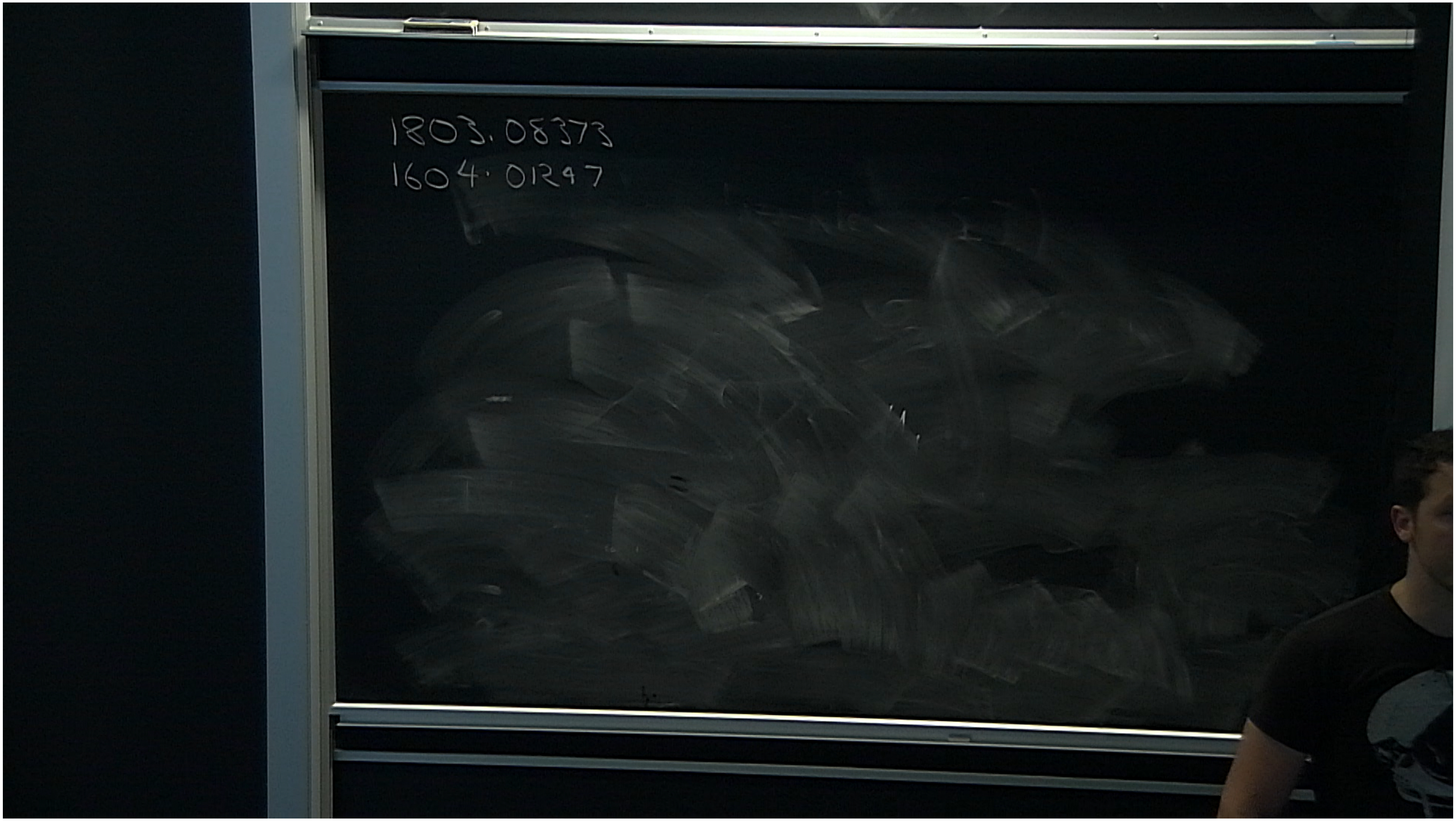
Title: Talk 8

Speakers:

Collection: Simplicity III

Date: September 10, 2019 - 11:45 AM

URL: http://pirsa.org/19090075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Particle theory from Jordan geometry

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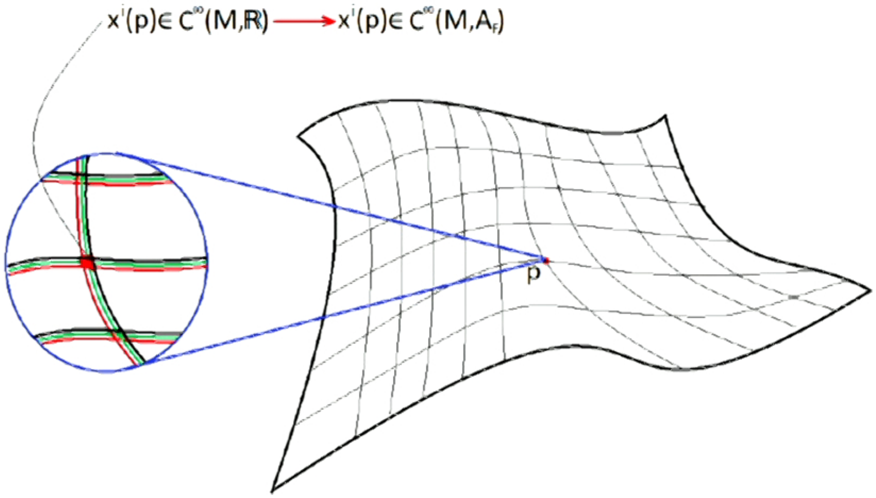
Coordinatizing a Manifold $C^\infty(M, \mathbb{R}) \rightarrow C^\infty(M, A_F)$



Constructing a geometry

So far we have a coordinate algebra: $C^\infty(M,\mathbb{R}) \otimes A_F$. **But how do we describe vector fields, differential forms, connections, spinor fields, Dirac operators, etc...?**

$$S = \kappa \int \epsilon_{IJKL} e^I e^J R^{KL} + \frac{i}{2.3!} \int \epsilon_{IJKL} e^I e^J e^K (\underbrace{\overline{\Psi}\gamma^L D\Psi} - \overline{D\Psi}\gamma^L \Psi)$$

