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Speakers:

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Interacting Topological Insulator and possible Connection to lattice-GUT

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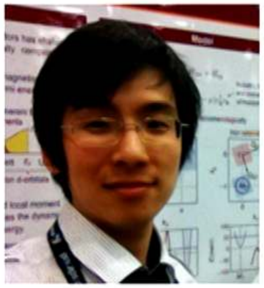
GET REAL!
BE
RATIONAL

Interacting Topological Insulator and possible Connection to lattice-GUT

Collaborators:
Yi-Zhuang You (now faculty at UCSD)



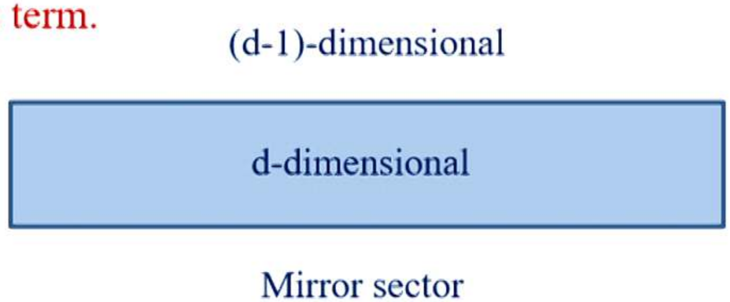
Very helpful discussions with
Joe Polchinski, Mark Srednicki, Robert Sugar, Xiao-Gang Wen, Alexei Kitaev, Tony Zee…….

Main reference: You, Xu, arXiv:1412.4784

Interacting Topological Insulator and possible Connection to lattice-GUT

Oversimplied summary of topological insulator:

d-dimensional bulk: massive/gapped spectrum without degeneracy; (d-1)-dimensional boundary: gapless Dirac/Weyl/Majorana fermions, gapless spectrum protected by symmetry, i.e. Symmetry forbids fermion mass term.



(d-1)-dimensional boundary cannot exist as a (d-1)-dimensional system without the bulk. i.e. Once symmetries are gauged, will have gauge anomaly. Full classification of noninteracting topological insulator: (Ryu, et.al., Kitaev, 2009)

With strong four-fermion interaction?

Current understanding of interacting TI:

Interaction can definitely "reduce" the classification of TI, i.e. interaction can drive some noninteracting TI trivial,

in other words, interaction can gap out the boundary of some noninteracting TI, without breaking any symmetry,
Or equivalently, interaction can gap out the boundary without generating a fermion mass term.

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Weyl fermions:

$$H_L = \psi_L^\dagger (i\vec{\sigma} \cdot \vec{\partial})\psi_L$$

$$H_R = \psi_R^\dagger (-i\vec{\sigma} \cdot \vec{\partial})\psi_R$$

$$\tilde{\psi}_L = \sigma^y \psi_R^\dagger$$

$$H_R = \tilde{\psi}_L^\dagger (i\vec{\sigma} \cdot \vec{\partial})\tilde{\psi}_L$$

Weyl fermions can be gapped out by Cooper pairing (Majorana mass):

$$H_m = m\psi_L \sigma^y \psi_L + h.c.$$

Interacting Topological Insulator and possible Connection to lattice-GUT

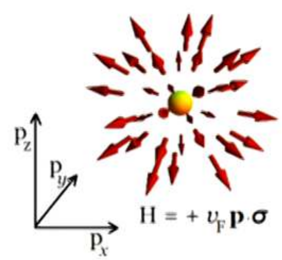
Very high energy In Standard Model (higher than EW unification energy), every generation has (effectively) 16 massless Left Weyl fermions coupled with gauge field: (spinor rep of SO(10))

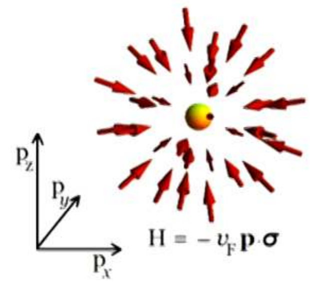
$$(u_\alpha, d_\alpha)_L,\ (u^\dagger_\alpha, d^\dagger_\alpha)_R,\ (e, \nu_e)_L,\ (e^\dagger, \nu^\dagger_e)_R$$

$$2 \times 3 + 2 \times 3 + 2 + 2 = 16$$

This theory is difficult to regularize on a 3d lattice. Because on a 3d lattice, if we want to realize left fermions, we also get right fermions coupled to the same gauge theory

For example: Weyl semimetal has both left, and right Weyl fermions in the 3d BZ:





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Popular alternative: Realize Weyl fermions on the 3d boundary of a 4d topological insulator/superconductor

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However, this approach requires a subtle adjustment of the fourth dimension. If the fourth dimension is too large, there will be gapless photons in the bulk; if the fourth dimension is too small, the mirror sector on the other boundary will interfere.

