Title: Discussion 1

Speakers:

Collection: Simplicity III

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2d boundary of 3d TSC

Consider a modified boundary Hamiltonian (Wang, Senthil 2014):

$$H = \int d^2x \sum_{a=1,2} \chi_a^t (i\tau^x \partial_x + i\tau^z \partial_y)\chi_a + \phi_x \chi^t \tau^y \sigma^x \chi + \phi_y \chi^t \tau^y \sigma^z \chi$$

Consider an enlarged O(2) symmetry.
When ϕ condenses/orders, it breaks T, breaks O(2), but keeps

$$T' = T \otimes (\pi\text{-rotation})$$

All the symmetries can be restored by condensing the vortices of the ϕ order parameter. A fully gapped, nondegenerate, symmetric state is only possible if the vortex is gapped, nondegenerate.
A vortex core has one Majorana mode, and

$$T' : \gamma_a \to \gamma_a$$

With $N = 16$, interaction can gap out the 2d boundary with no deg.

Numerical evidences of 2d massive Dirac fermions without fermion bilinear mass: Slagle, You, Xu, 2014, He, et.al. 2015

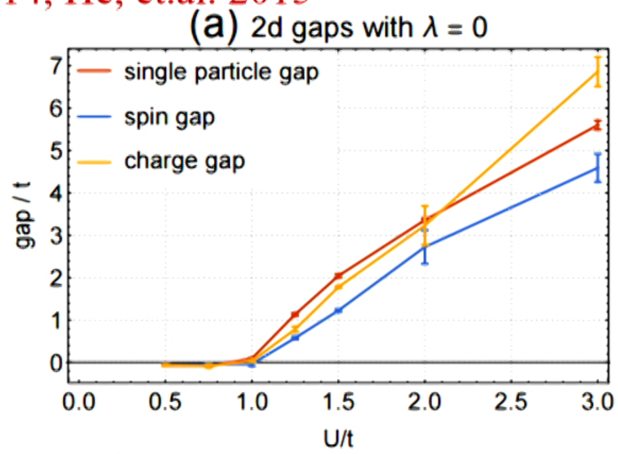
$$H = T + T' + W$$

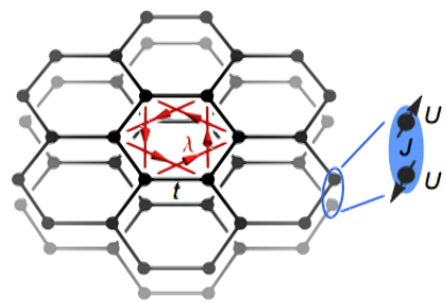
$$T = -t \sum_{\langle ij \rangle} \sum_{\ell,s} \left(c^\dagger_{i\ell s} c_{j\ell s} + h.c. \right)$$

$$T' = i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\ell} \nu_{ij} c^\dagger_{i\ell} \sigma^z c_{j\ell}$$

$$W = \frac{U}{2} \sum_{i,\ell} (n_{i\ell} - 1)^2$$

$$+ J \sum_i \left[\boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \frac{1}{4}(n_{i1} - 1)(n_{i2} - 1) - \frac{1}{4} \right]$$





More numerical evidences:

Ayyar, Chandrasekharan 2014, Catterall, 2014 (same number of Dirac fermions)

3d boundary of 4d TSC (Toy model of SM)

The 3d boundary of a 4d TSC with U(1) x T x Z2 symmetry:

$$H = \int d^3x \sum_{a=1}^{2} \psi_a^\dagger (i\vec{\sigma} \cdot \vec{\partial}) \psi_a$$

These symmetries guarantee that no quadratic mass terms are allowed at the 3d boundary. So without interaction the classification of this 4d TSC is **Z**.

We want to argue that, with interaction, the classification is reduced to $\mathbf{Z_8}$, namely the interaction can gap out **16** flavors of 3d left chiral fermions without generating any quadratic fermion mass.
You, BenTov, Xu, arXiv:1402.4151

3d boundary of 4d TSC (Toy model of SM)

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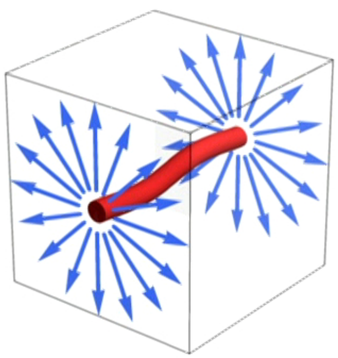
$$H = \int d^3x \sum_{a=1}^{2} \psi_a^\dagger (i\vec{\sigma}\cdot\vec{\partial})\psi_a$$

