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On the possible role of nilpotent internal symmetries in unification

*J.Phys.***A50**(2017)115401 and `arXiv1909.02208`

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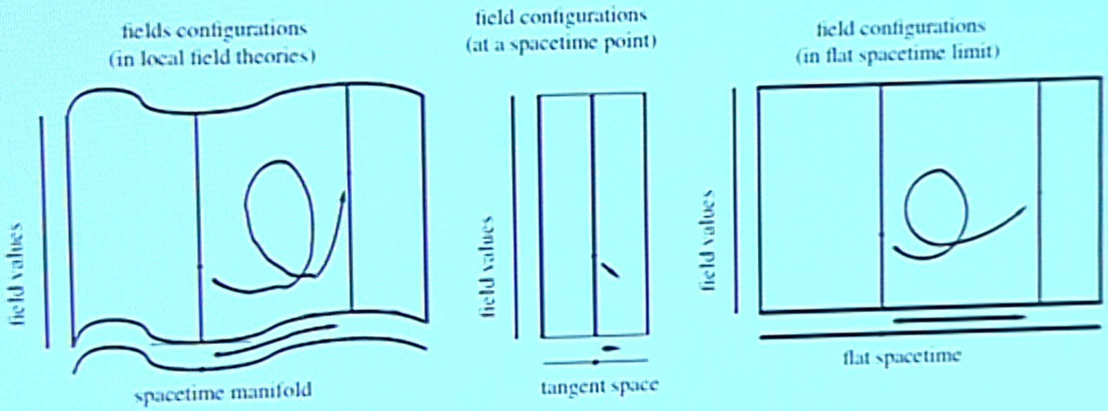
Introduction

- **Unification no-go theorems.** Spacetime symmetries (Poincaré group) and compact internal symmetries (compact gauge group) cannot be unified in an easy way (McGlinn1964, Coleman–Mandula1967).
- **Supersymmetry (SUSY).** The no-go theorems are circuimventable if some amount of "exotic" symmetries are allowed (Haag–Lopuszanski–Sohnius1975).
- **SUSY is not seen experimentally.** At present status (2019).
- **Do mathematical alternatives exist?** What are the most general (strongest) group theoretical obstacles and can one bypass them?
- **It seems, possibly yes.** We found a group theoretical idea for a mechanism to possibly substitute SUSY for symmetry unification.

(Will talk only in global symmetries limit, for brevity — making these local is not a problem.)

Assume that we are looking for a model which has a classical field theory limit.

In a (classical) field theory we have finite degrees of freedom at points of 4d spacetime.



Diffeo invariant Lagrangian is constrained by its *first order symmetries* at points of spacetime.

Such symmetry generators are finitely many, because they act on finite degrees of freedom.
(They have the same structure as if we looked at global symmetries limit.)

Such symmetries form always finite dimensional real Lie algs/groups. (Not a big surprise.)
So, looking at general structure of Lie groups / algs seems a wise strategy.

General structure of finite dim real Lie groups / algs

(*Will talk only about connected & simply connected Lie groups for simplicity.* $\leftrightarrow$ *Lie algebras.*)

- **Killing form signature:**
  On any finite dim real Lie algebra one has the *Killing form*, an invariant scalar product.

$$x \cdot y \quad = \quad \mathrm{Tr}\,(\mathrm{ad}_x\,\mathrm{ad}_y)$$

(It appears e.g. in the Yang-Mills Lagrangians:

$F_{\mu\nu} \cdot F^{\mu\nu}$ ← Killing form)

It may be definite, indefinite, or even can be degenerate:

$$\begin{pmatrix} (+1) & & \\ & (-1) & \\ & & (0) \end{pmatrix} \longleftarrow \text{(degenerate directions)}$$

- **Levi decomposition theorem**:

$$\underbrace{E}_{\text{finite dim real Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ (\textit{radical}, \text{ or } \textit{solvable part})}} \rtimes \underbrace{\overbrace{L_1}^{(\textit{simple})} \times \ldots \times \overbrace{L_n}^{(\textit{simple})}}_{\substack{\text{non-degenerate directions of Killing form} \\ (\textit{Levi factor}, \text{ or } \textit{semisimple part})}}$$

(Modulo global topology.)
E.g.: the symmetries of flat plane (translations $\rtimes$ rotations) is a typical example.