Key question: Can we gap out the mirror sector (Weyl fermions on the other boundary) without affecting the SM at all?

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Key question: Can we gap out the mirror sector (Weyl fermions on the other boundary) without affecting the SM at all?

This cannot be done in the standard way (spontaneous symmetry breaking, condense a boson that couples to the mirror fermion mass)

$$\phi\psi_R^t i\sigma^y\psi_R \longrightarrow \phi\psi_L^t i\sigma^y\psi_L$$

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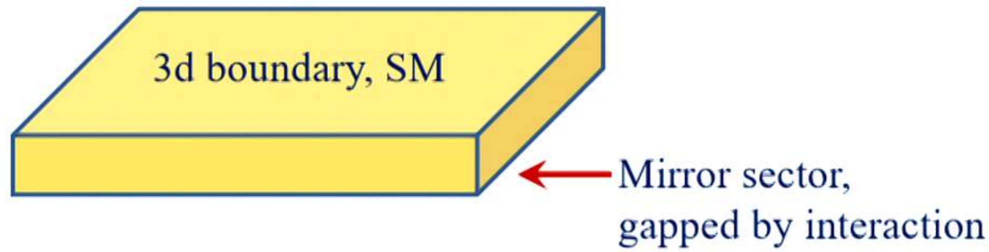

A different question: (Eichten, Preskill) perhaps we can gap out the mirror sector with short range interaction, while

$$\langle \psi_R^t i\sigma^y \psi_R \rangle = 0$$

If this is possible, then only the SM survives at low energy.

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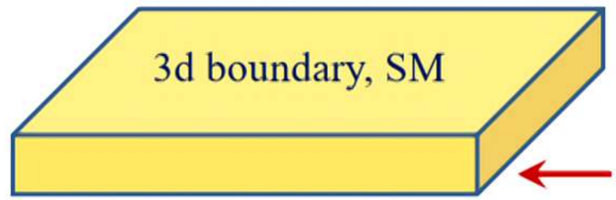

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Using CMT language: all we need to do, is to show that the 4d bulk topological insulator/superconductor is nontrivial without interaction, but trivialized by interaction.
But, now we understand that, for this mechanism to work, we need the correct flavor number, correct symmetry, and correct interaction.

ABSENCE OF NEUTRINOS ON A LATTICE
(I). Proof by homotopy theory

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The assumptions made for the charges Q (lepton number, say) of the theory are the following:

(i) Exact conservation of Q, even at scales where the lattice cutoff is relevant. Charge conservation means that the energy and momentum eigenstates are also charge eigenstates.

This means, in order to realize Weyl fermions without "mirror sector", we must break the (anomalous) U(1) symmetry explicitly. The U(1) symmetry becomes an emergent symmetry at IR.

0d boundary of 1d TSC

Consider N copies of 0d Majorana fermions with time-reversal symmetry (boundary of N copies of Kitaev's 1d TSC):

$$T : \gamma_a \rightarrow \gamma_a,$$

$$i\gamma_a\gamma_b \quad \text{Breaks time-reversal}$$

For N = 2, the only possible Hamiltonian is

$$H = Ki\gamma_1\gamma_2$$

But it breaks time-reversal symmetry, thus with time-reversal symmetry, H = 0, the state is 2-fold degenerate.

For N = 4, the only T invariant Hamiltonian is

$$H = K\gamma_1\gamma_2\gamma_3\gamma_4 = -\frac{K}{4}(2i\gamma_1\gamma_2)(2i\gamma_3\gamma_4) \ \sim -\frac{K}{4}\sigma_1^z\sigma_2^z$$

0d boundary of 1d TSC

Finally, when N = 8,

$\gamma_{1,2,3,4}$ doublet

$\gamma_{5,6,7,8}$ doublet

$$H = \vec{S}_1 \cdot \vec{S}_2$$

↓

GS fully gapped, nondegenerate

Thus, when N = 8, the Majorana fermions can be gapped out by interaction without degeneracy, and

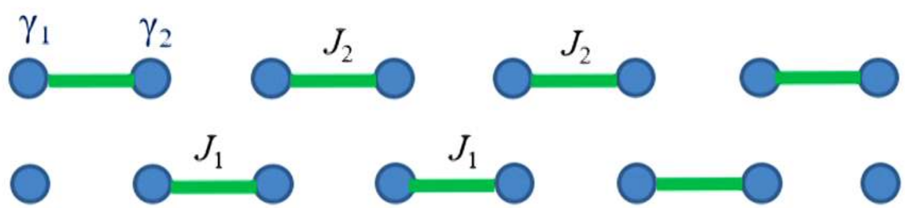
$$\langle i\gamma_a\gamma_b\rangle = 0$$