Metric

differential forms

Inner Product

Covariant exterior derivative

Clifford representation of forms

Outline

1. Vector fields
2. Differential forms
3. Connections
4. Spinor fields

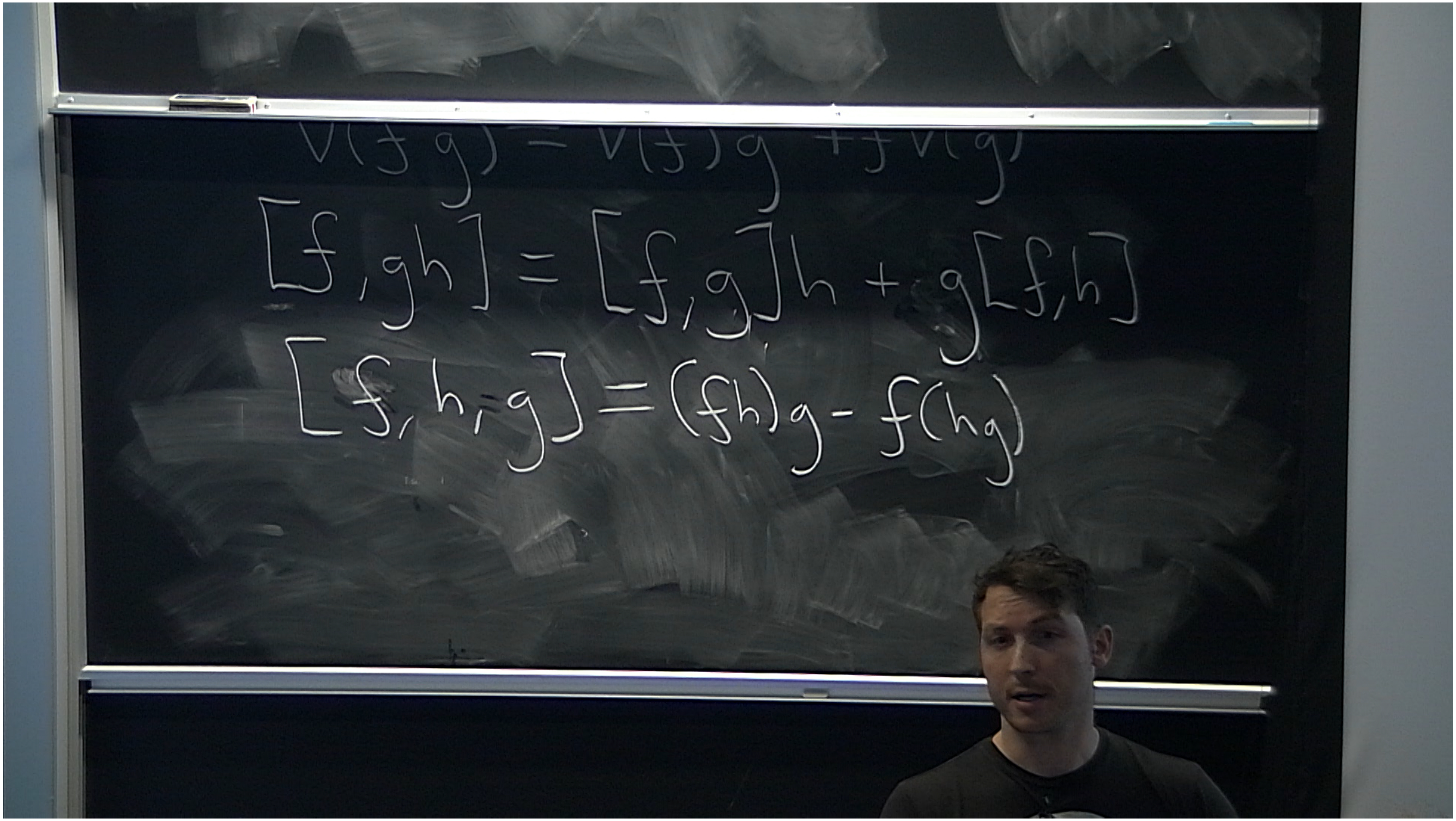
Vector Fields

Vector fields = derivations

Riemannian geometry: $V = V^{\mu}\partial_{\mu}$

Inner derivations:

$$\delta_a = [a, \_], \qquad \text{(non-commutative algebras)}$$
$$\delta_{ab} = [a, \_, b], \qquad \text{(Jordan algebras)}$$


V(fg) = V(f)g + fV(g)
[f, gh] = [f, g]h + g[f, h]
[f, h, g] = (fh)g - f(hg)

Vector fields = derivations

Riemannian geometry: $V = V^\mu \partial_\mu$

Inner derivations:

$$\delta_a = [a, \_], \qquad \text{(non-commutative algebras)}$$
$$\delta_{ab} = [a, \_, b], \qquad \text{(Jordan algebras)}$$

Properties:

$$[V, V']f = V''f, \qquad \alpha_V f = e^V$$

Example: $J_n(\mathbb{C})$ with basis $\{\mathbb{I}, \lambda_a\}$, $a = 1, ..., n^2 - 1$.

Jordan product: $ab = \frac{1}{2}(a \circ b + b \circ a)$, where:

$$\lambda_a \circ \lambda_b = \frac{1}{2}(\underbrace{\frac{1}{n}\hat{\delta}_{ab}\mathbb{I} + d_{abc}\lambda_c}_{\text{Jordan}} + \underbrace{if_{abc}\lambda_c}_{\text{Lie}}).$$

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Basis of derivations given by:

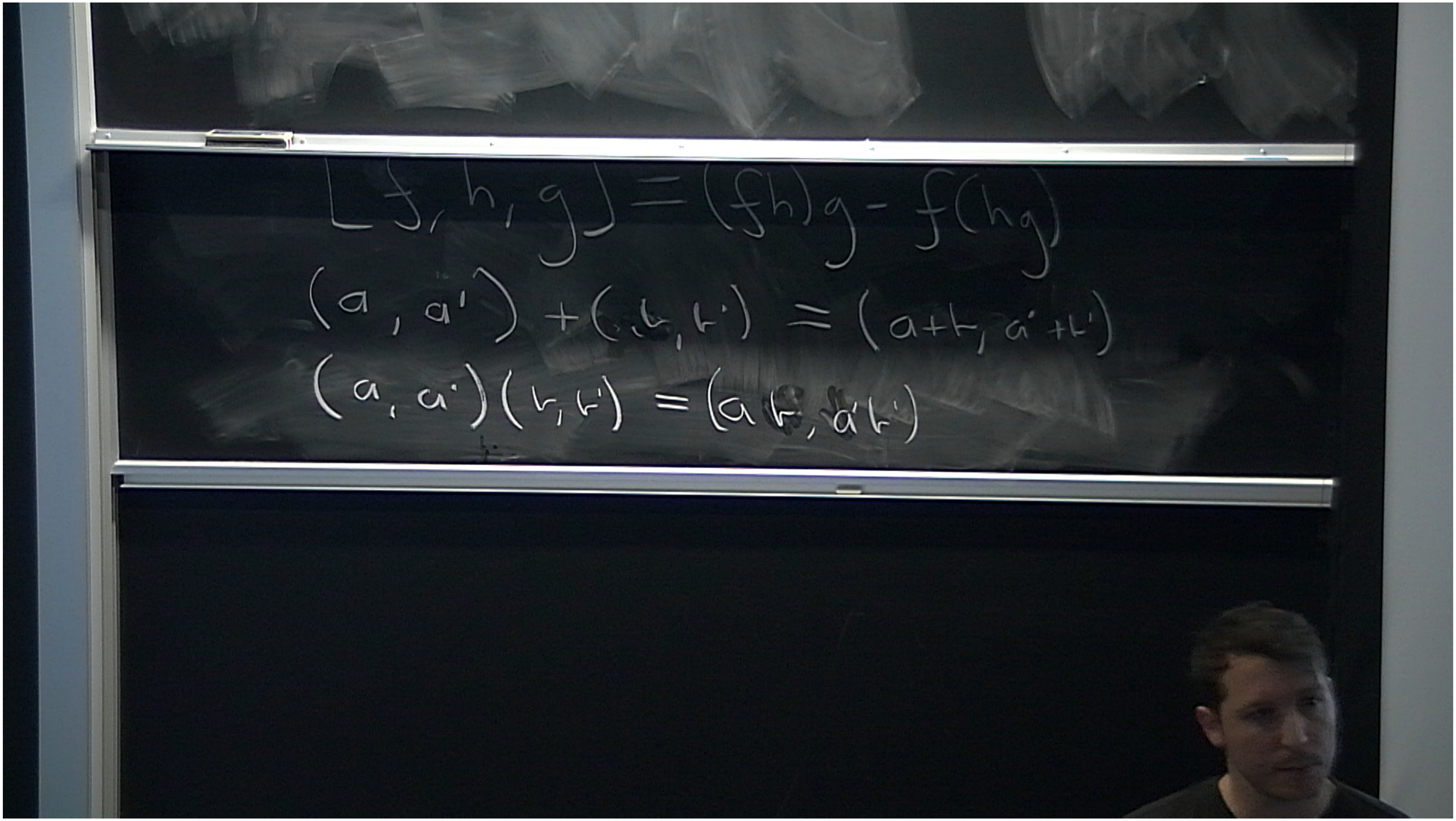
$$\delta_a = \frac{4}{n} f_a{}^{bc}[\lambda_c, \_, \lambda_b].$$

Action on basis λ_a given by:

$$\delta_a \lambda_b = -f_{ab}{}^c \lambda_c.$$

Example: Pati-Salam

Gauge group: $\underbrace{SU_L(2) \times SU_R(2)}_{\text{Let's look at this bit!}} \times SU(4).$


[f, h, g] = (fh)g - f(hg)
(a, a') + (b, b') = (a+b, a'+b')
(a, a')(b, b') = (ab, a'b')

Example: Pati-Salam

Gauge group: $\underbrace{SU_L(2) \times SU_R(2)}_{\text{Let's look at this bit!}} \times SU(4).$

Finite coordinate algebra: $A_F = J_2^L(\mathbb{C}) \oplus J_2^R(\mathbb{C})$

Two point space!

