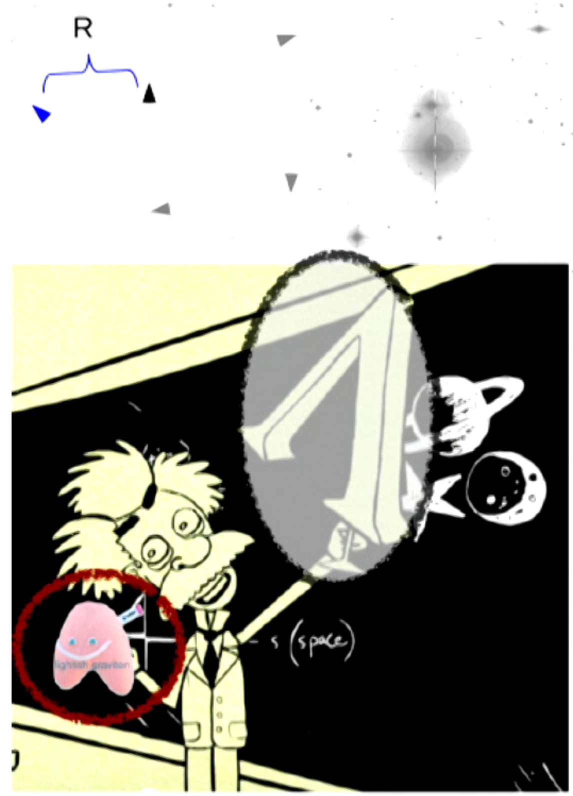
GR

(pseudo-) Riemanian manifold

$g_{\mu\nu}$ $\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}$

$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\lambda} (\partial_\nu g_{\lambda\mu} + \partial_\mu g_{\lambda\nu} - \partial_\lambda g_{\mu\nu})$$

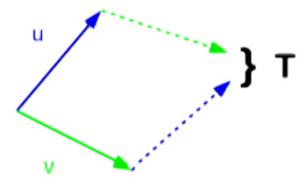
- **Riemann:** $R^{\alpha}_{\beta\mu\nu} \neq 0$
- **Torsion:** $T^{\alpha}_{\mu\nu} = 0$
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} = 0$




$$Q^{\alpha}_{\mu\nu} = \nabla_\alpha g_{\mu\nu}$$

# TEGR (Torsion)

**TEGR**




**a manifold based on torsion**

$\Gamma^a_{\mu\nu}$

$$\Gamma^a_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + K^\alpha_{\mu\nu}(T)$$

$$K^\alpha_{\mu\nu} = \frac{1}{2} T^\alpha_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)}$$ **contorsion tensor**

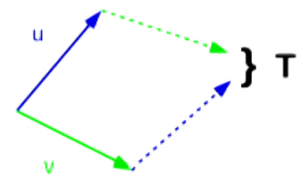
- **Riemann:** $R^\alpha_{\beta\mu\nu} = 0$
- **Torsion:** $T^\alpha_{\mu\nu} \neq 0$
- **Non-metricity:** $Q^\alpha_{\mu\nu} = 0$

$$T^\lambda_{\mu\nu} = (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})$$

**Torsion**

# TEGR (Torsion)

TEGR

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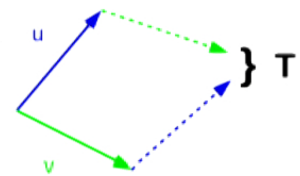
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} = 0$ → $g^{\lambda(\mu} \partial_{\alpha} \Lambda^{\nu)}_{\rho} (\Lambda^{-1})^{\rho}_{\lambda} = \frac{1}{2} \partial_{\alpha} g^{\mu\nu}$

$$T^{\alpha}_{\mu\nu} = 2(\Lambda^{-1})^{\alpha}_{\lambda} \partial_{[\mu} \Lambda^{\lambda}_{\nu]}$$