0d boundary of 1d TSC

These 0d fermions are realized at the boundary of 1d TSC:

Trivial

TSC



$$H = \sum_j iJ_1\gamma_{2j}\gamma_{2j+1} + iJ_2\gamma_{2j+1}\gamma_{2j+2}$$

$$J_1 > J_2, \quad \text{topological}, \qquad J_2 > J_1, \quad \text{trivial},$$

In the bulk: $T: \ \gamma_{2j} \to -\gamma_{2j}, \quad \gamma_{2j+1} \to \gamma_{2j+1}$

With N flavors, at the boundary $T: \gamma_a \to \gamma_a,$

This implies that, with interaction, 8 copies of such 1d TSC is trivial, i.e. interaction reduces the classification from $\mathbf{Z}$ to $\mathbf{Z_8}$.
Fidkowski, Kitaev, 2009

0d boundary of 1d TSC

$$H = \sum_j iJ_1\gamma_{2j}\gamma_{2j+1} + iJ_2\gamma_{2j+1}\gamma_{2j+2}$$

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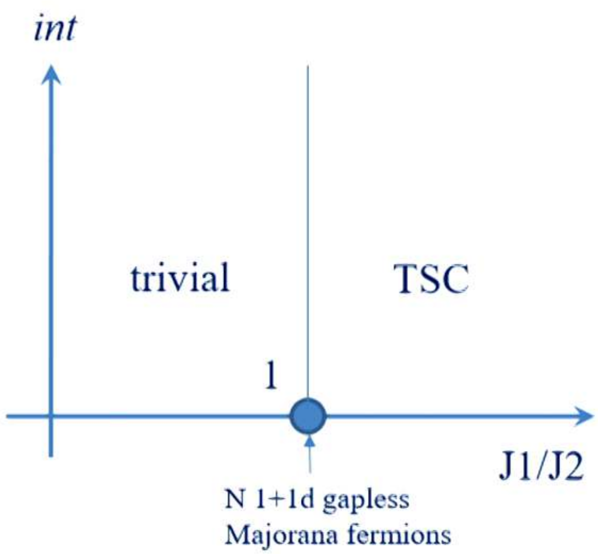
For N flavors with N = 8n+k



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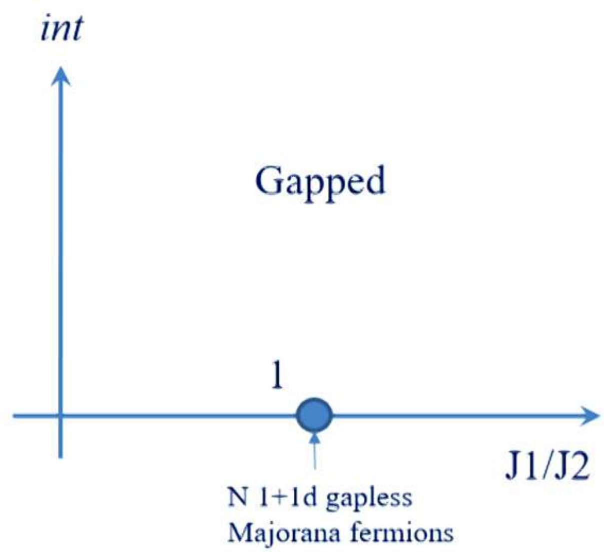
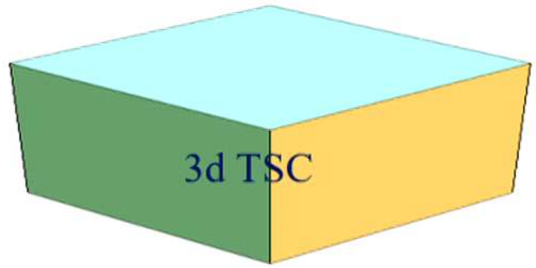


2d boundary of 3d TSC

$$H = \sum_{\vec{k}} \chi^t_{-\vec{k}} \left(\sum_{i=1}^{3} \sin(k_i)\Gamma^i + (m + 3 - \sum_{i=1}^{3} \cos(k_i))\Gamma^4 \right) \chi_{\vec{k}}$$

$$\Gamma^1 = \sigma^{30},\ \Gamma^2 = \sigma^{10},\ \Gamma^3 = \sigma^{22},\ \Gamma^4 = \sigma^{21},\ \Gamma^5 = \sigma^{23}$$



$$H_{edge} = \int d^2x \ \sum_a \chi^t_a (i\tau^x \partial_x + i\tau^z \partial_y)\chi_a \qquad T : \chi_a \to i\tau^y \chi_a$$