- **Traditional gauge theory folklore:** only (semi)simple groups are important. $\mathrm{SU}(N)$ etc.

- **Poincaré group**:

$$\underbrace{\mathcal{P}}_{\text{Poincaré group}} = \underbrace{\mathcal{T}}_{\text{translation group (radical)}} \rtimes \underbrace{\overbrace{\mathcal{L}}^{(\textit{simple})}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

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- **super-Poincaré group (SUSY):**

$$\underbrace{\mathcal{P}_s}_{\text{super-Poincaré group}} = \underbrace{\mathcal{S}}_{\text{supertranslation group (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

is a similar example, with a bit larger radical (so called: two-step nilpotent).

Supertranslations: a transformation group on the vector bundle of superfields. Action:

$$\begin{pmatrix} \theta^A \\ x^a \end{pmatrix} \xmapsto{\begin{pmatrix} \epsilon^A \\ d^a \end{pmatrix}} \begin{pmatrix} \theta^A + \epsilon^A \\ x^a + d^a + \sigma^a_{AA'}\,\mathrm{i}(\theta^A \bar{\epsilon}^{A'} - \epsilon^A \bar{\theta}^{A'}) \end{pmatrix}$$

on the "supercoordinates" and the affine spacetime coordinates.

Often, SUSY is presented as "super-Lie algebra":

$$
\begin{array}{lll}
\checkmark & [\, P_a \,, P_b \,] = 0, \\
\checkmark & [\, P_a \,, Q_A \,] = 0, \\
\checkmark & [\, P_a \,, \bar{Q}_{A'} \,] = 0, \\
!!! \rightarrow & \{\, Q_A \,, Q_B \,\} = 0, \\
!!! \rightarrow & \{\, \bar{Q}_{A'} \,, \bar{Q}_{B'} \,\} = 0, \\
!!! \rightarrow & \{\, Q_A \,, \bar{Q}_{A'} \,\} = 2\,\sigma^a_{AA'}\, P_a .
\end{array}
$$

If not a Lie algebra, how can it be a collection of infinitesimal transformations? Answer:

[Nucl.Phys.**B76**(1974)477, Phys.Lett.**B51**(1974)239]:
Take $\epsilon^A_{(i)}$ $(i = 1, 2)$ "supercoordinate" (Grassmann valued two-spinor) basis.
Introduce new generators $\delta_{(i)} = \epsilon^A_{(i)}\, Q_A$ instead of Q_A. (Infinitesimal change of superfields.)
⇒ SUSY has also an ordinary finite dim real Lie algebra presentation.

Exponentiating this Lie algebra: super-Poincaré Lie group is obtained. SUSY is not so exotic!
Ordinary Lie group / Lie algebra theory also applies!

All possible extensions of the Poincaré group

O'Raifeartaigh theorem (1965) — all possible finite dim extensions of the Poincaré group:

- **Either:**

$$\begin{aligned} E &= R \rtimes L_1 \times \ldots \times L_n \\ \mathcal{P} &= \mathcal{T} \rtimes \mathcal{L} \end{aligned}$$

(A) Trivial extension: ~Coleman–Mandula.
(*Other symmetries are independent from Poincaré.*)

(B) Extended radical: not (A), radical bigger than $\mathcal{T}$.
(SUSY, extended SUSY, and our new example.)