Now consider U(1) order parameter:

$$\vec{\phi} = (\mathrm{Re}[\psi^t \sigma^y \tau^x \psi], \quad \mathrm{Re}[\psi^t \sigma^y \tau^z \psi])$$

U(1) symmetry can be restored by proliferating vortex loop.

$$H_{vortex} = \int dx\ \chi_L i\partial_x \chi_L - \chi_R i\partial_x \chi_R \qquad Z_2 : \begin{array}{l} \chi_L \to \chi_L, \\ \chi_R \to -\chi_R \end{array}$$



For N=1 copy, the vortex line is a gapless 1+1d Majorana fermion with T and Z2 symmetry when and only when N=8 (16 chiral fermions at the 3d boundary), interaction can gap out vortex loop without degeneracy.

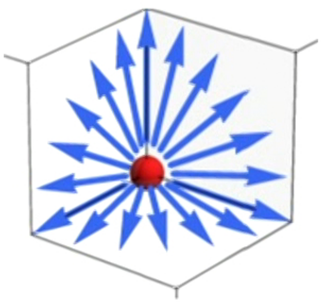
3d boundary of 4d TSC (Toy model of SM)

The 3d boundary of a 4d TSC with U(1) x T x Z2 symmetry:

$$H = \int d^3x \sum_{a=1}^{2} \psi_a^\dagger (i\vec{\sigma} \cdot \vec{\partial}) \psi_a$$

Now consider three component order parameter:

$$\vec{\phi} = \mathrm{Re}\left[\psi^t (\sigma^y \otimes i\tau^y \vec{\tau}) \psi\right]$$



All the symmetries can be restored by condensing the hedgehog monopole of the order parameter. For N=1 copy, the monopole is a 0d Majorana fermion with T symmetry

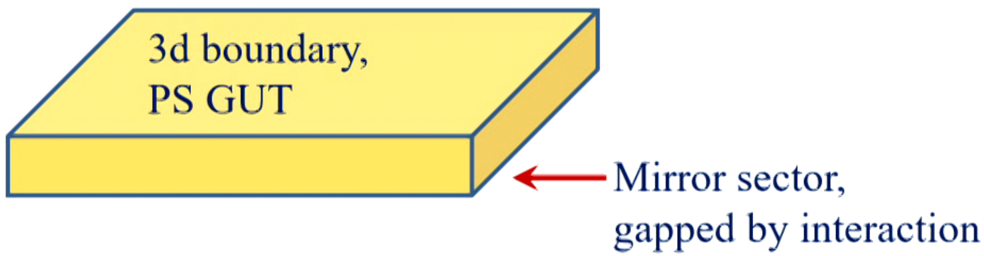
$$T : \gamma_a \to \gamma_a,$$

Then when N=8 (16 chiral fermions at the 3d boundary), interaction can gap out monopole.

Pati-Salam Grand Unified Theory

16 left handed chiral fermions decompose into representations of $SU(4) \times SU(2)_1 \times SU(2)_2$ gauge groups as (4, 2, 1) and (4*, 1, 2)
For more details, please wiki.

Our goal is to argue that, a 4+1d topological insulator (superconductor) with $SU(4) \times SU(2)_1 \times SU(2)_2$ symmetry, whose boundary has 16 chiral fermions, without interaction has Z classification, but under interaction becomes trivial.



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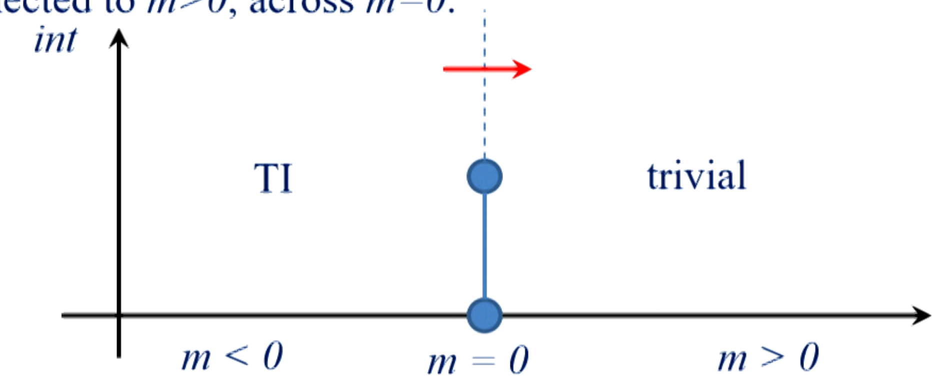
To make this statement, there are two strategies:
1, argue the boundary can be gapped out without generating any fermion bilinear mass;
2, argue the bulk quantum critical point between trivial and TI phases can be gapped out by interaction; i.e. there is only one trivial phase in the bulk.
We will take the second strategy.