# TEGR (Torsion)

**TEGR**

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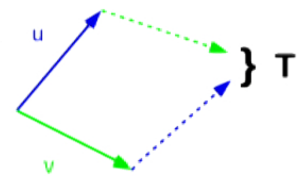
$$\mathcal{S} = \frac{1}{2}\sqrt{-g}\left(c_1 T_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu} + c_2 T_\alpha{}^{\mu\nu} T_\mu{}^\alpha{}_\nu + c_3 T_\mu T^\mu\right)$$
$$+\lambda_\alpha^{\beta\mu\nu} R^\alpha_{\beta\mu\nu} + \tilde{\lambda}^{\mu\nu}_\alpha Q_\alpha{}^{\mu\nu}$$

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1 \qquad \rightarrow \text{equivalent to GR!}$$

$$\Lambda^\alpha_\mu \ \text{(16 components)} - 2 \times 4 \ \text{(transl.gauge)} \ - 6 \ \text{(Lor. rot+boosts)} = 2 \cdot \text{dof}$$

# TEGR (Torsion)

TEGR

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$$+\lambda_\alpha^{\beta\mu\nu} R^\alpha_{\beta\mu\nu} + \tilde{\lambda}_\alpha^{\mu\nu} Q_\alpha{}^{\mu\nu}$$

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1 \qquad \rightarrow \text{equivalent to GR!}$$

$$R = \mathcal{R} + \mathring{\mathbb{T}} + 2\mathcal{D}_\alpha T^\alpha$$

$\uparrow$ $= 0$

$$\rightarrow \quad -\mathcal{R} = \mathring{\mathbb{T}} + 2\mathcal{D}_\alpha T^\alpha$$

# CGR (Non-metricity)

CGR

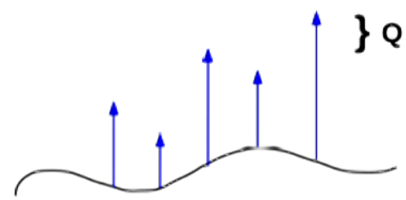
L.H & J.Beltran, T.Koivisto

PRD98 (2018) 4, 044048,
arXiv:1803.10185

a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q)$$




$$L^{\alpha}_{\mu\nu} = \frac{1}{2} Q^{\alpha}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)}$$ **disformation tensor**

- **Riemann:** $R^{\alpha}_{\beta\mu\nu} = 0$
- **Torsion:** $T^{\alpha}_{\mu\nu} = 0$
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} \neq 0$

$$\nabla_{\alpha} g_{\mu\nu} = Q_{\alpha\mu\nu}$$

**Non-metricity**

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- **Torsion:** $T^{\alpha}_{\mu\nu} = 0$ → $\Lambda^{\rho}_{\nu} = \partial_{\nu}\xi^{\rho}$

$$\Gamma^{\alpha}_{\mu\nu} = \left(\frac{\partial x^{\alpha}}{\partial \xi^{\rho}}\right)\partial_{\mu}\partial_{\nu}\xi^{\rho}$$ **Diffs**

the connection can be set to zero by means of a gauge choice

$$\Gamma^{\alpha}_{\mu\nu} = 0$$

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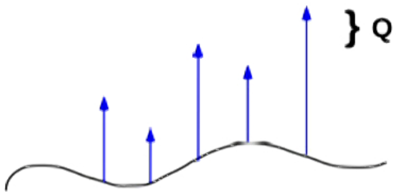


$$L^{\alpha}_{\mu\nu} = \frac{1}{2} Q^{\alpha}_{\mu\nu} - Q_{(\mu\ \ \nu)}^{\ \ \alpha}$$ **disformation tensor**