Differential Forms

'Jordan' One forms

Linear Map: $Vf = V^a \delta_a f$

$$V(f+g) = Vf + Vg,$$
$$(V+W)f = Vf + Wf.$$

Can think of Vf **as map from derivations to coordinate algebra. Call it:**

$$df[V] := Vf,$$

where the 'exterior derivative' d **satisfies:**

$$d(fg)[V] = V(f)g + fV(g) = ((df)g + fdg)[V].$$

Dual basis of 1-forms

Define df **by action on vector fields** $V^a\delta_a$**:**

$$df = E^a \delta_a f,$$

where $E^a[\delta_b] = \hat{\delta}^a_b$**. Write general one form as:**

$$\omega = \omega_a E^a,$$

$\omega_a \in A^0.$

Differential 'n' forms $A = \oplus_n A^n$

1. **Define product of forms $\omega \in A^n$, $\omega' \in A^m$:**

$$\omega\omega' = (\omega_{1,\ldots,n}\omega'_{1,\ldots,m})E^1 \wedge \ldots \wedge E^{n+m} \in A^{m+n}.$$

2. **Extend 'd' to higher order forms**

$$d^2 = 0,$$
$$d(\omega\omega') = (d\omega)\omega' + (-1)^{|\omega|}\omega d\omega'.$$

Example: $J_n(\mathbb{C})$ with basis $\{\mathbb{I}, \lambda_a\}$, $a = 1, ..., n^2 - 1$.

Dual forms satisfying $E^a(\delta_b) = \hat{\delta}^a_b$**:**

$$E^a = \frac{16}{n} f^{bca}(\lambda_c \lambda_d)(\lambda_b d\lambda_d), \qquad dE^a = \frac{1}{2} f^a{}_{bc} E^b \wedge E^c.$$

Simplest example $J_2(\mathbb{C})$**:**

$$\begin{aligned} E^a \delta_e &= 8\epsilon^{bca}(\lambda_c \lambda_d)(\lambda_b \delta_e \lambda_d) \\ &= 2\epsilon^{bca}(\lambda_b \delta_e \lambda_c) \\ &= -\frac{1}{2}\epsilon^{bca}(\epsilon_{ecb}) = \hat{\delta}^a_e. \end{aligned}$$

(Remember $\delta_a \lambda_b = -\epsilon_{ab}{}^c \lambda_c$.**)**

Connections

Connections on $A = C^\infty(M, \mathbb{R}) \otimes J_n(\mathbb{C})$

Local basis of derivations $\{\partial_\mu, \delta_a\}$:

$$\nabla_\mu V = \nabla_\mu (V^\nu \partial_\nu + V^a \delta_a)$$
$$= (\partial_\mu V^\nu - V^\tau \Gamma^\nu_{\mu\tau})\partial_\nu + (\partial_\mu V^a - V^b W^a_{\mu b})\delta_a,$$

where $\Gamma^\tau_{\mu\nu}$ is the usual Affine connection, while $W^b_{\mu a}$ is viewed as a local 'gauge' connection.