Pati-Salam Grand Unified Theory

Consider a 4+1d TI or TSC with SU(4) x SU(2)$_1$ x SU(2)$_2$ symmetry. Without interaction, it is 16 flavors of 4+1d integer quantum Hall state.

$$H = \sum_{a=1}^{16} \sum_{\vec{k}} \psi^\dagger_{\vec{k},a} \left(\sum_{i=1}^{4} \sin(k_i)\Gamma^i + (m + 4 - \sum_{i=1}^{4} \cos(k_i))\Gamma^5 \right) \psi_{\vec{k},a}$$

Goal: argue that the *m=0* line can be gapped by interaction with SU(4) x SU(2)$_1$ x SU(2)$_2$ symmetry, and state *m<0* can be smoothly connected to *m>0*, across *m=0*.



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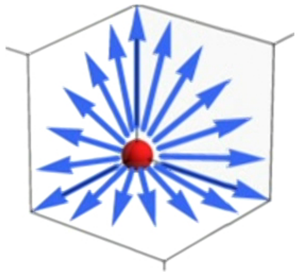
Strategy (sketch):
spontaneously break $SU(2)_1$ x $SU(2)_2$ symmetry by cooper pair. The cooper pair will be an SO(4) vector. Recall SO(4) ~ $SU(2)_1$ x $SU(2)_2$

$$(4, 2, 1) \otimes (\bar{4}, 1, 2) = (1, 2, 2) \oplus \cdots$$



The SO(4) symmetry can be restored by condensing the monopoles of the SO(4) vector. We need to argue, while tuning *m*, the spectrum of monopole never closes gap when and only when there is interaction.

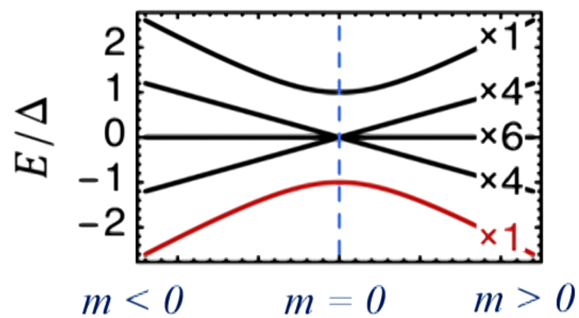
Pati-Salam Grand Unified Theory

The spectrum of monopole: four localized fermion modes, $f_1 \dots f_4$.

Effective Hamiltonian in the monopole core:

$$H_{eff} \sim m(\sum_{a=1}^{4} f_a^\dagger f_a - 2) - U(f_1 f_2 f_3 f_4 + H.c.)$$

The monopole spectrum changes smoothly without closing gap by tuning m, thus the system never closes gap after condensing monopoles, and restoring all the symmetries.



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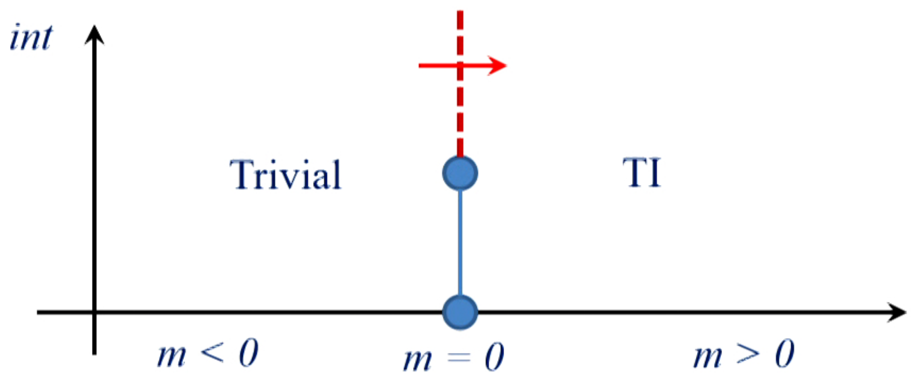

Comments:

1, A more mathematical study of whether the mirror sector can be gapped out, or a mathematical derivation of the classification of interacting 4+1d topological SC whose boundary mimics the GUT: Wang, Wen: arXiv:1809.11171 (1+1d analogue, arXiv:1307.7480)

2, The key point of our mechanism is that, the bulk TI/TSC is nontrivial without interaction, but trivialized by interaction. However, sometimes, even if the bulk is nontrivial, the boundary (which is anomalous) can be gapped out in to a "topological order".