- **Or:**

$$\begin{aligned} E &= R \rtimes L_1 \times \ldots \times L_n \\ \mathcal{P} &= \mathcal{T} \rtimes \mathcal{L} \end{aligned}$$

(C) Poincaré embedded into simple Lie group.
(*Conform* — $\mathrm{SO}(2,4)$ — *theories etc.*
Heavy symmetry breaking needed for a gauge-theory-like limit, i.e. to point out spacetime.)

Group theoretical constraints for any unification

- Collection of all symmetries:

$$\underbrace{E}_{\text{Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ (\textit{radical}, \text{ or } \textit{solvable part})}} \rtimes \underbrace{L_1 \times \cdots \times L_n}_{\substack{\text{non-degenerate directions of Killing form} \\ (\textit{Levi factor}, \text{ or } \textit{semisimple part})}}$$

- Poincaré group:

$$\underbrace{\mathcal{P}}_{\text{Poincaré group}} = \underbrace{\mathcal{T}}_{\text{translations (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

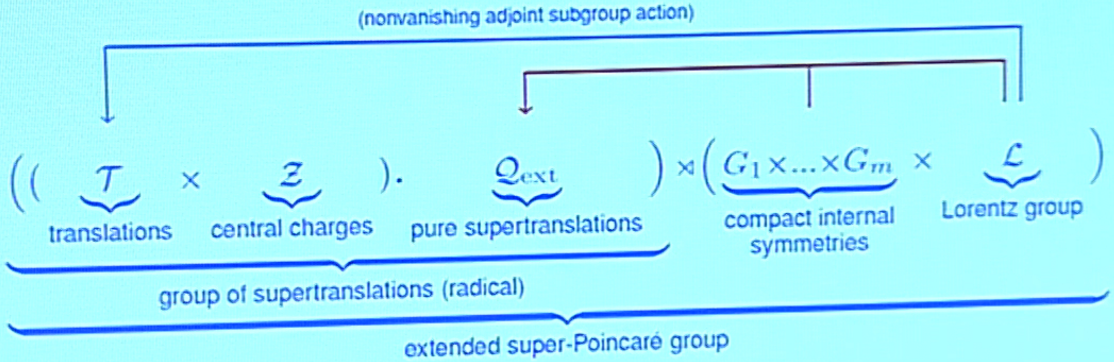
- Compact gauge group:

$$\underbrace{G}_{\text{compact Lie group}} = \underbrace{\mathrm{U}(1) \times \cdots \times \mathrm{U}(1)}_{\text{compact abelian part (radical)}} \times \underbrace{G_1 \times \cdots \times G_m}_{\text{compact non-abelian part (Levi factor)}}$$

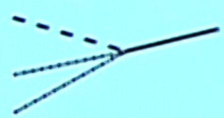
On the possible role of nilpotent internal symmetries in unification – p. 9/40

How (extended) SUSY works Lie group theoretically?

- **Unification via *extended super-Poincaré group*:**



- **The extended super-Poincaré group is direct-indecomposable (unified).**
  $\Rightarrow$ Connects spacetime symmetries with compact internal (gauge) symmetries.
  $\Rightarrow$ Connects potentially independent compact internal symmetries with each-other.
  $\Rightarrow$ Running of coupling factors do unify.