Discrete connections $A_F = J_n(\mathbb{C}) \oplus J_n(\mathbb{C})$

Basis of derivations:

$$\delta^L_a = \frac{4}{n}\epsilon_a{}^{bc}[\theta^L_c, \_, \theta^L_b], \qquad \delta^R_a = \frac{4}{n}\epsilon_a{}^{bc}[\theta^R_c, \_, \theta^R_b]$$

where

$$\theta^L_a = (\lambda_a, 0), \qquad \theta^R_a = (0, \lambda_a).$$

Discrete connection = gauge link:

$$\delta^L_a = P(L, R)^b_a \delta^R_b.$$

Spinor Fields

Spinor Representations (Pati-Salam)

Pati-Salam Gauge group: $\underbrace{SU_L(2) \times SU_R(2)}_{\text{Let's look at this bit!}} \times SU(4).$

$A_F = J_2(\mathbb{C}) \oplus J_2(\mathbb{C})$ **represented on** $H = \mathbb{C}^{32}$**.**

$$(q^L, q^R)\Psi = \frac{1}{2}\left\{ \left(\begin{array}{cc|cc} q_a^L \sigma^a & & & \\ & q_a^R \sigma^a & & \\ \hline & & 0 & \\ & & & 0 \end{array} \right), \left(\begin{array}{cccc|cccc} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \hline \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{\nu}_L & \overline{e}_L & \overline{\nu}_R & \overline{e}_R & & & & \end{array} \right) \right\}.$$

$\sigma^a = \{\mathbb{I}, \sigma^i\}$, $i = 1, 2, 3.$

Spinor Representations (Pati-Salam)

Pati-Salam derivation algebra: $\underbrace{su_L(2) \times su_R(2)}_{\text{Let's look at this bit!}} \times su(4)$.

Derivations on $B = A \oplus H$:

$$(V_L, V_R)\Psi = \frac{1}{2}\left[\left(\begin{array}{cc|cc} V_L^i i\sigma_i & & & \\ & V_R^j i\sigma_j & & \\ \hline & & 0 & \\ & & & 0 \end{array}\right), \left(\begin{array}{cccc|cccc} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \hline \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{\nu}_L & \overline{e}_L & \overline{\nu}_R & \overline{e}_R & & & & \end{array}\right)\right].$$

Spinor Representations (Pati-Salam)

Charge conjugation and grading:

$$J\Psi = \left(\begin{array}{cccc|cccc} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \hline \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{\nu}_L & \overline{e}_L & \overline{\nu}_R & \overline{e}_R & & & & \end{array}\right)^{\dagger}$$

$$\gamma\Psi = \left(\begin{array}{cccc|cccc} & & & & -u_L & -u_L & -u_L & -\nu_L \\ & & & & -d_L & -d_L & -d_L & -e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \hline \overline{u}_L & \overline{d}_L & -\overline{u}_R & -\overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & -\overline{u}_R & -\overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & -\overline{u}_R & -\overline{d}_R & & & & \\ \overline{\nu}_L & \overline{e}_L & -\overline{\nu}_R & -\overline{e}_R & & & & \end{array}\right)$$

These operators correspond to a 'geometry' with signature 6:

$$J^2 = \mathbb{I}, \qquad J\gamma = -\gamma J.$$

Spinor Representations (Pati-Salam)

'Local Lorentz'? $su_L(2) \times su_R(2) \times \underbrace{su(4)}_{\text{Let's look at this bit!}}$.

$$M\Psi = \left[\left(\begin{array}{cc|c} 0 & & \\ & 0 & \\ \hline & & M_i\Sigma^i \end{array} \right), \left(\begin{array}{cccc|cccc} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \hline \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{\nu}_L & \overline{e}_L & \overline{\nu}_R & \overline{e}_R & & & & \end{array} \right) \right].$$

$su(4) \simeq so(6)$

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$$
M\Psi = \left[\left(\begin{array}{cc|c} 0 & & \\ & 0 & \\ \hline & & M_i\Sigma^i \end{array} \right), \left(\begin{array}{cccc|cccc} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \hline \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{u}_L & \overline{d}_L & \overline{u}_R & \overline{d}_R & & & & \\ \overline{\nu}_L & \overline{e}_L & \overline{\nu}_R & \overline{e}_R & & & & \end{array} \right) \right].
$$

$su(4) \simeq so(6)$

To do/questions:

1. Construct a 'Clifford' action of forms.
2. Determine relationship between the 'discrete connection' and the 'discrete' Dirac operator.
3. What scalar representations are predicted?
4. How do three generations enter?
5. Construct gravitational action.
6. Quantization?