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↑

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$$\mathcal{S} = \int d^4x \sqrt{-g} \sum_{i=1}^{5} c_i Q_i^2 + \lambda_{\alpha}{}^{\beta\mu\nu} R^{\alpha}{}_{\beta\mu\nu} + \tilde{\lambda}_{\alpha}{}^{\mu\nu} T^{\alpha}{}_{\mu\nu}$$

$$c_1 = -c_3 = -\frac{1}{2} c_2 = -\frac{1}{2} c_5 = -\frac{1}{4} \text{ and } c_4 = 0 \quad \rightarrow \text{equivalent to GR!}$$

$$\sum_{i=1}^{5} c_i Q_i^2 = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q_{\beta\mu\alpha} - \frac{1}{4} Q_{\alpha} Q^{\alpha} + \frac{1}{2} Q_{\alpha} \tilde{Q}^{\alpha}$$

$$Q_{\mu} = Q_{\mu}{}^{\alpha}{}_{\alpha} \text{ and } \tilde{Q}^{\mu} = Q_{\alpha}{}^{\mu\alpha}$$

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**Diffs**

the connection can be set to zero by means of a gauge choice

$$\Gamma^{\alpha}_{\mu\nu} = 0$$

- All points are equivalent!
- No inertial effects at all!
- From the full GL(4,R) we have reduced it to Diffs.

# CGR (Non-metricity)

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$$\sum_{i=1}^{5} c_i Q_i^2 = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q_{\beta\mu\alpha} - \frac{1}{4} Q_\alpha Q^\alpha + \frac{1}{2} Q_\alpha \tilde{Q}^\alpha$$

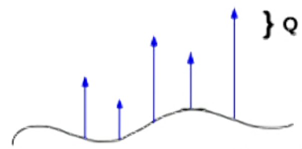
$$R = \mathcal{R} + \mathcal{Q} + \mathcal{D}_\alpha (Q^\alpha - \tilde{Q}^\alpha)$$

$= 0$

$$-\mathcal{R} = \mathcal{Q} + \mathcal{D}_\alpha (Q^\alpha - \tilde{Q}^\alpha)$$

# CGR (Non-metricity)

CGR

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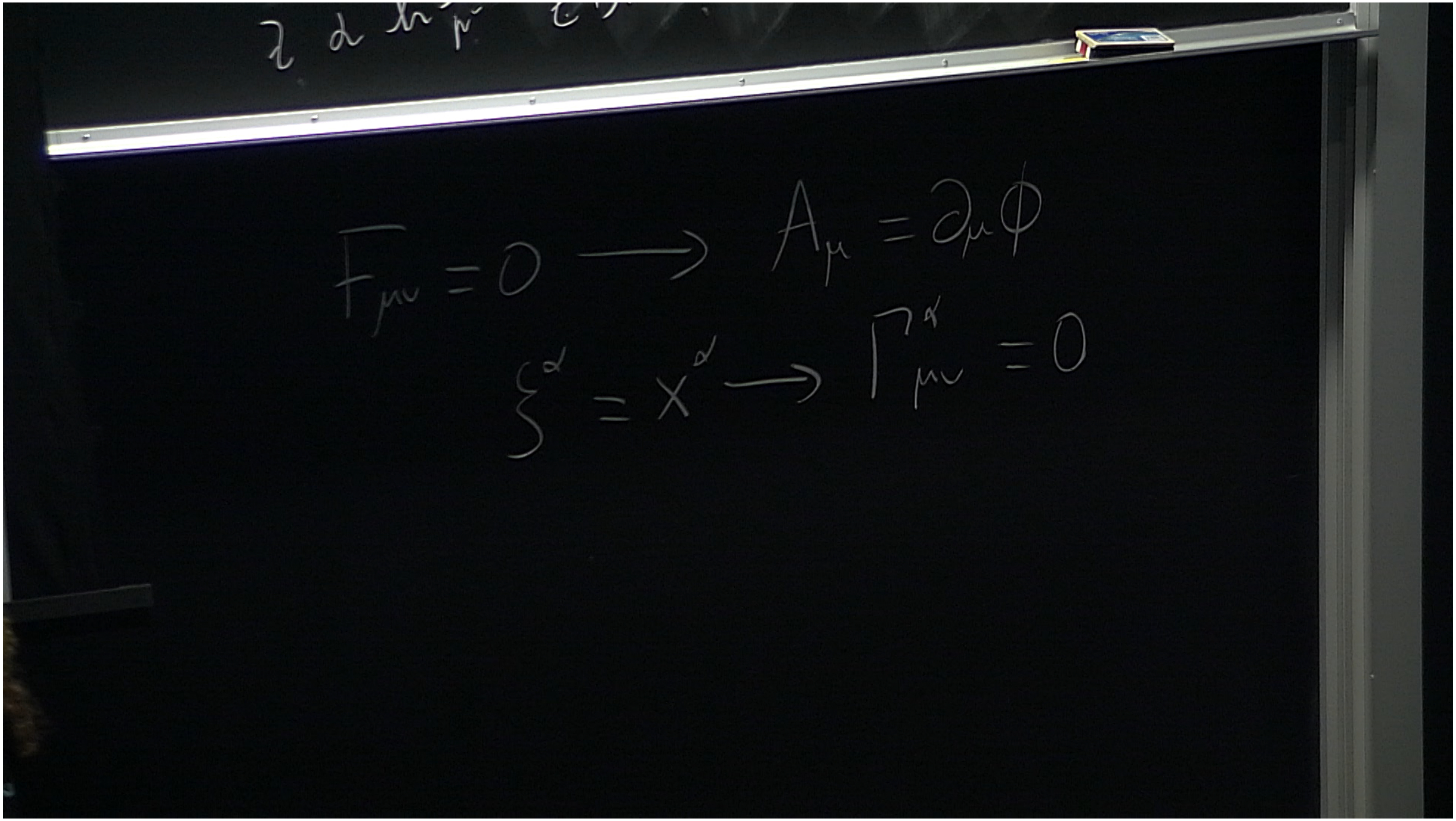
$$\sum_{i=1}^{5} c_i Q_i^2 = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q_{\beta\mu\alpha} - \frac{1}{4} Q_\alpha Q^\alpha + \frac{1}{2} Q_\alpha \tilde{Q}^\alpha$$