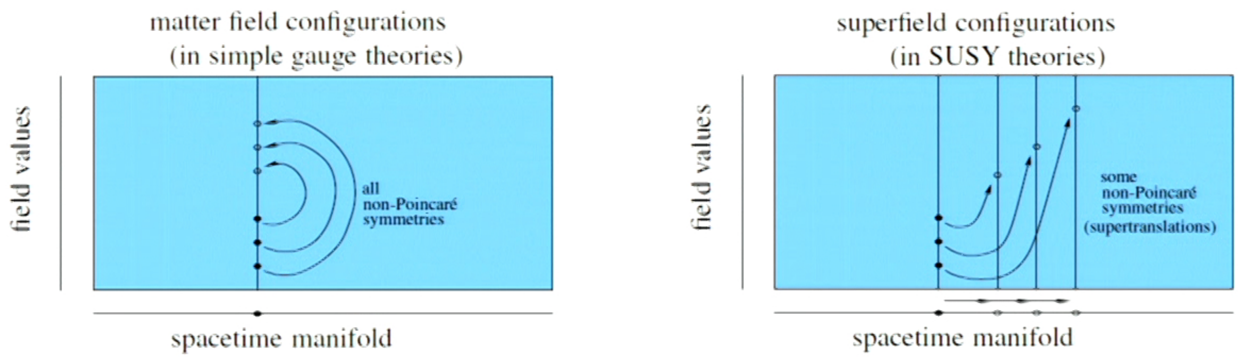


Running of gauge couplings

- **Operated by O'Raifeartaigh theorem case B.** Via the extension of the radical.

On the possible role of nilpotent internal symmetries in unification – p. 10/40

- **Symmetry breaking still needed.** Because in (extended) SUSY, the non-Poincaré symmetries couple too strongly to spacetime symmetries.

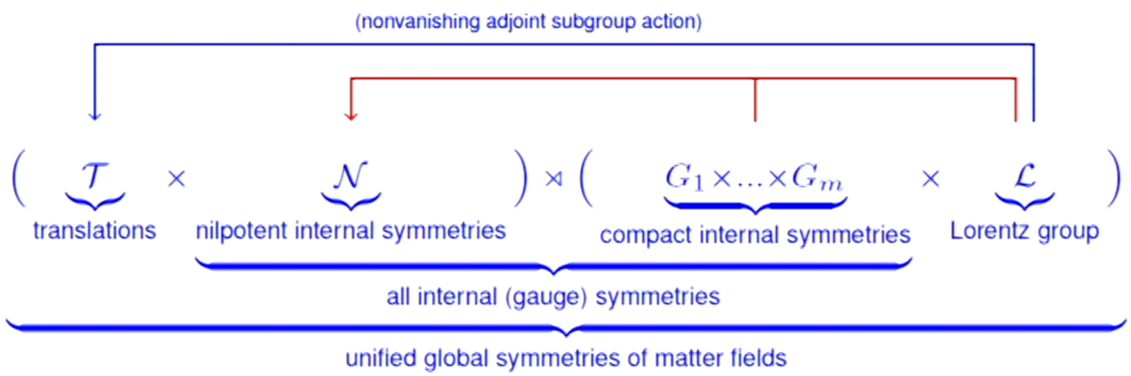


(Not a vector bundle automorphism group over spacetime.)

Experimental hint not seen for this, so symmetry breaking needed for gauge-theory-like limit. (bug? feature?)

A possible alternative mechanism to SUSY

- ***Conservative* extensions of the Poincaré group.**
  - $\Leftrightarrow$ The non-Poincaré symmetries are really all *internal*, i.e. do not act on spacetime.
  - $\Leftrightarrow$ There exists $\mathcal{P} \xrightarrow{i} E \xrightarrow{o} \mathcal{P}$ homomorphisms, such that $o \circ i =$ identity.
  - $\Leftrightarrow$ Symm. breaking not needed for gauge-theory-like limit (vector bundle automorphism).
- **The (extended) super-Poincaré is non-conservative extension of Poincaré group.**
- **All possible conservative extensions of the Poincaré group:**

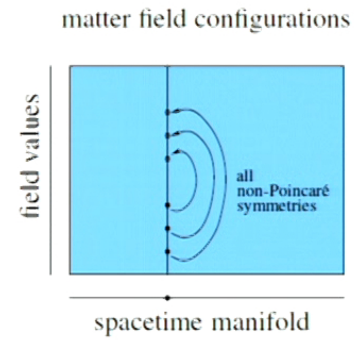


O'Raifeartaigh theorem + energy non-negativity $\Rightarrow$ these are only possible ones.
Similar gauge – spacetime symmetry unification to extended SUSY, via extended radical.
(notion: nilpotent $\approx$ solvable, means that ad_x for all x is nilpotent)

- **In a conservative Poincaré extension, non-Poincaré symmetries are all internal.**

any conservative extension of Poincaré group $=$

{all internal symmetries} $\rtimes$ {Poincaré} $\Longleftrightarrow$



(SUSY does not admit this property.)