Short range interactions reduce the classification of the 3d TSC from **Z** to $\mathbf{Z_{16}}$, namely its edge (16 copies of 2d Majorana fermions) can be gapped out by interaction, with $\langle \bar{\chi}\chi \rangle = 0$
(Vishwanath, et.al. 2014, Kitaev, and other groups)

2d boundary of 3d TSC

Consider a modified boundary Hamiltonian (Wang, Senthil 2014):

$$H = \int d^2x \sum_{a=1,2} \chi_a^t (i\tau^x \partial_x + i\tau^z \partial_y)\chi_a + \phi_x \chi^t \tau^y \sigma^x \chi + \phi_y \chi^t \tau^y \sigma^z \chi$$

Consider an enlarged O(2) symmetry.
When ϕ condenses/orders, it breaks T, breaks O(2), but keeps

$$T' = T \otimes (\pi\text{-rotation})$$

All the symmetries can be restored by condensing the vortices of the ϕ order parameter. A fully gapped, nondegenerate, symmetric state is only possible if the vortex is gapped, nondegenerate.
A vortex core has one Majorana mode, and

$$T' : \gamma_a \to \gamma_a$$

With $N = 16$, interaction can gap out the 2d boundary with no deg.

2d boundary of 3d TSC

Dual theory for vortices and U(1) Goldstone mode:

$$\mathcal{L}_{dual} = \sum_{a=1}^{n} |(\partial_\mu - i a_\mu) v_a| + r|v_a|^2 + \cdots$$

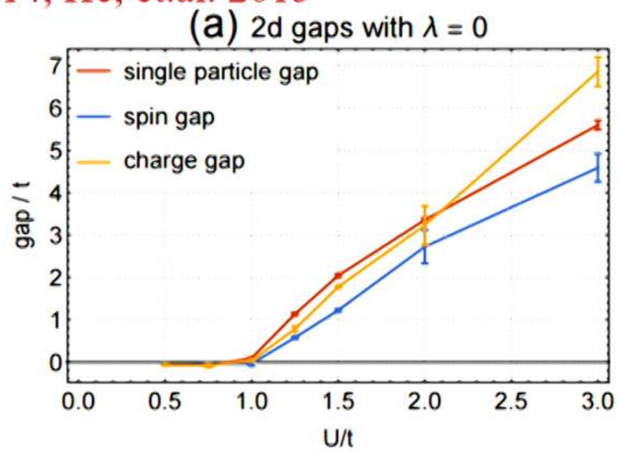
When vortices are gapped, gauge field gapless, dual to the the Goldstone mode of O(2) order parameter.

If vortex core is degenerate, dual theory is the CP^{n-1} theory, condensate of vortices must be degenerate.

With 16 copies of this 3d TSC, the vortex core at the boundary is gapped and nondegenerate. Thus we can condense the vortex, restore the symmetry, keep the spectrum gapped. The fermions acquire a local four-fermion interaction after integrating out ϕ.

2d boundary of 3d TSC

Numerical evidences of 2d massive Dirac fermions without fermion bilinear mass: Slagle, You, Xu, 2014, He, et.al. 2015



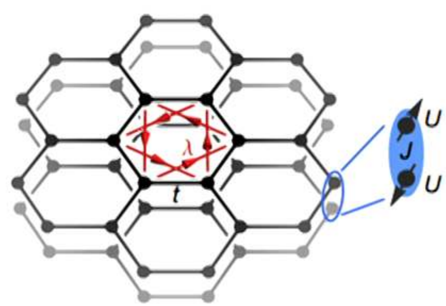
$$H = T + T' + W$$

$$T = -t \sum_{\langle ij \rangle} \sum_{\ell,s} \left(c^{\dagger}_{i\ell s} c_{j\ell s} + h.c. \right)$$

$$T' = i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\ell} \nu_{ij} c^{\dagger}_{i\ell} \sigma^z c_{j\ell}$$

$$W = \frac{U}{2} \sum_{i,\ell} (n_{i\ell} - 1)^2$$

$$+ J \sum_i \left[\boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \frac{1}{4}(n_{i1} - 1)(n_{i2} - 1) - \frac{1}{4} \right]$$



More numerical evidences:

Ayyar, Chandrasekharan 2014, Catterall, 2014 (same number of Dirac fermions)