in the coincident gauge

$$\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q) = 0 \;\; \rightarrow \;\; -\left\{ {\alpha \atop \mu\nu} \right\} = L^{\alpha}_{\mu\nu}$$

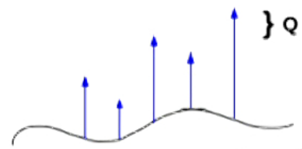
$$\sum_{i=1}^{5} c_i Q_i^2 = g^{\mu\nu} \left( \left\{ {\alpha \atop \beta\mu} \right\} \left\{ {\beta \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \beta\alpha} \right\} \left\{ {\beta \atop \mu\nu} \right\} \right)$$

the action is the {}{} part of R({})


$F_{\mu\nu} = 0 \longrightarrow A_\mu = \partial_\mu \phi$
$\xi^\alpha = x^\alpha \longrightarrow \Gamma^\alpha_{\mu\nu} = 0$

# CGR (Non-metricity)

CGR




**a manifold based on non-metricity**

$g_{\mu\nu}, \Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{{\alpha \atop \mu\nu}\right\} + L^{\alpha}_{\mu\nu}(Q) \;\; = 0$$

$$\sum_{i=1}^{5} c_i Q_i^2 = g^{\mu\nu}\Big( \left\{{\alpha \atop \beta\mu}\right\}\left\{{\beta \atop \nu\alpha}\right\} - \left\{{\alpha \atop \beta\alpha}\right\}\left\{{\beta \atop \mu\nu}\right\} \Big)$$

- No need for GHY boundary term for a well-defined variational principle
- More direct contact with (the most fundamental) field theory description (Deser's resummation approach)
- Improved and unambigous entropy of BHs
- New tool to canonically quantize GR?