- **Constructed an example:**

  See *J.Phys.***A50**(2017)115401, with $G = \mathrm{U}(1)$.

  It is the symmetry group acting on a QFT-inspired algebra valued fields.

  Price to pay: the full internal symmetry group is not purely {compact} but

  {nilpotent} $\rtimes$ {compact}

  (Issue: nilpotent generators $\to$ corresp. gauge fields have zero YM kinetic Lagrangian!!)

In a *conservative* Poincaré group extension: there exists a homomorphism of

$$\underbrace{\underbrace{\mathcal{N}}_{\text{nilpotent internal symmetries}} \rtimes \left(\underbrace{G_1\times\ldots\times G_m}_{\text{compact internal symmetries}} \times \underbrace{\mathcal{P}}_{\text{Poincaré symmetries}}\right)}_{\text{unified group, acting on fundamental field degrees of freedom}} \longrightarrow \underbrace{\underbrace{G_1\times\ldots\times G_m}_{\text{compact internal symmetries}} \times \underbrace{\mathcal{P}}_{\text{Poincaré symmetries}}}_{\text{observed symmetries, acting on some derived field quantities which are function of fundamental degrees of freedom}}$$

No immediate contradiction with experimental situation.
(Nilpotent internal symmetries can act "hidden" in some fundamental d.o.f.)

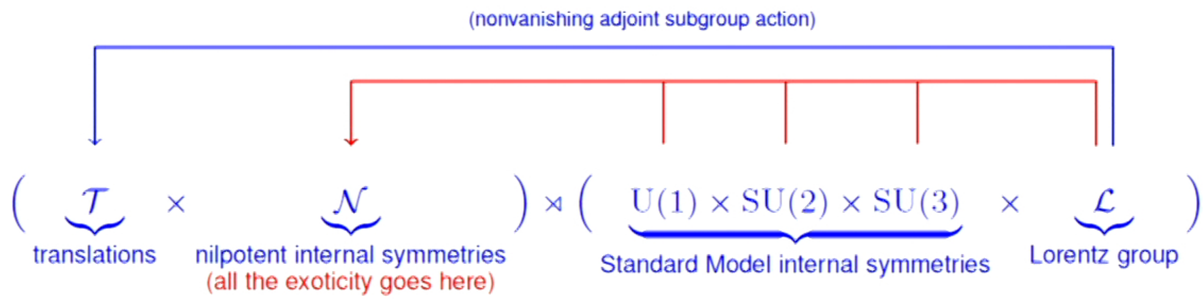
Distant analogy:

$$\underbrace{\Psi}_{\mathrm{U}(1)\times\text{Poincaré acts on it}} \longmapsto \underbrace{\text{Fierz bilinears of }\Psi}_{\text{only Poincaré acts on it}}$$

The Fierz bilinears forget the fundamental $\mathrm{U}(1)$ symmetry of Dirac bispinor fields.
(But such "forgetting function" mechanism works also for semi-direct product.)

Perspectives

Least exotic solution to gauge–spacetime and gauge–gauge symmetry unification: *conservative* unification pattern.



Unification happens not because of a heavy symmetry breaking.
But because of common adjoint subgroup action on "hidden" nilpotent internal symmetries.
Minimal exoticity: we inject subgroups where they naturally belong in Levi decomposition.
Unification achieved not by symmetry *breaking* but by symmetry *hiding*.

Bonus: L.Šnobl, *J.Phys.* **A43**(2010)505202 says: max 1 copy of $\mathrm{U}(1)$ can be present.

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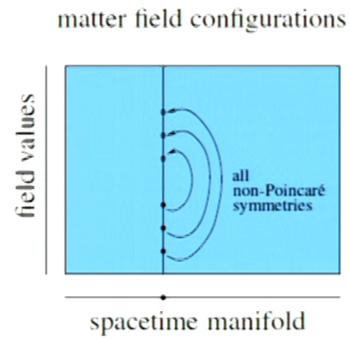

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Eliminating nilpotent gauge fields

Presence of some nilpotent generators acting on the matter field sector could be plausible.

But what to do with corresponding gauge fields?

They have vanishing Yang-Mills kinetic term $\Rightarrow$ don't have kinetic energy, don't propagate.

They eventually still could contribute to matter field Lagrangians.

But then, their Euler-Lagrange equations would be strange, wouldn't it?
(Some algebraic equations, without kinetic wave operator.)

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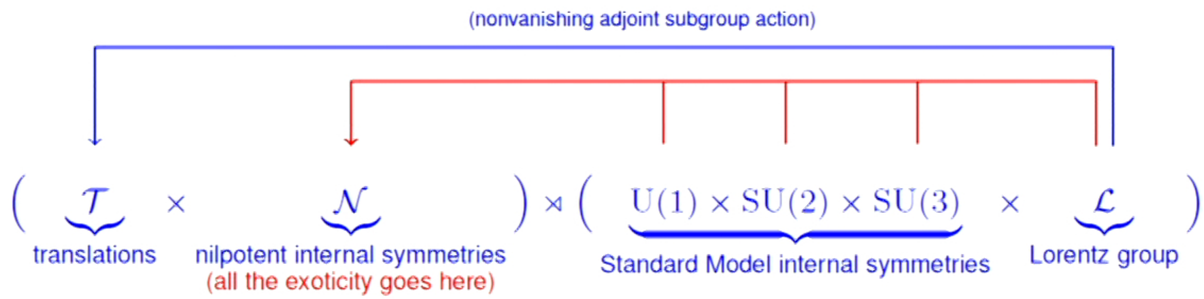

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Observation:

The ordinary Dirac kinetic Lagrangian can be regarded with a $\mathrm{D}(1)\times\mathrm{U}(1)$ internal group.

$$\mathbf{L}_{\text{Dirac}}(\gamma^a,\ \Psi,\ \nabla_b\Psi) \qquad = \qquad \mathbf{v}_\gamma\ \ \mathrm{Re}\left(\overline{\Psi}\,\gamma^c\,\mathrm{i}\nabla_c\Psi\right)$$

($\mathbf{v}_\gamma$ is the metric volume form, ∇_b is the metric + $\mathrm{D}(1)\times\mathrm{U}(1)$ gauge-covariant derivation.)

This, besides usual local $\mathrm{U}(1)$ gauge invariance, is also locally $\mathrm{D}(1)$ gauge invariant:

$$\begin{pmatrix} \Psi \\ \gamma^a \\ \nabla_b \end{pmatrix} \quad \overset{\Omega>0}{\longmapsto} \quad \begin{pmatrix} \Omega^{-\frac{3}{2}}\ \Psi \\ \Omega^{-1}\ \gamma^a \\ \Omega^{-\frac{3}{2}}\ \nabla_b\,\Omega^{\frac{3}{2}} \ = \ \nabla_b + \Omega^{-\frac{3}{2}}\ \mathrm{d}_b\,\Omega^{\frac{3}{2}} \end{pmatrix},$$

But has also an unusual hidden affine "shift" symmetry:

$$\begin{pmatrix} \Psi \\ \gamma^a \\ \nabla_b \end{pmatrix} \quad \overset{C_d}{\longmapsto} \quad \begin{pmatrix} \Psi \\ \gamma^a \\ \nabla_b + C_b \end{pmatrix}$$

(C_d is any real covector field, i.e. a $\mathrm{D}(1)$ gauge potential.)