# Modified gravity (geometrical perspective)

- Promote the scalars to general functions

$$\int \sqrt{-q}\mathcal{R} \quad \rightarrow \quad \int \sqrt{-q} f(\mathcal{R})$$

$$\int \sqrt{-q}\mathring{\mathbb{T}} \quad \rightarrow \quad \int \sqrt{-q} f(\mathring{\mathbb{T}})$$

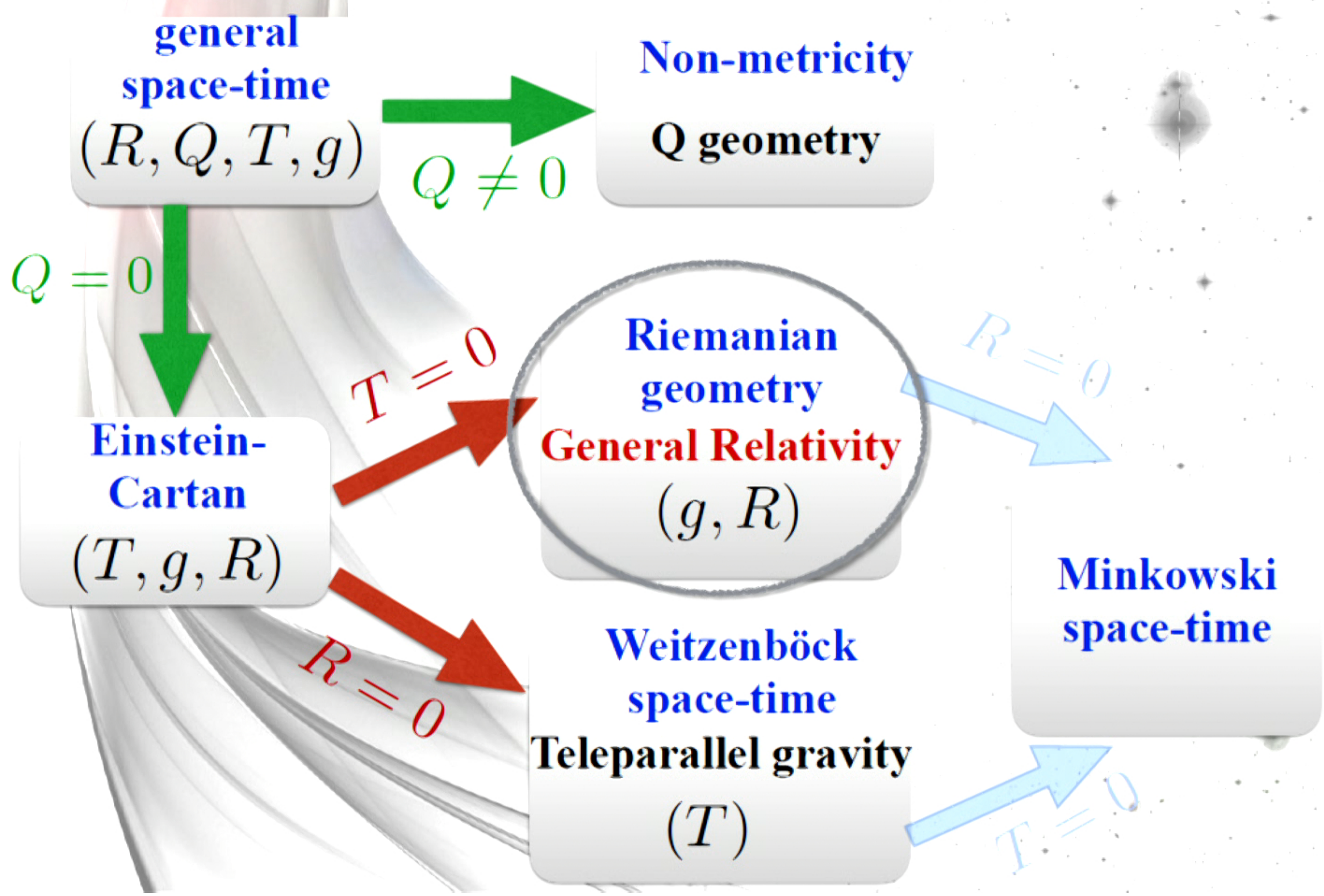
$$\int \sqrt{-q}\mathring{\mathcal{Q}} \quad \rightarrow \quad \int \sqrt{-q} f(\mathring{\mathcal{Q}})$$

- Other consistent quadratic actions? $c_i$

$$\mathcal{S} = \int d^4x \sqrt{-g} \sum_{i=1}^{5} c_i Q_i^2 + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \tilde{\lambda}_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu}$$

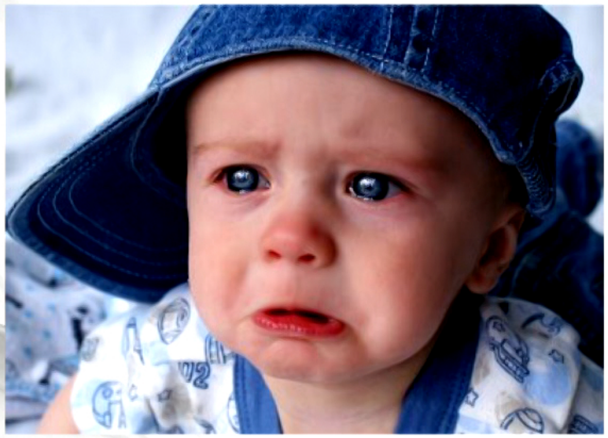
$$\mathcal{S} = \frac{1}{2}\sqrt{-g}\left(c_1 T_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu} + c_2 T_\alpha{}^{\mu\nu} T_\mu{}^\alpha{}_\nu + c_3 T_\mu T^\mu\right) + \lambda_\alpha^{\beta\mu\nu} R^\alpha_{\beta\mu\nu} + \tilde{\lambda}_\alpha^{\mu\nu} Q_\alpha{}^{\mu\nu}$$

# Geometrical setup



# General Relativity (field theory perspective)

**Imagine a universe in which Einstein did not exist!**



# General Relativity (field theory perspective)

## Imagine a modern version of Einstein

Who would have learned
all the standard techniques
of field theory description


We would have eventually constructed GR from
field theory perspective, just years later

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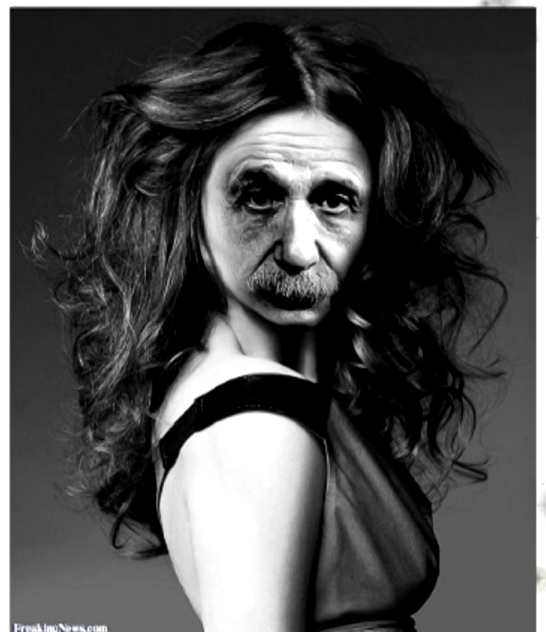