In total, this causes an unusual local internal symmetry:

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The $\mathrm{D}(1)$ group can act locally internally and faithfully on matter fields, but without a compensating $\mathrm{D}(1)$ gauge field!
(The $\mathrm{D}(1)$ gauge field is formally present, but can be transformed out from the Lagrangian.)

Can similar trick be used to get rid of nilpotent gauge bosons?
Answer: **yes.**

András László, Lars Andersson: `arXiv1909.02208` as a toy model.

Necessary condition for this: to reside in a normal sub-Lie algebra of internal symmetries.

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Our concrete example group

The smallest nilpotent Lie group is the 3 generator Heisenberg group H_3.
Generated by q, p, e, the only nonzero bracket being $[q,p] = K\,e$ (K arbitrary nonzero real).
The outer derivations of the Lie algebra of $\mathrm{H}_3(\mathbb{C})$ is nothing but $\mathrm{gl}(2,\mathbb{C})$.
Thus, one can form $\mathrm{H}_3(\mathbb{C}) \rtimes \mathrm{GL}(2,\mathbb{C})$ ($\mathrm{GL}(2,\mathbb{C})$ mixes q, p, while scales e by determinant).

$\Rightarrow$

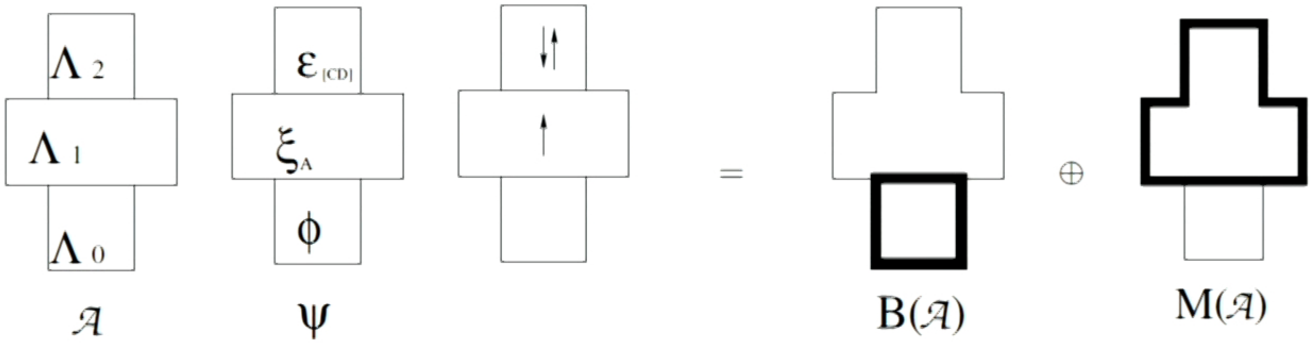
$$\Big(\underbrace{\mathcal{T}}_{\text{translations}} \times \underbrace{\mathrm{H}_3(\mathbb{C})}_{\text{Heisenberg group}}\Big) \rtimes \Big(\overbrace{\underbrace{\mathrm{U}(1)}_{\text{compact internal}} \times \underbrace{\mathrm{D}(1)}_{\text{dilations}} \times \underbrace{\mathrm{SL}(2,\mathbb{C})}_{\text{Lorentz}}}^{\mathrm{GL}(2,\mathbb{C})}\Big)$$

is indecomposable conservative Poincaré group extension with a compact component.
(**`arXiv1909.02208`**)

We construct a generally covariant Lagrangian, by taking a vector bundle with
$\mathrm{H}_3(\mathbb{C}) \rtimes \mathrm{GL}(2,\mathbb{C})$
as structure group.

First, we need to find nice defining representation and its invariants.

Let $\mathcal{A}$ be a 2 generator (4 dimensional) complex Grassmann algebra.
(Representation $\mathcal{A} \equiv \Lambda(S^*)$ is always possible, where S is two-spinor space.)