**We would have eventually constructed GR from field theory perspective, just years later**

# massless Spin 2 Field

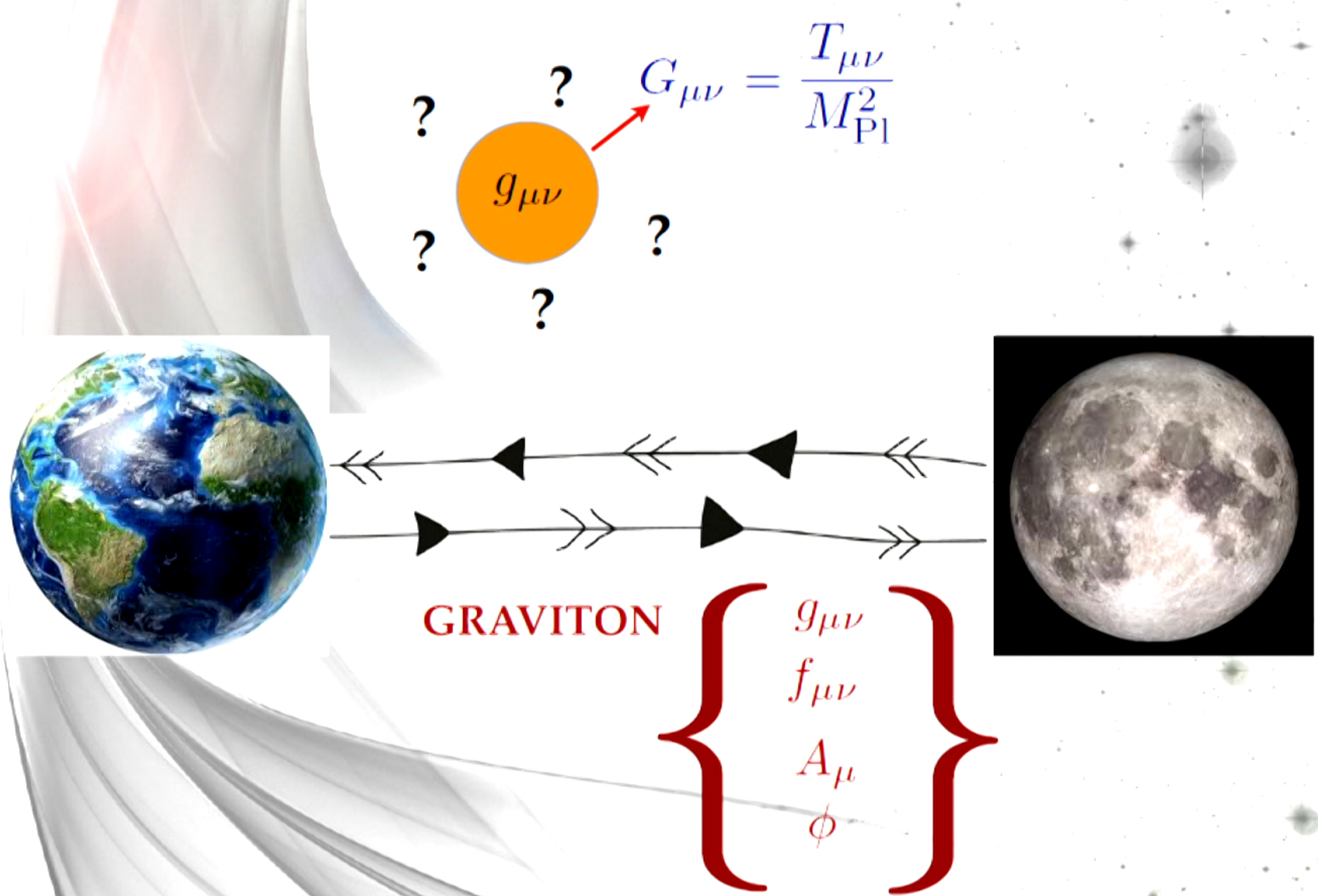
In 4 dimensions the only Lovelock invariants are

- **Cosmological constant** $\sqrt{-g}$
- **Ricci scalar** $R$
- **Gauss-Bonnet term** $\mathcal{L}_{GB}$ $\mathcal{L}_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

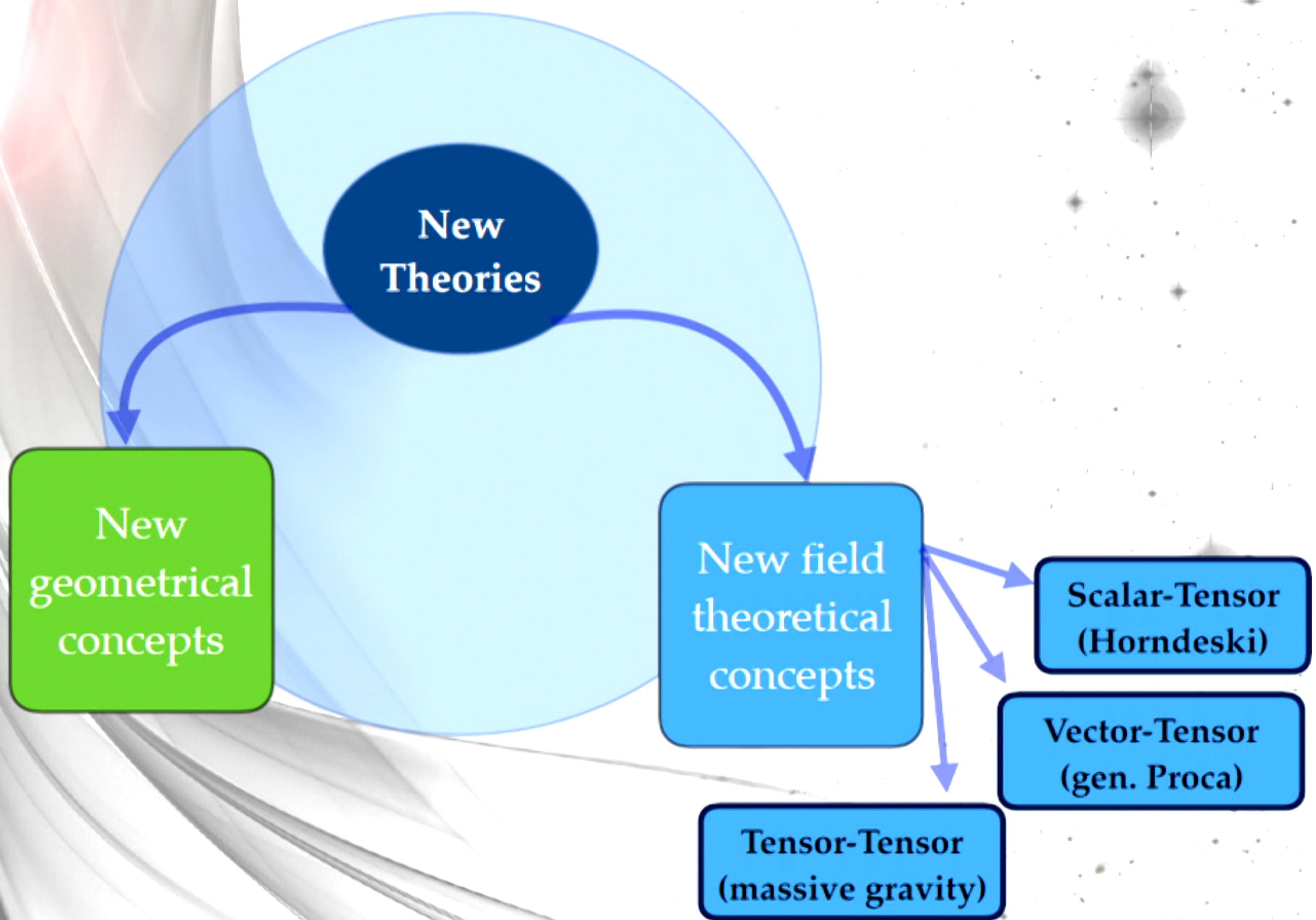
  **(topological)**

$$\mathcal{L}_g = \sqrt{-g}M_{\rm Pl}^2 R + \sqrt{-g}\Lambda$$ GR

# Modified Gravity



# Gravity theories



# Field Theoretical Trinity of Gravity