It turns out that $\mathrm{H}_3(\mathbb{C})$ is isomorphic to $L_{M(\mathcal{A})}$ (left multiplication, i.e. "particle insertion").

And of course, $\mathrm{GL}(2,\mathbb{C})$ naturally acts on $\mathcal{A} \equiv \Lambda(S^*)$ as $\mathrm{GL}(S^*) \equiv \mathrm{U}(1) \times \mathrm{D}(1) \times \mathrm{SL}(2,\mathbb{C})$.

Re-understanding Coleman–Mandula (knowing Levi decomp.th.)

Essence of Coleman–Mandula-like no-go theorems (assuming finite dimensionality):

- Full symmetry group is a Poincaré group extension ⇒ O'Raifeartaigh A or B or C. (no other possibilities exist Lie group theoretically)
- Complementing symmetries to Poincaré symmetries have positive definite invariant "internal" scalar product ⇒ no O'Raifeartaigh B.
- No symmetry breaking present ⇒ no O'Raifeartaigh C.

Our mechanism: internal group is $\mathcal{N} \rtimes G$ ⇒ internal scalar product degenerates on $\mathcal{N}$ ⇒ ✓
SUSY: similar degeneration over the pure supertranslations $\mathcal{Q}$ ⇒ ✓
(super-Lie algebra is a complicated way to say that we allow certain kind of nilradical)

Both are O'Raifeartaigh B mechanism.

Attention! Coleman-Mandula also has a hidden assumption:

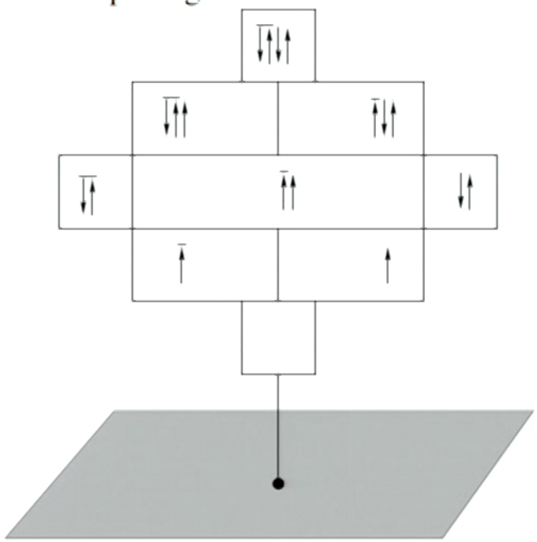
- Symmetry generators strictly conserve 1-particle subspaces.
  ⇒ No extra representation space for generators possibly stepping in the Fock hierarchy.
  (Are we sure on this assumption?!)

Summary

- **SUSY experimentally not visible at present.** As of 2019 status.
- **Mathematical alternatives to SUSY exist.** Are also O'Raifeartaigh B type, as SUSY.
- **The alternative: "conservative" extensions of the spacetime symmetries.** The complementing symmetries to spacetime symmetries are all strictly internal.
- **Example constructed.** At present, merely with $\mathrm{U}(1)$ as compact gauge group.
- **It connects gauge and spacetime symmetries.** Similarly to extended SUSY.
- **Harmonizes with present experimental situation.** Extra symmetries are "hidden".
- **Symmetry "hiding".** Symmetry breaking is not the only mechanism to get rid of exotics.
- **Testing on minimal model.** It seems we can construct a minimal model.

*J.Phys.***A50**(2017)115401, `arXiv1909.02208`.

We take A-valued vector bundle over 4d spacetime manifold, with discussed structure group.
(The invariant Lagrangians are based largely on the invariant form L.)



Encodes per spacetime point:
creation op. algebra for 2 fundamental d.o.f.,
Pauli principle,
charge conjugation.

Early attempt: R.M.Wald, S.Anco: *Phys.Rev.***D39**(1989)2297 with algebra valued fields.
(But they took too simple algebra for this.)